# Impact of Primary User Traffic on Adaptive Transmission for Cognitive Radio with Partial Relay Selection

# Impact of Primary User Traffic on Adaptive Transmission for Cognitive Radio with Partial Relay Selection

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**Abstract**

**In a cognitive relay system, the secondary user is permitted to transmit data via a relay when licensed frequency bands are detected to be free. Previous studies mainly focus on reducing or limiting the interference of the secondary transmission on the primary users. On the other hand, however, the primary user traffic will also affect the data transmission performance of the secondary users. In this paper, we investigate the impact of the primary user traffic on the bit error rate (BER) of the secondary transmission, when the secondary user adopts adaptive transmission with a relay partially selected. From the numerical results, we can see that the primary user traffic seriously degrades average BER. The worse-link partial selection can perform almost as well as the global selection when the channel conditions of the source-relay links and the relay-destination links differ a lot. In addition, although the relay selection improves the spectral efficiency of the secondary transmission, numerical results show that it only has slight impact on the overall average BER, so that the robustness of the system will not be affected by the relay selection.**

## I. Introduction

Due to the dramatic growth of wireless services over the last decades, cognitive radio has been proposed as a revolutionary technology to solve the conflict between spectrum scarcity and spectrum under-utilization [1]. When the licensed bands in the spectrum are detected to be unoccupied, the secondary users could utilize these free bands to perform their own transmission. In a multiuser environment, each secondary user has a chance to be chosen by the base station to transmit data over the free frequency bands. The selected secondary user, however, may unfortunately suffer from severe fading or shadowing. In this case, any other secondary user with better channel conditions may act as a relay for data transmission.

There are mainly two kinds of methods for relaying: amplify-and-forward (AF) and decode-and-forward (DF) methods. In the AF method, the relay receives the signals from the transmitter, and then simply amplifies the signals and re-transmits them. In the DF method, the relay first decodes the received signals and then re-encodes and transmits them [2]. The authors in [3] studied the outage probability of the cognitive relay networks, in which the secondary users cooperate based on the underlay approach with limited interference constraints on the primary users. In [4], the authors proposed a new cooperative transmission scheme for cognitive radio networks where a relay node is able to help both the primary and secondary transmissions. In the secondary network, the relay is acted by the secondary user. To simplify the implementation and decrease the complexity of relaying, the secondary user adopts the AF method to help the transmission of other secondary users in an interweave system.

In a multiuser environment, the secondary user with the best channel condition will be selected to act as the relay to assist the secondary transmission. The relay selection may be partial or global. For the partial selection scheme [5], [6], we can choose the relay with the best first-hop channel condition or the best second-hop channel condition. This selection scheme only requires one hop information (either the first hop or the second hop), and is easy to implement and extend. In [7], the authors analyzed the performance of the partial selection with feedback delay. In [8], the partial selection for DF relaying in Nakagami fading channels is studied. For the global selection proposed in [9], it requires the information of both hops, and yields high complexity and power consumption. The authors in [10] presented a low-complexity relay selection scheme to maximize the received signal-to-noise ratio (SNR) under the constraint of acceptable interference to the primary users. As the secondary user network is highly dynamic, we adopt the partial selection scheme to choose the best relay according to the instantaneous SNR of the relay link in this paper.

Adaptive modulation [11] is an effective method widely used to increase the link spectral efficiency. When the channel state information can be estimated and this estimation can be sent back to the transmitter, the transmitting rate and power can be adapted according to the channel characteristics to achieve higher throughput. With adaptive modulation, the secondary user could transmit with more flexibility and higher efficiency. One problem under this framework is that the primary user may randomly come or leave during the transmission period of the secondary users. This will certainly affect the performance of adaptive modulation due to the mismatch between the estimated and actual channel conditions. As the parameters for adaptive modulation are chosen before the transmission, the primary traffic will not affect the link spectral efficiency [1], but has an impact on the bit error rate (BER).

Previous studies mainly focus on how to reduce or limit the interference caused by the secondary user [10, 12], or the cooperation of the primary users and secondary users to achieve double wins [13]. In this paper, we study the effects of the primary user traffic on the BER of adaptive transmission of the secondary user in a cognitive relay system with partial relay selection. The rest of the paper is organized as follows. In Section II, we introduce the basic assumptions and models for adaptive cognitive transmission with the best relay chosen based on partial selection. Then in Section III, we examine the case of direct link transmission. In Section IV, different scenarios of the primary user status change for the adaptive data transmission with a relay are analyzed and the average BERs for these possible scenarios are presented. Finally, Section V shows numerical results and the conclusions are drawn in Section VI.

## II. System Model

In a cognitive radio network, the data transmission can be performed on a frame-by-frame basis. Before the data transmission, spectrum sensing is operated to detect free spectrum holes, and we assume that the false alarm probability and the miss detection probability are given by $P_{fa} = Pr\{H_1|H_0\}$ and $P_{md} = Pr\{H_0|H_1\}$, where $H_0$ and $H_1$ denote the absence and presence of the primary user, respectively. If the primary user is detected to be absent, one of the secondary users is selected to use the free bands for adaptive transmission.

[1]For adaptive modulation, the modulation parameters, such as the constellation size, the transmit power, are chosen according the channel conditions, so that the spectral efficiency is determined before the data transmission. During the transmission, the primary user traffic will not affect these modulation parameters, but only the instantaneous SNR, which have an impact of the instantaneous (average) BER.

If the channel conditions are good enough, the selected secondary user can transmit via a direct link between the source and the destination. Under severe fading and shadowing, however, it may not be appropriate to transmit via the direct link but with a relay. In an environment with multiple relays available, the best relay can be chosen to assist the secondary transmission, and we can use the partial selection scheme to choose the best relay.

We denote $\gamma_{1,j}$ and $\gamma_{2,j}$ as the SNRs for the channel between the source and the $j^{\text{th}}$ relay (the $j^{\text{th}}$ SR link), and for the channel between the $j^{\text{th}}$ relay and the destination (the $j^{\text{th}}$ RD link), respectively. The partial relay selection scheme is based either on the first hop (the SR link) or the second hop (the RD link). For the SR link based selection, the relay with the largest SR link SNR is chosen as the best, i.e. $k =$ arg max$\{\gamma_{1,j}, j = 1, 2, \cdots, N\}$. where $N$ is the number of available relays. For the RD link based selection, the best relay is chosen as the one with the largest RD link SNR by $k =$ arg max$\{\gamma_{2,j}, j = 1, 2, \cdots, N\}$.

For both partial selection schemes, the instantaneous SNR of the SR link and the RD link for the best relay are $\gamma_{1,k}$ (denoted as $\gamma_1$) and $\gamma_{2,k}$ (denoted as $\gamma_2$). By properly choosing the relay gain, the end-to-end SNR for the source-relay-destination link is given and approximated by $\frac{\gamma_1\gamma_2}{1+\gamma_1+\gamma_2} \approx \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}$ [14]. Since the theoretic analysis for the SR link based selection and the RD link based selection are almost the same, we focus on the SR based selection in the analysis, and compare these two selection schemes by numerical examples.

Given a threshold $\gamma_{th}$, we transmit via the direct link only when the instantaneous SNR of the direct link (denoted as $\gamma_0$) is larger than $\gamma_{th}$. Otherwise, the relay chosen by partial selection is used to assist the data transmission in the multi-relay environment, and the direct link is not combined with the relay link. In this way, the end-to-end SNR for the best link including both the direct link and relay link yields

$$\gamma_c = \begin{cases} \gamma_0 & \gamma_0 > \gamma_{th} \\ \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} & \gamma_0 < \gamma_{th}. \end{cases} \tag{1}$$

Assuming perfect estimation of channel conditions and reliable feedback link, we adopt adaptive modulation for the data transmission of the secondary user. With constant power allocation and uncoded M-ary quadrature amplitude modulation (M-QAM), the BER can be well approximated by $\mathrm{BER}(\gamma_c, M) \simeq 0.2\exp\left\{-\frac{3\gamma_c}{2(M-1)}\right\}$ [11]. When the target operation BER is maintained at $\mathrm{BER}_0$, the constellation size $M$ is adjusted according to the instantaneous end-to-end SNR $\gamma_c$ by

$$M = 1 + \frac{3\gamma_c}{2K_0}, \tag{2}$$

where $K_0 = -\log(5\mathrm{BER}_0)$. Generally, the constellation size $M$ is often restricted to integer values. The continuous rate means that the number of bits per symbol is also continuous, which is possible for M-QAM [15].

We also assume that the primary user traffic follows an independent and identically distributed ( i.i.d.) on-off process, and "$0$" and "$1$" represent the cases when the licensed channel is free and occupied, respectively. For each case, the holding time is assumed to be exponential distributed with mean $\lambda$ for "$0$" and $\mu$ for "$1$". So the status transition probability matrix is given by [16]

$$\boldsymbol{P} = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} = \frac{1}{\lambda+\mu} \begin{pmatrix} \mu + \lambda \mathrm{e}^{-(\lambda+\mu)t} & \lambda - \mu \mathrm{e}^{-(\lambda+\mu)t} \\ \mu - \lambda \mathrm{e}^{-(\lambda+\mu)t} & \lambda + \mu \mathrm{e}^{-(\lambda+\mu)t} \end{pmatrix}. \tag{3}$$

The initial probability is given as $p_b = \frac{\mu}{\lambda+\mu}$ for a busy licensed channel and $p_f = \frac{\lambda}{\lambda+\mu}$ for a free licensed channel. The frame for the secondary transmission takes a total time of $QT_s$, for either the direct link, or the SR link, or the RD link, where $Q$ is the number of symbols per secondary frame contains, and $T_s$ denotes symbol period time. We assume that the secondary frame is of approximately the same length as the primary frame such that the status of primary user changes at most once in each link [2].

*Notations*: For the convenience, we list some the symbols to avoid confusion.

- $\gamma_{1,j}$ and $\gamma_{2,j}$: the instantaneous SNR for the SR link and RD link of the $j^{\text{th}}$ relay;
- $\gamma_{1,k}$ and $\gamma_{2,k}$: the instantaneous SNR for the SR link and RD link of the best relay, and they are denoted as $\gamma_1$ and $\gamma_2$, i.e. $\gamma_1 = \gamma_{1,k}$ and $\gamma_2 = \gamma_{2,k}$;

[2]If the primary frame is much longer than the secondary frame, it is unlikely that the primary will change its status during both the RD link and the SR link, and this gives simpler situations only including IV-B2, IV-B3, IV-C2 and IV-C3. On the other hand, the secondary user must perform spectrum sensing periodically to avoid any transmission collision with the primary user as much as possible, so it is unlikely that the secondary frame will be much longer than the primary frame.

- $\gamma_0$ and $\gamma_{th}$: the instantaneous SNR of the direct link, and the SNR threshold that the secondary user transmit via the direct link only when $\gamma_0$ is larger than it;
- $\gamma_c$: the end-to-end SNR for the best link including both the relay link and the direct link;
- $\gamma_{p_0}$, $\gamma_{p_1}$ and $\gamma_{p_2}$: the SNR of primary user over the direct link, the SR links and the RD links, respectively.

## III. Direct Link Transmission

Before the data transmission, the secondary user estimates the channel conditions of the direct link. If the direct link is good enough, the secondary user transmits data via the direct link.

### A. Outage Probability

We assume that the direct link channel is subject to Nakagami fading, such that the probability density function (PDF) for $\gamma_0$ is given by $f_{\gamma_0}(\gamma) = \left(\frac{m_0}{\bar{\gamma}_0}\right)^{m_0} \frac{\gamma^{m_0-1}}{\Gamma(m_0)} \exp\left(-\frac{m_0}{\bar{\gamma}_0}\gamma\right)$ [17], where $\bar{\gamma}_0$ and $m_0$ are the first-order moment and Nakagami parameter for $\gamma_0$. The secondary user transmits via the direct link only when $\gamma_0 > \gamma_{\text{th}}$, the probability of which is given by

$$P_{out} = \int_{\gamma_{\text{th}}}^{\infty} f_{\gamma_0}(\gamma) d\gamma = \frac{\Gamma(m_0, m_0\gamma_{\text{th}}/\gamma_0)}{\Gamma(m_0)}, \tag{4}$$

where $\Gamma(\cdot)$ is the complete gamma function, and $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function.

### B. Primary User Initially Absent

The secondary user begins the adaptive data transmission when the primary user is absent, and the constellation size $M_0$ is chosen according to (2) with $\gamma_c = \gamma_0$. Unfortunately, the primary user may come back at any time during the data transmission period, and after that, the instantaneous SNR will change to $\gamma_0' = \frac{\gamma_0}{1+\gamma_{p_0}}$[3], where $\gamma_{p_0}$ is the SNR of the primary user signal. In this way, the instantaneous operating BER changes to $\text{BER}(\gamma_0', M_0) \simeq \frac{1}{5}\exp\left\{-\frac{3\gamma_0}{2(M_0-1)}\frac{\gamma_0'}{\gamma_0}\right\} = \frac{1}{5}(5\text{BER}_0)^{\frac{1}{1+\gamma_{p_0}}}$.

We suppose that the primary user comes back at the $k_0^{\text{th}}$ symbol ($k_0 = 1, 2, \cdots, Q$), and the probability for this case is given by

$$P_f\{k_0\} = p_f p_{00}^{k_0-1} p_{01} p_{11}^{Q-k_0}, \tag{5}$$

[3]Suppose the transmitting power of the primary user and secondary user are $E_1$ and $E_2$, and the noise power is $N_0$, so that the secondary user SNR $\gamma_0 = E_2/N_0$ and the primary user SNR $\gamma_p = E_1/N_0$. For the secondary transmission, the primary user traffic is regarded as noise, so the SNR for the secondary user with primary user traffic is $\gamma_0' = \frac{E_2}{E_1+N_0} = \frac{\gamma_0}{1+\gamma_p}$.

and the average BER over the whole transmission period can be derived as

$$\overline{\text{BER}}_{f(k_0)} = \frac{(k_0-1)\text{BER}_0 + \frac{1}{5}(Q-k_0+1)(5\text{BER}_0)^{\frac{1}{1+\gamma_{p0}}}}{Q}. \tag{6}$$

*C. Primary User Initially Present*

The secondary user may also begin the data transmission when the primary user is present but a miss detection happened. In this case, the instantaneous SNR is $\gamma_0' = \frac{\gamma_0}{1+\gamma_{p0}}$, and the constellation size $M_0'$ is chosen according to (2) with this new SNR $\gamma_0'$ to maintain the operating $\text{BER}_0$ as $M_0' = 1 + \frac{3\gamma_0}{2K_0(1+\gamma_{p0})}$.

After the primary user leaves at the $k_0^{\text{th}}$ symbol, the instantaneous SNR $\gamma_c$ then returns to $\gamma_0$. The instantaneous operating BER, however, will change with the new constellation size $M_0'$ to $\text{BER}(\gamma_0, M_0') \simeq \frac{1}{5}\text{exp}\left\{-\frac{3\gamma_0'}{2(M_0'-1)}\frac{\gamma_0}{\gamma_0'}\right\} = \frac{1}{5}(5\text{BER}_0)^{1+\gamma_{p0}}$.

The probability for the primary user's leaving at the $k_0^{\text{th}}$ symbol is

$$P_b\{k_0\} = p_b p_{11}^{k_0-1} p_{10} p_{00}^{Q-k_0}. \tag{7}$$

and the average BER over the whole transmission period is given by

$$\overline{\text{BER}}_{b(k_0)} = \frac{(k_0-1)\text{BER}_0 + \frac{1}{5}(Q-k_0+1)(5\text{BER}_0)^{1+\gamma_{p0}}}{Q}. \tag{8}$$

*D. Average BER for the direct link transmission*

When there is no primary user at the beginning, the secondary user is allowed to transmit only if the free spectrum bands are detected, the probability of which is $1-P_{fa}$. On the other hand, the secondary user can transmit with primary user present only if the miss detection happens, the probability of which yields $P_{md}$. So considering all the cases that the primary user may be present or absent at the beginning, and $k_0$ may be any number from $1$ to $K$, we can have the average BER for the direct link transmission as

$$\overline{\text{BER}}_d = \sum_{k_0=1}^{Q} \left\{(1-P_{fa})P_f\{k_0\}\overline{\text{BER}}_{f(k_0)} + P_{md}P_b\{k_0\}\overline{\text{BER}}_{b(k_0)}\right\}, \tag{9}$$

where $P_f\{k_0\}, \overline{\text{BER}}_{f(k_0)}, P_b\{k_0\}$ and $\overline{\text{BER}}_{b(k_0)}$ are given in (5), (6), (7) and (8), respectively.

## IV. Relay Assisted Transmission

When the direct link is not working ($\gamma_0 < \gamma_{\text{th}}$), the secondary user selects a relay to assist the transmission to achieve better performance. Similar to the direct link transmission, the initial status of the primary user for the relay transmission also consists of absence and presence. But with relay for the secondary transmission, the primary user may change its status either during the SR link or the RD link, or during both links during one secondary transmission.

### *A. Channel Statistics*

We assume that the channels for all the SR links and the RD links are subject to independent but not identically distributed (i.n.i.d.) Nakagami fading, so that the PDF for the SNR $\gamma_{i,j}$ $(i = 1,2; j = 1,2,\cdots,N)$ is given by

$$f_{\gamma_{i,j}}(\gamma) = \left(\frac{m_{i,j}}{\bar{\gamma}_{i,j}}\right)^{m_{i,j}} \frac{\gamma^{m_{i,j}-1}}{\Gamma(m_{i,j})} \exp\left(-\frac{m_{i,j}}{\bar{\gamma}_{i,j}}\gamma\right), \quad \gamma \geq 0, \tag{10}$$

where $\bar{\gamma}_{i,j}$ and $m_{i,j}$ are the first-order moment and Nakagami parameter for $\gamma_{i,j}$.

Suppose that the best relay is chosen based on the SR link partial selection, so the PDF of the RD link SNR $\gamma_2$ for the best carrier is still (10) with $i = 2$ and $j = k$, where $k$ is the index of the best relay, and the PDF of the SR link SNR $\gamma_1$ is given by [18]

$$f_{\gamma_1}(\gamma) = \sum_S \sum_L (-1)^q C_{S,L} \left(l_S - E_S\gamma\right) \gamma^{l_S-1} \mathrm{e}^{-E_S\gamma}, \gamma \geq 0, \tag{11}$$

where $S = \{s_1, s_2, \cdots, s_q\}$ is a subset of $\{1, 2, \cdots, N\}$, $\sum_S$ is the sum over all possible $S$, and $\sum_L$ is the sum defined by $\sum_L = \sum_{l_{s_1}=0}^{m_{1s_1}-1} \sum_{l_{s_2}=0}^{m_{1s_2}-1} \cdots \sum_{l_{s_q}=0}^{m_{1s_q}-1}$. In (11), the parameters $l_s$, $E_S$ and $C_{S,L}$ are given by $l_S = l_{s_1} + l_{s_2} + \cdots + l_{s_q}$, $E_S = \frac{m_{1s_1}}{\bar{\gamma}_{1s_1}} + \frac{m_{1s_2}}{\bar{\gamma}_{1s_2}} + \cdots + \frac{m_{1s_q}}{\bar{\gamma}_{1s_q}}$, and $C_{S,L} = \prod_{i=1}^{q} \frac{1}{l_{s_i}!} \left(\frac{m_{1s_i}}{\bar{\gamma}_{1s_i}}\right)^{l_{s_i}}$. When $S = \phi$ is an empty set, we have $l_S = 0, E_S = 0$ and $C_{S,L} = 1$.

For any real-positive values of $a$ and $b$ $(a \neq b)$, we define the random variable $Z(a, b)$ as

$$Z(a,b) = \frac{\gamma_1 + a\gamma_2}{\gamma_1 + b\gamma_2}, \tag{12}$$

Given $\gamma_1$ subject to (11) and $\gamma_2$ subject to (10) with parameter $(m_{2,k}, \gamma_{2,k})$, the moment generating function (MGF) of $Z(a,b)$ is given by

$$M_Z(z;a,b) = \sum_{S,L} (-1)^q C_{S,L} \frac{\Gamma(l_S+1)}{E_S^{l_S}} \left\{ \Psi_z\left(l_S, E_S, m_{2,k}, \frac{\bar{\gamma}_{2,k}}{m_{2,k}}\right) - \Psi_z\left(l_S+1, E_S, m_{2,k}, \frac{\bar{\gamma}_{2,k}}{m_{2,k}}\right) \right\} \tag{13}$$

where $\sum_{S,L} = \sum_S \sum_L$, and $\Psi_z(n,w,\alpha,\beta)$ is given by

$$\Psi_z(n,w,\alpha,\beta) = \begin{cases} \mathrm{e}^{\frac{a}{b}z} (\beta b w)^n \, \Phi_1\left(n, \alpha+n, \alpha+n; 1-\beta b w, \frac{b-a}{b} z\right), & 0 < w < \frac{2}{b\beta} \\ \mathrm{e}^{z} (\beta b w)^{-\alpha} \, \Phi_1\left(\alpha, \alpha+n, \alpha+n; 1-\frac{1}{\beta b w}, \frac{a-b}{b} z\right), & w > \frac{1}{2b\beta} \end{cases}. \tag{14}$$

where $\Phi_1(\cdot,\cdot,\cdot;\cdot,\cdot)$ is the Humbert series [19, Chapter 9.26].

Especially, when the number of available relays is just $N=1$, the partial selection simplifies to the fixed relay problem. In this case, there is no relay selection, and $\gamma_1$ and $\gamma_2$ are Gamma-distributed with $(m_i, \bar{\gamma}_i), i=1,2$. The MGF of the random variable $Z = \frac{\gamma_1 + a\gamma_2}{\gamma_1 + b\gamma_2}$ is given by

$$M_Z(z;a,b) = \begin{cases} \mathrm{e}^{\frac{a}{b}z} \left(\frac{\beta_2 b}{\beta_1}\right)^{\alpha_1} \Phi_1\left(\alpha_1, \alpha_1+\alpha_2, \alpha_1+\alpha_2; 1-\frac{\beta_2 b}{\beta_1}, \frac{b-a}{b} z\right), & 0 < \frac{\beta_2 b}{\beta_1} < 2, \\ \mathrm{e}^{z} \left(\frac{\beta_1}{\beta_2 b}\right)^{\alpha_2} \Phi_1\left(\alpha_2, \alpha_1+\alpha_2, \alpha_1+\alpha_2; 1-\frac{\beta_1}{\beta_2 b}, \frac{a-b}{b} z\right), & 0 < \frac{\beta_1}{\beta_2 b} < 2. \end{cases} \tag{15}$$

where $(\alpha_i, \beta_i) = (m_i, \bar{\gamma}_i/m_i), i=1,2$.

### *B. Situations with the Primary User Initially Absent*

*1) Status Changing:* Without the presence of the primary user at the beginning, the constellation size for the adaptive transmission is chosen from (2), and the end-to-end SNR is given by $\gamma_{c_1} = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}$. In this case, the primary user may comes in the SR link, or in the RD link, or comes in the SR link and leaves in the RD link. If the primary user arrives in the SR link, then the secondary user suffers from the primary user traffic after its arrival, and the SR link SNR changes from $\gamma_1$ to $\gamma_1' = \frac{\gamma_1}{1+\gamma_{p_1}}$, where $\gamma_{p_1}$ is the primary user traffic present over the SR link. In this case, the end-to-end SNR is thus given by

$$\gamma_A' = \frac{\gamma_1'\gamma_2}{\gamma_1' + \gamma_2} = \frac{\gamma_1\gamma_2}{\gamma_1 + (1+\gamma_{p_1})\gamma_2}. \tag{16}$$

and the instantaneous operating BER yields $\frac{1}{5}(5\text{BER}_0)^{\gamma'_A/\gamma_{c_1}}$. Note that $\frac{\gamma'_A}{\gamma_{c_1}} = \frac{\gamma_1+\gamma_2}{\gamma_1+(1+\gamma_{p_1})\gamma_2} = Z(1, 1+\gamma_{p_1})$, so the BER with primary user traffic in the SR link is thus given by

$$\overline{\text{BER}}_1 = \int \frac{1}{5}(5\text{BER}_0)^z f_1(z)dz = \frac{1}{5}M_Z(-K_0; 1, 1+\gamma_{p_1}), \tag{17}$$

where $f_1(z)$ is used to denote the PDF of the random variable $Z(1, 1+\gamma_{p_1})$.

If the primary user is present over some time of the RD link, however, the RD link SNR during these periods may change from $\gamma_2$ to $\gamma'_2 = \frac{\gamma_2}{1+\gamma_{p2}}$, where $\gamma_{p_2}$ is the primary user traffic present over the RD link. and the end-to-end link SNR is then given by

$$\gamma'_B = \frac{\gamma_1\gamma'_2}{\gamma_1+\gamma'_2} = \frac{\gamma_1\gamma_2}{(1+\gamma_{p2})\gamma_1+\gamma_2}. \tag{18}$$

Similarly, the instantaneous operating BER in this case yields $\frac{1}{5}(5\text{BER}_0)^{\gamma'_B/\gamma_{c_1}}$ with $\frac{\gamma'_B}{\gamma_{c_1}} = \frac{\gamma_1+\gamma_2}{(1+\gamma_{p_2})\gamma_1+\gamma_2} = \frac{1}{1+\gamma_{p2}}Z\left(1, \frac{1}{1+\gamma_{p2}}\right)$. Then the BER when there is primary user traffic in the RD link is thus given by

$$\overline{\text{BER}}_2 = \int \frac{1}{5}(5\text{BER}_0)^{\frac{z}{1+\gamma_{p2}}} f_2(z)dz = \frac{1}{5}M_Z\left(-\frac{K_0}{1+\gamma_{p2}}; 1, \frac{1}{1+\gamma_{p2}}\right), \tag{19}$$

where $f_2(z)$ is used to denote the PDF of $Z\left(1, \frac{1}{1+\gamma_{p2}}\right)$.

*2) Primary User Arrives in SR link:* In this case, there is no primary-user traffic at the beginning. The primary user comes at the $k_1^{\text{th}}$ symbol in the SR link, and keeps its presence in the whole RD link. The probability for this case is given by

$$P_f\{k_1\} = p_f p_{00}^{k_1-1} p_{01} p_{11}^{2Q-k_1}. \tag{20}$$

The whole source-relay-destination link can be divided into three periods:

- $0 \sim k_1^{\text{th}}$ symbols in the SR link: There is no primary user traffic during this period, and the instantaneous SNR is $\gamma_{c_1} = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}$, and the operating BER is $\text{BER}_0$;
- $k_1^{\text{th}} \sim Q$ symbols in the SR link: The primary user traffic is present over this period, and the instantaneous SNR is $\gamma'_A$ in (16), and the operating BER is $\overline{\text{BER}}_1$ in (17);
- the RD link: The primary user is present over the whole RD link so that the instantaneous

SNR is $\gamma'_B$ in (18), and the operating BER is $\overline{\text{BER}}_2$ in (19);

The average BER for all the periods with primary user arriving in SR link is the BER of each period weighted by the time that each period lasts

$$\overline{\text{BER}}_{f(k_1)} = \frac{(k_1-1)\text{BER}_0 + (Q-k_1+1)\overline{\text{BER}}_1 + Q\overline{\text{BER}}_2}{2Q}. \quad (21)$$

*3) Primary User Arrives in RD link:* Before the primary user comes at the $k_2^{\text{th}}$ symbol in the RD link, there is no primary-user traffic in the SR link and the $1 \sim k_2^{\text{th}}$ symbols in the RD link. The probability with which this case happens is

$$P_f\{k_2\} = p_f p_{00}^{Q+k_2-1} p_{01} p_{11}^{Q-k_2}. \quad (22)$$

Similarly, the whole source-relay-destination link can be divided into two periods:

- the SR link and $0 \sim k_2^{\text{th}}$ symbols in the RD link: With no primary user traffic, the instantaneous SNR is $\gamma_{c_1} = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}$, and the operating BER is $\text{BER}_0$;
- $k_2^{\text{th}} \sim Q$ symbols in the RD link : After the primary user arrives, the instantaneous SNR changes to $\gamma'_B$ in (18), and the operating BER changes to $\overline{\text{BER}}_2$ in (19);

In this case, the average BER for the whole period is thus given by

$$\overline{\text{BER}}_{f(k_2)} = \frac{(Q+k_2-1)\text{BER}_0 + (Q-k_2+1)\overline{\text{BER}}_2}{2Q}. \quad (23)$$

*4) Primary User Arrives in SR link and Leaves in RD link:* If the primary arrives at the $k_1^{\text{th}}$ symbol in the SR link but leaves at the $k_2^{\text{th}}$ symbol in the RD link, the probability yields

$$P_f\{k_1, k_2\} = p_f p_{00}^{k_1-1} p_{01} p_{11}^{Q-k_1+k_2-1} p_{10} p_{00}^{Q-k_2}. \quad (24)$$

The whole source-relay-destination link is then divided into three periods:

- the $0 \sim k_1^{\text{th}}$ symbols in the SR link and the $k_2^{\text{th}} \sim Q$ symbols in the RD link: This period includes the time before the primary user arrives and the time after the primary user leaves. The instantaneous SNR is $\gamma_{c_1} = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}$, and the operating BER is $\text{BER}_0$;
- the $k_1^{\text{th}} \sim Q$ symbols in the SR link: With primary user traffic in the SR link, the instanta-

neous SNR is $\gamma'_A$ in (16), and the operating BER is $\overline{\text{BER}}_1$ in (17);

- the $0 \sim k_2^{\text{th}}$ symbols in the RD link: With primary user traffic in the RD link, the instantaneous SNR is $\gamma'_B$ in (18), and the operating BER is $\overline{\text{BER}}_2$ in (19);

The average BER for the whole source-relay-destination link period is

$$\overline{\text{BER}}_{f(k_1,k_2)} = \frac{(k_1+Q-k_2)\text{BER}_0 + (Q-k_1+1)\overline{\text{BER}}_1 + (k_2-1)\overline{\text{BER}}_2}{2Q}. \tag{25}$$

### C. Situations with the Primary User Initially Present

*1) Status Changing:* If the primary user is present at the beginning of the time slot, the initial SNR for the SR link and the RD link should be $\gamma'_1 = \frac{\gamma_1}{1+\gamma_{p_1}}$ and $\gamma'_2 = \frac{\gamma_2}{1+\gamma_{p_2}}$. The end-to-end link SNR then yields $\gamma_{c_2} = \frac{\gamma'_1\gamma'_2}{\gamma'_1+\gamma'_2} = \frac{\gamma_1\gamma_2}{(1+\gamma_{p_2})\gamma_1+(1+\gamma_{p_1})\gamma_2}$. In this case, the secondary user is allowed to transmit only when a miss detection happens for the spectrum sensing. Under the assumption of miss detection, the constellation size for secondary transmission is thus chosen to maintain the operating BER unchanged as $M' = 1 + \frac{3\gamma_{c_2}}{2K_0}$. The primary user in this case may leaves in the SR link or the RD link, or leaves in the SR link but comes back in the RD link.

When the primary user leaves in the SR link, the instantaneous SNR for the SR link will change back to $\gamma_1$, and the end-to-end SNR becomes $\gamma'_B$. In this case, the instantaneous BER yields $\text{BER}(\gamma'_B, M') = \frac{1}{5}(5\text{BER}_0)^{\gamma'_B/\gamma_{c_2}}$ with $\frac{\gamma'_B}{\gamma_{c_2}} = \frac{(1+\gamma_{p_2})\gamma_1+(1+\gamma_{p_1})\gamma_2}{(1+\gamma_{p_2})\gamma_1+\gamma_2} = Z\left(\frac{1+\gamma_{p_1}}{1+\gamma_{p_2}}, \frac{1}{1+\gamma_{p_2}}\right)$. The BER without primary user traffic in the SR link is thus given by

$$\overline{\text{BER}}_3 = \int \frac{1}{5}(5\text{BER}_0)^z f_3(z)dz = \frac{1}{5}M_Z\left(-K_0; \frac{1+\gamma_{p_1}}{1+\gamma_{p_2}}, \frac{1}{1+\gamma_{p_2}}\right), \tag{26}$$

where $f_3(z)$ is used to denote the PDF of $Z\left(\frac{1+\gamma_{p_1}}{1+\gamma_{p_2}}, \frac{1}{1+\gamma_{p_2}}\right)$.

When the primary user leaves in the RD link, the instantaneous SNR for the RD link will change back to $\gamma_2$, and the end-to-end SNR becomes $\gamma'_A$. Then we have $\frac{\gamma'_A}{\gamma_{c_2}} = \frac{(1+\gamma_{p_2})\gamma_1+(1+\gamma_{p_1})\gamma_2}{\gamma_1+(1+\gamma_{p_1})\gamma_2} = (1+\gamma_{p_2})Z\left(\frac{1+\gamma_{p_1}}{1+\gamma_{p_2}}, 1+\gamma_{p_1}\right)$. Similarly, the BER without primary user traffic in the RD link yields

$$\overline{\text{BER}}_4 = \int \frac{1}{5}(5\text{BER}_0)^{(1+\gamma_{p_2})z} f_4(z)dz = \frac{1}{5}M_Z\left(-(1+\gamma_{p_2})K_0; \frac{1+\gamma_{p_1}}{1+\gamma_{p_2}}, 1+\gamma_{p_1}\right), \tag{27}$$

where $f_4(z)$ is used to denote the PDF of $Z\left(\frac{1+\gamma_{p_1}}{1+\gamma_{p_2}}, 1+\gamma_{p_1}\right)$.

*2) Primary User Leaves in SR link:* At the beginning, the primary user is present over the SR link, but it leaves at the $k_1^{\text{th}}$ symbol and keeps absent over the RD link. The probability of this case should be

$$P_b\{k_1\} = p_b p_{11}^{k_1-1} p_{10} p_{00}^{2Q-k_1}. \tag{28}$$

Similar to the first case, the status of the SD period can also be divided into three periods:

- $0 \sim k_1^{\text{th}}$ symbols in the SR link: The primary user traffic is present in this period, and the instantaneous SNR is $\gamma_{c_2}$, and the operating BER are both $\text{BER}_0$;
- $k_1^{\text{th}} \sim Q$ symbols in the SR link: After the primary user leaves, the instantaneous SNR becomes $\gamma'_B$ in (18), and the operating BER is $\overline{\text{BER}}_3$ in (26);
- the RD link: Without the primary user traffic in the whole RD link, the instantaneous SNR is $\gamma'_A$ in (16), and the operating BER is $\overline{\text{BER}}_4$ in (27);

Thus the average BER for all the periods is given by

$$\overline{\text{BER}}_{b(k_1)} = \frac{(k_1-1)\text{BER}_0 + (Q-k_1+1)\overline{\text{BER}}_3 + Q\overline{\text{BER}}_4}{2Q}, \tag{29}$$

*3) Primary User Leaves in RD link:* If the primary user is present in the whole SR link and leaves at the $k_2^{\text{th}}$ symbol in the RD link, the probability of its happening yields

$$P_b\{k_2\} = p_b p_{11}^{Q+k_2-1} p_{10} p_{00}^{Q-k_2}. \tag{30}$$

The status of the two divided periods are:

- the whole SR link and $0 \sim k_2^{\text{th}}$ symbols in the RD link: With the primary user traffic, the instantaneous SNR is $\gamma_{c_2}$, and the operating BER is also $\text{BER}_0$;
- $k_2^{\text{th}} \sim Q$ symbols in the RD link: After the primary user leaves, the instantaneous SNR changes to $\gamma'_A$ in (16), and the operating BER is $\overline{\text{BER}}_4$ in (27);

The the average BER of the whole period can be written as

$$\overline{\text{BER}}_{b(k_2)} = \frac{(Q+k_2-1)\text{BER}_0 + (Q-k_2+1)\overline{\text{BER}}_4}{2Q}, \tag{31}$$

*4) Primary User Leaves in SR link and Arrives in RD link:* In this case, the primary user leaves at the $k_1^{\text{th}}$ symbol in the SR link, and after a short break, it returns at the $k_2^{\text{th}}$ symbol in the RD link. The probability this case happens is given as

$$P_b\{k_1, k_2\} = p_b p_{11}^{k_1-1} p_{10} p_{00}^{Q-k_1+k_2-1} p_{01} p_{11}^{Q-k_2}. \tag{32}$$

The status of the whole time can be divided into three periods as:

- the $0 \sim k_1^{\text{th}}$ symbols in the SR link and the $k_2^{\text{th}} \sim Q$ symbols in the RD link: During these time, the instantaneous SNR is $\gamma_{c_2}$, and the operating BER is $\text{BER}_0$;
- the $k_1^{\text{th}} \sim Q$ symbols in the SR link: After the primary user's leave, the instantaneous SNR becomes $\gamma'_B$ in (18), and the operating BER becomes $\overline{\text{BER}}_3$ in (26);
- the $0 \sim k_2^{\text{th}}$ symbols in the RD link: After the primary user's return, the instantaneous SNR changes to $\gamma'_A$ in (16), and the average BER changes to $\overline{\text{BER}}_4$ in (27);

The average BER for the whole link can be written as

$$\overline{\text{BER}}_{b(k_1,k_2)} = \frac{(k_1 + Q - k_2)\text{BER}_0 + (Q - k_1 + 1)\overline{\text{BER}}_3 + (k_2 - 1)\overline{\text{BER}}_4}{2Q}. \tag{33}$$

### D. *Average BER for the relay assisted transmission*

Considering all the cases of spectrum bands detected and missing detection, of the six scenarios given in the above subsections, and of all the possible $k_1$ and $k_2$, the average BER for relay assisted transmission is thus given by

$$\begin{aligned}
\overline{\text{BER}}_r = &\sum_{k_1=1}^{Q} \left((1 - P_{fa})\overline{\text{BER}}_{f(k_1)} P_f\{k_1\} + P_{md}\overline{\text{BER}}_{b(k_1)} P_b\{k_1\}\right) \\
&+ \sum_{k_2=1}^{Q} \left((1 - P_{fa})\overline{\text{BER}}_{f(k_2)} P_f\{k_2\} + P_{md}\overline{\text{BER}}_{b(k_2)} P_b\{k_2\}\right) \\
&+ \sum_{k_1=1}^{Q} \sum_{k_2=1}^{Q} \left((1 - P_{fa})\overline{\text{BER}}_{f(k_1,k_2)} P_f\{k_1, k_2\} + P_{md}\overline{\text{BER}}_{b(k_1,k_2)} P_b\{k_1, k_2\}\right)
\end{aligned} \tag{34}$$

## V. Numerical Results

In this section, the false alarm and miss detection probability are set as $P_{fa} = 0.1$ and $P_{md} = 0.1$. The number of symbols per secondary frame contains is chosen as $Q = 20$. The operating BER for the adaptive modulation is chosen as $\text{BER}_0 = 10^{-6}$. For simplicity of the simulation, we choose identical primary user SNR over the SR link, the RD link and the direct link, i.e. $\gamma_{p_0} = \gamma_{p_1} = \gamma_{p_2} = \gamma_p$.

In Fig. 1, Fig. 2 and Fig. 3, four relay selection schemes, including the SR selection, the RD selection, the global selection and the random selection (randomly select a relay to assist transmission), are investigated for their impact on the average BER and the spectral efficiency. The channels for all SR links and all RD links are assumed to be i.i.d. separately with the identical average SNR $\bar{\gamma}_1$ and $\bar{\gamma}_2$, i.e. $\bar{\gamma}_{1j} = \bar{\gamma}_1$ and $\bar{\gamma}_{2j} = \bar{\gamma}_2$ for all $j = 1, 2, \cdots, N$. The SNR of the primary user traffic is set as $\gamma_p = 10$dB. Since we focus on the impact of relay selection, the direct link transmission is ignored in these three figures.

In Fig. 1, we set $\bar{\gamma}_1 = 5$ dB and $\bar{\gamma}_2 = 20$ dB so that channel conditions of the RD links are much better than that of the SR links. The simulation results show that the SR selection and the global selection almost have the same effects such that they both improve the spectral efficiency considerably and decrease the $\overline{\text{BER}}_r$. The RD selection has little impact on both the spectral efficiency and $\overline{\text{BER}}_r$, and its performance is close to the random selection. In Fig. 2, $\bar{\gamma}_1 = 20$ dB and $\bar{\gamma}_1 = 5$ dB so that the SR links have much better channel conditions. The results just show the effects of relay selection are opposite to the the results in Fig. 1. In Fig. 3, we set $\bar{\gamma}_1 = \bar{\gamma}_2 = 10$ dB. In this case, the $\overline{\text{BER}}_r$ changes little as the number of user increases, while the global selection outperforms both the SR selection and RD selection on spectral efficiency.

In all these three figures, we can see that the $\overline{\text{BER}}_r$ remains almost within the same order of magnitude, no matter how the relay is selected and how many relays are available. When the channel conditions of the SR links and RD links differ a lot, the worse-link selection achieves almost the same performance as the global selection, and the better-link selection approaches to the random selection. When the channel conditions for the SR links and RD links are close, on the other hand, the global selection obviously outperforms the partial selection schemes.

Keep in mind that our initial data transmission scheme in (1) that the direct link is used only when $\gamma_0 > \gamma_{\text{th}}$, so the over-all average BER is given by

$$\overline{\text{BER}} = P_{out}\overline{\text{BER}}_d + (1 - P_{out})\overline{\text{BER}}_r, \tag{35}$$

where $P_{out}$ is given in (4). In the following figures, we include the direct link to evaluate the overall average BER, and the average SNR and threshold for the direct link are chosen as $\bar{\gamma}_0 = 5$ dB and $\bar{\gamma}_{th} = 10$ dB with $m_0 = 4$.

In Fig. 4, the impact of the primary user traffic on the $\overline{\text{BER}}$ is studied for the SR selection, the RD selection and the global selection with different pairs of $(\bar{\gamma}_1, \bar{\gamma}_2)$. The number of users available for selection is $N = 5$. This figure shows that the primary user traffic degrades the average BER greatly. The performance of the worse-link selection approaches that of the global selection when the SR links and RD links have significantly different channel conditions, as we can see in the second and third subfigure. For the cases of the same average SNR as in the first and forth subfigure, the SR selection has larger $\overline{\text{BER}}$ and the RD selection has smaller $\overline{\text{BER}}$ than the global selection, even though the difference is very small. In addition, the $\overline{\text{BER}}$ goes to the magnitude of $10^{-2}$ under the high SNR of primary user traffic with the $\overline{\text{BER}}_0 = 10^{-6}$, and the secondary transmission under this circumstance is seriously jeopardized.

In Fig. 5, we compare the SR selection, the RD selection and the global selection for different values of the average SNRs. The number of users available for selection is also set to be $N = 5$. In the upper subfigure, the $\bar{\gamma}_2$ is fixed at $10$ dB and we vary $\bar{\gamma}_1$ for different values. In the lower subfigure, we do the opposite to fix $\bar{\gamma}_1$ at $10$ dB and vary $\bar{\gamma}_2$.

Comparing the two subfigures, we see that the $\overline{\text{BER}}$ increases as we rise the average SNR of the SR link or decrease the average SNR of the RD link. Reconsider the two situations of the initial absence and presence of the primary user, in the latter situation the instantaneous BER is far less than that of the former one, since the leave of the primary user will significantly reduce the instantaneous BER, while the arrive of the primary user greatly increase it. So the situation with the primary user initially absent plays a far more important role in determining the $\overline{\text{BER}}$. In the former situation, the RD links are more likely to be affected by the primary user traffic, which means that the average BER is more susceptible and negatively correlated to $\frac{\gamma_1+\gamma_2}{(1+\gamma_{p_2})\gamma_1+\gamma_2}$. Note $\frac{\gamma_1+\gamma_2}{(1+\gamma_{p_2})\gamma_1+\gamma_2}$ is monotonically increasing with $\gamma_2$, so increasing $\bar{\gamma}_2$ will decrease the $\overline{\text{BER}}$ ultimately, as we see in the lower subfigures. Considering that the $\overline{\text{BER}}$ is related only to the relative SNR $\frac{\gamma_1+a\gamma_2}{\gamma_1+b\gamma_2}$, but not to the single SNR $\gamma_1$ or $\gamma_2$, it is not surprising to see that increasing $\bar{\gamma}_1$ and decreasing $\bar{\gamma}_2$ have the same effects. By comparing different selection schemes, we see that the SR selection always has greater and the RD selection always has less average BER than the global selection under the same channel conditions. That is because that the partial selection is equivalent to increase the average SNR of the selected link.

## VI. Conclusion

In this paper, we have examined the impact of the primary user traffic on the BER of adaptive data transmission for the secondary user with the relay chosen based on the partial selection. Six situations of the primary-user status change have been analyzed for the relay link transmission, and two for the direct link transmission. The instantaneous BER for each situation, as well as the average BER over all possible situations, have been derived with closed-form expressions. Different relay selection schemes are compared by the numerical results, and we find that as the relay selection increases the spectral efficiency of the system, while the average BER only changes slightly within the same order of magnitude. In addition, we also find that the performance on average BER of the worse-link selection approaches to the global selection, while the better-link selection only performs as bad as the random selection when there is a great difference between the channel conditions of the SR links and RD links. When the channels conditions of the SR links and the RD links are close, the global selection outperforms both the SR and the RD selection.

## Appendix: The PDF and MGF of $Z$

In this appendix, we derive the PDF and MGF of the random varialbe $Z(a,b)$ defined in (12).

### A. *Gamma Distributed Random Variables*

First of all, we suppose that $\gamma_i$'s are Gamma-distributed random variables with shape parameter $\alpha_i$ and a scale parameter $\beta_i$, i.e. $\gamma_i \sim \mathcal{G}(\alpha_i, \beta_i), i = 1, 2$. The PDF of $\gamma_i$ is given by $f_{\gamma_i}(\gamma) =$

$\frac{1}{\Gamma(\alpha_i)\beta_i^{\alpha_i}}\gamma^{\alpha_i-1}\mathrm{e}^{-\gamma/\beta_i}$ with $\gamma > 0$. Denote two random variables $X = \gamma_1 + a\gamma_2$ and $Y = \gamma_1 + b\gamma_2$, then it is not difficult to show that the joint PDF of $(X, Y)$ is given by

$$f_{X,Y}(x,y) = J\frac{\beta_1^{-\alpha_1}\beta_2^{-\alpha_2}}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\left(\frac{ay-bx}{a-b}\right)^{\alpha_1-1}\left(\frac{x-y}{a-b}\right)^{\alpha_2-1}\exp\left(-\frac{1}{\beta_1}\frac{ay-bx}{a-b}-\frac{1}{\beta_2}\frac{x-y}{a-b}\right),$$

where $J = \frac{1}{|a-b|}$. Note that the random variable $Z$ is the ratio of $X$ and $Y$, then the PDF of $Z$ can be calculated via $f_Z(z) = \int_{-\infty}^{+\infty}|y|f_{X,Y}(yz,y)dy$, with which we have the PDF of $Z$ as

$$f_Z(z) = \frac{J}{\beta_1^{\alpha_1}\beta_2^{\alpha_2}}\frac{\Gamma(\alpha_1+\alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\left(\frac{a-bz}{a-b}\right)^{\alpha_1-1}\left(\frac{z-1}{a-b}\right)^{\alpha_2-1}\left(\frac{1}{\beta_1}\frac{a-bz}{a-b}+\frac{1}{\beta_2}\frac{z-1}{a-b}\right)^{-(\alpha_1+\alpha_2)}. \tag{36}$$

The MGF of $Z$ is calculated by its definition $M_Z(s) = \mathbb{E}(\mathrm{e}^{sz}) = \int_{-\infty}^{+\infty}\mathrm{e}^{sz}f_Z(z)dz$. Denote $t = b\frac{z-1}{a-b}$, so that $\frac{z-1}{a-b} = \frac{t}{b}$ and $\frac{a-bz}{a-b} = 1-t$, then we have

$$M_Z(s) = \mathrm{e}^s(1-\mu)^{\alpha_2}\frac{\Gamma(\alpha_1+\alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\int_0^1\mathrm{e}^{\eta st}t^{\alpha_2-1}(1-t)^{\alpha_1-1}(1-\mu t)^{-(\alpha_1+\alpha_2)}dt,$$

where $\mu = 1-\frac{\beta_1}{\beta_2 b}$ and $\eta = \frac{a-b}{b}$. Note that for any real $u, v, \epsilon, x, y$ satisfying $u > 0, v > 0, |x| < 1$, we have [19, 3.385]

$$\int_0^1 t^{u-1}(1-t)^{v-1}(1-xt)^{-\epsilon}\mathrm{e}^{yt}dt = B(u,v)\Phi_1(u,\epsilon,u+v;x,y), \tag{37}$$

where $B(u,v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}$ is the Beta function, and $\Phi_1(\cdot,\cdot,\cdot;\cdot,\cdot)$ is the Humbert series. So when $0 < |\mu| < 1$, or $0 < \frac{\beta_1}{\beta_2 b} < 2$, the MGF of $Z$ can thus be written as

$$M_Z(s) = \mathrm{e}^s\left(\frac{\beta_1}{\beta_2 b}\right)^{\alpha_2}\Phi_1\left(\alpha_2,\alpha_1+\alpha_2,\alpha_1+\alpha_2;1-\frac{\beta_1}{\beta_2 b},\frac{a-b}{b}s\right), 0 < \frac{\beta_1}{\beta_2 b} < 2. \tag{38}$$

On the other hand, we could also denote $t = \frac{a-bz}{a-b}$, so that $\frac{z-1}{a-b} = \frac{1-t}{b}$. In the same way, we get the MGF of $Z$ with the help of (37) as

$$M_Z(s) = \mathrm{e}^{\frac{a}{b}s}\left(\frac{\beta_2 b}{\beta_1}\right)^{\alpha_1}\Phi_1\left(\alpha_1,\alpha_1+\alpha_2,\alpha_1+\alpha_2;1-\frac{\beta_2 b}{\beta_1},\frac{b-a}{b}s\right), 0 < \frac{\beta_2 b}{\beta_1} < 2. \tag{39}$$

Especially when $\beta_1 = b\beta_2$, we have $\mu = 1-\frac{\beta_2 b}{\beta_1} = 0$, and the integral (37) with $x = 0$ will simplify to $\int_0^1 t^{u-1}(1-t)^{v-1}\mathrm{e}^{\theta t}dt = B(u,v)\ {}_1F_1(u;u+v;\theta)$ [19, 3.383], and we can have $\Phi_1(u,\epsilon,u+v;0,y) = {}_1F_1(u;u+v;y)$, where ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function. The MGF of $Z$ in this case will also simplify to $M_Z(s) = \mathrm{e}^s\ {}_1F_1(\alpha_2;\alpha_1+\alpha_2;\eta s) = \mathrm{e}^{\frac{a}{b}s}\ {}_1F_1(\alpha_1;\alpha_1+\alpha_2;-\eta s)$.

### *B. Partial Selection*

We still suppose $\gamma_2$ is Gamma-distributed with parameter $(\alpha,\beta)$, but the PDF of $\gamma_1$ is subject to (11). Similar to the process above, we can calculate the PDF of $Z = \frac{\gamma_1+a\gamma_2}{\gamma_1+b\gamma_2}$ via $X$ and $Y$, which is given by

$$f_Z(z) = \sum_S\sum_L(-1)^q C_{S,L}\frac{\Gamma(l_S+1)}{E_S^{l_S}}\times \Bigg\{\underbrace{\frac{JE_S^{l_S}}{\beta^\alpha}\frac{\Gamma(\alpha+l_S)}{\Gamma(\alpha)\Gamma(l_S)}\left(\frac{a-bz}{a-b}\right)^{l_S-1}\left(\frac{z-1}{a-b}\right)^{\alpha-1}\left(E_S\frac{(a-bz)}{a-b}+\frac{1}{\beta}\frac{(z-1)}{a-b}\right)^{-(\alpha+l_S)}}_{\mathbf{A}}$$
$$-\underbrace{\frac{JE_S^{l_S+1}}{\beta^\alpha}\frac{\Gamma(\alpha+l_S+1)}{\Gamma(\alpha)\Gamma(l_S+1)}\left(\frac{a-bz}{a-b}\right)^{l_S}\left(\frac{z-1}{a-b}\right)^{\alpha-1}\left(E_S\frac{(a-bz)}{a-b}+\frac{1}{\beta}\frac{(z-1)}{a-b}\right)^{-(\alpha+l_S+1)}}_{\mathbf{B}}\Bigg\}. \quad (40)$$

Compare part **A** and **B** in (40) with (36), we can derive the MGF of $Z$ for this parts as $\Psi(l_S,E_S,\alpha,\beta)$ and $\Psi(l_S+1,E_S,\alpha,\beta)$, respectively, where

$$\Psi(l_S,E_S,\alpha,\beta) = \begin{cases} \mathrm{e}^{\frac{a}{b}s}\left(\beta bE_S\right)^{l_S}\Phi_1\left(l_S,\alpha+l_S,\alpha+l_S;1-\beta bE_S,\frac{b-a}{b}s\right), & 0<E_S<\frac{2}{b\beta} \\ \mathrm{e}^{s}\left(\beta bE_S\right)^{-\alpha}\Phi_1\left(\alpha,\alpha+l_S,\alpha+l_S;1-\frac{1}{\beta b}E_S^{-1},\frac{a-b}{b}s\right), & E_S>\frac{1}{2b\beta} \end{cases}$$

So the MGF of $Z = \frac{\gamma_1+a\gamma_2}{\gamma_1+b\gamma_2}$ with $\gamma_1$ subject to (11) and $\gamma_2 \sim \mathcal{G}(\alpha,\beta)$ is given by

$$M_Z(s) = \sum_S\sum_L(-1)^q C_{S,L}\frac{\Gamma(l_S+1)}{E_S^{l_S}}\Big(\Phi(l_S,E_S,\alpha,\beta)-\Phi(l_S+1,E_S,\alpha,\beta)\Big). \quad (41)$$

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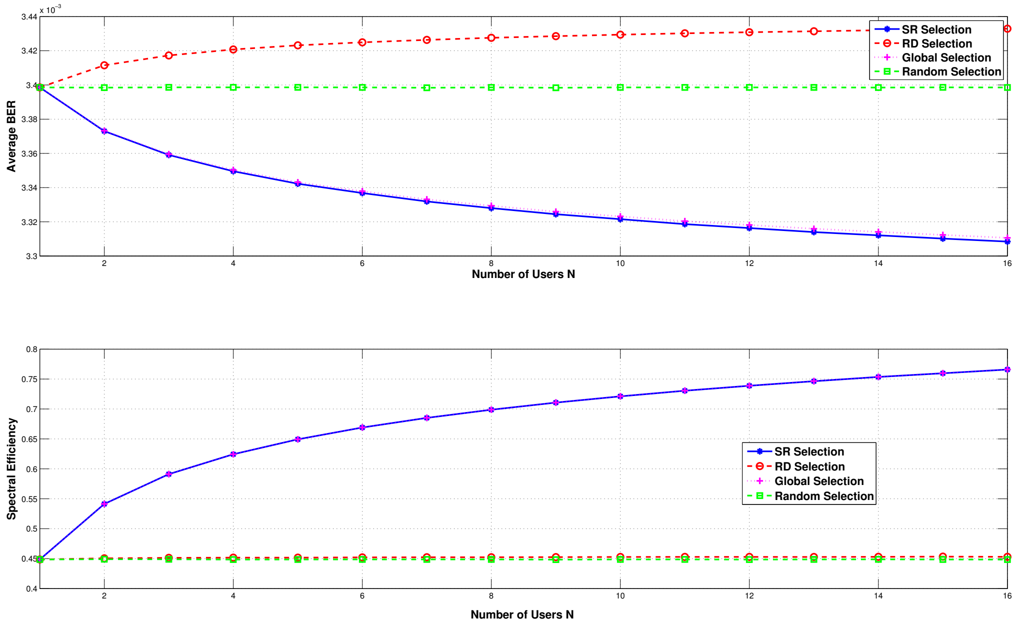


Figure 1: The average BER and spectral efficiency for different numbers of users $N$ with $\lambda = \mu = \frac{1}{200T_s}$ ($m_1 = 5$, $m_2 = 3$, $\gamma_p = 10$ dB, $\bar{\gamma}_1 = 5$ dB, $\bar{\gamma}_2 = 20$ dB).

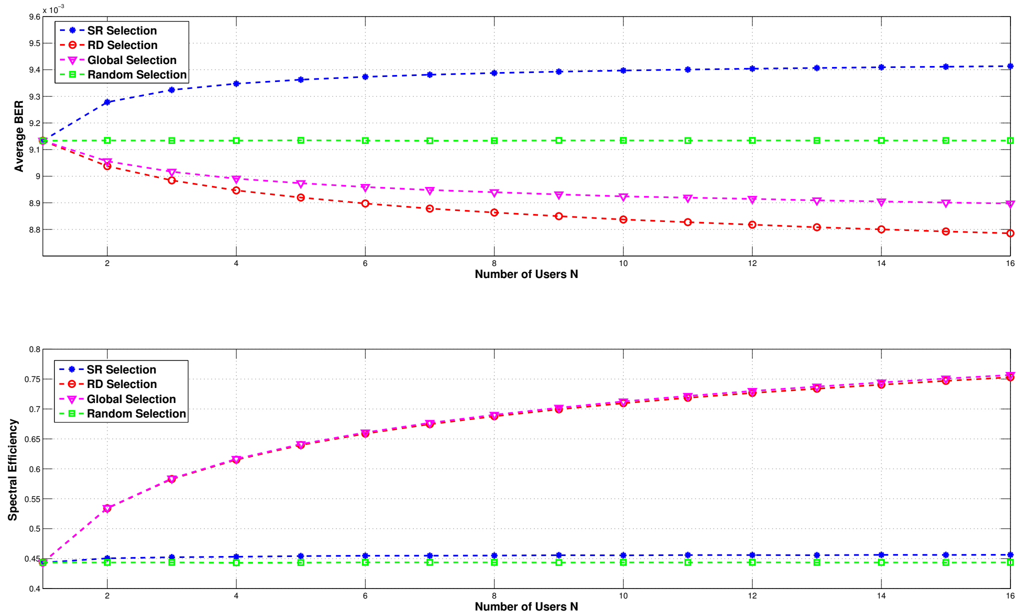


Figure 2: The average BER and spectral efficiency for different numbers of users $N$ with $\lambda = \mu = \frac{1}{200T_s}$ ($m_1 = 3$, $m_2 = 5$, $\gamma_p = 10$ dB, $\bar{\gamma}_1 = 20$ dB, $\bar{\gamma}_2 = 5$ dB).

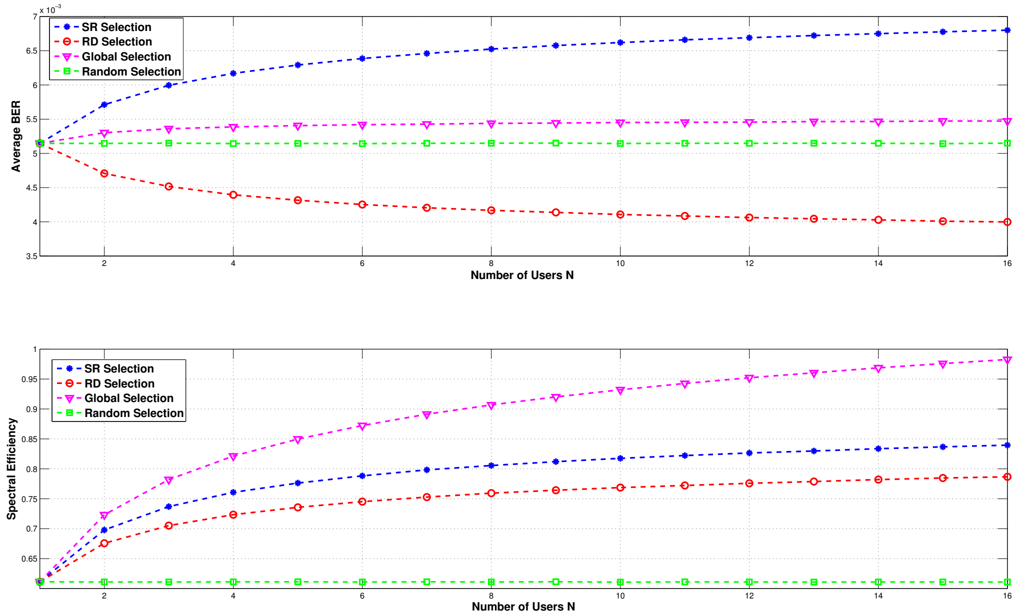


Figure 3: The average BER and spectral efficiency for different numbers of users $N$ with $\lambda = \mu = \frac{1}{200T_s}$ ($m_1 = 3$, $m_2 = 5$, $\gamma_p = 10$ dB, $\bar{\gamma}_1 = 10$ dB, $\bar{\gamma}_2 = 10$ dB).

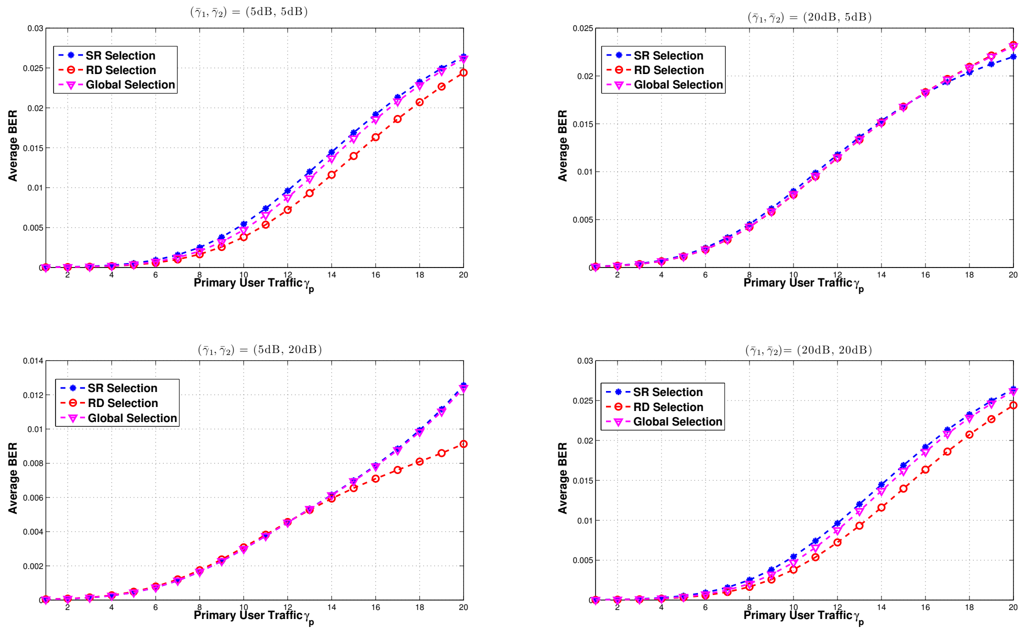


Figure 4: The average BER for different values of primary user SNR $\gamma_p$ with $\lambda = \mu = \frac{1}{200T_s}$ ($m_1 = 3$, $m_2 = 5$, $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are set as different value pairs).

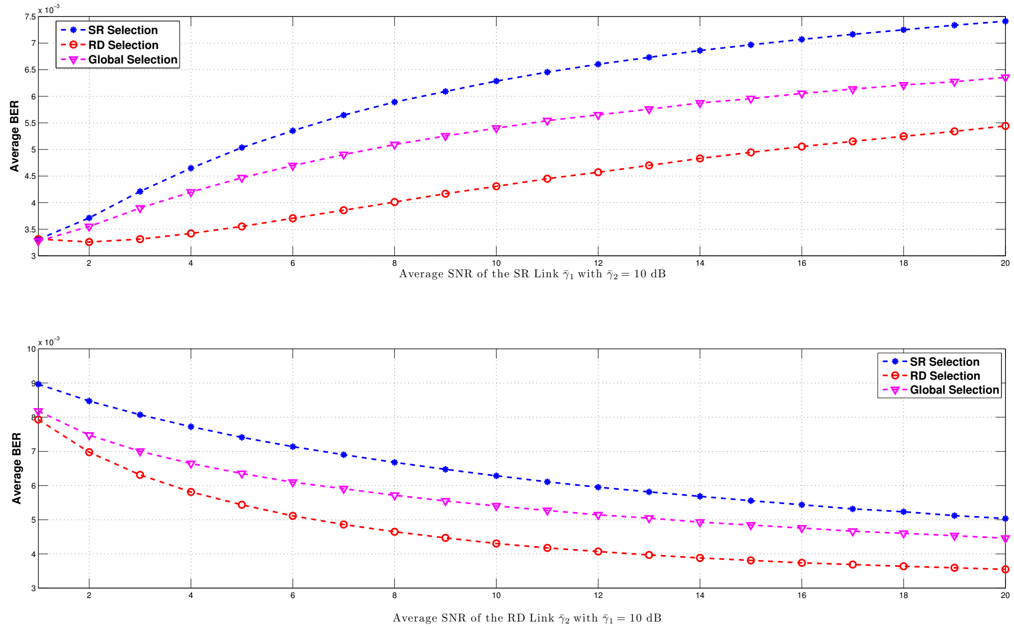


Figure 5: The average BER for different values of average SNR $\bar{\gamma}_1$ and $\bar{\gamma}_2$ with $\lambda = \mu = \frac{1}{200T_s}$ ($m_1 = 3$, $m_2 = 5$, $\gamma_p = 10$ dB)