Recent advances in coherent perfect absorber-lasers and their future applications ######-#################

Item Type	Article
Authors	Yang, Minye;Ye, Zhi lu;Zhu, Liang;Farhat, Mohamed;Chen, Pai Yen
Citation	Yang, M., Ye, Z., Zhu, L., Farhat, M., & Chen, P.-Y. (2022). Recent advances in coherent perfect absorber-lasers and their future applications. Journal of Central South University, 29(10), 3203–3216. https://doi.org/10.1007/s11771-022-5160-0
Eprint version	Post-print
DOI	10.1007/s11771-022-5160-0
Publisher	Springer Science and Business Media LLC
Journal	Journal of Central South University
Rights	This is an accepted manuscript version of a paper before final publisher editing and formatting. Archived with thanks to Springer Science and Business Media LLC. The version of record is available from Journal of Central South University.
Download date	2023-11-30 07:20:31
Link to Item	http://hdl.handle.net/10754/685957

J. Cent. South Univ. (2022) 29: –
DOI:

Springer

Recent Advances in Development of Coherent Perfect Absorber-Laser and Their Future Applications

Minye Yang[1*], Zhilu Ye[1], Liang Zhu[1], Mohamed Farhat[2], and Pai-Yen Chen[1]

1. Department of Electrical and Computer Engineering, University of Illinois at Chicago, Chicago, IL 60607, United States
2. Computer, Electrical, and Mathematical Science and Engineering Division, King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900, Saudi Arabia

---

Abstract: In recent years, peculiar physical phenomena enabled by non-Hermitian systems, especially the parity-time (*PT*)-symmetric systems, have drawn tremendous research interests. Particularly, special spectral degeneracies known as exceptional points (EPs) and coherent perfect absorber-laser (CPAL) points where zero and infinite large eigenvalues coexist are the most popular topics to be studied. To date, the discussions of EPs that serve as transition boundaries between broken *PT*-symmetry phase and exact *PT*-symmetry phase have been intensively presented. However, the theoretical analysis and experimental validations of CPAL points are inadequate. Different from EPs, CPAL points, as a special solution of broken *PT*-symmetry phase, may exhibit even further counterintuitive physical features, which may have significant implications to study non-Hermitian physics. Here we review some recent advances of CPAL phenomena in different sub-disciplines of physics, including optics, electronics and electromagnetics, and acoustics. Additionally, we also provide an envision of future directions and applications of CPAL systems.

Key words: parity-time symmetry; coherent-perfect absorber-laser; spectral singularities; non-Hermitian physics

Cite this article as: [J]. Journal of Central South University, 2022, 29(): . DOI:

---

1 Introduction

In quantum mechanics, the observables with real eigenvalues may intuitively manifest measurable physical realities, which, in a commonly held view, are only associated with Hermitian Hamiltonians in closed systems exhibiting real eigenenergies [1]. However, several important pioneer works on non-Hermitian physics in open systems [1–3] have raised an active debate that non-Hermitian Hamiltonians commuting with parity (*P*) and time (*T*) operators may counterintuitively possess purely real eigenspectra [4], known as *PT*-symmetry. In addition to such peculiar eigenspectra characteristics as a non-Hermitian system, *PT*-symmetry has also been

---

Foundation item:
Received date: 0000-00-00; **Accepted date:** 0000-00-00
Corresponding author: Minye Yang; Email: myang66@uic.edu; ORCID: https://orcid.org/0000-0002-4659-7063.

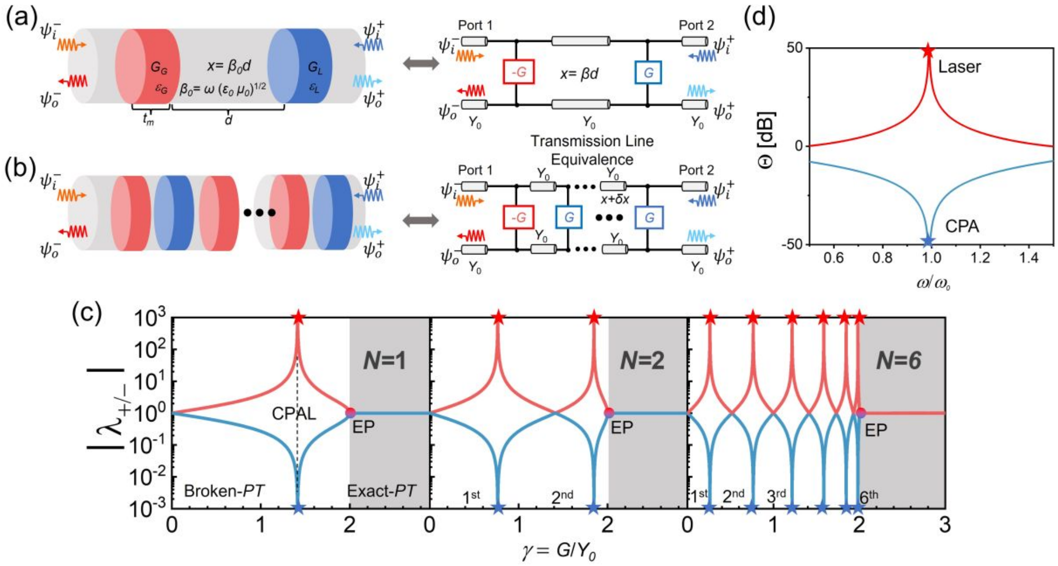

Figure 1 (a) Diagram of a standard *PT*-symmetric optical cavity and its equivalent transmission line (TL) model. Gain and loss components are made of active and passive metasurfaces having the opposite surface conductances $-G$ and G. Dielectric media placed in between gain-loss pair have a characteristic admittance of Y_0 and electrical length $x=\beta_0 d$. (b) Periodical *PT* chain structure and its equivalent TL model. (c) Evolution of the eigenvalues of periodical *PT* chain structures when $N=1$, $N=2$ and $N=6$. (d) Output power intensity of a standard *PT*-symmetric system operating at the CPAL point. The system behaves as a laser when the complex amplitude ratio between input light is $i(\sqrt{2}-1)$. CPA is obtained at any other value.

theoretically and experimentally proved to have more exotic physical phenomena, of which the most famous ones are respectively exceptional points (EPs), or so-called branch-point singularities [5–7], and coherent perfect absorber-laser (CPAL) points [8–10]. EPs are special spectral degeneracies where not only both the real and imaginary eigenvalues coalesce, but also their eigenvectors become identical. Therefore, EPs also serve as transition boundaries between broken *PT*-symmetry phase where eigenvalues are non-unimodular and exact *PT*-symmetry phase where eigenvalues are unimodular [11–13]. On the other hand, CPAL point is another type of spectral anomaly occurring in broken *PT*-symmetry phase where infinite and zero eigenvalues may coexist [14,15]. In other words, zeros and poles of eigenvalues transfer functions may concurrently occur. Generally, a system solely exhibits perfect absorptions of all incoming modes that does not respect *PT*-symmetry is termed as a coherent perfect absorber (CPA) [16]. Conversely, a system may be defined as a laser when producing coherent electromagnetic radiations [17]. In line with this description, by virtue of the coincidence of infinite and zero eigenvalues, a *PT*-symmetric system at CPAL points may indeed perform simultaneously as a CPA and a laser [18].

While the theoretical debates on mathematical progress still remains after the first proposal by Bender and Boettcher [1], practical observations of *PT*-symmetry and its corresponding features, e.g., EPs and CPAL points, have been widely presented in optics and photonics [19–27], radio-frequency (RF) electronics [28–34], and acoustics [35–38], to mention a few. In particular, the extraordinary eigenspace degeneracies of EPs have been proved to have great potentials for a variety of applications, such as the next-generation ultra-sensitive sensors [31,39], unidirectional reflectionless invisibility

[40,41], invisibility cloaking [42–44], and loss-induced transparency [45–47], to name a few. Despite that EPs have been intensively studied in different electromagnetic regimes, yet, another unusual kind of singularities, CPAL points, are lacking discussions. The coexistence of CPA and laser in a *PT*-symmetric system was first proposed by Longhi [48], in which an optic cavity was taken into consideration. It has been reported that optics can be a fertile ground where the *PT*-symmetry associated features may be fruitfully investigated [49], since optical metasurfaces may significantly provide the opportunities to engineer the electromagnetic waves, including the polarity [50], reflections [51], permitivities [52] and permeabilities [53]. In the optical regime shown in Fig. 1(a), systems respecting the standard *PT*-symmetry must have one pair of judiciously balanced gain and loss contributions which are made of two mediums (e.g., active and/or passive metasurfaces) with complex permitivities having spatial profiles as: $\varepsilon(\mathbf{r}) = \varepsilon^*(-\mathbf{r})$, where $\mathbf{r}$ represents the position vector. In other words, these two mediums having the same absolute conductance, but opposite signs are spatially located in two sides of a dielectric media, satisfying *PT*-symmetry. Such scheme can be modeled as a transmission line network depicted in the right panel of Fig. 1(a), where the gain-loss pair is represented by shunt conductances, and the dielectric media is denoted by a transmission line segment. This topology may also be extended to a periodical distribution of N gain-loss pairs as depicted in Fig. 1(b). By contrast with a conventional point of view that a lossy component in a system may be perceived as a foe, introducing loss to its time-reversed image to establish *PT*-symmetry may enable exotic signatures that are not valid in Hermitian systems. In this vein, CPAL phenomenon may be readily observed by fine tuning the gain-to-loss ratio and the system's excitation modes. In this article, we will first introduce a fundamental analysis of CPAL phenomenon in accordance to an optical *PT*-symmetric cavity. Then we primarily focus on recent advances in theoretical and experimental explorations of CPAL points in different frequency spectra and discuss their foreseeable future directions and applications.

2 CPAL in Optics

Constructions of *PT*-symmetry in optical regions rely on the engineering of signs and profiles of permitivities or permeabilities [54]. Figure 1(a) shows a one-dimensional optical cavity satisfying *PT*-symmetry by introducing a dielectric medium with thickness d $(\beta_0 = \omega\sqrt{\varepsilon_0\mu_0}$, where ω is angular frequency) sandwiched by two spatially distributed and isotropic gain $(\varepsilon_G = \varepsilon_r - i\varepsilon_i)$ and loss $(\varepsilon_L = \varepsilon_G^*)$ media; here we may point out that time harmonic notation of $e^{i\omega t}$ is adopted throughout this article. By embracing a premise that thickness of gain and loss mediums is of subwavelength scale, i.e., $t_m = \lambda$, the transmission line (TL) theory can be applied as described by the right panel of Fig. 1(a). Such TL equivalence comprises two shunt conductances with opposite signs attributed to the surface conductances of two sheet media as $G_G = G_{G,r} - iG_{G,i} \approx -i\omega\varepsilon_G t_m$, $G_L = (G_G)^*$, and a TL segment with electrical length of $x = \beta_0 d$ representing the dielectric medium. Thereafter, a

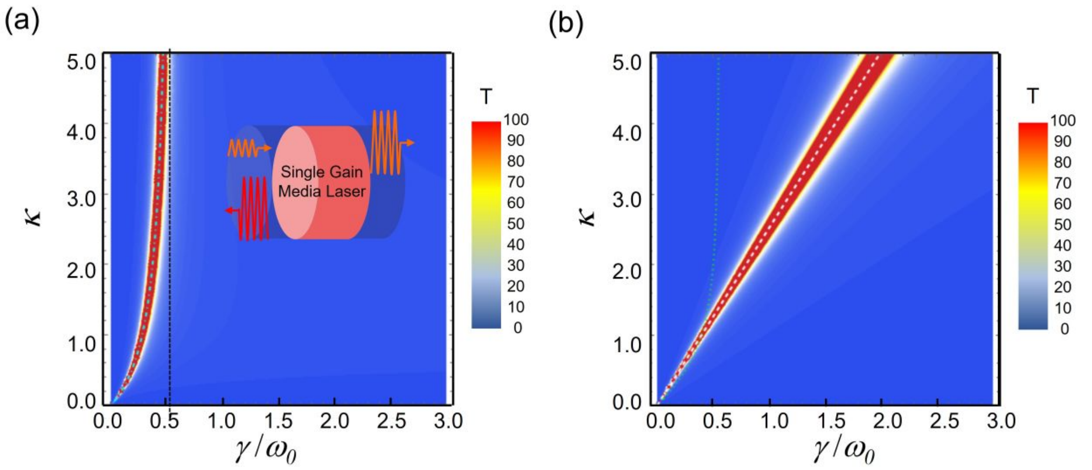

Figure 2 (a) Contours of transmittance as functions of frequency-normalized non-Hermiticity γ/ω_0 and reciprocal scaling factor κ for individual active metasurface enabled laser, as schematized in the inset of (a). (b) Similar to (a) but for a *GPT*-symmetric system. When beyond certain critical point of non-Hermiticity, the lasing property may only be found in *GPT*-symmetric optical system. All figures are reproduced under permissions.

two-port transmission line network is constructed and CPAL phenomenon may be studied by exploiting its scattering matrix $\mathbf{S}$ [4,55]. A scattering matrix stringently connects the incoming and outgoing light waves in two ports as $|\psi_{out}\rangle = \mathbf{S}\,|\psi_{in}\rangle$ [see Fig. 1(a)]; here, $|\psi_{in}\rangle = (\psi_f^-, \psi_b^+)^T$, $|\psi_{out}\rangle = (\psi_f^+, \psi_b^-)^T$, and $\mathbf{S} = \begin{pmatrix} t & r_2 \\ r_1 & t \end{pmatrix}$ where t is the transmission coefficient and r_1, r_2 denote the reflection coefficients of port 1 and 2, respectively. The scattering matrix of an optical cavity obeying *PT*-symmetry yields the conservation relation of: $\mathbf{S}^*(\omega) = \mathcal{PT}\mathbf{S}(\omega)\mathcal{PT} = \mathbf{S}^{-1}(\omega)$ [56], where $\mathcal{P} \equiv \sigma_1$ signifies the space-inverse operator, $\mathcal{T} \equiv \sigma_1\mathcal{K}$ presents the time-reversal operator, σ_1 and $\mathcal{K}$ are Pauli matrix and complex conjugation operator, respectively. This unique CPAL phenomenon may also be extended to a periodical *PT* chain structure, as depicted in Fig. 1(b). The corresponding eigenvalues are shown in panels of Fig. 1(c) where N is the number of *PT* unit cells (gain-loss pair) [57]. We can straightforwardly observe from Fig. 1(c) that the number of EP remains the same with different N, while the number of CPAL points increases with increase in the number of *PT* unit cell.

After the *PT*-symmetric optical cavity is established, CPAL phenomenon may be readily perceived by exploiting eigenvalues of such two-port network in non-Hermiticity parameter space where $\gamma = G/Y_0$ is defined as the non-Hermiticity of a *PT*-symmetric system and $Y_0 = \sqrt{\varepsilon_0/\mu_0}$ is the characteristic admittance of the dielectric medium. The evolution of the eigenvalues of a standard *PT*-symmetric system versus γ can be found in the first panel of Fig. 1(c), from which it can be clearly observed that the eigenvalues may experience a spontaneous phase transition at EP. When the system is beyond the critical point of non-Hermiticity, its eigenvalues may become unimodular, i.e., $|\lambda_\pm| = 1$, leading to so-called exact *PT*-symmetric phase. On the contrary, the eigenvalues will remain non-unimodular $(\lambda_+ = (\lambda_-^*)^{-1})$ in the broken

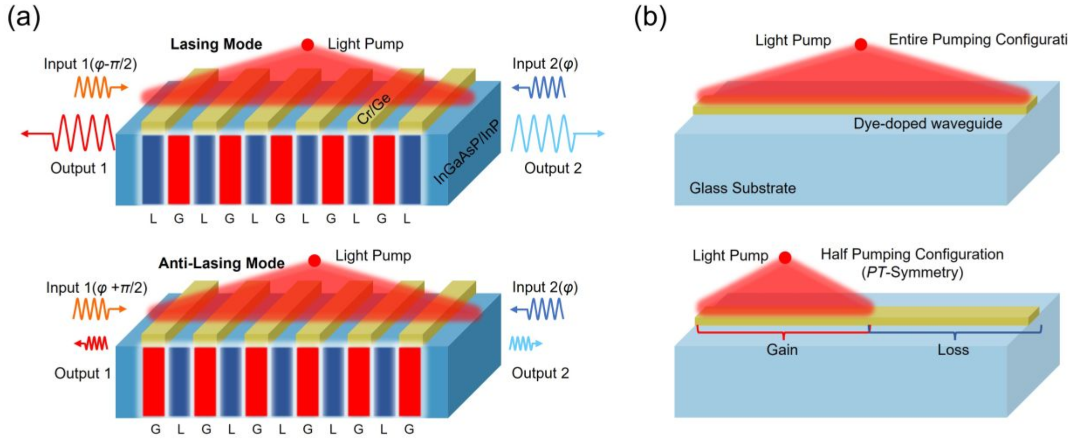

Figure 3 (a) Schematics of a periodical *PT* chain structure showing both lasing and anti-lasing properties. The gain (G) and loss (L) components are made by topping periodic loss structures on a semiconductor InGaAsP/InP gain waveguide. Lasing and anti-lasing modes may be obtained by tuning the phase shift between two input light waves. (b) Schematics of a stripe waveguide laser. The dye-doped waveguide is on top of a glass substrate for achieving high confinement of light. The gain-loss ratio is determined by the pumping area and pumping power intensity.

PT-symmetry phase. In addition to this peculiar singularity of eigenvalue degeneracy, the zero-to-infinite bifurcation effect of eigenvalues in the broken *PT*-symmetry phase naturally stands for coherent perfect absorber $(\lambda_- = 0)$ and laser $(\lambda_+ = \infty)$. Further, the periodical *PT*-symmetric system having *N* pairs of gain-loss components may have *N* CPAL points as plotted in the rest panels of Fig. 1(c). In addition to the eigenvalues point of view, the lasing and CPA properties of a system may also be evaluated by the system's output power intensity in frequency spectrum, defined as: $\Theta = |\psi_{out}|^2 / |\psi_{in}|^2$. By fine tuning the non-Hermiticity parameters to make the cavity exited at CPAL state, it may behave as lasing mode $(\Theta \to \infty)$ or anti-lasing mode $(\Theta \to 0)$ via adjustment of excitation modes, i.e., complex amplitude ratio between two incoming light waves, as can be seen in Fig. 1(d).

An important and distinct property that a *PT*-symmetric absorber-laser has is its low lasing threshold. It is reported in Ref. [58] that a generalized *PT* (*GPT*)-symmetric optical cavity $(N = 1)$ having an extra degree of freedom on gain-loss parameters may have lower lasing thresholds compared to a traditional laser structure made of a single gain medium. This *GPT* structure can essentially have a relaxed requirement that the stringent balance between gain and loss media is not mandatory anymore, instead, a scaled gain-loss components pair may still exhibit signatures of *PT*-symmetry. In this paper, the system's transmittance is taken into consideration for presenting the lasing behavior. A specific scaling factor κ applied to gain and loss contributions, e.g., $G_G' = -G_G / \kappa$, $G_L' = \kappa G_G$, may introduce an interesting scenario that a low-threshold laser may be realize with a low-gain and high-loss system while an anti-lasing phenomenon may be achieved with a high-gain and low-loss structure. In traditional laser cavities, lasing behavior is not attainable beyond a certain threshold, as can be seen in Fig. 2(a). Whereas by sharp contrast to it, a *GPT*-symmetric optical cavity can always perform as a laser, as illustrated in Fig. 2(b), significantly reducing the lasing thresholds.

Beyond the theoretical progress on investigation of CPAL phenomenon, milestone experimental advances are also proposed recently. Zi Jing Wong *et al.* have successfully constructed an optical cavity achieving coherent switching between lasing and anti-lasing modes based upon the CPAL singular points [59]. An extracted experimental setup of such an optical system is demonstrated in Fig. 3(a), from which we can see that a periodical *PT* chain structure is employed. Those gain-loss unit cells are introduced by toping periodical loss structures (Cr/Ge) on a semiconductor InGaAsP/InP waveguide. This time the lasing property is evaluated by the system's output power intensity, Θ. Under external light pump, the waveguide may offer considerable gain to the system to satisfy the *PT*-symmetry. Since the eigenstates of lasing and anti-lasing are orthogonal with a certain spatial phase shift of π, spontaneous mode transitions are achieved by tailoring phase offset (φ) of two incoming light waves. To probe the output power intensity of designed cavity, a unique experimental setup is employed: four couplers are applied to couple to two ports of the *PT* chain structure. Such device setup allows two input and two output light intensities to be individually measured by different couplers. When energy pumping is below the lasing threshold, anti-lasing mode is ensured where $\Theta \approx -15$ dB is observed. On the other hand, when energy pumping exceeds the lasing threshold, a distinct lasing mode is obtained where $\Theta \approx 15$ dB. The experimental results demonstrate that such amplification and absorption modes can be obtained at the same frequency, which validates the prediction of CPAL singularities. Additionally, the authors have also reported the responsivity of mode switching with respect to phase shift, which may provide promising opportunities of practical applications, such as ultrasensitive optical sensors [60].

Another experimental effort on *PT*-symmetric CPA-laser was presented by Zhiyuan Gu *et al.* [54]. Different from the previous one, a stripe waveguide satisfying *PT*-symmetry was proposed, which is similar to $N = 1$ scenario discussed beforehand. The entire system was fabricated upon a commercial glass sheet for desired light confinement in the vertical direction. The dye-doped stripe waveguide is intrinsically lossy and may provide gain property under light pumping. Hence, by engineering the pumped light intensity and pumping area of dye-doped stripe waveguide, the *PT*-symmetry can be achieved. As the attenuation rate of the stripe waveguide without pumping is fixed, this reported *PT*-symmetric waveguide system is a gain modulated structure, as described in Fig. 3(b). To make a comparison, the authors have compared the results of emission spectra obtained by pumping the full and half area of stripe waveguide. It is reported that the occurrences of lasing modes lie in different eigenfrequencies of these two scenarios. Interestingly, the number of lasing modes of half-pumped *PT*-symmetric waveguide is also half of that of a full-pumped waveguide, as predicted by their theoretical analysis. Further, when increasing the pumping energy for *PT*-symmetric waveguide, a bifurcation effect on light intensities between two adjacent lasing modes was observed, which in turn confirms the existence of CPA-laser. This experimental exploration of CPAL phenomenon

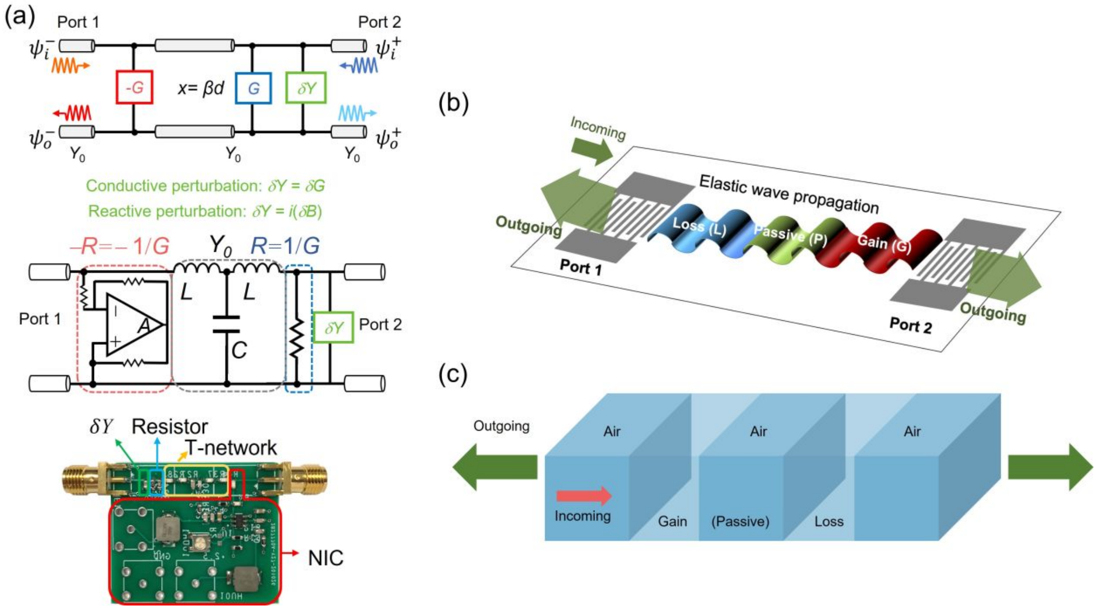

Figure 4 (a) Electromagnetic realization of CPAL-enabled sensor. The TL model is established by lumped elements and fabricated by printed circuit board techniques. Small perturbations can be introduced by electric sensors transducers which may translate physical fluctuations to resistive/reactive variances. (b) Thin-elastic plate (TEP) model of *PT*-symmetry in acoustics exhibiting CPAL feature. (c) *PT*-symmetry in acoustics with an airborne structure for pressure sensing. All figures are reproduced under permissions.

using a dye-doped stripe waveguide has widened the investigation platform of *PT*-symmetry and opened new avenues for loss-induced lasing [62], coupled optical resonators [63], topological insulator [64], etc.

There are also many other ramifications of *PT*-symmetry in optics, as optical cavities and resonators are perfect platforms to construct such systems. Hence, in addition to the investigations of CPAL phenomena discussed above, *PT*-symmetry has been widely utilized to study other characteristics and applications of non-Hermitian physics, for instance, negative refraction [65,66], subdiffractional imaging [22,23], optical isolators and circulators [25,67,68]. However, despite that, fabrication imperfections and costs, added to complex experimental setups remain the major obstacles preventing their practical implementations. To this end, the discussions and attempts to attain *PT*-symmetry and its corresponding exotic signatures in other frequency regimes, such as RF region and even the acoustic region, have emerged [69–72].

3 CPAL in Radio-Frequency and Acoustics

Whereas when tremendous research efforts on CPAL phenomena of *PT*-symmetry in optics are centered on observations of such self-dual singularity, the realizations of CPAL points in the RF region have indeed stepped forward for practical implementations [73–79]. Although the electronic and electromagnetic systems seem to be irrelevant to quantum mechanics originated *PT*-symmetry, they essentially possess formal similarities between Helmholtz and Schrödinger equations, as Helmholtz equation describes the electromagnetic wave propagation and time-invariant Schrödinger equation renders the wave function of massive particles [80]. Specifically, the TL equivalence of a standard *PT*-symmetric structure may be realized by lumped elements in the RF region, as can be seen in Fig. 4(a). In this RF topology, the gain element consists of a negative-impedance-converter (NIC), which, in contrast with a lossy component continuously dissipating system's energy, may constantly inject power to the system. Furthermore, the TL segment, as constitutive coupling between gain and loss elements, are made of "T" shaped transformer, denoting dielectric media in optical *PT*-symmetric cavity. Subsequently, the electronic system in the RF region described by the telegraph equations and Kirchhoff's Laws [80] is capable of providing more feasible methods (e.g., lumped elements) to establish and engineer the balanced gain-loss elements. A major challenge for this electronic *PT*-symmetric system is the design of the NIC, typically made of feedback loops, comprising operational amplifier (OPAMP) [77] or a single transistor enabled Colpitts-type oscillator [30]. The inevitable noises accompanying active circuitries may inevitably hinder the observation of the CPAL phenomenon. Additionally, the imaginary parts of negative impedance which appear as parasitic inductances or capacitances of the NIC may also hinder the establishment of *PT*-symmetry. Therefore, deliberate impedance matching is mandatory for obtaining judiciously balanced negative and positive impedances for achieving the standard *PT*-symmetry.

The theoretical basis of a standard *PT*-symmetric electronic system is analogous to the one in a single optical cavity discussed before. The self-dual laser-absorber phenomenon have been reported to be successfully observed by using a printed circuit board (PCB) technique as schematized in Fig. 4(a), of which the lasing and anti-lasing (CPA) modes are extracted by scattering matrix obtained by a vector network analyzer (VNA). In this RF setup, CPAL point is obtained at 13.48 MHz which is a sufficiently low frequency range for reducing extra loss taking place at the conducting wires on PCB. Nonetheless, instability of active module in NIC still exhibits unavoidable imaginary part of negative impedance, which, as reported in Ref. [77], is partially compensated by matching network. Moreover, other parasitic effects such as fabrication tolerances of lumped elements and imperfections of PCB also contribute to the performance degradation that lasing mode may be hard to observe. Even so, this proof-of-concept still manifest the outstanding sensing potentials of systems operating at CPAL points. It has been validated that such electronic *PT*-structure when initially locked at CPA state by delicately designing the non-Hermiticity parameter and excitation modes may have remarkable responsivity with respect to external disturbances applied near lossy component. Their experimental results show that even a perturbation on the order of 10^{-2} can dramatically modulate the output intensity of this ultrasensitive CPAL-enabled sensor over 30 dB. Interestingly, when the sensing behavior of an EP-based sensor have been intensively studied, the presence of CPAL-based sensor may demonstrate more superiorities. It is well-known that EP-based sensors reside on the frequency or phase manipulations which comes at the price of significant phase or flicker noises[81]. On the contrary, CPAL-enabled sensors are indeed monochromatic (i.e., no frequency or phase modulations), effectively avoiding various noise sources and leading to a larger signal-to-noise ratio. Compared to its counterparts in optics, the *PT*-symmetry and its peculiar spectral features in

Table 1 Investigations on CPAL Phenomena in Different Sub-Disciplines

	Project Type	Regime	Gain Realizations	Applications
GPT-Symmetry [58]	Theoretical	Optic	Active metasurface	Low-threshold lasing
Periodic chain *PT*-Symmetry [59]	Experimental	Optic	InGaAsP/InP Substrate	Observation of CPAL
Dye-doped *PT*-Symmetry [54]	Experimental	Optic	Dye-doped strip waveguide	Observation of CPAL
PCB-based *PT*-Symmetry [77]	Experimental	Radio-Frequency	Negative-impedance Converter	Ultrasensitive sensor
Thin-Elastic Plate *PT*-symmetry [82]	Theoretical	Acoustic	Piezoelectric TEP	Observation of CPAL
Acoustic *PT*-symmetry [89]	Theoretical	Acoustic	Imaginary part of mass density	Acoustic Pressure sensor

RF regions may not only convey physical significances, but also express practical implications.

Parallel to the development of *PT*-symmetry in electromagnetics and optics, the discussions of realizations of such non-Hermitian physics have been also extended to the acoustic regions [56]. Farhat *et al.* have first proposed the possibilities to extend the nature of *PT*-symmetry to thin-elastic plate (TEP) system in Ref. [82], of which the conceptual demonstration is rendered in Fig. 4(b). TEP has attracted increasing research and practical interests in wave physics [83]. The flexural modes are of major considerations in TEP structure since the thickness of a plate may be neglected compared to its lateral dimensions and corresponding wavelength [84]. Thereafter, discussions on TEP have drawn considerable research outcomes for flexural waves in the past decades. Examples include cloaking [85], scattering cancellation [86], localized surface plate modes [87], and elastic plate crystals [88]. The propagation of flexural waves along TEPs can be described by a fourth-order partial differential equation, the Kirchhoff-Love equation. By which the interplay between propagating waves and evanescent waves may be presented which can be translated to *PT*-symmetry characteristics in an elastic plate system. In this TEP system, a positive imaginary part of Young's modulus accounts for loss and a negative imaginary part may indicate gain as can be seen in Fig. 4(b). The proposed *PT*-symmetric TEP system takes use of a shunted piezoelectric TEP for tailoring the effective Young's modulus by use of positive and negative resistors. Similarly, the negative resistor is still achieved by an NIC. The theoretical investigations and simulation results have been presented in Ref. [82] where the existence of lasing, i.e., referred to as FLASER in elastic plates, and anti-lasing may be observed in the frequency band of a few hundred hertz.

Later, an acoustic pressure sensor based on the duality of CPA and laser is proposed in Ref. [89], where an airborne structure is assumed. As depicted in Fig. 4(c), the gain and loss media are separated by a passive layer where the gain or loss layers are controlled by tailoring the imaginary part of density of two layers in unit of air density. This time, the observation of CPAL phenomenon takes place at several kilohertz. The output intensity of the system may undergo unprecedented shift when tiny pressure perturbations are applied to the passive layer. A noise analysis is also included which shows that the inherently existed background white noise may slightly affect the eigenvalues or phase of the acoustic CPAL pressure sensor, while the

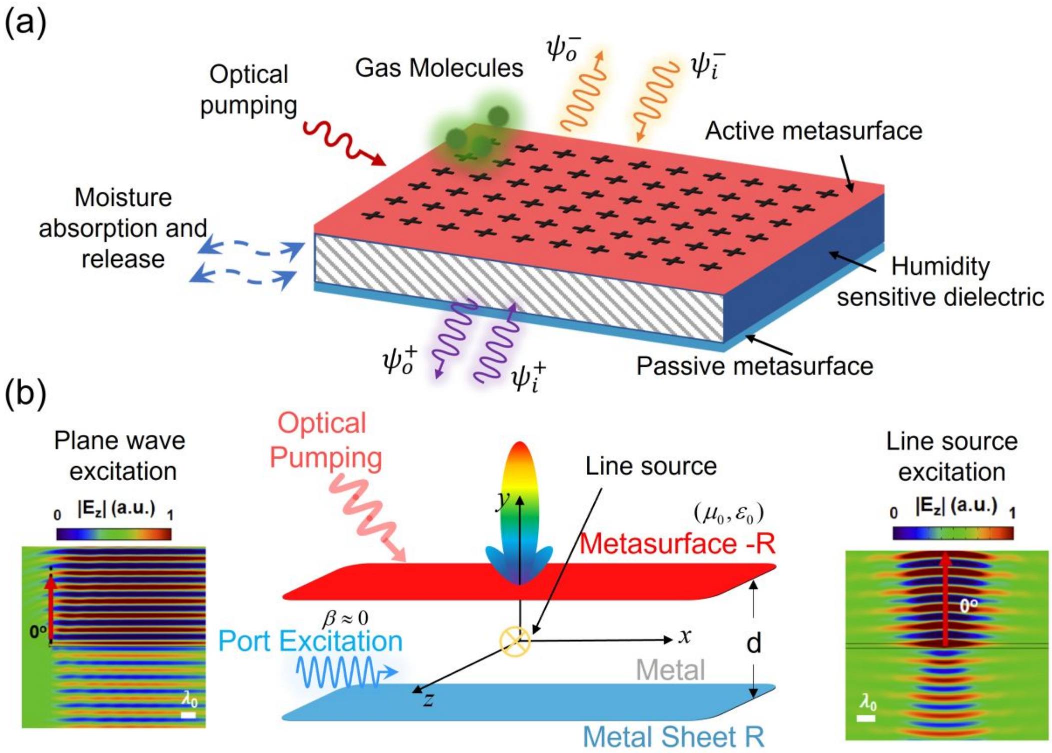

Figure 5 (a) Generalized sensing scheme of an optical CPAL sensor. Either the active metasurface or dielectric media in-between active and passive layers may perceive perturbations which may cause drastic shifts of the system's output intensity. (b) *PT* metachannel that is excited by plane wave or line source. The wave propagating along the channel may exhibit fast-wave property which introduces a surperdirective leaky wave antenna for both excitation modes. All figures are reproduced under permissions.

sensitivity of such sensor may not be deteriorated much. Thanks to the monochronic sensing behavior, this CPAL-based acoustic pressure sensor does not track the resonant frequency or phase shift, instead, it monitors the power modulation at a fixed operating frequency (e.g., CPAL point in spectrum), effectively maintaining unprecedented sensitivity under the implications of noises. Such investigations may significantly benefit for developing the next-generation acoustic sensors [90].

In Table 1, we summarize the above discussed progresses of CPAL phenomena in different sub-disciplines. Both the theoretical and experimental demonstrations of self-dual laser-absorber in electromagnetics and acoustics have opened new avenues that shapes our understanding of *PT*-symmetry and its unique physical features. We next provide an outlook of future directions and possible applications of CPAL system.

4 Future Directions and Applications of CPAL System

The past decades have witnessed a rapidly growing development of *PT*-symmetry, however, the applications of CPAL system are remaining exclusive. The previous reviews of recent advances of CPAL system in different regimes have illustrated its great potential for the next-generation ultrasensitive monochromatic sensors. Hence, one of the promising future directions of CPAL systems may be centered on building the unprecedent electromagnetic or optic sensors. The sensing schemes reported in Ref. [77] rely on a

CPA-initiated state providing remarkable sensing resolutions, which, however, requires stringent complex amplitude ration between two incoming excitations (voltages or light waves). On the contrary, a CPAL system initially locked on lasing state does not require two-port excitation which significantly reduces the system's complexity. Nevertheless, when small disturbances applied to the gain and/or loss components or TL segment, the system may experience a disappearance of lasing state, and thus may still perform excellent sensitivity. As shown in Fig. 5(a), an optical sensing scheme is proposed in Ref. [91] where an active metasurface and a dielectric medium may be utilized as a sensor transducer. With respect to small disturbances applied to them (e.g., gas molecules or aqueous solutions), one can expect the system's output intensities under single excitation may shift from lasing $\Theta\sim\infty$ to unitary $\Theta\sim1$ thanks to the ultrahigh Q-factor resulted near zero-linewidth lasing characteristic. Therefore, such lasing-initiated optical CPAL sensing scheme may be implemented to many environmental monitoring scenarios, such as temperature [92], humidity [93,94], and air quality monitoring [95], as those parameters may tailor the effective permittivity of dielectric media placed in-between gain-loss pairs, and the surface conductances of gain media made of metamaterials (e.g., photo-pumped graphene active metasurface [96–98]). Thereafter, the optical CPAL sensors utilizing graphene metasurfaces may also be outstanding candidates for biomedical sensing, such as bacterial control [99,100], protein or glucose level monitoring [101,102], and pH sensing [103]. Further, by introducing piezoelectric components, which may translate small pressure and/or stress variations to electrical signals, to RF CPAL sensors, they may be applied to electrocardiogram monitoring [104], intraocular pressure monitoring [105], and other vital signs monitoring.

In light of high Q-factor that CPAL system may display, it can also be essentially implemented to construct coherent lasing meta-structures as another possible future direction. In Fig. 5(b), a *PT* metachannel is proposed to form a superdirective leaky-wave antenna [106]. Either a small line sourced placed in the middle of the channel or a plane wave traveling through the metachannel may indeed generate a leaky-wave antenna with ultrahigh directivity and gain. Interestingly, the plane wave traveling through the metachannel may display a fast wave property, which indicates that the channel can be regarded as an epsilon-near-zero (ENZ) media. Therefore, we may envision that such CPAL phenomenon may be applied to construct a new class of superdirective leaky-wave antenna with tunable beam angle. Such coherent amplification of energy source may also benefit to other applications, for example, metasurface laser [107], hologram imaging [108], and ENZ applications [109], etc. In addition, there may be other possible fields waiting for the combination with CPAL systems to conduct groundbreaking ramifications, such as wireless power transfer, next-generation physically unclonable functions, ultrahigh Q channelizer, to name a few.

We have reviewed a few recent progresses on theoretical discussions and experimental validations of CPAL phenomena in a number of sub-disciplines of physics, including optics, electronics and electromagnetics, and acoustics. Various findings based on *PT*-symmetric realms have shown its magnificent potentials on studying new classes of applied physics. In summary, although this non-Hermitian concept have attracted a growing research efforts, more unique physical properties and practical implementations are still in their infancy, with many of possibilities waiting for discovery.

Contributors

Pai-Yen Chen and Mohamed Farhat perceived the idea. Minye Yang, Zhilu Ye and Liang Zhu drafted the manuscript. Pai-Yen Chen and Mohamed Farhat provided theoretical support. Zhilu Ye and Liang Zhu polish the manuscript.

Conflict of interest

Minye Yang, Zhilu Ye, Liang Zhu, Mohamed Farhat, and Pai-Yen Chen declare that they have no conflict of interest.

References

[1] C.M. Bender, S. Boettcher, Real Spectra in Non-Hermitian Hamiltonians Having PT Symmetry, Physical Review Letters. 80 (1998) 5243–5246. DOI: https://doi.org/10.1103/ PhysRevLett.80.5243.

[2] C.M. Bender, D.C. Brody, H.F. Jones, Complex Extension of Quantum Mechanics, Physical Review Letters. 89 (2002) 270401. DOI: https://doi.org/10.1103/PhysRevLett. 89.270401.

[3] C.M. Bender, S. Boettcher, P.N. Meisinger, PT-symmetric quantum mechanics, Journal of Mathematical Physics. 40 (1999) 2201–2229. DOI: https://doi.org/10.1063/1.532860.

[4] G. Lévai, M. Znojil, Systematic search for $\mathcal{PT}$-symmetric potentials with real energy spectra, Journal of Physics A: Mathematical and General. 33 (2000) 7165–7180. DOI: https://doi.org/10.1088/0305-4470/33/40/313.

[5] W. Heiss, Exceptional points of non-Hermitian operators, Journal of Physics A: Mathematical and General. 37 (2004) 2455. DOI: 10.1088/0305-4470/37/6/034

[6] M.V. Berry, Physics of nonhermitian degeneracies, Czechoslovak Journal of Physics. 54 (2004) 1039–1047. DOI: https://doi.org/10.1023/B:CJOP.0000044002.05657.04

[7] N. Moiseyev, Non-Hermitian quantum mechanics, Cambridge University Press, 2011. DOI: https://doi.org/10.1017/CBO9780511976186

[8] Y. Chong, L. Ge, H. Cao, A.D. Stone, Coherent perfect absorbers: time-reversed lasers, Physical Review Letters. 105 (2010) 053901. DOI: https://doi.org/10.1103/PhysRevLett.105.053901

[9] Y. Sun, W. Tan, H. Li, J. Li, H. Chen, Experimental demonstration of a coherent perfect absorber with PT phase transition, Physical Review Letters. 112 (2014) 143903. DOI: https://doi.org/10.1103/PhysRevLett.112.143903

[10] H. Schomerus, Quantum Noise and Self-Sustained Radiation of P T -Symmetric Systems, Physical Review Letters. 104 (2010) 233601. DOI: https://doi.org/10.1103/PhysRevLett.104.233601.

[11] A. Guo, G. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. Siviloglou, D. Christodoulides, Observation of P T-symmetry breaking in complex optical potentials, Physical Review Letters. 103 (2009) 093902. DOI: 10.1103/PhysRevLett.103.093902

[12] S. Bittner, B. Dietz, U. Günther, H. Harney, M. Miski-Oglu, A. Richter, F. Schäfer, P t symmetry and spontaneous symmetry breaking in a microwave billiard, Physical Review Letters. 108 (2012) 024101. DOI: https://doi.org/10.1103/PhysRevLett.108.024101

[13] C.M. Bender, B.K. Berntson, D. Parker, E. Samuel, Observation of PT phase transition in a simple mechanical system, American Journal of Physics. 81 (2013) 173–179. DOI: https://doi.org/10.1119/1.4789549

[14] S. Longhi, L. Feng, PT-symmetric microring laser-absorber, Optics Letters. 39 (2014) 5026–5029. DOI: https://doi.org/10.1364/OL.39.005026

[15] Y. Fu, Y. Xu, H. Chen, Zero index metamaterials with PT symmetry in a waveguide system, Optics Express. 24 (2016) 1648–1657. DOI: https://doi.org/10.1364/OE.24.001648

[16] Y. Chong, L. Ge, A.D. Stone, P t-symmetry breaking and laser-absorber modes in optical scattering systems , Physical Review Letters. 106 (2011) 093902. DOI: https://doi.org/10.1103/PhysRevLett.106.093902

[17] W.T. Silfvast, Laser fundamentals, Cambridge university press, 2004. DOI: https://doi.org/10.1017/CBO9780511616426

[18] P. Ambichl, K.G. Makris, L. Ge, Y. Chong, A.D. Stone, S. Rotter, Breaking of PT symmetry in bounded and unbounded scattering systems, Physical Review X . 3 (2013) 041030. DOI: https://doi.org/10.1103/PhysRevX.3.041030

[19] M.C. Rechtsman, Optical sensing gets exceptional, Nature. 548 (2017) 161–162. https://doi.org/10.1038/548161a.

[20] W. Chen, Ş. Kaya Özdemir, G. Zhao, J. Wiersig, L. Yang, Exceptional points enhance sensing in an optical microcavity, Nature. 548 (2017) 192–196. https://doi.org/10.1038/nature23281.

[21] P.-Y. Chen, J. Jung, P T Symmetry and Singularity-Enhanced Sensing Based on Photoexcited Graphene Metasurfaces, Physical Review Applied. 5 (2016) 064018 . https://doi.org/10.1103/PhysRevApplied.5.064018.

[22] X. Lin, Y. Yang, N. Rivera, J.J. López, Y. Shen, I. Kaminer, H. Chen, B. Zhang, J.D. Joannopoulos, M. Soljačić, All-angle negative refraction of highly squeezed plasmon and phonon polaritons in graphene–boron nitride heterostructures, Proceedings of the National Academy of Sciences of the United States of America. 114 (2017) 6717–6721. https://doi.org/10.1073/pnas.1701830114.

[23] R. Fleury, D.L. Sounas, A. Alù, Negative Refraction and Planar Focusing Based on Parity-Time Symmetric Metasurfaces, Physical Review Letters. 113 (2014) 023903. https://doi.org/10.1103/PhysRevLett.113.023903.

[24] Z. Lin, H. Ramezani, T. Eichelkraut, T. Kottos, H. Cao, D.N. Christodoulides, Unidirectional Invisibility Induced by P T -Symmetric Periodic Structures, Physical Review Letters. 106 (2011) 213901. https://doi.org/10.1103/PhysRevLett.106.213901.

[25] L. Chang, X. Jiang, S. Hua, C. Yang, J. Wen, L. Jiang, G. Li, G. Wang, M. Xiao, Parity–time symmetry and variable optical isolation in active–passive-coupled microresonators, Nature Photonics. 8 (2014) 524–529. https://doi.org/10.1038/nphoton.2014.133.

[26] L. Feng, Y.-L. Xu, W.S. Fegadolli, M.-H. Lu, J.E.B. Oliveira, V.R. Almeida, Y.-F. Chen, A. Scherer, Experimental demonstration of a unidirectional reflectionless parity-time metamaterial at optical frequencies, Nature Materials. 12 (2013) 108–113. https://doi.org/10.1038/nmat3495.

[27] M. Farhat, M. Yang, Z. Ye, P.-Y. Chen, PT-Symmetric Absorber-Laser Enables Electromagnetic Sensors with Unprecedented Sensitivity, ACS Photonics. 7 (2020) 2080–2088. https://doi.org/10.1021/acsphotonics.0c00514.

[28] Z. Ye, M. Farhat, P.-Y. Chen, Tunability and switching of Fano and Lorentz resonances in *PTX* -symmetric electronic systems, Applied Physics Letters. 117 (2020) 031101. https://doi.org/10.1063/5.0014919.

[29] P.-Y. Chen, M. Sakhdari, M. Hajizadegan, Q. Cui, M.M.-C. Cheng, R. El-Ganainy, A. Alù, Generalized parity–time symmetry condition for enhanced sensor telemetry, Nature Electronics. 1 (2018) 297–304. https://doi.org/10.1038/s41928-018-0072-6.

[30] M. Sakhdari, M. Hajizadegan, Q. Zhong, D.N. Christodoulides, R. El-Ganainy, P.-Y. Chen, Experimental Observation of P T Symmetry Breaking near Divergent Exceptional Points, Physical Review Letters. 123 (2019) 193901. https://doi.org/10.1103/PhysRevLett.123.193901.

[31] Z. Dong, Z. Li, F. Yang, C.-W. Qiu, J.S. Ho, Sensitive readout of implantable microsensors using a wireless system locked to an exceptional point, Nature Electronics. 2 (2019) 335–342. https://doi.org/10.1038/s41928-019-0284-4.

[32] M. Sakhdari, M. Hajizadegan, Y. Li, M.M.-C. Cheng, J.C.H. Hung, P.-Y. Chen, Ultrasensitive, Parity–Time-Symmetric Wireless Reactive and Resistive Sensors, IEEE Sensors Journal. 18 (2018) 9548–9555. https://doi.org/10.1109/JSEN.2018.2870322.

[33] M. Hajizadegan, M. Sakhdari, S. Liao, P.-Y. Chen, High-Sensitivity Wireless Displacement Sensing Enabled by PT-Symmetric Telemetry, IEEE Transactions on Antennas and Propagation. 67 (2019) 3445–3449. https://doi.org/10.1109/TAP.2019.2905892.

[34] J. Schindler, A. Li, M.C. Zheng, F.M. Ellis, T. Kottos, Experimental study of active *LRC* circuits with PT symmetries, Physical Review A. 84 (2011) 040101. https://doi.org/10.1103/PhysRevA.84.040101.

[35] C. Shi, M. Dubois, Y. Chen, L. Cheng, H. Ramezani, Y. Wang, X. Zhang, Accessing the exceptional points of parity-time symmetric acoustics, Nature Communications. 7 (2016) 1–5. DOI: https://doi.org/10.1038/ncomms11110

[36] R. Fleury, D. Sounas, A. Alu, An invisible acoustic sensor based on parity-time symmetry, Nature Communications. 6 (2015) 1–7. DOI: https://doi.org/10.1038/ncomms6905

[37] X. Zhu, H. Ramezani, C. Shi, J. Zhu, X. Zhang, P t-symmetric acoustics, Physical Review X. 4 (2014) 031042. DOI: https://doi.org/10.1103/PhysRevX.4.031042

[38] Y. Aurégan, V. Pagneux, P T-symmetric scattering in flow duct acoustics, Physical Review Letters. 118 (2017) 174301. DOI: https://doi.org/10.1103/PhysRevLett.118.174301

[39] H. Hodaei, A.U. Hassan, S. Wittek, H. Garcia-Gracia, R. El-Ganainy, D.N. Christodoulides, M. Khajavikhan, Enhanced sensitivity at higher-order exceptional points, Nature. 548 (2017) 187–191. https://doi.org/10.1038/nature23280.

[40] A. Mostafazadeh, Invisibility and PT symmetry, Physical Review A. 87 (2013) 012103. DOI: https://doi.org/10.1103/PhysRevA.87.012103

[41] T. Liu, X. Zhu, F. Chen, S. Liang, J. Zhu, Unidirectional wave vector manipulation in two-dimensional space with an all passive acoustic parity-time-symmetric metamaterials crystal, Physical Review Letters. 120 (2018) 124502. DOI: https://doi.org/10.1103/PhysRevLett.120.124502

[42] R. Fleury, F. Monticone, A. Alù, Invisibility and cloaking: Origins, present, and future perspectives, Physical Review Applied. 4 (2015) 037001. DOI: https://doi.org/10.1103/PhysRevApplied.4.037001

[43] X. Zhu, L. Feng, P. Zhang, X. Yin, X. Zhang, One-way invisible cloak using parity-time symmetric transformation optics, Optics Letters. 38 (2013) 2821–2824. DOI: https://doi.org/10.1364/OL.38.002821

[44] H. Li, M. Rosendo-López, Y. Zhu, X. Fan, D. Torrent, B. Liang, J. Cheng, J. Christensen, Ultrathin acoustic parity-time symmetric metasurface cloak, Research. 2019 (2019). DOI: https://doi.org/10.34133/2019/8345683

[45] A.A. Zyablovsky, A.P. Vinogradov, A.A. Pukhov, A.V. Dorofeenko, A.A. Lisyansky, PT-symmetry in optics, Physics-Uspekhi. 57 (2014) 1063. DOI: 10.3367/UFNe.0184.201411b.1177

[46] H. Jing, Ş.K. Özdemir, Z. Geng, J. Zhang, X.-Y. Lü, B. Peng, L. Yang, F. Nori, Optomechanically-induced transparency in parity-time-symmetric microresonators, Scientific Reports. 5 (2015) 1–7. DOI: https://doi.org/10.1038/srep09663

[47] H. Zhang, F. Saif, Y. Jiao, H. Jing, Loss-induced transparency in optomechanics, Optics Express. 26 (2018) 25199–25210.

[48] S. Longhi, PT-symmetric laser absorber, Physical Review A. 82 (2010) 031801. DOI: https://doi.org/10.1103/PhysRevA.82.031801

[49] Ş.K. Özdemir, S. Rotter, F. Nori, L. Yang, Parity–time symmetry and exceptional points in photonics, Nature Materials. 18 (2019) 783–798. DOI: https://doi.org/10.1038/s41563-019-0304-9

[50] Y. Zhao, A. Qing, Y. Meng, Z. Song, C. Lin, Dual-band Circular Polarizer Based on Simultaneous Anisotropy and Chirality in Planar Metamaterial, Scientific Report. 8 (2018) 1729. https://doi.org/10.1038/s41598-017-17976-w.

[51] L. Chen, Y. Li, M. Hong, Total Reflection Metasurface with Pure Modulated Signal, Advanced Optical Materials. 7 (2019) 1801130. https://doi.org/10.1002/adom.201801130.

[52] A. Forouzmand, M.M. Salary, G.K. Shirmanesh, R. Sokhoyan, H.A. Atwater, H. Mosallaei, Tunable all-dielectric metasurface for phase modulation of the reflected and transmitted light via permittivity tuning of indium tin oxide, Nanophotonics. 8 (2019) 415–427. DOI: https://doi.org/10.1515/nanoph-2018-0176

[53] A. Radkovskaya, P. Petrov, S. Kiriushechkina, A. Satskiy, M. Ivanyukovich, A. Vakulenko, V. Prudnikov, O. Kotelnikova, A. Korolev, P. Zakharov, Magnetic metamaterials: Coupling and permeability, Journal of Magnetism and Magnetic Materials. 459 (2018) 187–190. DOI: http://dx.doi.org/10.1016/j.jmmm.2017.11.031

[54] V.M. Shalaev, Optical negative-index metamaterials, Nature Photonics. 1 (2007) 41–48. https://doi.org/10.1038/nphoton.2006.49.

[55] V.S. Gerdjikov, G.G. Grahovski, R.I. Ivanov, The N-wave equations with PT symmetry, Theoretical and Mathematical Physics. 188 (2016) 1305–1321. DOI: https://doi.org/10.1134/S0040577916090038

[56] R. El-Ganainy, K.G. Makris, M. Khajavikhan, Z.H. Musslimani, S. Rotter, D.N. Christodoulides, Non-Hermitian physics and PT symmetry, Nature Physics. 14 (2018) 11–19. https://doi.org/10.1038/nphys4323.

[57] M. Yang, Z. Ye, M. Farhat, P.-Y. Chen, Cascaded PT-symmetric artificial sheets: multimodal manipulation of self-dual emitter-absorber singularities, and unidirectional and bidirectional reflectionless transparencies, Journal of Physics D: Applied Physics. 55 (2021) 085301. https://doi.org/10.1088/1361-6463/ac3300.

[58] M. Sakhdari, N.M. Estakhri, H. Bagci, P.-Y. Chen, Low-Threshold Lasing and Coherent Perfect Absorption in Generalized P T -Symmetric Optical Structures, Physical Review Applied. 10 (2018) 024030. https://doi.org/10.1103/PhysRevApplied.10.024030.

[59] Z.J. Wong, Y.-L. Xu, J. Kim, K. O'Brien, Y. Wang, L. Feng, X. Zhang, Lasing and anti-lasing in a single cavity, Nature Photonics. 10 (2016) 796–801. https://doi.org/10.1038/nphoton.2016.216.

[60] J. Li, R. Yu, C. Ding, Y. Wu, PT-symmetry-induced evolution of sharp asymmetric line shapes and high-sensitivity refractive index sensors in a three-cavity array, Physical Review A. 93 (2016) 023814. DOI: https://doi.org/10.1103/PhysRevA.93.023814

[61] Z. Gu, N. Zhang, Q. Lyu, M. Li, S. Xiao, Q. Song, Experimental demonstration of PT-symmetric stripe lasers: Experimental demonstration of PT-symmetric stripe lasers, Laser & Photonics Reviews. 10 (2016) 588–594. https://doi.org/10.1002/lpor.201500114.

[62] B. Peng, Ş. Özdemir, S. Rotter, H. Yilmaz, M. Liertzer, F. Monifi, C. Bender, F. Nori, L. Yang, Loss-induced suppression and revival of lasing, Science. 346 (2014) 328–332. DOI: 10.1126/science.1258004

[63] L. Feng, Z.J. Wong, R.-M. Ma, Y. Wang, X. Zhang, Single-mode laser by parity-time symmetry breaking, Science. 346 (2014) 972–975. DOI: L. Feng, Z.J. Wong, R.-M. Ma, Y. Wang, X. Zhang, Single-mode laser by parity-time symmetry breaking, Science. 346 (2014) 972–975. DOI: 10.1126/science.1258479

[64] G. Liang, Y. Chong, Optical resonator analog of a two-dimensional topological insulator, Physical Review Letters. 110 (2013) 203904. DOI: https://doi.org/10.1103/PhysRevLett.110.203904

[65] Z. Hou, H. Ni, B. Assouar, P T-symmetry for elastic negative refraction, Physical Review Applied. 10 (2018) 044071. DOI: https://doi.org/10.1103/PhysRevApplied.10.044071

[66] F. Monticone, C.A. Valagiannopoulos, A. Alu, Parity-time symmetric nonlocal metasurfaces: all-angle negative refraction and volumetric imaging, Physical Review X. 6 (2016) 041018. DOI: https://doi.org/10.1103/PhysRevX.6.041018

[67] C.A. Downing, D. Zueco, L. Martin-Moreno, Chiral current circulation and PT symmetry in a trimer of oscillators, ACS Photonics. 7 (2020) 3401–3414. DOI: https://doi.org/10.1021/acsphotonics.0c01208

[68] J. Ma, J. Wen, S. Ding, S. Li, Y. Hu, X. Jiang, L. Jiang, M. Xiao, Chip-Based Optical Isolator and Nonreciprocal Parity-Time Symmetry Induced by Stimulated Brillouin Scattering, Laser & Photonics Reviews. 14 (2020) 1900278. DOI: https://doi.org/10.1002/lpor.201900278

[69] M. Sakhdari, M. Hajizadegan, P.-Y. Chen, Robust extended-range wireless power transfer using a higher-order PT-symmetric platform, Physical Review Research. 2 (2020) 013152. DOI: https://doi.org/10.1103/PhysRevResearch.2.013152

[70] X. Yang, J. Li, Y. Ding, M. Xu, X.-F. Zhu, J. Zhu, Observation of Transient Parity-Time Symmetry in Electronic Systems, Physical Review Letters. 128 (2022) 065701. DOI: https://doi.org/10.1103/PhysRevLett.128.065701

[71] H. Ramezani, J. Schindler, F.M. Ellis, U. Günther, T. Kottos, Bypassing the bandwidth theorem with PT symmetry, Physical Review A. 85 (2012) 062122. DOI: https://doi.org/10.1103/PhysRevA.85.062122

[72] T. Liu, X. Zhu, F. Chen, S. Liang, J. Zhu, Unidirectional wave vector manipulation in two-dimensional space with an all passive acoustic parity-time-symmetric metamaterials crystal, Physical Review Letters. 120 (2018) 124502. DOI: https://doi.org/10.1103/PhysRevLett.120.124502

[73] Z. Wei, B. Zhang, Transmission Range Extension of PT-Symmetry-Based Wireless Power Transfer System, IEEE Transactions on Power Electronics. 36 (2021) 11135–11147. DOI: 10.1109/TPEL.2021.3066988

[74] S. Assawaworrarit, X. Yu, S. Fan, Robust wireless power transfer using a nonlinear parity–time-symmetric circuit, Nature. 546 (2017) 387–390. DOI: 10.1038/nature22404

[75] M. Yang, Z. Ye, N. Alsaab, M. Farhat, P.-Y. Chen, In-Vitro Demonstration of Ultra-Reliable, Wireless and Batteryless Implanted Intracranial Sensors Operated on Loci of Exceptional Points, IEEE Transactions on Biomedical Circuits and Systems. 16 (2022) 287–295. https://doi.org/10.1109/TBCAS.2022.3164697.

[76] M. Yang, Z. Ye, P.-Y. Chen, A Quantum-Inspired Biotelemetry System for Robust and Ultrasensitive Wireless Intracranial Pressure Monitoring, in: IEEE, n.d.: pp. 1–4. DOI: 10.1109/SENSORS47087.2021.9639684

[77] M. Yang, Z. Ye, M. Farhat, P.-Y. Chen, Enhanced radio-frequency sensors based on a self-dual emitter-absorber, Physical Review Applied. 15 (2021) 014026. DOI: https://doi.org/10.1103/PhysRevApplied.15.014026

[78] Z. Ye, M. Yang, P.-Y. Chen, Multi-Band Parity-Time-Symmetric Wireless Power Transfer Systems, in: 2021 IEEE Wireless Power Transfer Conference (WPTC), IEEE, San Diego, CA, USA, 2021: pp. 1–4. https://doi.org/10.1109/WPTC51349.2021.9457925.

[79] Z. Ye, M. Yang, P.-Y. Chen, Multi-Band Parity-Time-Symmetric Wireless Power Transfer Systems for ISM-Band Bio-Implantable Applications, IEEE Journal of Electromagnetics, RF and Microwaves in Medicine and Biology. 6 (2022) 196–203. https://doi.org/10.1109/JERM.2021.3120621.

[80] M. Yang, Z. Ye, M. Farhat, P.-Y. Chen, Ultrarobust wireless interrogation for sensors and transducers: a non-Hermitian telemetry technique, IEEE Transactions on Instrumentation and Measurement. 70 (2021) 1–9. DOI: 10.1109/tim.2021.3107057

[81] W. Langbein, No exceptional precision of exceptional-point sensors, Physical Review A. 98 (2018) 023805. DOI: https://doi.org/10.1103/PhysRevA.98.023805

[82] M. Farhat, P.-Y. Chen, S. Guenneau, Y. Wu, Self-dual singularity through lasing and antilasing in thin elastic plates, Physical Review B 103 (2021) 134101. https://doi.org/10.1103/PhysRevB.103.134101.

[83] A.N. Norris, C. Vemula, Scattering of flexural waves on thin plates, Journal of Sound and Vibration. 181 (1995) 115–125. https://doi.org/10.1006/jsvi.1995.0129.

[84] S. Timoshenko, S. Woinowsky-Krieger, Theory of plates and shells, McGraw-hill New York, 1959. DOI: https://doi.org/10.1201/9781315104621

[85] M. Farhat, S. Guenneau, S. Enoch, Ultrabroadband Elastic Cloaking in Thin Plates, Physical Review Letters. 103 (2009) 024301. https://doi.org/10.1103/PhysRevLett.103.024301.

[86] M. Farhat, P.-Y. Chen, H. Bağcı, S. Enoch, S. Guenneau, A. Alù, Platonic Scattering Cancellation for Bending Waves in a Thin Plate, Scientific Report. 4 (2015) 4644. https://doi.org/10.1038/srep04644.

[87] M. Farhat, P.-Y. Chen, S. Guenneau, K.N. Salama, H. Bağcı, Localized surface plate modes via flexural Mie resonances, Physical Review B. 95 (2017) 174201. https://doi.org/10.1103/PhysRevB.95.174201.

[88] A.B. Movchan, N.V. Movchan, R.C. McPhedran, Bloch–Floquet bending waves in perforated thin plates, Proceedings of The Royal Society A. 463 (2007) 2505–2518. https://doi.org/10.1098/rspa.2007.1886.

[89] M. Farhat, W.W. Ahmad, A. Khelif, K.N. Salama, Y. Wu, Enhanced acoustic pressure sensors based on coherent perfect absorber-laser effect, Journal of Applied Physics. 129 (2021) 104902. https://doi.org/10.1063/5.0041771.

[90] Yang LY, Li YP, Fang F, Li LY, Yan ZJ et al. Highly sensitive and miniature microfiber-based ultrasound sensor for photoacoustic tomography. Opto-Electron Advances. 5, 200076 (2022). DOI: 10.29026/oea.2022.200076

[91] Z. Ye, M. Yang, L. Zhu, P.-Y. Chen, PTX-symmetric metasurfaces for sensing applications, Frontiers of Optoelectronics. 14 (2021) 211–220. DOI: https://doi.org/10.1007/s12200-021-1204-6

[92] S. Kananian, G. Alexopoulos, A.S. Poon, Coupling-Independent Real-Time Wireless Resistive Sensing Through Nonlinear P T Symmetry, Physical Review Applied. 14 (2020) 064072. DOI: https://doi.org/10.1103/PhysRevApplied.14.064072

[93] B.-B. Zhou, W.-J. Deng, L.-F. Wang, L. Dong, Q.-A. Huang, Enhancing the remote distance of LC passive wireless sensors by parity-time symmetry breaking, Physical Review Applied. 13 (2020) 064022. DOI: https://doi.org/10.1103/PhysRevApplied.13.064022

[94] Z. Ye, M. Yang, P.-Y. Chen, Metasurface Absorber-Emitter for Humidity Sensing, Radio Science Letters. 3 (2021). https://doi.org/10.46620/21-0029.

[95] Y. Fang, X. Li, J. Xia, Z. Xu, Sensing gases by the pole effect of parity-time symmetric coupled resonators, IEEE Sensors Journal. 19 (2018) 2533–2539. DOI: 10.1109/JSEN.2018.2887084

[96] P.-Y. Chen, A. Alù, Terahertz metamaterial devices based on graphene nanostructures, IEEE Transactions on Terahertz Science and Technology. 3 (2013) 748–756. DOI: 10.1109/TTHZ.2013.2285629

[97] T. Low, P.-Y. Chen, D. Basov, Superluminal plasmons with resonant gain in population inverted bilayer graphene, Physical Review B. 98 (2018) 041403. DOI: https://doi.org/10.1103/PhysRevB.98.041403

[98] T. Guo, L. Zhu, P.-Y. Chen, C. Argyropoulos, Tunable terahertz amplification based on photoexcited active graphene hyperbolic metamaterials, Optical Materials Express. 8 (2018) 3941–3952. DOI: https://doi.org/10.1364/OME.8.003941

[99] X. Tan, M. Yang, L. Zhu, G. Gunathilaka, Z. Zhou, P.-Y. Chen, Y. Zhang, M.M.-C. Cheng, Ultrasensitive and Selective Bacteria Sensors Based on Functionalized Graphene Transistors, IEEE Sensors Journal. 22 (2022) 5514–5520. https://doi.org/10.1109/JSEN.2022.3147229.

[100] Y. Liu, X. Dong, P. Chen, Biological and chemical sensors based on graphene materials, Chemical Society Reviews. 41 (2012) 2283–2307. DOI: https://doi.org/10.1039/C1CS15270J

[101] S. Viswanathan, T.N. Narayanan, K. Aran, K.D. Fink, J. Paredes, P.M. Ajayan, S. Filipek, P. Miszta, H.C. Tekin, F. Inci, Graphene–protein field effect biosensors: glucose sensing, Materials Today. 18 (2015) 513–522. DOI: https://doi.org/10.1016/j.mattod.2015.04.003

[102] Y. Ohno, K. Maehashi, Y. Yamashiro, K. Matsumoto, Electrolyte-gated graphene field-effect transistors for detecting pH and protein adsorption, Nano Letters. 9 (2009) 3318–3322. DOI: https://doi.org/10.1021/nl901596m

[103] Z.L. Wu, M.X. Gao, T.T. Wang, X.Y. Wan, L.L. Zheng, C.Z. Huang, A general quantitative pH sensor developed with dicyandiamide N-doped high quantum yield graphene quantum dots, Nanoscale. 6 (2014) 3868–3874. DOI: https://doi.org/10.1039/C3NR06353D

[104] E. Nemati, M.J. Deen, T. Mondal, A wireless wearable ECG sensor for long-term applications, IEEE Communications Magazine. 50 (2012) 36–43. DOI: 10.1109/MCOM.2012.6122530

[105] J. Kim, J. Park, Y.-G. Park, E. Cha, M. Ku, H.S. An, K.-P. Lee, M.-I. Huh, J. Kim, T.-S. Kim, D.W. Kim, H.K. Kim, J.-U. Park, A soft and transparent contact lens for the wireless quantitative monitoring of intraocular pressure, Nature Biomedical Engineering. 5 (2021) 772–782. https://doi.org/10.1038/s41551-021-00719-8.

[106] M. Hajizadegan, L. Zhu, P.-Y. Chen, Superdirective leaky radiation from a PT-synthetic metachannel, Optics. Express. 29 (2021) 12330–12343. https://doi.org/10.1364/OE.419775.

[107] X. Chen, Y. Zhang, L. Huang, S. Zhang, Ultrathin metasurface laser beam shaper, Advanced Optical Materials. 2 (2014) 978–982. DOI: ttps://doi.org/10.1002/adom.201400186

[108] C. Hahn, Y. Choi, J.W. Yoon, S.H. Song, C.H. Oh, P. Berini, Observation of exceptional points in reconfigurable non-Hermitian vector-field holographic lattices, Nature Communications. 7 (2016) 1–6. DOI: https://doi.org/10.1038/ncomms12201

[109] S. Savoia, G. Castaldi, V. Galdi, A. Alu, N. Engheta, Tunneling of obliquely incident waves through PT-symmetric epsilon-near-zero bilayers, Physical Review B. 89 (2014) 085105. DOI: https://doi.org/10.1103/PhysRevB.89.085105