Low-Complexity Full-Diversity Detection in Two-User MIMO X Channels

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***Abstract*—Several interference cancellation (IC) schemes have been recently proposed to suppress multi-user interference for various network configurations (e.g., multiple access and X channels). However, most of these schemes trade-off diversity for implementation complexity or vice-versa. In this paper, we propose a full-diversity interference cancellation scheme in a multiple-input multiple-output (MIMO) X channel with two sources and two destinations while maintaining low decoding complexity. We provide sufficient conditions for a wide range of space-time block codes (STBCs) to achieve full-diversity gain under the so-called *partial interference cancellation group decoding* (PICGD) in the configuration of interest. A systematic construction is then proposed to achieve full-diversity. The constructed scheme is compared to recently proposed IC scheme in terms of performance and decoding complexity. Our IC scheme outperforms the recently proposed scheme in the case it provides higher transmission rate, while it loses slightly in the case of equal rates. In terms of decoding complexity, both schemes are equivalent.**

***Index Terms*—Interference cancellation, full-diversity, decoding complexity, partial interference cancellation group decoding.**

I. INTRODUCTION

Multi-user interference cancellation has become a key factor to improve both reliability and capacity of nowadays wireless networks. An efficient way, besides conventional multiple access approaches (e.g., TDMA, FDMA, and DS/FH-CDMA) is the use of multiple-input multiple-output (MIMO) techniques to alleviate the multi-user interference inherent in wireless systems. In fact, multiple-antenna arrays are becoming commonplace in recent wireless communication systems as they can dramatically improve both the capacity via the multiplexing gain (channel degrees of freedom) and the reliability via the channel diversity gain. This newly-added dimension (i.e, space) can be efficiently exploited to design *space-based* interference cancellation (IC) techniques that can be employed alongside classical techniques. Recently, many space-based IC schemes have been proposed to suppress multi-user interference for several network configurations (e.g., multiple access and X channels). However, most of these schemes trade-off diversity for implementation complexity or vice-versa. For instance, beamforming techniques can be employed to create orthogonal subspaces for distinct users but they require perfect or limited channel state information at the transmitter side (CSIT). On the other hand, in the absence of CSIT, space-time coding techniques may be used to cancel the multi-user interference at the expense of a substantial loss of diversity gain. A straightforward solution to overcome the diversity loss of space-time coding techniques when CSI is only available at the receiver side is to resort to maximum-likelihood (ML) joint detection of all transmitted symbols. However, this approach suffers from two major drawbacks, namely, the induced high decoding complexity, and moreover, for some wireless network configurations the destination may be forced to even decode unintended messages (e.g., the X channel). Our main focus in this paper is to design full-diversity, low-complexity interference cancellation scheme for the case of two blind sources and two destinations in a MIMO X channel.

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A. Prior Work

In the last few years, the X channel has attracted significant interests since it models several real scenarios; for instance, the soft handover feature in CDMA and W-CDMA standards, where a mobile user may be connected simultaneously to several base stations. In [1], the authors investigated the degrees of freedom region of a MIMO X channel comprised of two sources and two destinations, each equipped with M antennas. The authors proved that $\frac{4}{3}M$ degrees of freedom (DOFs) are achievable through *interference alignment* combined with zero-forcing (ZF). However, the aforementioned IC suffers from two drawbacks. First, it assumes global CSIT knowledge, i.e., not only each source needs to know its individual channels, but it needs to know the other source's channels as well. The second drawback is that the proposed scheme suffers form a substantial diversity gain loss. In [2], the authors restricted themselves to the case of two transmit antennas ($M = 2$) and proposed an IC scheme that elegantly exploit the fact that the Alamouti codewords are closed under matrix multiplication and addition to suppress the interference while providing a diversity gain of two with a negligible decoding complexity cost. The above scheme achieves a total rate of $8/3$, thus in compliance with the theoretical total DOFs, while only assuming individual CSIT instead of the global CSIT assumption in [1]. In [3], the authors constructed precoders to achieve full-diversity under the assumption of global CSIT knowledge. The main idea behind this scheme is to design the precoders such that the useful messages and the undesired messages at each destination span orthogonal subspaces, therefore ZF can be employed to reduce the decoding complexity without sacrificing the diversity gain. Recently, L. Shi *et al.* proposed a two-user MIMO X channel IC scheme [4], [5] that achieves full-diversity at an affordable complexity cost that requires CSI at the receiving side only.

B. Our Contribution

In this paper, we focus on the MIMO X channel comprised of two sources and two destinations and propose sufficient conditions for a wide range of STBCs to achieve full-diversity under partial interference cancellation group decoding (PICGD) with no prior knowledge of the CSI at the sources. We then provide an IC scheme for this X channel configuration and prove that it indeed achieves the full-diversity gain under PICGD. Our IC scheme is then compared to the recently proposed IC scheme in [4], [5] in terms of the codeword error rate (CER) and the worst-case decoding complexity order. Our IC scheme is shown to be superior in terms of CER for cases it provides a higher rate than its counterpart, while slightly losing in cases of equal rates. It is worth noting that, in terms of worst-case decoding complexity, both schemes are equivalent.

The rest of the paper is organized as follows. In Section II, the system model is introduced, then the PICGD approach is outlined. The full-diversity criteria are provided in Section III. In Section IV, the proposed IC scheme for the X channel comprised of two sources and two destinations is presented and proven to achieve the full-diversity gain offered by the network. Simulation results are provided in Section V, and the paper is concluded in Section VI.

Notations

Throughout the paper, small letters, bold small letters, bold capital letters, and calligraphic letters will designate scalars, vectors, matrices, and sets respectively. If $\mathbf{A}$ is a matrix, then $\mathbf{A}^{\mathsf{H}}$, $\mathbf{A}^{\mathsf{T}}$, and $\mathbf{A}^{\dagger}$ denote the Hermitian, the transpose, and the pseudo-inverse of $\mathbf{A}$, respectively. We define $\text{vec}(\mathbf{A})$ as the operator which, when applied to an $m \times n$ matrix $\mathbf{A}$, transforms it into an $mn \times 1$ vector by simply concatenating vertically the columns of the corresponding matrix. The $\otimes$ operator is the Kronecker product. $\mathbf{I}_n$ and $\mathbf{0}$ denote the $n \times n$ identity and null matrix with the appropriate size, respectively. Finally, $\mathbb{P}\left[A\right]$ denotes the probability of the event A to occur, and $\mathbb{E}_{\mathbf{x}}\left[f\left(\mathbf{x}\right)\right]$ denotes the statistical average of an arbitrary function $f\left(\mathbf{x}\right)$ w.r.t the random vector $\mathbf{x}$.

II. System model

Our general X channel system model depicted in Fig. 1 consists of K sources, each equipped with N_t transmit antennas, that are communicating with M destinations, each equipped with N_r receive antennas. We will refer to this configuration afterwards by $\left(N_t^K, N_r^M\right)$. During the signalling period T, the message $\mathbf{u}_{ij}$ needs to be conveyed from source i to destination j, $\forall (i,j) \in \{1,\ldots,K\} \times \{1,\ldots,M\}$. The baseband received signal at the j-th destination (D_j), is given by

$$\mathbf{Y}_j = \sum_{k=1}^{K}\sum_{m=1}^{M}\mathbf{X}_{km}\mathbf{H}_{kj} + \mathbf{W}_j, \tag{1}$$

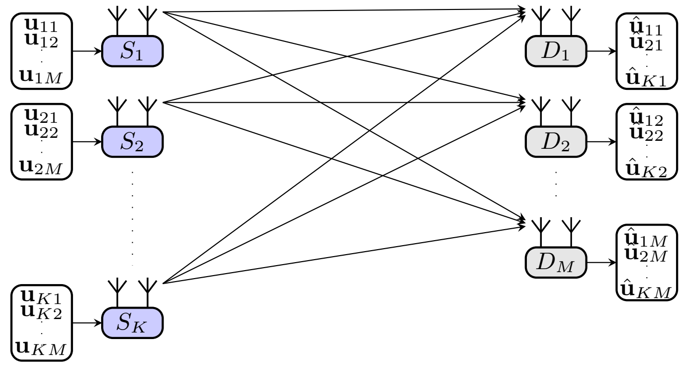

Fig. 1: General $K \times M$ MIMO X-channel

where $\mathbf{Y}_j \in \mathbb{C}^{T\times N_r}$ is the received signal at D_j, T is the signalling period (channel uses), and $\mathbf{X}_{km} \in \mathbb{C}^{T\times N_t}$ is the space-time mapping of the k-th source (S_k) message addressed to the m-th destination (i.e., $\mathbf{X}_{km} = f\left(\mathbf{u}_{km}\right)$, where $f(\cdot)$ is an injective function for the code to be uniquely decodable, that completely defines the coding scheme). The channel coefficients matrix from the k-th source to the j-th destination is denoted by $\mathbf{H}_{kj} \in \mathbb{C}^{N_t\times N_r}$ with independent and identically distributed (i.i.d.) entries drawn from a circularly symmetric complex Gaussian distribution with zero mean and unit variance, and $\mathbf{W}_j \in \mathbb{C}^{T\times N_r}$ is the noise matrix at the j-th destination with i.i.d. entries drawn from a circular symmetric complex Gaussian distribution with zero mean and variance N_0. According to the above model, the t-th row of the matrix $\mathbf{X}_{km}$ denotes the symbols transmitted through the N_t transmit antennas during the t-th channel use while the n-th column denotes the symbols transmitted through the n-th transmit antenna during the signalling period T. We assume perfect synchronization among distinct users in the network and perfect CSI is available only at the destination. Applying the $\text{vec}(\cdot)$ operator to (1), we obtain

$$\underbrace{\text{vec}\left(\mathbf{Y}_j\right)}_{\mathbf{y}_j} = \sum_{k=1}^{K}\sum_{m=1}^{M}\underbrace{\mathbf{I}_{Nr}\otimes\mathbf{X}_{km}}_{\widetilde{\mathbf{X}}_{km}}\underbrace{\text{vec}\left(\mathbf{H}_{kj}\right)}_{\mathbf{h}_{kj}} + \underbrace{\text{vec}\left(\mathbf{W}_j\right)}_{\mathbf{w}_j}. \tag{2}$$

For a large class of STBCs ([6]–[8], among others), the code matrices take the form

$$\mathbf{X}_{km} = \sum_{i=1}^{n_{km}}\mathbf{A}_{km,i}s_{km,i},\ \forall\, k = 1,\ldots,K, m = 1,\ldots,M, \tag{3}$$

with $s_{km,i} \in \mathbb{C}$ and $\mathbf{A}_{km,i} \in \mathbb{C}^{T\times N_t}$. Replacing $\mathbf{X}_{km}$ by its expression in Eq. (3), the system model in Eq. (2) becomes

$$\mathbf{y}_j = \sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{i=1}^{n_{km}}\left(\mathbf{I}_{N_r}\otimes\mathbf{A}_{km,i}\right)\mathbf{h}_{kj}s_{km,i} + \mathbf{w}_j,$$

or equivalently

$$\mathbf{y}_j = \sum_{k=1}^{K}\sum_{m=1}^{M}\widetilde{\boldsymbol{\mathcal{H}}}_{kmj}\mathbf{s}_{km} + \mathbf{w}_j, \tag{4}$$

with $\widetilde{\boldsymbol{\mathcal{H}}}_{kmj} = \left[\boldsymbol{\mathcal{H}}_{kmj,1}^{\mathsf{T}}\cdots\boldsymbol{\mathcal{H}}_{kmj,N_r}^{\mathsf{T}}\right]^{\mathsf{T}}$, where $\boldsymbol{\mathcal{H}}_{kmj,l} = \left[\mathbf{A}_{km,1}\mathbf{h}_{kj,l}\ldots\mathbf{A}_{km,n_{km}}\mathbf{h}_{kj,l}\right]$ for all $l = 1, 2, \cdots, N_r$, $\mathbf{h}_{kj,l}$ denotes the channel coefficients vector from the k-th user to the l-th antenna of the j-th destination, and $\mathbf{s}_{km} = \left[s_{km,1}\ldots s_{km,n_{km}}\right]^{\mathsf{T}}$. Accordingly, the received signal model in (4) can be re-written as

$$\mathbf{y}_j = \underbrace{\sum_{k=1}^{K} \widetilde{\boldsymbol{\mathcal{H}}}_{kjj}\mathbf{s}_{kj}}_{\text{useful signal}} + \underbrace{\sum_{k=1}^{K}\sum_{\substack{m=1\\ m\neq j}}^{M} \widetilde{\boldsymbol{\mathcal{H}}}_{kmj}\mathbf{s}_{km}}_{\text{interference}} + \mathbf{w}_j.$$

If we take $\widetilde{\boldsymbol{\mathcal{H}}}_{jj} = \left[\widetilde{\boldsymbol{\mathcal{H}}}_{1jj}\ldots\widetilde{\boldsymbol{\mathcal{H}}}_{Kjj}\right]$, and $\mathbf{s}_j = \left[\mathbf{s}_{1j}^{\mathsf{T}}\ldots\mathbf{s}_{Kj}^{\mathsf{T}}\right]^{\mathsf{T}}$, then the above equation can be re-written in a more compact form as

$$\mathbf{y}_j = \widetilde{\boldsymbol{\mathcal{H}}}_{jj}\mathbf{s}_j + \sum_{k=1}^{K}\sum_{\substack{m=1\\ m\neq j}}^{M} \widetilde{\boldsymbol{\mathcal{H}}}_{kmj}\mathbf{s}_{km} + \mathbf{w}_j. \tag{5}$$

Interference Cancellation

The objective herein is to enable each destination $D_j, \forall j \in \{1,\ldots,M\}$ to separately decode its desired messages $\{\mathbf{s}_{kj} : k = 1,\ldots,K\}$ at a low-complexity cost while achieving the full-diversity gain offered by the network configuration (i.e., N_tN_r). For this purpose, we provide sufficient conditions to achieve full-diversity under PICGD for the considered system model. The PICGD is a decoding algorithm that generalizes the zero-forcing receiver. Specifically, it separates the transmitted symbols into disjoint sets and decodes these sets independently. In the case of X channels, if for instance, we consider the j-th destination, the set of symbols $\{\mathbf{s}_{kj} : k = 1,\ldots,K\}$ needs only to be decoded. Let $\widetilde{\boldsymbol{\mathcal{H}}}_{\overline{jj}}$ denote a basis of the subspace spanned by the undesired messages. Therefore, the required projection matrix $\mathbf{P}_j$ needs to satisfy $\mathbf{P}_j\widetilde{\boldsymbol{\mathcal{H}}}_{\overline{jj}} = \mathbf{0}$. This condition has a general solution described by

$$\mathbf{P}_j = \left(\mathbf{I}_{N_rT} - \widetilde{\boldsymbol{\mathcal{H}}}_{\overline{jj}}\widetilde{\boldsymbol{\mathcal{H}}}_{\overline{jj}}^{\dagger}\right).$$

Left multiplying Eq. (5) by $\mathbf{P}_j$, one obtains

$$\mathbf{P}_j\mathbf{y}_j = \mathbf{P}_j\widetilde{\boldsymbol{\mathcal{H}}}_{jj}\mathbf{s}_j + \mathbf{P}_j\mathbf{w}_j.$$

The ML estimate of $\{\mathbf{s}_{kj} : k = 1,\ldots,K\}$ under PICGD is then given by

$$\mathbf{s}_j^{\text{ML}|\text{PICGD}} = \arg\min_{\mathbf{s}_j\in\mathcal{A}_j} \left\|\mathbf{P}_j\mathbf{y}_j - \mathbf{P}_j\widetilde{\boldsymbol{\mathcal{H}}}_{jj}\mathbf{s}_j\right\|,$$

where $\mathcal{A}_j$ denotes the codebook of $\mathbf{s}_j$. For the considered class of STBCs, one has $\widetilde{\boldsymbol{\mathcal{H}}}_{kjj}\mathbf{s}_{kj} = \widetilde{\mathbf{X}}_{kj}\mathbf{h}_{kj}$, therefore the ML detection under PICGD reduces to

$$\widetilde{\mathbf{X}}_j^{\text{ML}|\text{PICGD}} = \arg\min_{\widetilde{\mathbf{X}}_{kj}\in\mathcal{C}_{kj}} \left\|\mathbf{P}_j\mathbf{y}_j - \mathbf{P}_j\sum_{k=1}^{K}\widetilde{\mathbf{X}}_{kj}\mathbf{h}_{kj}\right\| \tag{6}$$

where $\widetilde{\mathbf{X}}_j = \sum_{k=1}^{K}\widetilde{\mathbf{X}}_{kj}$, and $\widetilde{\mathcal{C}}_{kj}$ denotes the codebook spanned by $\widetilde{\mathbf{X}}_{kj}$.

III. Full-diversity criteria

In what follows, we will derive sufficient conditions for the family of STBCs in (3) to achieve the full-diversity under PICGD in the $\left(N_t^2, N_r^2\right)$ X channels setting. It is worth noting that in the case of the X channel, both the equivalent channel and projection matrices depend upon the same channel coefficients in contrast to the case of multiple-access channels [9]–[11] where the equivalent channel and projection matrices are totally independent. Consequently, the followed procedure to derive sufficient condition to achieve full-diversity in multiple-access channels is not applicable in our case. In order to work around the aforementioned hurdle, we resort to the following lemma.

Lemma 1. *If a matrix $\mathbf{A} \in \mathbb{C}^{n\times m}$ is of full column rank, $\mathbf{x} \in \mathbb{C}^{m\times 1}$ is independent from $\mathbf{A}$, thus we have*

$$\|\mathbf{A}\mathbf{x}\|^2 \geq c_{\mathbf{A}}\|\mathbf{x}\|^2,$$

where $c_{\mathbf{A}}$ is a positive constant.

Proof: The matrix $\mathbf{A}$ is of full column rank, thus we have

$$\mathbf{A}\mathbf{x} \neq \mathbf{0},\ \forall\ \mathbf{x} \in \mathbb{C}^{m\times 1}\setminus\{\mathbf{0}\}.$$

Restricting ourselves to the vectors $\mathbf{x}$ of unit norm results in

$$\left\|\mathbf{A}\mathbf{x}/\|\mathbf{x}\|\right\|^2 > 0,\ \forall\ \mathbf{x} \in \mathbb{C}^{m\times 1}\setminus\{\mathbf{0}\} \tag{7}$$

Now, recall the Heine-Borel theorem which completely characterizes compact sets in Euclidean space $\mathbb{R}^n$.

Theorem 1. (Theorem 2.4 in [12]) *The compact subsets of a Euclidean space $\mathbb{R}^n$ are precisely those that are closed and bounded.*

Moreover, for continuous functions from a topological space X to another one Y, one has the following theorem.

Theorem 2. (Theorem 2.10 in [12]) *Let X and Y be topological spaces, and let $f : X \to Y$ be continuous. If $\mathcal{K}$ is a compact subset of X, then $f(\mathcal{K})$ is compact.*

The set $\mathcal{X} = \left\{\mathbf{x}/\|\mathbf{x}\| : \mathbf{x} \in \mathbb{C}^{m\times 1}\setminus\{\mathbf{0}\}\right\}$ is compact according to theorem 1, as the unit sphere is closed and bounded. As the norm in (7) is continuous, according to theorems 1, 2, there must exist a positive constant $c_{\mathbf{A}}$ such that

$$\left\|\mathbf{A}\mathbf{x}/\|\mathbf{x}\|\right\|^2 \geq c_{\mathbf{A}}, \forall\ \mathbf{x} \in \mathbb{C}^{m\times 1}\setminus\{\mathbf{0}\}$$

or equivalently

$$\|\mathbf{A}\mathbf{x}\|^2 \geq c_{\mathbf{A}}\|\mathbf{x}\|^2, \forall\ \mathbf{x} \in \mathbb{C}^{m\times 1}\setminus\{\mathbf{0}\}.$$

Finally, the above inequality holds for the case of $\mathbf{x} = \mathbf{0}$, thus completing the proof. ■

Now, we proceed towards the main theorem.

Theorem 3. *An IC scheme achieves the full-diversity under PICGD at the j-th destination in the $\left(N_t^2, N_r^2\right)$ X channel if $\forall\ 1 \leq l \neq k \leq 2$ one has*

$$\|\mathbf{P}_j\Delta\widetilde{\mathbf{X}}_{kj}\mathbf{h}_{kj}\|^2 \geq \alpha\|\mathbf{h}_{kj}\|^2,$$

$$\forall\ \mathbf{h}_{lj},\ \mathbf{h}_{kj} \in \mathbb{C}^{N_tN_r}, \Delta\widetilde{\mathbf{X}}_{kj} \in \Delta\widetilde{\mathcal{C}}_{kj}\setminus\{\mathbf{0}\}$$

where α is a positive number independent from $\mathbf{h}_{kj}$, $\mathbf{X}_{kj}$ are expressed as in (3), and $\Delta\widetilde{\mathbf{X}}_{kj}$ (resp. $\Delta\widetilde{\mathcal{C}}_{kj}$) denote the codeword difference (resp. codeword difference codebook).

Proof: see Appendix.

It is worth noting that a prerequisite of the above theorem is that the STBCs $\{\mathbf{X}_{kj} : k, j = 1, 2\}$ should achieve the full-diversity under ML decoding.

IV. Proposed IC schemes for the $\left(N_t^2, N_r^2\right)$ X Channel

In this section, we provide a systematic IC scheme designed for the $\left(N_t^2, N_r^2\right)$ X channel which satisfies the full-diversity criteria in Theorem 1. Let $\mathbf{s}'_{km} = \mathbf{U}_{n_{km}}\mathbf{s}_{km}$, where $\mathbf{U}_{n_{km}}$ is the $n_{km}\times n_{km}$ full-diversity rotation matrix [13], $\mathbf{s}_{km}, \forall\ k = 1, 2, m = 1, 2$ are drawn form a conventional QAM constellation $\mathcal{A}_{km}$, and $n_{11} = n_{22} = n, n_{12} = n_{21} = m$. Hence, the proposed IC scheme takes the following form

$$\mathbf{X}_{k1} = \begin{bmatrix} \mathbf{C}\left(\mathbf{s}'_{k1}, N_t\right) \\ \underset{(n_{k2}+\tau)\times N_t}{\mathbf{0}} \end{bmatrix}, \mathbf{X}_{k2} = \begin{bmatrix} \underset{(n_{k1}+\tau)\times N_t}{\mathbf{0}} \\ \mathbf{C}\left(\mathbf{s}'_{k2}, N_t\right) \end{bmatrix} \tag{8}$$

where

$$\mathbf{C}\left(\mathbf{s}'_{kj}, N_t\right) = \begin{bmatrix} s'_{kj,1} & 0 & \dots & 0 & 0 \\ s'_{kj,2} & s'_{kj,2} & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ s'_{kj,N_t-1} & s'_{kj,N_t-1} & \dots & s'_{kj,N_t-1} & 0 \\ s'_{kj,N_t} & s'_{kj,N_t} & \dots & s'_{kj,N_t} & s'_{kj,N_t} \\ \vdots & \vdots & \dots & \dots & \vdots \\ s'_{kj,n_{kj}} & s'_{kj,n_{kj}} & \dots & s'_{kj,n_{kj}} & s'_{kj,n_{kj}} \\ 0 & s'_{kj,1} & \dots & s'_{kj,1} & s'_{kj,1} \\ 0 & 0 & \ddots & s'_{kj,2} & s'_{kj,2} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & s'_{kj,N_t-1} \end{bmatrix},$$

and τ is given by

$$\tau = \max\left\{\max\left\{0, N_t - n\right\}, \max\left\{0, N_t - m\right\}\right\}.$$

For the proposed IC scheme, the rate per source is equal to $\frac{n+m}{n+m+N_t+\tau-1}$, which approaches unity for $n + m \gg N_t$.

Lemma 2. *The IC scheme for the* $\left(N_t^2, N_r^2\right)$ *X channel in* (8) *enables separate decoding for* $\mathbf{X}_1$ *and* $\mathbf{X}_2$ *while achieving the full-diversity.*

Proof: Without loss of generality, we only consider the decoding of $\mathbf{X}_1$, since $\mathbf{X}_2$ is decoded similarly. According to Theorem 3, the full-diversity condition can be checked for $\mathbf{X}_{11}$ and $\mathbf{X}_{21}$ separately. It can be easily noticed from (8) that $\mathbf{X}_{11}$ and $\mathbf{X}_{21}$ have similar structure, thus in what follows we will only consider $\mathbf{X}_{11}$. For the sake of simplicity, we will focus on the case where each destination is equipped with a single receive antenna; the case of arbitrary number of receive antennas follows similarly and is therefore omitted.

Thanks to the structure of the proposed IC scheme (8), the first N_t components of $\Delta\mathbf{X}_{11}\mathbf{h}_{11}$ (which we denote by $\mathbf{v}$) are orthogonal to the subspace spanned by the interfering messages $\boldsymbol{\mathcal{H}}_{\overline{11}}$ irrespectively of the X channel realization. The projection matrix $\mathbf{P}_j$ projects the received signal into the subspace orthogonal to the one spanned by the interfering messages, thus the first N_t components of $\mathbf{P}_1\Delta\mathbf{X}_{11}\mathbf{h}_{11}$ and $\Delta\mathbf{X}_{11}\mathbf{h}_{11}$ are identical. Consequently, one has

$$\|\mathbf{P}_1\Delta\mathbf{X}_{11}\mathbf{h}_{11}\|^2 \geq \|\mathbf{v}\|^2, \forall\ \mathbf{h}_{11},\ \mathbf{h}_{21} \in \mathbb{C}^{N_t\times 1}. \tag{9}$$

According to (8), one has

$$\begin{aligned} \|\mathbf{v}\|^2 &= \sum_{i=1}^{N_t}\Big|\sum_{j=1}^{i}\left[\mathbf{h}_{11}\right]_j\Big|^2 |\Delta s'_{11,i}|^2 \\ &\geq \delta_{\Delta\mathbf{X}_{11}} \sum_{i=1}^{N_t}\Big|\sum_{j=1}^{i}\left[\mathbf{h}_{11}\right]_j\Big|^2 \\ &\geq \underbrace{\min_{\Delta\mathbf{X}_{11}\in\Delta\mathcal{C}_{11}}\left\{\delta_{\Delta\mathbf{X}_{11}}\right\}}_{\delta_{\min}} \sum_{i=1}^{N_t}\Big|\sum_{j=1}^{i}\left[\mathbf{h}_{11}\right]_j\Big|^2 \end{aligned}$$

where $\left[\mathbf{h}_{11}\right]_j$ denotes the j-th entry of $\mathbf{h}_{11}$, and $\delta_{\Delta\mathbf{X}_{11}} \triangleq \min\left\{|\Delta s'_{11,i}|^2 : i = 1, \dots, N_t\right\}$. Recall that the full-diversity algebraic rotations [13] are designed to maximize the minimum product distance $d_{p,\min}$ defined as:

$$d_{p,\min} \triangleq \min_{\Delta\mathbf{s}'=\mathbf{U}\Delta\mathbf{s}|\Delta\mathbf{s}\in\mathbb{Z}[i]^n\setminus\{\mathbf{0}\}} \left\{\prod_{i=1}^{n}|\Delta s'_i|\right\}.$$

Hence, $\delta_{\Delta\mathbf{X}_{11}} > 0$, $\forall\ \Delta\mathbf{X}_{11} \in \Delta\mathcal{C}_{11}\setminus\{\mathbf{0}\}$. Note that $\Delta\mathcal{C}_{11}$ is finite, thus $\delta_{\min}$ is strictly positive. Moreover, it can be easily verified that

$$\sum_{i=1}^{N_t}\Big|\sum_{j=1}^{i}\left[\mathbf{h}_{11}\right]_j\Big|^2 = \|\mathbf{M}\mathbf{h}_{11}\|^2$$

where $\mathbf{M}$ is a lower triangular matrix with $[\mathbf{M}]_{ij}$ is set to unity $\forall\ i \geq j$. The matrix $\mathbf{M}$ is of full colum rank, thus applying Lemma 1, the inequality (9) reduces to

$$\|\mathbf{P}_1\Delta\mathbf{X}_{11}\mathbf{h}_{11}\|^2 \geq c_{\mathbf{M}}\|\mathbf{h}_{11}\|^2$$

which concludes the proof. ■

In the following examples, the symbols intended for the first destination are lightly shaded while the symbols intended for the second destination are darkly shaded.

Example 1. *Consider the following rate*-$4/7$ *IC scheme* $(n_{11} = n_{12} = n_{21} = n_{22} = 2)$ *for the* $\left(3^2, N_r^2\right)$ *X channel:*

$$\begin{aligned} \mathbf{X}_{11}^{\mathsf{T}} + \mathbf{X}_{12}^{\mathsf{T}} &= \begin{bmatrix} s'_{11,1} & s'_{11,2} & 0 & s'_{12,1} & s'_{12,2} & 0 & 0 \\ 0 & s'_{11,2} & s'_{11,1} & 0 & s'_{12,2} & s'_{12,1} & 0 \\ 0 & 0 & s'_{11,1} & s'_{11,2} & 0 & s'_{12,1} & s'_{12,2} \end{bmatrix}, \\ \mathbf{X}_{21}^{\mathsf{T}} + \mathbf{X}_{22}^{\mathsf{T}} &= \begin{bmatrix} s'_{21,1} & s'_{21,2} & 0 & s'_{22,1} & s'_{22,2} & 0 & 0 \\ 0 & s'_{21,2} & s'_{21,1} & 0 & s'_{22,2} & s'_{22,1} & 0 \\ 0 & 0 & s'_{21,1} & s'_{21,2} & 0 & s'_{22,1} & s'_{22,2} \end{bmatrix}, \end{aligned} \tag{10}$$

It is worth noting that the proof of Lemma 2 relies on the fact that we have N_t entries of $\Delta\mathbf{X}_{kj}\mathbf{h}_{kj}, \forall\ k, j \in 1, 2$, that are orthogonal to the subspace spanned by the interfering messages $\boldsymbol{\mathcal{H}}_{\overline{jj}}$. This observation may be efficiently used to enhance the rate of the proposed IC scheme as illustrated in the following example.

Example 2. *Consider the following rate*-$5/7$ *IC scheme* $(n_{11} = n_{21} = 3, n_{12} = n_{22} = 2)$ *for the* $\left(3^2, N_r^2\right)$ *X channel:*

$$\begin{aligned} \mathbf{X}_{11}^{\mathsf{T}} + \mathbf{X}_{12}^{\mathsf{T}} &= \begin{bmatrix} s'_{11,1} & s'_{11,2} & s'_{11,3} & s'_{12,1} & s'_{12,2} & 0 & 0 \\ 0 & s'_{11,2} & s'_{11,3} & s'_{11,1} & s'_{12,2} & s'_{12,1} & 0 \\ 0 & 0 & s'_{11,3} & s'_{11,1} & 0 & s'_{12,1} & s'_{12,2} \end{bmatrix}, \\ \mathbf{X}_{21}^{\mathsf{T}} + \mathbf{X}_{22}^{\mathsf{T}} &= \begin{bmatrix} s'_{21,1} & s'_{21,2} & s'_{21,3} & s'_{22,1} & s'_{22,2} & 0 \\ 0 & s'_{21,2} & s'_{21,3} & s'_{21,1} & s'_{22,2} & s'_{22,1} & 0 \\ 0 & 0 & s'_{21,3} & s'_{21,1} & 0 & s'_{22,1} & s'_{22,2} \end{bmatrix}, \end{aligned} \tag{11}$$

V. Simulations results

our theoretical claims are verified via numerical simulations. In the first part, the proposed two-user IC schemes for the MIMO X channel are shown to achieve the full diversity gain offered by the channel configuration. For this purpose, the CER performance of the proposed two-user IC schemes are compared to the reference diversity slopes in two MIMO X channel configurations, namely $\left(2^2, 1^2\right)$ and $\left(3^2, 1^2\right)$. The CER performance for our rate-$4/5$, $6/7$ IC schemes in the MIMO X channel $\left(2^2, 1^2\right)$ configuration is depicted in Fig. 2, while the CER performance for our rate-$5/7$ IC scheme in Eq. (11) for the MIMO X channel $\left(3^2, 1^2\right)$ configuration is depicted in Fig. 3. As can be easily verified, our proposed two-user IC schemes achieve the full-diversity gain as predicted by Lemma 2.

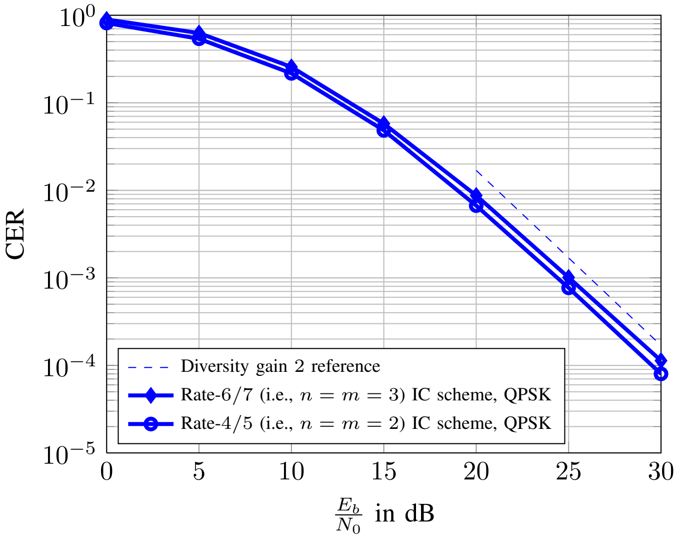

Fig. 2: Codeword error rate performance for the $(2^2, 1^2)$ MIMO X channel.

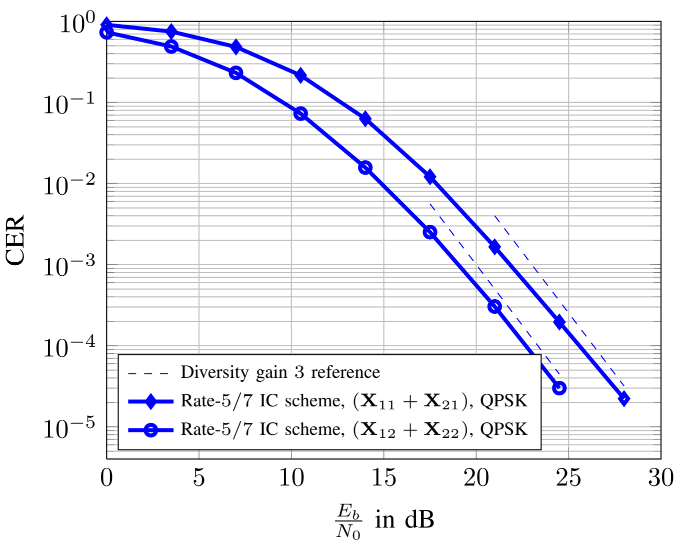

Fig. 3: Codeword error rate performance for the $(3^2, 1^2)$ MIMO X channel.

In the second part, we compare the CER performance of our rate-$5/7$ two-user IC scheme in Eq. (11) and the rate-$4/7$ interference cancellation scheme [5] in the $(3^2, 2^2)$ MIMO X channel. According to Eq. (11), the message intended for the first destination (i.e., $\mathbf{X}_{11} + \mathbf{X}_{21}$) encodes 6 complex symbols while the message for the second destination (i.e., $\mathbf{X}_{12} + \mathbf{X}_{22}$) encodes 4 complex symbols. The messages intended for the first and the second destination in [5] encode 4 complex symbols each. For fairness of comparison, the spectral efficiency at each destination is fixed for both schemes, hence the underlying constellation for the rate-$4/7$ L. Shi *et al.* IC scheme is 64-QAM, while for our IC scheme in Eq. (11), the underlying constellation for the messages intended for the first and second destinations are 16-QAM, and 64-QAM respectively. One can easily notice that our IC scheme provides a gain of 2 dB in the case it has higher rate than the IC scheme in [5], while losing only about 0.5 dB in the case it has equal rate than the IC scheme in [5].

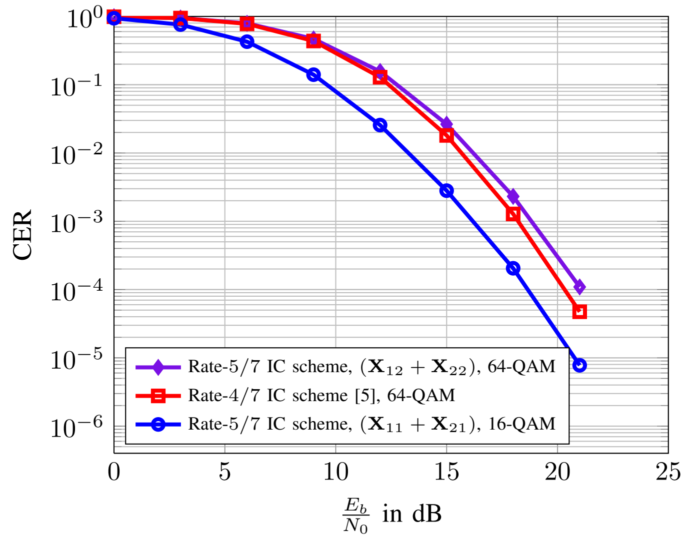

Fig. 4: Codeword error rate performance for the $(3^2, 2^2)$ MIMO X channel.

Worst-Case Decoding Complexity Analysis

The worst-case decoding complexity order is a widely recognized measure of decoding complexity. It is defined as the minimum number a decoder needs to evaluate the Euclidean distance to optimally estimate the sent codeword [14]. According to this definition, one can easily verify that both schemes have the same worst-case decoding complexity order. For instance, if we consider our rate-$4/7$ and the L. Shi *et al.* rate-$4/7$ scheme, the decoding complexity is of $\mathcal{O}\left(q^4\right)$ where q denotes the size of the underlying constellation.

VI. Conclusion

In this paper we focused on the $\left(N_t^2, N_r^2\right)$ X channel and provided sufficient conditions for a large family of STBCs to achieve the full-diversity gain offered by the network under PICGD in the case of blind sources. We provided a systematic construction of IC schemes for the configuration of interest that enables each destination to decode only the desired messages through PICGD while retaining the full-diversity gain. We compared the proposed scheme to the recently proposed MIMO $\left(N_t^2, N_r^2\right)$ X channel IC scheme in terms of CER and worst-case decoding complexity order. Our scheme provided significant gains in the cases it offers higher rates while losing slightly in the case of equal rates. It was observed that both schemes have the same worst-case decoding complexity order.

Appendix

Proof: A MIMO system is said to achieve a diversity gain d if the probability of error P_e can be upper-bounded in the high SNR regime as

$$\mathrm{P}_e \lesssim \beta \ \mathrm{SNR}^{-d}$$

where β is a positive constant. Now consider without loss of generality the decoding at the j-th destination. The pairwise error probability (PEP) of the $\left(N_t^2, N_r^2\right)$ X channel may be decomposed as follows

$$\begin{aligned}\text{PEP} =&\mathbb{P}\left[\Delta\widetilde{\mathbf{X}}_{1j} \neq \mathbf{0}, \Delta\widetilde{\mathbf{X}}_{2j} = \mathbf{0}\right] + \mathbb{P}\left[\Delta\widetilde{\mathbf{X}}_{1j} = \mathbf{0}, \Delta\widetilde{\mathbf{X}}_{2j} \neq \mathbf{0}\right] \\ &+\mathbb{P}\left[\Delta\widetilde{\mathbf{X}}_{1j} \neq \mathbf{0}, \Delta\widetilde{\mathbf{X}}_{2j} \neq \mathbf{0}\right].\end{aligned} \tag{12}$$

From (6), the first term in (12) can be expressed as [15]

$$\mathbb{P}\left[\Delta\widetilde{\mathbf{X}}_{1j} \neq \mathbf{0}, \Delta\widetilde{\mathbf{X}}_{2j} = \mathbf{0}\right] = \mathbb{E}_{\mathbf{h}_{2j}}\left[\mathbb{E}_{\mathbf{h}_{1j}|\mathbf{h}_{2j}}\left[Q\left(\sqrt{\frac{\text{SNR}\|\mathbf{P}_j\Delta\widetilde{\mathbf{X}}_{1j}\mathbf{h}_{1j}\|^2}{2}}\right)\right]\right].$$

Assuming that the design criteria in Theorem 3 are satisfied, the above can be upper-bounded as

$$\begin{aligned}&\leq \mathbb{E}_{\mathbf{h}_{2j}}\left[\mathbb{E}_{\mathbf{h}_{1j}|\mathbf{h}_{2j}}\left[Q\left(\sqrt{\frac{\alpha\ \text{SNR}\|\mathbf{h}_{1j}\|^2}{2}}\right)\right]\right] \\ &\leq \mathbb{E}_{\mathbf{h}_{2j}}\left[\mathbb{E}_{\mathbf{h}_{1j}|\mathbf{h}_{2j}}\left[\exp\left(-\frac{\alpha\ \text{SNR}\|\mathbf{h}_{1j}\|^2}{4}\right)\right]\right]\end{aligned}$$

where SNR denotes the energy per symbol to noise ratio. But we have

$$\mathbb{E}_{\mathbf{h}_{1j}|\mathbf{h}_{2j}}\left[\exp\left(-\frac{\alpha\ \text{SNR}\|\mathbf{h}_{1j}\|^2}{4}\right)\right] = \left(\frac{1}{1+\alpha\frac{\text{SNR}}{4}}\right)^{N_tN_r}.$$

Therefore, in the high SNR regime, one has

$$\mathbb{P}\left[\widetilde{\mathbf{X}}_{1j} \neq \mathbf{0}, \Delta\widetilde{\mathbf{X}}_{2j} = \mathbf{0}\right] \lesssim \underbrace{\left(\frac{4}{\alpha}\right)^{N_tN_r}}_{\beta}\text{SNR}^{-N_tN_r}$$

where β is finite as $\alpha > 0$. Proceeding similarly, one obtains

$$\mathbb{P}\left[\Delta\widetilde{\mathbf{X}}_{1j} = \mathbf{0}, \Delta\widetilde{\mathbf{X}}_{2j} \neq \mathbf{0}\right] \lesssim \gamma\ \text{SNR}^{-N_tN_r}$$

where $\gamma < \infty$. The remaining term in (12) can be evaluated as

$$\mathbb{P}\left[\Delta\widetilde{\mathbf{X}}_{1j} \neq \mathbf{0}, \Delta\widetilde{\mathbf{X}}_{2j} \neq \mathbf{0}\right] = \mathbb{E}_{\mathbf{h}_{1j},\mathbf{h}_{2j}}\left[Q\left(\sqrt{\frac{\text{SNR}\left\|\mathbf{P}_j\left(\Delta\widetilde{\mathbf{X}}_{1j}\mathbf{h}_{1j} + \Delta\widetilde{\mathbf{X}}_{2j}\mathbf{h}_{2j}\right)\right\|^2}{2}}\right)\right].$$

Now, let $\mathbf{h}'_{2j} = \Delta\widetilde{\mathbf{X}}_{1j}^{-1}\Delta\widetilde{\mathbf{X}}_{2j}\mathbf{h}_{2j}$. Therefore,

$$\begin{aligned}&\mathbb{E}_{\mathbf{h}_{1j}|\mathbf{h}_{2j}}\left[Q\left(\sqrt{\frac{\text{SNR}\left\|\mathbf{P}_j\Delta\widetilde{\mathbf{X}}_{1j}\left(\mathbf{h}_{1j}+\mathbf{h}'_{2j}\right)\right\|^2}{2}}\right)\right] \\ &\overset{(a)}{\leq}\mathbb{E}_{\mathbf{h}_{1j}|\mathbf{h}_{2j}}\left[Q\left(\sqrt{\frac{\alpha\ \text{SNR}\|\mathbf{h}_{1j}+\mathbf{h}'_{2j}\|^2}{2}}\right)\right] \\ &\leq\mathbb{E}_{\mathbf{h}_{1j}|\mathbf{h}'_{2j}}\left[\exp\left(-\xi\|\mathbf{h}_{1j}+\mathbf{h}'_{2j}\|^2\right)\right]\end{aligned}$$

where (a) follows from the assumption that the full-diversity criteria in Theorem 3 are satisfied, and $\xi = \alpha\frac{\text{SNR}}{4}$. As the entries of $\mathbf{h}_{1j}$ are independent, one gets

$$\begin{aligned}&\mathbb{E}_{\mathbf{h}_{1j}|\mathbf{h}'_{2j}}\left[\exp\left(-\xi\|\mathbf{h}_{1j}+\mathbf{h}'_{2j}\|^2\right)\right] = \\ &\prod_{i=1}^{N_tN_r}\mathbb{E}_{[\mathbf{h}_{1j}]_i|[\mathbf{h}'_{2j}]_i}\left[\exp\left(-\xi|\left[\mathbf{h}_{1j}\right]_i + \left[\mathbf{h}'_{2j}\right]_i|^2\right)\right].\end{aligned} \tag{13}$$

Recall that the probability density function of a noncentral chi-square random variable $X = X_1^2+X_2^2$, where, $X_i \sim \mathcal{N}\left(\mu_i, \sigma\right)$ is given in [16] by

$$P_X\left(x\right) = \frac{1}{2\sigma^2}\exp\left(-\frac{x+a^2}{2\sigma^2}\right)I_o\left(\sqrt{\frac{a^2x}{\sigma^4}}\right); x \geq 0$$

where $I_o\left(y\right)$ denotes the zeroth-order modified Bessel function of the first kind and $a^2 = \mu_1^2 + \mu_2^2$. Consequently, it can be verified that

$$\mathbb{E}_X\left[\exp\left(-bX\right)\right] = \frac{\exp\left(-\frac{a^2b}{1+2b\sigma^2}\right)}{1+2b\sigma^2}.$$

It is then straightforward to verify that (13) reduces to

$$\prod_{i=1}^{N_tN_r}\frac{\exp\left(-\frac{\xi|\mathbf{h}'_{2j}(i)|^2}{1+\xi}\right)}{1+\xi} = \left(\frac{1}{1+\xi}\right)^{N_tN_r}\exp\left(-\frac{\xi\|\mathbf{h}'_{2j}\|^2}{1+\xi}\right). \tag{14}$$

Note that for $\Delta\widetilde{\mathbf{X}}_{1j}^{-1} \neq \mathbf{0},\ \Delta\widetilde{\mathbf{X}}_{2j} \neq \mathbf{0}$, one has

$$\|\mathbf{h}'_{2j}\| = \|\Delta\widetilde{\mathbf{X}}_{1j}^{-1}\Delta\widetilde{\mathbf{X}}_{2j}\mathbf{h}_{2j}\| > 0,\ \forall\ \mathbf{h}_{2j} \neq \mathbf{0}$$

which can be lower-bounded according to Lemma 1 as

$$\|\mathbf{h}'_{2j}\|^2 \geq c_{\Delta\widetilde{\mathbf{X}}_{1j}^{-1}\Delta\widetilde{\mathbf{X}}_{2j}}\|\mathbf{h}_{2j}\|^2 \geq \underbrace{\min_{\substack{\Delta\widetilde{\mathbf{X}}_{1j}\in\Delta\widetilde{\mathcal{C}}_{1j} \\ \Delta\widetilde{\mathbf{X}}_{2j}\in\Delta\widetilde{\mathcal{C}}_{2j}}} c_{\Delta\widetilde{\mathbf{X}}_{1j}^{-1}\Delta\widetilde{\mathbf{X}}_{2j}}}_{c_{\min}}\|\mathbf{h}_{2j}\|^2$$

or equivalently

$$\|\mathbf{h}'_{2j}\|^2 \geq c_{\min}\|\mathbf{h}_{2j}\|^2.$$

Hence, (14) can be upper-bounded in the high SNR regime as

$$\begin{aligned}&\left(\frac{1}{1+\xi}\right)^{N_tN_r}\exp\left(-\frac{\xi\|\mathbf{h}'_{2j}\|^2}{1+\xi}\right) \lesssim \\ &\qquad\left(\frac{4}{\alpha}\right)^{N_tN_t}\exp\left(-c_{\min}\|\mathbf{h}_{2j}\|^2\right)\text{SNR}^{-N_tN_r}.\end{aligned}$$

Averaging over the distribution of $\mathbf{h}_{2j}$, one obtains

$$\mathbb{P}\left[\Delta\widetilde{\mathbf{X}}_{1j} \neq \mathbf{0}, \Delta\widetilde{\mathbf{X}}_{2j} \neq \mathbf{0}\right] \lesssim \left(\frac{4}{\alpha\left(1+c_{\min}\right)}\right)^{N_tN_r}\text{SNR}^{-N_tN_r}$$

which concludes the proof. ∎

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