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Low-SNR Capacity of Parallel IM-DD Optical Wireless Channels

Anas Chaaban, Zouheir Rezki, and Mohamed-Slim Alouini

***Abstract*—The capacity of parallel intensity-modulation and direct-detection (IM-DD) optical wireless channels with total average intensity and per-channel peak intensity constraints is studied. The optimal intensity allocation at low signal-to-noise ratio (SNR) is derived, leading to the capacity-achieving on-off keying (OOK) distribution. Interestingly, while activating the strongest channel is optimal if (i) the peak intensity is fixed, this is not the case if (ii) the peak intensity is proportional to the average intensity. The minimum average optical intensity per bit is also studied, and is characterized for case (i) where it is achievable at low SNR. However, in case (ii), the average optical intensity per bit grows indefinitely as SNR decreases, indicating that lower optical intensity per bit can be achieved at moderate SNR than at low SNR.**

***Index Terms*—Parallel IM-DD channels; Capacity; Intensity allocation; Intensity per bit.**

I. Introduction

Optical wireless communication (OWC) is a promising technique for supporting high data-rate communications as an alternative or a backup for radio-frequency (RF) communications. It has numerous applications such as wireless backhaul or visible-light communications [1], [2], and enjoys abundant unlicensed spectrum. Like in RF communications, studying parallel OWC is important due to its applicability in many scenarios such as multiple input multiple output (MIMO) systems [3]–[5], multi-wavelength OWC (e.g. color-shift keying or wave-division multiplexing) [6]–[8], and TDMA multi-user systems. Indeed, some of those systems cannot be directly modeled as parallel channels (e.g. MIMO). However, pre/post-processing techniques (e.g. inversion or SVD [4], [9]) can be used to transform them to parallel channels. These might incur some performance losses but are still practically appealing due to their simplicity.

In this paper, we focus on the capacity and minimum intensity per bit (IPB) of parallel IM-DD OWC systems. These performance criteria are of significant importance as they provide fundamental theoretical limits on transmission efficiency that cannot be surpassed. At a given time instant, the parallel IM-DD OWC system can be described by

$$Y_i = h_i X_i + Z_i, \quad i \in \mathcal{N} \triangleq \{1, \dots, N\}, \tag{1}$$

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where Y_i is the received signal of channel i, $X_i \in [0, \mathcal{A}]$ represents light intensity, Z_i is Gaussian noise with zero mean and unit variance independent over time, and $\boldsymbol{h} = (h_1, \dots, h_N) \in \mathbb{R}_+^N$ is the channel gain.[1] We assume that $h_i \geq h_{i+1}$ without loss of generality, and that $\boldsymbol{h}$ is available at the transmitter and the receiver through a feedback mechanism, and remains fixed throughout a transmission block due to the slow fading property of OWC channels [1].

Here, $\mathcal{A}$ is the peak intensity constraint. The average optical intensity of channel i is $\mathcal{E}_i = \mathbb{E}[X_i] \in [0, \mathcal{A}]$ which should satisfy an ℓ_1-norm constraint $\|\boldsymbol{\mathcal{E}}\|_1 \leq \mathcal{E}$. We focus on the low-SNR ($\mathcal{E} \to 0$), and consider two cases: (i) fixed $\mathcal{A}$, and (ii) $\mathcal{A} = \frac{\mathcal{E}}{\alpha}$ for some $\alpha > 0$ as in [12], [13]. Low SNR models situations with low average intensity requirement in the former, and situations with strong attenuation or short pulse duration in the latter. Owing to those constraints, the capacity and minimum IPB of this system are different from those of classical RF parallel channels [10, Sec. 9.4]. Next, we focus on the low-SNR capacity (high-SNR was studied in [11]).

II. Low-SNR Capacity

We denote the capacity by $C_N(\boldsymbol{h}, \mathcal{E}, \mathcal{A})$, and define it in the standard Shannon sense [10]. We start by reviewing some bounds on $C_1(h_1, \mathcal{E}, \mathcal{A})$, the capacity for $N = 1$.

Lemma 1: [12] $C_1(h_1, \mathcal{E}, \mathcal{A})$ satisfies $C_{1,0}(h_1, \mathcal{E}, \mathcal{A}) \leq C_1(h_1, \mathcal{E}, \mathcal{A}) \leq C_{1,1}(h_1, \mathcal{E}, \mathcal{A}) \leq C_{1,2}(h_1, \mathcal{E}, \mathcal{A})$ where

$$C_{1,0}(h_1, \mathcal{E}, \mathcal{A}) = I(X_1; Y_1), \ X_1 = \mathcal{A} \cdot \text{Bern}(\mathcal{E}/\mathcal{A}), \tag{2}$$

$$C_{1,1}(h_1, \mathcal{E}, \mathcal{A}) = \frac{1}{2} \log\left(1 + h_1^2 \mathcal{E}(\mathcal{A} - \mathcal{E})\right), \tag{3}$$

$$C_{1,2}(h_1, \mathcal{E}, \mathcal{A}) = \frac{1}{2} h_1^2 \mathcal{E}(\mathcal{A} - \mathcal{E}). \tag{4}$$

Here, $\text{Bern}(a)$ denotes a Bernoulli random variable with $\mathbb{P}(1) = 1 - \mathbb{P}(0) = a$ and $I(\cdot;\cdot)$ is the mutual information. The lower bound is achievable using on-off keying (OOK). The bounds coincide at low SNR as shown in [12] since[2]

$$\lim_{\mathcal{E} \to 0} \frac{C_{1,0}(h_1, \mathcal{E}, \mathcal{A})}{C_{1,2}(h_1, \mathcal{E}, \mathcal{A})} = 1. \tag{5}$$

For a system with $N > 1$, the capacity $C_N(\boldsymbol{h}, \mathcal{E}, \mathcal{A})$ is equal to the sum of the capacities of the N channels $C_1(h_i, \mathcal{E}_i, \mathcal{A})$, $i = 1, \dots, N$, optimized over the set of feasible $\boldsymbol{\mathcal{E}}$, i.e.,

$$C_N(\boldsymbol{h}, \mathcal{E}, \mathcal{A}) = \max_{\boldsymbol{\mathcal{E}} \in \mathcal{S}} \sum_{i \in \mathcal{N}} C_1(h_i, \mathcal{E}_i, \mathcal{A}). \tag{6}$$

[1] Throughout the paper, we use bold symbols $\mathbf{x}$ to denote $(x_1, \dots, x_N)$.

[2] This convergence was proved in [12] for $\mathcal{E} = \alpha\mathcal{A}$, but can be shown similarly for fixed $\mathcal{A}$ by invoking [14, Th. 1] instead of [14, Th. 2] and noting that $X = \mathcal{A} \cdot \text{Bern}(\mathcal{E}/\mathcal{A}) \Rightarrow \text{Var}(X) = \mathcal{E}(\mathcal{A} - \mathcal{E}) \xrightarrow{\mathcal{E} \to 0} 0$.

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This can be proved as in [10, Sec. 9.4]. Since $\mathcal{E}_i$ can be restricted to $[0, \frac{\mathcal{A}}{2}]$ without any loss in capacity [12], then $\mathcal{S} = \left\{ \boldsymbol{\mathcal{E}} \in [0, {}^{\mathcal{A}}\!/_2]^N : \|\boldsymbol{\mathcal{E}}\|_1 \leq \mathcal{E} \right\}$ leading to the following.

Theorem 1: $C_N(\boldsymbol{h}, \mathcal{E}, \mathcal{A})$ satisfies $C_{N,0}(\boldsymbol{h}, \mathcal{E}, \mathcal{A}) \leq C_N(\boldsymbol{h}, \mathcal{E}, \mathcal{A}) \leq C_{N,1}(\boldsymbol{h}, \mathcal{E}, \mathcal{A}) \leq C_{N,2}(\boldsymbol{h}, \mathcal{E}, \mathcal{A})$ where

$$C_{N,j}(\boldsymbol{h}, \mathcal{E}, \mathcal{A}) = \max_{\boldsymbol{\mathcal{E}} \in \mathcal{S}} \sum_{i \in \mathcal{N}} C_{1,j}(h_i, \mathcal{E}_i, \mathcal{A}), \; j = 0, 1, 2. \quad (7)$$

Proof: The bounds follow using Lemma 1 and (6). ■

Those bounds are tight at low SNR as given next.

Theorem 2: The bounds in Theorem 1 converge at low SNR.

Proof: Let the solution of the maximization in (7) for $j = 2$ be $\boldsymbol{\mathcal{E}}^*$. Then, we can write $C_{N,2}(\boldsymbol{h}, \mathcal{E}, \mathcal{A}) = \sum_{i \in \mathcal{N}} \widehat{C}(h_i, \mathcal{E}_i^*, \mathcal{A})$. Since $\|\boldsymbol{\mathcal{E}}^*\|_1 \leq \mathcal{E}$ and $\mathcal{E} \to 0$, then $\mathcal{E}_i^* \to 0$ and hence $\lim_{\mathcal{E} \to 0} \frac{C_{1,0}(h_i, \mathcal{E}_i^*, \mathcal{A})}{C_{1,2}(h_i, \mathcal{E}_i^*, \mathcal{A})} = 1$ by (5). Therefore, $C_{1,0}(h_i, \mathcal{E}_i^*, \mathcal{A}) = C_{1,2}(h_i, \mathcal{E}_i^*, \mathcal{A}) + o(C_{1,2}(h_i, \mathcal{E}_i^*, \mathcal{A}))$ where $\lim_{\mathcal{E} \to 0} \frac{o(C_{1,2}(h_i, \mathcal{E}_i^*, \mathcal{A}))}{C_{1,2}(h_i, \mathcal{E}_i^*, \mathcal{A})} = 0$. But $C_{N,2}(\boldsymbol{h}, \mathcal{E}, \mathcal{A}) \geq C_{N,0}(\boldsymbol{h}, \mathcal{E}, \mathcal{A}) \geq \sum_{i \in \mathcal{N}} C_{1,0}(h_i, \mathcal{E}_i^*, \mathcal{A})$. Thus

$$1 \geq \lim_{\mathcal{E} \to 0} \frac{C_{N,0}(\boldsymbol{h}, \mathcal{E}, \mathcal{A})}{C_{N,2}(\boldsymbol{h}, \mathcal{E}, \mathcal{A})} \geq \lim_{\mathcal{E} \to 0} \frac{\sum_{i \in \mathcal{N}} C_{1,0}(h_i, \mathcal{E}_i^*, \mathcal{A})}{\sum_{i \in \mathcal{N}} C_{1,2}(h_i, \mathcal{E}_i^*, \mathcal{A})}$$
$$= \lim_{\mathcal{E} \to 0} \frac{\sum_{i \in \mathcal{N}} [C_{1,2}(h_i, \mathcal{E}_i^*, \mathcal{A}) + o(C_{1,2}(h_i, \mathcal{E}_i^*, \mathcal{A}))]}{\sum_{i \in \mathcal{N}} C_{1,2}(h_i, \mathcal{E}_i^*, \mathcal{A})} = 1,$$

by definition of $o(C_{1,2}(h_i, \mathcal{E}_i^*, \mathcal{A}))$. Thus, $\lim_{\mathcal{E} \to 0} \frac{C_{N,0}(\boldsymbol{h}, \mathcal{E}, \mathcal{A})}{C_{N,2}(\boldsymbol{h}, \mathcal{E}, \mathcal{A})}$ equals 1 which completes the proof. ■

Based on Theorem 2, $C_{N,2}(\boldsymbol{h}, \mathcal{E}, \mathcal{A})$ is asymptotically achievable by OOK at low SNR. It remains to find the optimal allocation $\boldsymbol{\mathcal{E}}^*$ at low SNR. This would lead to the low-SNR capacity and would provide the optimal OOK distribution which depends on $\mathcal{E}_i$. This is considered next.

III. Intensity Allocation

Since $C_{N,0}(\boldsymbol{h}, \mathcal{E}, \mathcal{A})$ (OOK) and $C_{N,2}(\boldsymbol{h}, \mathcal{E}, \mathcal{A})$ (upper bound) coincide at low SNR, we consider the maximization

$$\max_{\boldsymbol{\mathcal{E}}} \quad \sum_{i \in \mathcal{N}} C_{1,2}(h_i, \mathcal{E}_i, \mathcal{A}) \quad (8)$$
$$\text{s.t.} \quad \boldsymbol{\mathcal{E}} \in [0, {}^{\mathcal{A}}\!/_2]^N, \; \|\boldsymbol{\mathcal{E}}\|_1 \leq \mathcal{E}.$$

Since the objective is concave and the constraints are linear in $\boldsymbol{\mathcal{E}}$, this problem is convex and can be solved efficiently [15]. Its Lagrangian is given by

$$\mathcal{L}(\boldsymbol{\mathcal{E}}, \lambda, \boldsymbol{\nu}, \boldsymbol{\eta}) = \sum_{i \in \mathcal{N}} (-C_{1,2}(h_i, \mathcal{E}_i, \mathcal{A}) + \nu_i (\mathcal{E}_i - {}^{\mathcal{A}}\!/_2) - \eta_i \mathcal{E}_i)$$
$$+ \lambda(\|\boldsymbol{\mathcal{E}}\|_1 - \mathcal{E}), \quad (9)$$

with $\lambda, \nu_i, \eta_i \geq 0 \; \forall i \in \mathcal{N}$. Due to convexity, the KKT conditions are necessary and sufficient for optimality, and the optimal solution must satisfy $\frac{\partial}{\partial \mathcal{E}_i} \mathcal{L}(\boldsymbol{\mathcal{E}}, \lambda, \boldsymbol{\nu}, \boldsymbol{\eta}) = 0$, $\lambda(\|\boldsymbol{\mathcal{E}}\|_1 - \mathcal{E}) = 0$, $\nu_i (\mathcal{E}_i - {}^{\mathcal{A}}\!/_2) = 0$ and $\eta_i \mathcal{E}_i = 0 \; \forall i \in \mathcal{N}$, in addition to the constraints of (8). The solutions of these conditions are summarized in Table I. Those can be combined into the optimal solution at low SNR given by

$$\mathcal{E}_i^* = \left[\frac{\mathcal{A}}{2} - \frac{\lambda}{h_i^2} \right]^+, \quad (10)$$

<table>
<tr><th></th><th colspan="2">$\lambda > 0$ and $\|\boldsymbol{\mathcal{E}}\|_1 = \mathcal{E}$</th><th>$\lambda = 0$</th></tr>
<tr><td>$\nu_i > 0$ for some i</td><td colspan="2">$\mathcal{E}_i = {}^{\mathcal{A}}\!/_2$, $\eta_i = 0$ and $\lambda = -\nu_i < 0$ (a contradiction)</td><td rowspan="3">$\mathcal{E}_i = {}^{\mathcal{A}}\!/_2 \; \forall i$, $\eta_i = 0 \; \forall i$, and ${}^{N\mathcal{A}}\!/_2 \leq \mathcal{E}$</td></tr>
<tr><td rowspan="2">$\nu_i = 0$, $\forall i$</td><td>$\eta_i > 0$</td><td>$\mathcal{E}_i = 0, \; \lambda > {}^{h_i^2 \mathcal{A}}\!/_2$</td></tr>
<tr><td>$\eta_i = 0$</td><td>$\mathcal{E}_i = {}^{\mathcal{A}}\!/_2 - {}^{\lambda}\!/_{h_i^2}, \; \lambda \leq {}^{h_i^2 \mathcal{A}}\!/_2$</td></tr>
</table>

TABLE I: Solutions of the KKT conditions of (8). The shaded cell is infeasible.

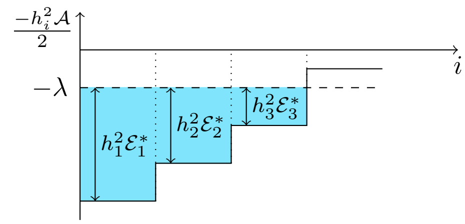

Fig. 1: Inverted water-filling solution of problem (8).

where $x^+ = \max\{0, x\}$ and λ is chosen such that $\sum_{i \in \mathcal{N}} \mathcal{E}_i^* = \min\{\mathcal{E}, {}^{N\mathcal{A}}\!/_2\}$. The corresponding optimal OOK distribution over channel i is $X_i \sim \mathcal{A} \cdot \text{Bern}({}^{\mathcal{E}_i^*}\!/_{\mathcal{A}})$.

This solution is different from standard water-filling [16, Ch. 5], but still has such an interpretation. To emphasize its water-filling flavor, we write it alternatively as

$$h_i^2 \mathcal{E}_i^* = \left[-\lambda - \frac{(-h_i^2 \mathcal{A})}{2} \right]^+. \quad (11)$$

This can be described as *inverted water-filling*, where ${}^{-h_i^2 \mathcal{A}}\!/_2$ represents the bottom of the container, $-\lambda$ the water level, and $h_i^2 \mathcal{E}_i^*$ the water poured in channel i (Fig. 1).

Interestingly, (10) does not saturate a channel ($\mathcal{E}_i = \frac{\mathcal{A}}{2}$ for some i) unless $\mathcal{E} \geq \frac{N\mathcal{A}}{2}$. Thus, *either all or none of the channels* use symmetric OOK ($X_i \sim \mathcal{A} \cdot \text{Bern}({}^1\!/_2)$).

In the RF context, activating the strongest channel is optimal at low SNR [16, p. 208]. Thus, it is interesting to compare it with inverted water-filling (10). However, if $\mathcal{E} > {}^{\mathcal{A}}\!/_2$, then activating the strongest channel only in our case is wasteful of $\mathcal{E}$ (cf. (8)).[3] Alternatively, we can activate the $\kappa+1$ strongest channels, where $\kappa = \lfloor {}^{2\mathcal{E}}\!/_{\mathcal{A}} \rfloor$, as follows

$$\mathcal{E}_i = {}^{\mathcal{A}}\!/_2 \text{ for } i = 1, \ldots, \kappa, \text{ and } \mathcal{E}_{\kappa+1} = \mathcal{E} - {}^{\kappa\mathcal{A}}\!/_2. \quad (12)$$

We denote the OOK scheme using this allocation by 'OOK-strongest', the OOK scheme using (10) by 'OOK-optimal' (low-SNR optimal), and that using the allocation $\mathcal{E}_i = \min\{{}^{\mathcal{E}}\!/_N, {}^{\mathcal{A}}\!/_2\} \; \forall i \in \mathcal{N}$ by 'OOK-equal'.

Fig. 2 shows a numerical comparison for an exemplary system and confirms Theorem 2. For fixed $\mathcal{A}$, OOK-equal is sub-optimal, and OOK-optimal coincides with OOK-strongest at low SNR. In particular, as $\mathcal{E} \to 0$, λ increases to maintain $\sum_{i \in \mathcal{N}} \mathcal{E}_i^* = \mathcal{E}$ (cf. Fig 1), until a point where only $\mathcal{E}_1^* \neq 0$ where the two coincide. This observation does not hold when $\mathcal{E} = \alpha \mathcal{A}$, and interestingly *multiple channels might be activated at low SNR* depending on $\boldsymbol{h}$. In this case, $\mathcal{E} \to 0 \Rightarrow \mathcal{A} \to 0$, and the stair-shape in Fig. 1 flattens,

[3]The objective of (8) is decreasing in $\mathcal{E}_i$ for $\mathcal{E}_i \in [{}^{\mathcal{A}}\!/_2, \mathcal{A}]$. Hence, if $\mathcal{E} > {}^{\mathcal{A}}\!/_2$, then activating the strongest channel only is strictly sub-optimal.

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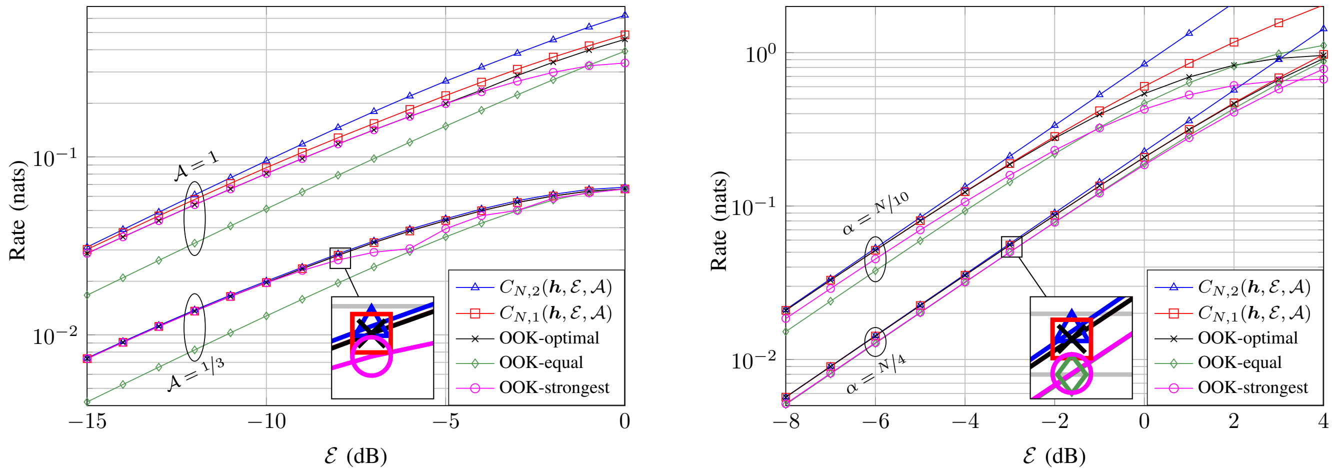

(a) Fixed $\mathcal{A}$

(b) $\mathcal{A} = \mathcal{E}/\alpha$

Fig. 2: Bounds vs. $\mathcal{E}$ for $N = 4$ and $\boldsymbol{h} = (1, 0.8, 0.6, 0.4)$.

preserving a nonzero water level in multiple channels as $\mathcal{E} \to 0$. Both OOK-strongest and OOK-equal are sub-optimal in this case. Note that the gap between OOK-equal and OOK-optimal is smaller if α is larger. For $\alpha \geq \frac{N}{2}$, the two coincide, since (10) becomes $\mathcal{E}_i = \frac{\mathcal{A}}{2}$.

IV. Minimum Average Intensity per Bit

The average optical IPB $\mathcal{E}_b(\mathcal{E})$ of a scheme achieving a rate $r(\mathcal{E})$ nats under a total intensity constraint $\mathcal{E}$ can be written as $\mathcal{E}_b(\mathcal{E}) = \frac{\mathcal{E}\log(2)}{r(\mathcal{E})}$ where $\log(2)$ is used to convert nats to bits. Here, we study the minimum $\mathcal{E}_b(\mathcal{E})$ which can be defined similar to [17] as

$$\mathcal{E}_{b,min} = \min_{\mathcal{E}\in(0,N\mathcal{A}]} \frac{\mathcal{E}\log(2)}{C_N(\boldsymbol{h}, \mathcal{E}, \mathcal{A})}. \quad (13)$$

If the channel capacity is monotonically increasing and concave in $\mathcal{E}$, then $\mathcal{E}_{b,min}$ can be obtained as [17]

$$\mathcal{E}_{b,min} = \lim_{\mathcal{E}\to 0} \frac{\mathcal{E}\log(2)}{C_N(\boldsymbol{h}, \mathcal{E}, \mathcal{A})}. \quad (14)$$

We start with fixed $\mathcal{A}$ for which we have the following.

Theorem 3: The minimum average optical IPB for a system of N parallel channels with channel $\boldsymbol{h}$ and fixed $\mathcal{A}$ is given by $\mathcal{E}_{b,min} = \frac{2\log(2)}{h_1^2\mathcal{A}}$.

Proof: Since the capacity with $\mathcal{E} > \frac{N\mathcal{A}}{2}$ is equal to that with $\mathcal{E} = \frac{N\mathcal{A}}{2}$, we focus on $\mathcal{E} \leq \frac{N\mathcal{A}}{2}$. Consider an input $\boldsymbol{X} \in [0, \mathcal{A}]^N$ distributed according to $f_1(\mathbf{x}) = \prod_{i\in\mathcal{N}} f_{1i}(x_i)$ with $\sum_{i\in\mathcal{N}} \mathbb{E}[X_i] = \mu_1 < \frac{N\mathcal{A}}{2}$ and denote the corresponding achievable rate by $I_1 = I(\boldsymbol{X}; \boldsymbol{Y})$.[4] Consider also another input distributed according to $f_2(\mathbf{x}) = \prod_{i\in\mathcal{N}} f_{2i}(x_i)$ with $f_{2i}(x_i) = f_{1i}(\mathcal{A} - x_i)$. This input satisfies $\sum_{i\in\mathcal{N}} \mathbb{E}[X_i] = \mu_2 = N\mathcal{A} - \mu_1 > \frac{N\mathcal{A}}{2}$, and has an achievable rate $I_2 = I_1$ by symmetry of the Gaussian noise. Now construct a new input distributed according to

$$f_3(\mathbf{x}) = \tau f_1(\mathbf{x}) + (1-\tau) f_2(\mathbf{x}) \quad (15)$$

[4]A product distribution is considered since capacity can be achieved by independent coding over the N channels.

for some $\tau \in [0, 1]$. This input satisfies

$$\sum_{i\in\mathcal{N}} \mathbb{E}[X_i] = \mu_3 = \tau\mu_1 + (1-\tau)\mu_2 \in [\mu_1, N\mathcal{A} - \mu_1],$$

and leads to an achievable rate

$$I_3 = I(\boldsymbol{X}; \boldsymbol{Y}) \geq \tau I_1 + (1-\tau) I_2 = I_1 \quad (16)$$

by Jensen's inequality and the concavity of mutual information in $f(\mathbf{x})$ for a given $f(\mathbf{y}|\mathbf{x})$ [10]. Thus, $I(\boldsymbol{X}; \boldsymbol{Y})$ is concave and increasing for $\sum_{i\in\mathcal{N}} \mathbb{E}[X_i] \in (0, \frac{N\mathcal{A}}{2}]$. Thus, (14) applies. By Theorem 2, capacity satisfies $\lim_{\mathcal{E}\to 0} \frac{C_N(\boldsymbol{h},\mathcal{E},\mathcal{A})}{C_{N,2}(\boldsymbol{h},\mathcal{E},\mathcal{A})} = 1$. Moreover, we have that $C_{N,2}(\boldsymbol{h}, \mathcal{E}, \mathcal{A}) = \frac{1}{2} h_1^2 \mathcal{E}(\mathcal{A} - \mathcal{E})$ since the optimal allocation solution of this maximization at low SNR is $\mathcal{E}_1 = \mathcal{E}$ and $\mathcal{E}_i = 0$ for $i \neq 1$ as given in (10). Therefore, $\lim_{\mathcal{E}\to 0} \frac{C_N(\boldsymbol{h},\mathcal{E},\mathcal{A})}{\frac{1}{2}h_1^2\mathcal{E}(\mathcal{A}-\mathcal{E})} = 1$ leading to the desired result. ■

Thus, *the minimum $\mathcal{E}_b(\mathcal{E})$ is achieved at low SNR for fixed $\mathcal{A}$*. Note that $\mathcal{E}_{b,min}$ decreases with increasing h_1^2 or $\mathcal{A}$, which is consistent with intuition. Fig. 3a shows a plot of capacity bounds versus $\mathcal{E}_b(\mathcal{E})$ obtained by plotting the pair $\left(\frac{\mathcal{E}\log(2)}{C_{N,j}(\boldsymbol{h},\mathcal{E},\mathcal{A})}, C_{N,j}(\boldsymbol{h}, \mathcal{E}, \mathcal{A})\right)$ for $j \in \{0, 1, 2\}$ and $\mathcal{E} > 0$. Note that capacity approaches zero as $\mathcal{E}_b(\mathcal{E}) \to \mathcal{E}_{b,min}$ which is 1.41 dB for $\mathcal{A} = 1$ in this example.

Now we consider $\mathcal{A} = \mathcal{E}/\alpha$. In this case, capacity is not concave in $\mathcal{E}$. For $N = 1$ for instance, the low-SNR capacity is $\frac{1}{2}h_1^2\mathcal{E}(\mathcal{A} - \mathcal{E}) = \frac{1}{2}h_1^2\mathcal{E}^2(\frac{1}{\alpha} - 1)$ which is convex, and the upper bound $C_{1,1}(\boldsymbol{h}, \mathcal{E}, \mathcal{A})$ in Lemma 1 implies

$$\mathcal{E}_b(\mathcal{E}) \geq \frac{2\mathcal{E}\log(2)}{\log(1 + h_1^2\mathcal{E}^2(\frac{1}{\alpha} - 1))}. \quad (17)$$

This lower bound grows indefinitely as $\mathcal{E} \to 0$, which can be verified using L'Hôpital's rule. The same behavior holds for $N > 1$ as shown in Fig. 3b where it is clear that $\mathcal{E}_b(\mathcal{E})$ increases as $\mathcal{E} \to 0$. This indicates that *the minimum $\mathcal{E}_b(\mathcal{E})$ is not achieved at low SNR*, and that *capacity is nonzero at the minimum $\mathcal{E}_b(\mathcal{E})$*, contrary to fixed $\mathcal{A}$. Hence, operating the system at moderate SNR is more efficient in terms of average IPB than at low SNR.

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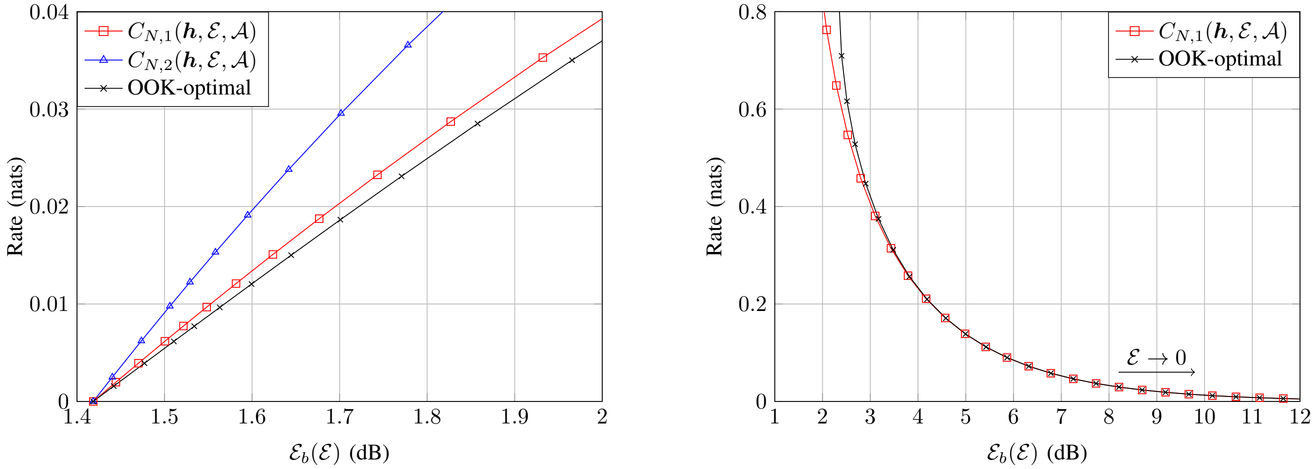

(a) Fixed $\mathcal{A}$

(b) $\mathcal{A} = {}^{\mathcal{E}}/_{\alpha}$

Fig. 3: Bounds vs. $\mathcal{E}_b$ for $N = 2$ and $\boldsymbol{h} = (1, 0.7)$.

V. Conclusion

We characterized the low-SNR capacity of parallel OWC channels employing IM-DD with peak and a total average optical intensity constraints. Low SNR can take place due to dimming for instance, in which the peak constraint remains constant and the average constraint decreases, or due to attenuation or shortening the pulse duration in which both peak and average constraints decrease proportionally. For both cases, the capacity achieving OOK distribution is obtained by deriving the optimal intensity allocation over the channels, which has an inverted water-filling representation. While it is optimal to activate the strongest channel (similar to RF) as the average constraint decreases with a fixed peak constraint, this is sub-optimal if the peak and average constraints decrease proportionally. In this latter case, all channels might need to be activated (contrary to RF) in order to achieve capacity. The performance of equal intensity allocation can be significantly lower than the optimal performance, which demonstrates the importance of this solution. This result provides a guideline for optimal code design for this channel. The two cases also differ in terms of their minimum intensity per bit. For the first case, the lowest average optical intensity per bit is achievable at low SNR. This is in sharp contrast with the second case for which we demonstrated that using moderate SNR can be more efficient in terms of intensity per bit, contrary to RF.

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