Mapping of moveout approximations in TI media

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Summary

Moveout approximations play very important role in seismic modeling, inversion and scanning for parameters in complex media. We propose to map the moveout approximations from VTI to TTI medium by introducing the effective tilt angle. As a result, we obtain highly accurate TTI moveout equations synonymous with their VTI counterparts.

Introduction

The theory of moveout approximations in anisotropic media with a tilted symmetry axis (TTI medium) is developed in recent years (Pech, Tsvankin and Grechka, 2003; Alkhalifah, 2011b). In order to derive the moveout approximation in a TTI medium, Stovas and Alkhalifah (2011) used the perturbation method with anelliptic parameter η being a small parameter. They derive a trial solution of the TTI eikonal equation, and applied the Shanks transform in order to stabilize the truncated power series. Generally, the presence of tilt results in significant complications in moveout expressions and traveltime parameters.

In this abstract, we use the moveout mapping concept proposed earlier by the authors (Stovas and Alkhalifah, 2012) to derive the TTI moveout approximations from the ones defined for a VTI medium.

The moveout mapping in TI media

In a homogeneous 2D TI medium, the mapping of moveout from VTI to TTI model is given by rational equations (Stovas and Alkhalifah, 2012)

$$x_n(p) = z\frac{x(p)\cos\theta + z\sin\theta}{z\cos\theta - x(p)\sin\theta}, \qquad t_n(p) = t(p)\frac{z}{z\cos\theta - x(p)\sin\theta} \tag{1}$$

where (x,t) and (x_n,t_n) are the offset-traveltime points in a VTI and TTI model, respectively, p is the horizontal slowness in a VTI model, z is the layer thickness and θ is the tilt angle. One can see that $(x=0, t=t_0) \Leftrightarrow (x_n = z\tan\theta, t_n = t_0/\cos\theta)$ and $(x=-z\tan\theta, t=t_{n0}/\cos\theta) \Leftrightarrow (x_n=0, t_n=t_{n0})$.

The inverse transform is defined as

$$x(p) = z\frac{x_n(p)\cos\theta - z\sin\theta}{z\cos\theta + x_n(p)\sin\theta}, \qquad t(p) = t_n(p)\frac{z}{z\cos\theta + x_n(p)\sin\theta}. \tag{2}$$

The function $t_n(x_n)$ is no longer symmetric and has a minimum at $x_n = x_{n0}$, which is defined by equation $dt_n/dx_n = 0$ with two solutions: $dx/dp = 0$ (the caustic condition) and $p_n = p\cos\theta + q\sin\theta = 0$ (the zero horizontal slowness in p_n, q_n space).

Mapping of moveout approximations

There exists many different moveout approximations developed recently (Malovichko, 1978; Alkhalifah and Tsvankin, 1995; Fomel and Stovas, 2010; Alkhalifah, 2011a). To derive the moveout approximation for TTI media is non-trivial task. Recently, Alkhalifah (2011a) and Stovas and Alkhalifah (2011) proposed the moveout approximations based on the linearization of eikonal equation in θ and in η, respectively, and following use of the Shanks transform, which allowed for an inhomogeneous background medium treatment.

Another approach, we propose in this abstract, is to map the moveout approximation from a VTI to TTI medium based on the mapping equations (1). We start with an approximation for a VTI medium in the form $t(x)$ as a function of also the vertical traveltime t_0, the normal moveout velocity v_n and the anelliptic parameter η. Using equation (1), we obtain a generalized approach to transform VTI approximations to TTI ones using the following formula,

$$t_n(x_n) = t\left(x = z\frac{x_n\cos\theta - z\sin\theta}{z\cos\theta + x_n\sin\theta}\right)\left(\cos\theta + \frac{x_n}{z}\sin\theta\right). \tag{3}$$

Let us illustrate the transformation (3) on a few known moveout approximations. In all following equations, we will use $z = v_0 t_0$.

The Taylor series for traveltime squared that is given by

$$t^2(x) = t_0^2 + \frac{x^2}{v_n^2} - \frac{2\eta x^4}{v_n^4 t_0^2} + \ldots \tag{4}$$

DOI http://dx.doi.org/10.1190/segam2012-0050.1

transforms, using equation 3, into the series

$$t_n^2(x_n) = t_0^2 \cos^2\theta \left[1 + \frac{v_0^2}{v_n^2}\tan^2\theta - \frac{2\eta v_0^4}{v_n^4}\tan^4\theta \right]$$
$$+ \frac{v_0 t_0 x_n}{v_n^2}\left[\left(\frac{v_n^2}{v_0^2} - 1 \right)\sin 2\theta + 2\eta \frac{v_0^2}{v_n^2}\left(3 + \cos 2\theta\right)\tan^3\theta \right]$$
$$+ \frac{x_n^2}{v_n^2}\left[\cos^2\theta + \sin^2\theta \left(\frac{v_n^2}{v_0^2} - 2\eta \frac{v_0^2}{v_n^2}\left(6 + 8\tan^2\theta + 3\tan^4\theta\right) \right) \right]$$
$$+ \frac{8\eta v_0 \tan\theta}{t_0 v_n^4 \cos^4\theta} x_n^3 \qquad (5)$$
$$- \frac{2\eta\left(3 - 2\cos 2\theta\right)x_n^4}{v_n^4 t_0^2 \cos^6\theta} + \ldots$$

Note that the series (5) for a TTI medium contains both even and odd power terms. Each Taylor series coefficient in (5) is given by the series with the number of terms that corresponds to the number of terms in the truncated Taylor series (4).

Applying the transformation on the hyperbolic approximation

$$t^2(x) = t_0^2 + \frac{x^2}{v_n^2}, \qquad (6)$$

yields the hyperbola with shifted apex position,

$$t_n^2(x_n) = t_{0n}^2 + \frac{\left(x_n - x_{0n}\right)^2}{v_{n,n}^2}, \qquad (7)$$

where the new traveltime parameters are given by

$$t_{0n}^2 = t_0^2 \frac{v_0^2}{v_0^2\cos^2\theta + v_n^2\sin^2\theta}$$
$$x_{0n} = \frac{t_0 v_0\left(v_0^2 - v_n^2\right)\sin\theta\cos\theta}{v_0^2\cos^2\theta + v_n^2\sin^2\theta}. \qquad (8)$$
$$v_{n,n}^2 = \frac{v_0^2 v_n^2}{v_0^2\cos^2\theta + v_n^2\sin^2\theta}$$

The moveout approximation (7) is valid for elliptical TTI media (Golikov and Stovas, 2012).

Applying the transformation on the rational approximation shown in Alkhalifah and Tsvankin (1995) as,

$$t^2(x) = t_0^2 + \frac{x^2}{v_n^2} - \frac{2\eta x^4}{v_n^4 t_0^2\left(1 + \left(1 + 2\eta\right)\frac{x^2}{v_n^2 t_0^2}\right)}, \qquad (9)$$

yields another rational approximation

$$t_n^2(x_n) = \frac{a_0 + a_1 x_n + a_2 x_n^2 + a_3 x_n^3 + a_4 x_n^4}{b_0 + b_1 x_n + b_2 x_n^2} \qquad (10)$$

with parameters

$$a_0 = v_0^4 t_0^4\left(v_n^4\cos^4\theta + 2v_0^2 v_n^2\left(1+\eta\right)\sin^2\theta\cos^2\theta + v_0^4\sin^4\theta\right)$$
$$a_1 = 4v_0^3 t_0^3\cos\theta\sin\theta\left(v_n^4\cos^2\theta - v_0^2 v_n^2\left(1+\eta\right)\cos 2\theta - v_0^4\sin^2\theta\right)$$
$$a_2 = 2v_0^2 t_0^2\left(3v_n^4\sin^2\theta\cos^2\theta + v_0^2 v_n^2\left(1+\eta\right)\left(1 - 6\sin^2\theta\cos^2\theta\right)\right.$$
$$\left. + 3v_0^4\sin^2\theta\cos^2\theta\right)$$
$$a_3 = 4v_0 t_0\cos\theta\sin\theta\left(v_n^4\sin^2\theta + v_0^2 v_n^2\left(1+\eta\right)\cos 2\theta - v_0^4\cos^2\theta\right)$$
$$a_4 = v_n^4\sin^4\theta + 2v_0^2 v_n^2\left(1+\eta\right)\sin^2\theta\cos^2\theta + v_0^4\cos^4\theta$$
$$b_0 = v_0^4 v_n^2 t_0^2\left(v_n^2\cos^2\theta + \left(1+2\eta\right)v_0^2\sin^2\theta\right) \qquad (11)$$
$$b_1 = v_0^3 v_n^2 t_0\sin 2\theta\left(v_n^2 - \left(1+2\eta\right)v_0^2\right)$$
$$b_2 = v_0^2 v_n^2\left(v_n^2\sin^2\theta + \left(1+2\eta\right)v_0^2\cos^2\theta\right)$$

For the shifted hyperbola equation (Malovichko, 1978)

$$t(x) = t_0\left[1 + \frac{1}{S_2}\left(\sqrt{1 + S_2\frac{x^2}{v_n^2 t_0^2}} - 1\right)\right], \qquad (12)$$

we obtain

$$t_n(x_n) = \frac{1}{S_2 v_0 v_n}\left[\left(S_2 - 1\right)v_n\left(x_n\cos\theta + v_0 t_0\sin\theta\right)\right. \qquad (13)$$
$$\left. + \sqrt{S_2 v_0^2\left(x_n\cos\theta - v_0 t_0\sin\theta\right)^2 + v_{nmo}^2\left(x_n\sin\theta + v_0 t_0\cos\theta\right)^2}\right]$$

We can map any of analytical moveout approximation from VTI to TTI media including the most accurate one, the generalized approximation.

Analysis of the approximations

Now we can analyze the vertical ($x_n = 0$) and horizontal ($x_n \to \infty$) velocities from the mapped moveout approximations.

From the hyperbolic approximation, we obtain

$$\frac{1}{V_0^2} = \frac{\cos^2\theta}{v_0^2} + \frac{\sin^2\theta}{v_n^2}$$
$$\frac{1}{V_x^2} = \frac{\cos^2\theta}{v_n^2} + \frac{\sin^2\theta}{v_0^2} \qquad (14)$$

From the rational approximation, we have

DOI http://dx.doi.org/10.1190/segam2012-0050.1

$$\frac{1}{V_0^2}=\frac{\frac{\cos^4\theta}{v_0^4}+2\left(1+\eta\right)\frac{\sin^2\theta\cos^2\theta}{v_0^2v_n^2}+\frac{\sin^4\theta}{v_n^4}}{\frac{\cos^2\theta}{v_0^2}+\left(1+2\eta\right)\frac{\sin^2\theta}{v_n^2}}$$

$$\frac{1}{V_x^2}=\frac{\frac{\sin^4\theta}{v_0^4}+2\left(1+\eta\right)\frac{\sin^2\theta\cos^2\theta}{v_0^2v_n^2}+\frac{\cos^4\theta}{v_n^4}}{\frac{\sin^2\theta}{v_0^2}+\left(1+2\eta\right)\frac{\cos^2\theta}{v_n^2}} \tag{15}$$

For shifted hyperbola approximation, we have

$$\frac{1}{V_0}=\frac{1}{S_2}\left[\frac{\left(S_2-1\right)\sin\theta}{v_0}+\sqrt{\frac{\cos^2\theta}{v_0^2}+\frac{S_2\sin^2\theta}{v_n^2}}\right]$$

$$\frac{1}{V_x}=\frac{1}{S_2}\left[\frac{\left(S_2-1\right)\cos\theta}{v_0}+\sqrt{\frac{\sin^2\theta}{v_0^2}+\frac{S_2\cos^2\theta}{v_n^2}}\right] \tag{16}$$

For the generalized moveout approximation, we obtain

$$\frac{1}{V_0^2}=\frac{\cos^2\theta}{v_0^2}+\frac{\sin^2\theta}{v_n^2}-\frac{4\eta\frac{\sin^4\theta}{v_n^4}}{\frac{\cos^2\theta}{v_0^2}+B\frac{\sin^2\theta}{v_n^2}+\sqrt{\frac{\cos^4\theta}{v_0^4}+2B\frac{\sin^2\theta\cos^2\theta}{v_0^2v_n^2}+C\frac{\sin^4\theta}{v_n^4}}}$$

$$\frac{1}{V_x^2}=\frac{\cos^2\theta}{v_n^2}+\frac{\sin^2\theta}{v_0^2}-\frac{4\eta\frac{\cos^4\theta}{v_n^4}}{\frac{\cos^2\theta}{v_n^2}+B\frac{\sin^2\theta}{v_0^2}+\sqrt{\frac{\cos^4\theta}{v_n^4}+2B\frac{\sin^2\theta\cos^2\theta}{v_0^2v_n^2}+C\frac{\sin^4\theta}{v_0^4}}} \tag{17}$$

The parameters B and C are calculated from a horizontal ray in a homogeneous VTI medium and equal to $\left(1+8\eta+8\eta^2\right)/\left(1+2\eta\right)$ and $1/\left(1+2\eta\right)^2$, respectively (Fomel and Stovas, 2010).

Due to the symmetry of the TTI model over $\pi/2$ all equations for the horizontal velocity can be obtained from the equations for the vertical velocity by interchanging of sine and cosine functions. The normal moveout velocities, which are crucial for surface seismic data, defined at $x_n=0$, can also be easily computed from the mapped moveout approximations.

The approximated vertical, horizontal and normal moveout velocities are plotted in Figure 1 versus the tilt angle for anisotropic medium with parameters: $v_0=2.0\,km/s$, $\delta=0.1$ and $\eta=0.2$. We can see that the results from the hyperbolic and the shifted hyperbola approximations, especially for the normal moveout velocity, are very different from the ones obtained from the rational and generalized approximations.

Numerical examples

To illustrate the accuracy of the proposed approximation, we select a homogeneous TTI model with parameters mentioned above and $\theta=30^o$. In Figure 2, we display the relative error in traveltime obtained from selected moveout approximations being mapped from a VTI medium. The generalized approximation gives the best results with a maximum relative error of about 0.0002 on the offset/depth spread up to 5. The worst results are obtained from a hyperbolic approximation. Note that at offset $x_n=z\tan\theta$, which corresponds to $x=0$, all moveout approximations predict the exact traveltime. In Figure 3, we show the relative error in traveltime for mapped generalized approximation for different values of symmetry tilt angles. We can see that the accuracy of the generalized moveout approximation remains the same regardless to the symmetry direction.

Layered media

For layered media with a constant tilt in the symmetry axis (factorized anisotropic in the tilt), effective media approximations used for the VTI equations holds here as well. Nevertheless, the parameters now are those corresponding to the symmetry axis direction. For example the vertical velocity would be given by

$$v_{0eff}(t_0)=\frac{1}{t_0}\int_0^{t_0}v_0(\tau)d\tau\ , \tag{18}$$

where $v_0(t_0)$ is the velocity along the constant symmetry direction. The same applies for other parameters.

Conclusions

We develop an analytical general formula to map moveout approximations from VTI to TTI media. The method is applied on the hyperbolic, the shifted hyperbola, the rational and the generalized approximations for a vertical symmetry axis direction. The mapped moveout approximations are also used to derive approximations for vertical and horizontal velocities.

DOI http://dx.doi.org/10.1190/segam2012-0050.1

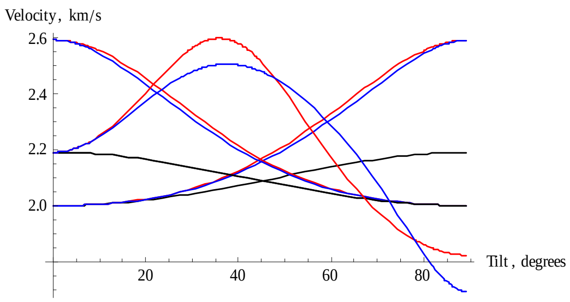

Figure 1: The vertical, horizontal and normal moveout velocities computed from the mapped moveout approximations for a homogeneous TTI model is given by $v_0 = 2.0\, km/s$, $\delta = 0.1$ and $\eta = 0.2$. The hyperbolic, rational and generalized moveout approximations are indicated by black, red and blue lines, respectively. Note, that for hyperbolic moveout approximation, the horizontal and normal moveout velocities coincide. No results are shown for the shifted hyperbola approximation since it is not suitable for a homogeneous VTI or TTI model.

The accuracy of the mapped moveout approximations is tested for a homogeneous TTI medium example. We show that the generalized moveout approximation being mapped to a TTI medium remains the most accurate one.

Acknowledgments

Alexey Stovas would like to acknowledge the ROSE project for financial support. Tariq Alkhalifah would like to thank KAUST for its financial support.

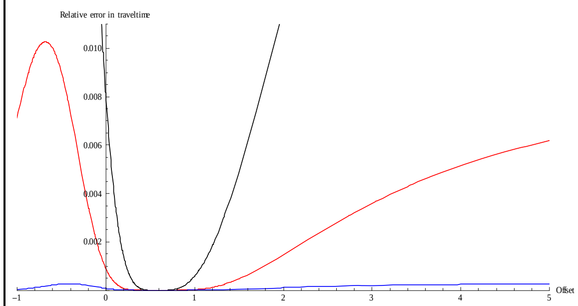

Figure 2: The relative error in traveltime by using the mapped moveout approximations for a homogeneous TTI model is given by $v_0 = 2.0\, km/s$, $\delta = 0.1$, $\eta = 0.2$ and $\theta = 30^o$. The hyperbolic, rational and generalized moveout approximations are indicated by black, red and blue lines, respectively. No results are shown for the shifted hyperbola approximation since it is not suitable for a homogeneous VTI or TTI model. Note that at offset $x_n = z \tan\theta$ which corresponds to $x = 0$, all moveout approximations predict the exact traveltime.

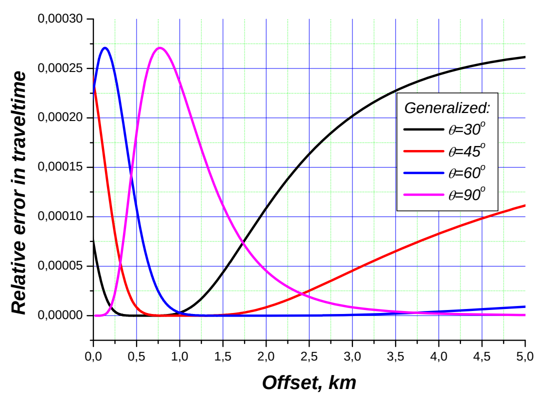

Figure 3: The relative error in traveltime by using the mapped generalized moveout approximations for a homogeneous TTI model is given by $v_0 = 2.0\, km/s$, $\delta = 0.1$, $\eta = 0.2$ and different values of tilt angle.

DOI http://dx.doi.org/10.1190/segam2012-0050.1

http://dx.doi.org/10.1190/segam2012-0050.1

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