Performance Analysis of RIS-Aided THz Wireless Systems over $\alpha - \mu$ Fading: An Approximate Closed-Form Approach

Ngoc Phuc Le, Member, IEEE, and Mohamed-Slim Alouini, Fellow, IEEE

Abstract—In this paper, we study a reconfigurable intelligent surfaces (RIS)-assisted Terahertz (THz) wireless systems with hardware impairments, where $\alpha - \mu$ small-scale fading is considered for THz links in accordance with a recent measurement campaign. Firstly, we propose an accurate closed-form approximation of a weighted sum of cascaded non-identical $\alpha - \mu$ variates based on the Gauss-Laguerre quadrature and a moment-matching method. This approximate approach facilitates analysis of the RIS-THz system over $\alpha - \mu$ fading channels. To demonstrate, we derived closed-form expressions of the outage probability (OP), the ergodic capacity (EC), and the energy-efficiency (EE) of the system based on the proposed approximation. Secondly, we approximately characterize the end-to-end channel of the RIS-THz system when the number of RIS elements is large in scenarios with or without the presence of phase-shift errors. Based on this statistical characterization, the closed-form expressions of the OP, the EC, and the EE of the large-size RIS-THz system are obtained. Furthermore, we devise a low-complexity algorithm that jointly optimizes the transmit power and RIS element activation (i.e., ON/OFF RIS) to maximize the EE in the RIS-THz systems. This algorithm adopts an iterative dynamic programming approach for a maximum subarray problem (i.e., Kadane’s algorithm). Finally, simulations are provided to validate the accuracy of the theoretical analysis as well as demonstrate the efficacy of the devised algorithm.

Index Terms—Reconfigurable intelligent surface, Terahertz communications, $\alpha - \mu$ fading, outage probability, ergodic capacity, energy efficiency.

I. INTRODUCTION

Terahertz communications has been identified as one of the key technologies that meets the requirements for 6G for 2030 and beyond [1]. With the huge available bandwidth, THz networks can accommodate a massive number of connected users/devices as well as feature high data-rate in the order of terabit-per-second (Tbit/s). Therefore, it is expected to support emerging wireless applications, such as virtual/ augmented reality, nano communications, and ultra-large capacity data backhaul. Unfortunately, transmissions at THz bands suffer from some major issues. In particular, high propagation loss (i.e., due to high spreading loss, high molecular absorption loss, high reflection and scattering losses) results in a very short transmission distance. Also, impacts of blockages and misalignment become more severe in THz systems.

Several techniques and approaches can be considered to tackle the above challenges. In particular, multiple antenna schemes, i.e., massive multi-input multi-output (massive MIMO), are able to achieve large beamforming gains for compensating propagation loss [2]. Another solution is amplify-and-forward relaying [3]. However, the deployment of massive MIMO might complicate the signal processing and scale-up hardware costs as well as power consumption since massive associated radio frequency (RF) chains are required. Also, amplify-and-forward relaying systems usually operate with high power consumption. While active relays in half-duplex suffers from low spectral efficiency, relays in full-duplex demand efficient self-interference cancellation techniques.

Recently, reconfigurable intelligent surfaces-RIS (also known as intelligent reflecting surface-IRS) has been identified as a promising solution to alleviate the short-range and blockage-sensitive bottlenecks in THz communication systems [4]. RIS typically consists of an array of passive and low-cost reflectors that are able to reflect or absorb the impinging electromagnetic waves. In RIS-assisted systems, the phases and/or the amplitudes of transmission signals can be adjusted via a collaboration among elements through a microcontroller to enhance the overall signal-to-noise ratio (SNR) at a receiver. As a result, the key system performance metrics such as outage probability and transmission capacity can be improved. Compared to the above techniques, RIS owns some distinct advantages. First, the reflecting elements of RIS are passive and no active RF chains are required. Thus, RIS has low-hardware complexity (i.e., low-cost deployment), as well as low-power consumption (i.e., energy-efficient solution). Second, the deployment of RIS can create virtual line-of-sight (LoS) links to deal with blockage issues as well as extend transmission coverage. Note that RIS is light weight with arbitrary shapes, which can be conveniently installed on environment objects. Additionally, RIS is beneficial in terms of simple processing and coding, well-suited for full-duplex mode operation, as well as great flexibility and compatibility with existing wireless systems [5].

A. Related Research on RIS-THz Wireless Systems

Many researchers have recently considered RIS for THz wireless systems. Topics range from hardware design of RIS for THz bands, modeling and characterizing channels, to conducting performance analysis and beamforming optimization [6]. Several system models have been proposed, such as RIS-THz systems with multiple RISs, multi-input multi-output (MIMO), multiuser scenarios, or non-terrestrial networks. Some RIS-THz works also took into account practical constraints (e.g., transceiver’s impairments, RIS phase-shift errors, or beam misalignment) as well as emerging services. In what follows, we highlight related papers on RIS-THz.

Multiple RISs-Assisted THz: The idea of using multiple RIS panels to improve transmission performance and coverage of THz systems were reported in [7]-[10]. In general, multiple RISs can be deployed as either a cascaded (i.e., multi-hops) RIS or distributed (i.e., one-hop) RIS configuration. For multi-hop RIS case, the authors in [7] derived the outage probability...
expression of multi-RIS-THz systems subject to turbulence and stochastic beam misalignment. Efficient phase-shift optimization to maximize the received signal-to-noise ratio (SNR) in multi-hop RIS-THz with highly directional antennas and spatially correlated channels was considered in [8]. Also, a joint design of digital beamforming at the base station and analog beamforming at RISs to improve the ergodic sum-rate in multi-hop RIS-THz based on deep reinforcement learning (DRL) was proposed [9]. For distributed RIS-THz case, the authors in [10] proposed a distributed RIS framework associated with a 3D ray-tracing method for realistic indoor environments with the existence of human blockers to improve transmission coverage, received SNR, as well as energy efficiency.

**RIS-THz with MIMO/Multiuser:** Besides proposals of multiple RISs, some studies have combined RIS-THz with MIMO to take advantages of these technologies. Two main challenges when realizing this combination are channel estimation (i.e., due to passive nature of RIS elements for receiving/sending pilot signals) and low-cost beamforming (i.e., due to large-scale usages of radio-frequency chains in MIMO). In [11], a low-complexity scheme for channel estimation is proposed based on an iterative atom pruning based subspace pursuit (IAP-SP) scheme. Moreover, the authors developed a deep neural network-based phase-shift search to maximize the transmission rate. A deployment of massive MIMO in RIS-THz systems for both single and multiuser scenarios was studied in [12]. In particular, the authors proposed a joint beam training and hybrid beamforming scheme that can achieve similar performance as the optimal fully digital beamforming. Going further with multiuser RIS-THz, the authors in [13] proposed a block coordinate searching algorithm to jointly optimize the coordinates and phase-shifts of RISs, THz sub-band allocation, and transmit power such that the sum-rate of all users is maximized. In addition, coverage analysis (i.e., derivation of the closed-form expression of the coverage probability) of RIS-THz systems was carried out in [14], where users are assumed located randomly within a circular cluster.

**RIS-THz for Non-Terrestrial Networks:** Following increasing interests on non-terrestrial networks (NTNs), some researchers have examined RIS-THz for NTNs [15]-[17]. In particular, the authors in [15] studied RIS-THz for intersatellite links (ISLs) in low-Earth orbit (LEO) networks. They evaluated the error-rate performance of RIS-assisted ISLs in the presence of misalignment fading. In [16], RISs are used to enhance unmanned aerial vehicle (UAV)-based THz communication systems. A joint optimization of UAV’s trajectory, RIS phase-shifts, sub-band allocation, and power control was formulated and solved to maximize the minimum achievable rate among ground users. In addition, a problem of proactive handoff and beam selection in UAV-based RIS-THz networks was considered in [17]. Here, the authors proposed an efficient deep learning solution that potentially extends the coverage of UAVs and enhances the reliability of the networks, which is vital for time-critical applications. A NTN network that consists of LEO, an RIS-assisted high-altitude platform (HAP), and UAVs operating at THz bands was proposed in [18]. This work focuses on deriving an approximate expression of the ergodic secrecy rate as well as optimizing the RIS phase-shifts to maximize the lower bound of the secrecy rate.

**RIS-THz for Emerging Services:** In addition to RIS-THz air-interface models, exploiting advantages of RIS-THz for new services in B5G/6G networks has attracted attention. In [19], RIS-THz systems were considered for the quality-of-experience (QoE) requirements of wireless virtual reality (VR) in an indoor scenario. An efficient machine learning method was proposed, which performs predictions of locations and LoS/non-LoS statuses of VR users in the uplink and RIS phase-shift optimization under latency constraints in the downlink. Meanwhile, in [20], RIS-THz was used to support the coexistence of ultra-reliable low-latency communication (URLLC) and enhanced mobile broadband (eMBB) services. This work proposed a supervised learning-based resource management framework to jointly optimize the transmit power, the RIS phase-shifts, and resource-block allocation subject to the URLLC and eMBB services requirements. In addition, a convergence between communications and localization services in RIS-assisted mmWave/THz B5G was investigated in [21]. This requires a joint design for efficient resource usages and flexible trade-off given that one will rely on the other for optimized performance.

**Small-Scale Fading α−μ in THz Bands:** It is worth noting that small-scale fading was ignored in the aforementioned studies. In [22], the fluctuating two-ray distribution was used to model small-scale fading in RIS-THz systems. Based on this, analytical expressions for the outage probability and ergodic capacity were derived. A recent measurement campaign reported in [23] shows that α−μ distribution offers an excellent fitting to practical measurements for THz links. Therefore, when it comes to performance assessment of THz systems, it is essential to include α−μ small-scale fading modeling, rather than other types of fading models, e.g., Nakagami-\(m\) or Rayleigh\(^1\). To this end, a RIS-THz system over α−μ fading was analyzed in terms of outage probability, error-rate and capacity in [25]. However, the exact analytical expressions of these metrics heavily depend the multivariate Fox’s H function that is very complicated by its definition [26]. Moreover, this function is currently not a built-in function in software packages, e.g., Matlab or Mathematica. In addition, energy-efficiency performance, large-size RIS, as well as impacts of phase-shifts errors were not considered in [25].

**B. Contributions**

Motivated by the discussions above, we proposed in this paper an accurate closed-form approximation approach to analyze performance of a RIS-assisted THz wireless system with hardware impairments over α−μ fading channels. The main contributions of this work are summarized below.

1) An accurate closed-form approximation of the weighted sum of cascaded non-identical α−μ variates is proposed based on the Gauss-Laguerre quadrature and moment-matching. This novel result facilitates statistical characterization of the end-to-end (e2e) channels in the RIS-THz system with optimal RIS phase-shifts.

\(^1\)Note that the α−μ distribution is a general fading distribution that includes the Gamma, exponential, Weibull, Nakagami-\(m\), Rayleigh, and one-sided Gaussian distributions as special cases [24].
2) For the RIS-THz system with a large number of RIS elements, the e2e channels are approximately characterized by using the central limit theorem (CLT) under different cases, including optimal phase-shifts, phase estimation errors with von Mises distribution and phase quantization errors following uniform distribution.

3) Closed-form approximate and asymptotic expressions of the outage probability, the ergodic capacity, and the energy efficiency in the RIS-THz wireless systems with hardware impairments are derived. These expressions are obtained based on the proposed approximate approaches, which do not contain the complicated multivariate Fox’s H function.

4) The transmit power and activation of RIS elements are jointly optimized for maximal energy efficiency in the RIS-THz systems. An algorithm is devised that can achieve similar performance as exhaustive search while requiring much lower computational complexity.

5) Impacts of hardware impairments, a number of RIS elements, as well as types of phase-shift errors on the performance of the RIS-THz system are evaluated.

C. Organization of the Paper

The rest of this paper is organized as follows. In Section II, we describe the RIS-aided THz system model with hardware impairments over $\alpha - \mu$ fading. Section III first performs the approximation of the weighted sum of cascaded non-identical $\alpha - \mu$ variates, and then derives the OP, the EC, and the EE expressions. Meanwhile, approximate analysis in the system with large number of RIS elements is carried out in Section IV. A proposed algorithm to maximize the EE in the large-size RIS-THz system with ON/OFF RISs is provided in Section V. Simulation results and discussions are presented in Section VI. Finally, Section VII concludes the paper.

II. RIS-ASSISTED THz WIRELESS SYSTEM MODEL

A. System Model

We consider a RIS-aided THz system model with hardware impairments that consists of one base station (BS), one passive RIS of $M$ elements, and one user. The direct link between the BS and the user is assumed unavailable due to severe blockage, which is similar to several works, e.g., [7], [11], [13]-[15], [17]-[19], [25]. The received signal at the user can be expressed as (cf. [7], [25], [27]-[28])

$$r = \left( \sum_{m=1}^{M} h_{1,m} e^{j\theta_m} h_{2,m} \right) (\sqrt{P_0} s + \phi) + n,$$  

(1)

where $s$ is the transmit signal with $\mathbb{E}\{|s|^2\} = 1$, $P_0$ is the transmit power, $h_{1,m}$ denotes the link from the BS to the $m^{th}$ RIS element, $h_{2,m}$ denotes the link from the $m^{th}$ RIS element to the user, $\theta_m$ is the phase-shift of the $m^{th}$ RIS element, $\phi$ is the aggregated distortion noise due to non-ideal hardware at the transceiver, and $n$ is the additive white Gaussian noise (AWGN) with zero-mean and variance of $N_0$, i.e., $n \sim \mathcal{CN}(0,N_0)$. Note that $\phi$ can be modeled as $\phi \sim \mathcal{CN}(0,\kappa^2P_0)$, where $\kappa$ denotes the level of residual hardware impairments (RHI) [27]-[29].

The channel coefficient at THz bands can be expressed as

$$h = \bar{h}h_1,$$  

(2)

where $\bar{h}$ represents the path gain and $h_1$ denotes the small-scale fading coefficient (i.e., $h_{1,m} = \bar{h}_{1,m}h_{1,m}$ and $h_{2,m} = \bar{h}_{2,m}h_{2,m}$). Following a measurement report in [23], we model the small-scale fading $h$ as an $\alpha - \mu$ random variable. Also, the path gain $h$ consists of a propagation gain and molecular absorption gain, which is expressed as [7]-[9], [12]-[14],

$$\bar{h} = \frac{c^2 G_t G_r}{4\pi^2 fd} e^{-\frac{\kappa}{4}(f)d},$$  

(3)

where $c$ is the speed of light, $G_t$ and $G_r$ are the antenna gains at the transmitter and the receiver sides, $f$ is the operating frequency, $d$ is the transmission distance, and $\kappa(f)$ is the molecular absorption coefficient. Note that molecular absorption causes loss to the signal since the electromagnetic energy is transformed in part into the internal energy of the molecules. Also, this absorption coefficient highly depends on the operating frequencies. Specifically, for the 275-400 GHz band, the coefficient can be evaluated from an absorption loss model developed in [30]. Meanwhile, for frequencies higher than 400GHz, it can be obtained from the high resolution transmission (HITRAN) database [31]. Further details about molecular absorption can be found in [13]-[14], [30]-[31].

Let us denote the overall e2e channel coefficient between the BS and the user by $g = \sum_{m=1}^{M} h_{1,m} e^{j\theta_m} h_{2,m}$. Then, the received signal-to-distortion-plus-noise ratio (SDNR) value can be expressed by (cf. (1))

$$\gamma = \frac{P_0|g|^2}{\kappa^2 P_0|g|^2 + N_0} = \frac{\tilde{\gamma}|g|^2}{\kappa^2\tilde{\gamma}|g|^2 + 1},$$  

(4)

where $\tilde{\gamma} \triangleq P_0/N_0$.

B. RIS Phase-Shift Cases

In this work, we consider different cases of RIS phase-shifts.

1) Optimal Phase-Shifts: Assuming that perfect channel state information (CSI) is availableootnote{Discussions about channel estimation for RIS-assisted mm-wave/sub-THz communication systems can be found in [32].}, the optimal phase-shift $\theta_m$ is given by $\theta_m = -\angle h_{1,m} - \angle h_{2,m}$. Thus, the phases of all links between the BS and user are co-phased. Then, we can express the overall link between the BS and the user as

$$|g| = \left| \sum_{m=1}^{M} h_{1,m} e^{j\theta_m} h_{2,m} \right| = \sum_{m=1}^{M} |h_{1,m}| |h_{2,m}|.$$

(5)

2) Phase Estimation Errors: In the system with continuous phase adjustment, we can model phase estimation errors by the von Mises distribution [33] or the Gaussian distribution [34]. Thus, we can express

$$|g| = \left| \sum_{m=1}^{M} h_{1,m} e^{j\theta_m} h_{2,m} \right| = \sum_{m=1}^{M} |h_{1,m}| |h_{2,m}| e^{j\varphi_m},$$

(6)
where $\varphi_m$ is the error that is assumed as either a zero-mean von Mises variable or a zero-mean Gaussian variable. Note that for the von Mises distribution, the probability density function (PDF) of $\varphi_m$ is given by [35]

$$f_{\varphi_m}(\varphi) = \frac{e^{\theta \cos(\varphi)}}{2\pi I_0(\theta)}, -\pi < \varphi < \pi,$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero and $\theta$ is a concentration parameter.

3) Phase Quantization Errors: In the system with discrete phase-shift adjustment, there are only $2^B$ distinct values of phases for each RIS element when $B$ bits are used to uniformly quantize the phase. In this case, the expression of $|g|$ is similar to (6), excepting that $\varphi_m$ is now the quantization error that is assumed following uniform distribution, i.e., $\varphi_m \sim U(-\Delta, \Delta)$, where $\Delta = \pi/2^B$ [36].

4) No Phase-Shift Estimation (i.e., ON/OFF RIS): In this case, all phase-shifts are fixed (e.g., $\varphi_m = 0, \forall m$), and the BS only determines which RIS elements are ON (i.e., activated) and which are OFF (i.e., idle status). Thus, RIS phase-shift estimation is not required and overhead signaling transmission is reduced. The overall link can be expressed as

$$|g| = \sum_{m=1}^{M} \delta_m h_{1,m} h_{2,m},$$

where $\delta_m = 1$ if the $m^{th}$ RIS element is ON, and $\delta_m = 0$ if the $m^{th}$ RIS element is OFF.

III. ANALYSIS OF RIS-TTH WIRELESS SYSTEMS WITH OPTIMAL PHASE-SHIFTS

A. Closed-Form Approximation of Weighted Sum of Cascaded $\alpha - \mu$ Variates

In this section, we characterize the statistical distribution of the weighted sum of cascaded non-identical $\alpha - \mu$ variates. According to [24], the PDF and cumulative distribution function (CDF) of a $\alpha - \mu$ random variable can be expressed as

$$f_X(x) = \frac{\alpha \beta^{\alpha \mu}}{\Gamma(\mu)} x^{\alpha \mu - 1} e^{-(\beta / \mu) x} \leq f(x; \alpha, \mu, \Omega),$$

and

$$F_X(x) = \frac{\gamma (\mu, (\beta / \mu) x)}{\Gamma(\mu)} \leq F(x; \alpha, \mu, \Omega),$$

where $\alpha > 0$ and $\mu > 0$ represent the non-linearity and the number of multipath components of the propagation medium, respectively, $\beta = \frac{\Gamma(\mu + \frac{1}{2})}{\Gamma(\mu)}$, $\Omega = \mathbb{E}\{X\}$ is the average of $X$, and $\Gamma(.)$ and $\gamma(.,.)$ denote the gamma function and the lower incomplete gamma function, respectively. Also, the $k^{th}$ moment of $X$ can be computed as

$$\mathbb{E}\{X^k\} = \frac{\Gamma(k - 1)(\mu + k/\alpha)}{\Gamma(k/\mu + 1/\alpha)} \mathbb{E}\{X^k\}.$$

Let $S = \sum_{m=1}^{M} \tau_m X_m Y_m$, where $\tau_m > 0$ is the weighted coefficient, and $X_m$ and $Y_m$ are independent $\alpha - \mu$ random variables with parameters of $(\alpha_1, \mu_1, \Omega_1)$ and $(\alpha_2, \mu_2, \Omega_2)$, respectively. By using the Gauss-Laguerre quadrature and moment-matching, we obtain approximate closed-form expressions for the PDF and CDF of the random variable $S$.

**Theorem 1:** The approximate closed-form expressions of the PDF and CDF of the variable $S$ are given by

$$f_S(x) \approx \sum_{n=\{n_1, n_2, \ldots, n_M\}} \Phi_n f(x; \alpha_n, \mu_n, \Omega_n),$$

and

$$F_S(x) \approx \sum_{n=\{n_1, n_2, \ldots, n_M\}} \Phi_n F(x; \alpha_n, \mu_n, \Omega_n),$$

where

$$\sum_{n=\{n_1, n_2, \ldots, n_M\}} \Phi_n = M, \quad \sum_{n=\{n_1, n_2, \ldots, n_M\}} \mu_n = \mu, \quad \sum_{n=\{n_1, n_2, \ldots, n_M\}} \Omega_n = \Omega, \quad \sum_{n=\{n_1, n_2, \ldots, n_M\}} \alpha_n = \alpha.$$
the approximation error becomes smaller with an increasing number of terms \( N \).

**Statistical distribution of the overall channel \( g \):** The overall channel \( g \) with optimal phase-shifts in (5) can be rewritten as

\[
|g| = \sum_{m=1}^{M} \tilde{h}_{1,m} \tilde{h}_{2,m} |\tilde{h}_{1,m}| |\tilde{h}_{2,m}|, \tag{15}
\]

where \( \tau_{m} = \tilde{h}_{1,m} \tilde{h}_{2,m} \), and \( \tilde{h}_{1,m} \) and \( \tilde{h}_{2,m} \) follow \( \alpha - \mu \) distribution. Thus, by using the result of *Theorem 1*, we can obtain the PDF and the CDF of \( |g| \).

**B. Analysis of Outage Probability**

The outage probability (OP) is defined as the probability that the SDNR value falls below a certain threshold \( \gamma_{th} \), i.e.,

\[
P_{out} = Pr(\gamma < \gamma_{th}). \tag{16}
\]

By using the approximation result in Section III.A, we obtain the following result.

**Theorem 2:** The approximate closed-form expression of the outage probability \( P_{out} \) is given by

\[
P_{out} \approx \sum_{n=\{n_{1}, n_{2}, \ldots, n_{M}\}} \frac{\Phi_{n}}{\Gamma(\mu_{n})} \times \gamma_{\mu_{n},n} \left( \frac{\gamma_{th}}{\gamma(1-\kappa^{2}\gamma_{th})} \right)^{\frac{\alpha_{n}}{2}}, \quad \gamma_{th} < 1/\kappa^{2}, \tag{17}
\]

where \( \gamma_{\mu_{n},n} = \Omega_{n}^{2}/\beta_{n}^{2} \). If \( \gamma_{th} \geq 1/\kappa^{2} \), we have \( P_{out} = 1 \).

**Proof:** By substituting (4) into (16), we obtain

\[
P_{out} = \begin{cases} Pr(\gamma|g|^{2} < \frac{\gamma_{th}}{\gamma(1-\kappa^{2}\gamma_{th})}) \times \frac{\gamma_{th}}{1} \quad \gamma_{th} < 1/\kappa^{2}, \\ F_{|g|^{2}}(x) = Pr(\gamma|g|^{2} < x) = F_{|g|}(\sqrt{x}), \quad \gamma_{th} \geq 1/\kappa^{2}. \end{cases} \tag{18}
\]

On the other hand, we have

\[
F_{|g|^{2}}(x) = Pr(\gamma|g|^{2} < x) = F_{|g|}(\sqrt{x}), \tag{19}
\]

where the CDF of the overall channel gain \( |g| \) can be obtained from (13). The result in (17) is thus obtained.

**High SNR:** At the high SNR regime (\( \tilde{\gamma} \to \infty \)), by using an approximation of \( \gamma(c,x) \approx x^c/c \), we obtain a simpler expression of the outage probability when \( \gamma_{th} < 1/\kappa^{2} \) as

\[
P_{out}^{\infty} \approx \sum_{n} \frac{\Phi_{n}}{\Gamma(\mu_{n}+1)} \left( \frac{\gamma_{th}}{\Gamma_{n}^{\gamma_{th}}(1-\kappa^{2}\gamma_{th})} \right)^{\frac{\alpha_{n}+\mu_{n}}{2}}. \tag{20}
\]

**C. Analysis of Ergodic Capacity**

In wireless communications, the (Shannon) channel capacity (in bits/Hz) can be evaluated by \( C_{\text{ins}} = \log_{2}(1+\gamma) \), where \( \gamma \) is the instantaneous SNR. Assuming that the channel fading process is ergodic, the time-average capacity, which is referred to as the ergodic capacity, is obtained by averaging \( C_{\text{ins}} \) over all states of the time-varying channel. Thus, the ergodic capacity in our system can be evaluated as

\[
C = \mathbb{E}\{\log_{2}(1+\gamma)\} = \int_{0}^{\infty} \log_{2}(1+x) f_{\gamma}(x) dx, \tag{21}
\]

where \( f_{\gamma}(x) \) is the PDF of the SDNR \( \gamma \). We obtain the following result regarding the capacity \( C \).

**Theorem 3:** The approximate closed-form expression of the ergodic capacity \( C \) is given by

\[
C \approx \sum_{n=\{n_{1}, n_{2}, \ldots, n_{M}\}} \frac{\Phi_{n}}{2\Gamma(\mu_{n}+1)} \log_{2} \left( \frac{\tilde{\gamma}_{q} \Gamma_{n}^{\gamma_{th}}}{1+\kappa^{2}\tilde{\gamma}_{q} \Gamma_{n}^{\gamma_{th}}} \right), \quad \tilde{\gamma}_{q} \quad (q = 1, 2, \ldots, Q) \text{ are the weights and abscissas of the Q-point Gauss-Laguerre quadrature.} \tag{22}
\]

**Proof:** See Appendix B.

**High SNR:** At the high SNR regime, by performing some mathematical manipulations with the help of \([38, \text{Eq.}(3.381.4)]\), we obtain

\[
C_{\kappa^{2} \neq 0}^{\infty} \approx \sum_{n} \Phi_{n} \log_{2} \left( 1 + \frac{1}{\kappa^{2}} \right). \tag{23}
\]

It can be seen from (23) that the capacity is saturated at high SNR region. The saturation level is governed by the RHI level \( \kappa^{2} \) and the number of multipath components \( \mu \).

When there is no RHI, by using an approximation of \( \log(1+x) \approx \ln(x) \) and the integral formula in \([38, \text{Eq.}(4.352.1)]\), we can simplify the capacity expression to

\[
C_{\kappa^{2}=0}^{\infty} \approx \log_{2} e \sum_{n} \Phi_{n} \left[ \frac{2}{\gamma_{\mu_{n},n}} \psi(\mu_{n} \gamma) + \ln \left( \Gamma_{n} \gamma \right) \right] \equiv \mathcal{A} + \mathcal{B} \log_{2}(\tilde{\gamma}), \tag{24}
\]

where \( \mathcal{A} \equiv \sum_{n} \Phi_{n} \left[ 2 \log_{2} e \psi(\mu_{n} \gamma) + \log_{2} \Gamma_{n} \gamma \right], \mathcal{B} \equiv \sum_{n} \Phi_{n} \psi(\gamma) \) is the psi function \([38, \text{Eq.}(8.360.1)]\). This expression clearly shows the increase of the capacity with respect to \( \tilde{\gamma} \).
D. Analysis of Energy Efficiency

In this work, we consider an energy-efficiency metric defined as a ratio between the rate and the total power consumption. Specifically, we can express [39]-[40]

$$EE = \frac{W_BC}{P_{tot}}, \quad (\text{bits/Joule})$$  \hspace{1cm} (25)$$

where $W_B$ is the transmission bandwidth (in Hertz), and $P_{tot}$ is the total power consumption (in Watts). This power can be calculated as $P_{tot} = P_0/\eta + P_c$, where $\eta$ is the efficiency of the power amplifier (PA), and $P_c$ is the total circuit power consumption excluding the PA, i.e., $P_c = MP_{c,r} + P_{c,b} + P_{c,w}$, where $P_{c,r}, P_{c,b}$, and $P_{c,w}$ denotes the circuit power consumption of the BS, one RIS element, and the user, respectively.

By substituting the result in (22) into (25), we obtain the approximate closed-form expression of the average EE. Also, at the high SNR regime, the average EE expressions are obtained based on (23), (24) and (25).

IV. ANALYSIS OF RIS-THz WIRELESS SYSTEMS WITH A LARGE NUMBER OF RIS ELEMENTS

In this section, we focus on analyzing the RIS-THz system when the number of RIS elements are large. Also, different cases of phase-shifts described Section II are taken into the analysis. Since the path gains of all links associated with each RIS element are assumed equal (i.e., $h_{1,m}, h_{2,m} = h_1 h_2 \triangleq \tilde{g}, \forall m \in (1, M)$), we can rewrite the overall link between the BS and the user as

$$\tilde{g} = \tilde{h}_1 \tilde{h}_2 \sum_{m=1}^{M} \tilde{h}_{1,m} e^{j\phi_m} \tilde{h}_{2,m} \triangleq \tilde{gg'},$$  \hspace{1cm} (26)$$

where $\tilde{g'} \triangleq \sum_{m=1}^{M} \tilde{h}_{1,m} e^{j\phi_m} \tilde{h}_{2,m}$. To analyze the system performance, we need to characterize the distribution of $|\tilde{g}|^2$.

A. Statistical Characterization of the End-to-End Channels

1) Optimal Phase-Shifts: In case of the optimal phase-shifts, we can express the small-scale fading as $|\tilde{g}| = \sum_{m=1}^{M} |h_{1,m}||h_{2,m}|$ (cf. (4)). It is noted that $|\tilde{g}|$ is a sum of i.i.d. random variables. Hence, the CLT can be used to statistically characterize $|\tilde{g}|$ when $M$ becomes large.

**Theorem 4:** When $M$ is large, $|\tilde{g}|$ can be approximated as a Gaussian random variable, i.e.,

$$|\tilde{g}| \xrightarrow{M \to \infty} N(M\sigma_0, M\sigma_0^2),$$  \hspace{1cm} (27)$$

where $\sigma_0 = \frac{\Omega_{O_2}}{\Omega_{O_2} + \frac{\Gamma(\nu/2+\nu/2)}{\Gamma(\nu/2)}}, \quad \sigma_0^2 = \Psi - \sigma_o^2,$ and $\Psi = \frac{M^{\nu/2+1/\alpha_o}}{\Gamma(\nu/2+1/\alpha_o)}$.

**Proof:** Following the CLT, the mean value $\sigma_0$ of $|\tilde{g}|$ can be evaluated as $\sigma_0 = M \mathbb{E}\{ |h_{1,m}||h_{2,m}| \} = M \mathbb{E}\{ |h_{1,m}| \} \mathbb{E}\{ |h_{2,m}| \} = M \Omega_{O_2}$. Meanwhile, the variance $\sigma_0^2$ is obtained as

$$\sigma_0^2 = M \left[ \mathbb{E}\{|h_{1,m}|^2|h_{2,m}|^2\} - \mathbb{E}\{|h_{1,m}|\}^2 \mathbb{E}\{|h_{2,m}|\}^2 \right]$$

$$= M \left[ \mathbb{E}\{|h_{1,m}|^2\} \mathbb{E}\{|h_{2,m}|^2\} - \mathbb{E}\{|h_{1,m}|\}^2 \mathbb{E}\{|h_{2,m}|\}^2 \right].$$  \hspace{1cm} (28)$$

By plugging the result in (11) into (28), we have the expression of $\sigma_0^2$. This completes the proof.

2) Phase Estimation Errors: The modulus squared of $\tilde{g}$ when phase errors follow von Mises distribution can be expressed as (cf. (6))

$$|\tilde{g}|^2 = \left( \sum_{m=1}^{M} \tilde{h}_{1,m} |h_{2,m}| e^{j\varphi_m} \right)^2$$

$$= \left( \sum_{m=1}^{M} \tilde{h}_{1,m} |h_{2,m}| \cos \varphi_m \right)^2 + \left( \sum_{m=1}^{M} \tilde{h}_{1,m} |h_{2,m}| \sin \varphi_m \right)^2.$$  \hspace{1cm} (29)$$

**Theorem 5:** When $M$ is large, $|\tilde{g}|^2$ can be approximated as a gamma random variable. In particular, its PDF and CDF expressions are given by

$$f_{|\tilde{g}|^2}(x) = \frac{D_c}{\Gamma(C_c)} x^{C_c-1} e^{-D_c x} \Leftrightarrow f_{\Gamma, C_c, D_c}(x),$$  \hspace{1cm} (30)$$

and

$$F_{|\tilde{g}|^2}(x) = \frac{1}{\Gamma(C_c)} \gamma(C_c, D_c x) \Leftrightarrow F_{\Gamma, C_c, D_c}(x),$$  \hspace{1cm} (31)$$

where $C_c = \frac{x_0^2}{\sigma_0^2} + 1$ and $D_c = \frac{x_0^2}{\sigma_0^2} + 1$. When the phase errors follow von Mises distribution, $|\tilde{g}|^2$ is a gamma random variable.

**Proof:** See Appendix C.

3) Phase Quantization Errors: In this case, the quantization errors follow von Mises distribution, i.e., $\varphi_m \sim \mathcal{U}(-\Delta, \Delta)$. We obtain the following result.

**Theorem 6:** When $M$ is large and the phase quantization errors follow von Mises distribution, $|\tilde{g}|^2$ can be approximated as a gamma random variable. In particular, the PDF and CDF expressions are given by

$$f_{|\tilde{g}|^2}(x) = f_{\Gamma, C_q, D_q}(x),$$  \hspace{1cm} (32)$$

and

$$F_{|\tilde{g}|^2}(x) = F_{\Gamma, C_q, D_q}(x),$$  \hspace{1cm} (33)$$

where $C_q = \frac{\sigma_q}{\sigma_0^2}$, $D_q = \frac{\sigma_q}{\sigma_0^2} - 1$, $\sigma_q^2 = M(\sigma^2_\phi + \sigma^2_\theta) + (M\sigma^2_\varphi)$, $\sigma_q^2 = 2M^2(\sigma^4_\theta + \sigma^4_\phi) + 4M^3(\sigma^2_\phi^2 + \sigma^2_\theta^2)$, $\sigma_\phi = \frac{\Omega_{O_2} \sin \Delta}{\Omega_{O_2} + \frac{\Gamma(\nu/2+\nu/2)}{\Gamma(\nu/2)}}$, and $\sigma_\theta = \frac{\Omega_{O_2} (1 - \cos \Delta)}{\Omega_{O_2} + \frac{\Gamma(\nu/2+\nu/2)}{\Gamma(\nu/2)}}$.

**Proof:** The proof is similar to that of Theorem 5, which is omitted for brevity.

**Remarks:**

1) From Theorem 4, it is straightforward to express the PDF and CDF of $|\tilde{g}|^2$ with optimal phase-shifts as

$$f_{|\tilde{g}|^2}(x) = \frac{1}{2\pi} \frac{1}{\sqrt{1 - x^2}} \left( \frac{x^2 - M\sigma_0^2}{M\sigma_0^2} \right)^{1/2}$$

and

$$F_{|\tilde{g}|^2}(x) = \frac{1}{2} \left[ 1 + e^{rf\left( \frac{x^2 - M\sigma_0^2}{M\sigma_0^2} \right) \right],$$

where $rf(\cdot)$ is the error function. However, these expressions are not useful for tractable analysis. Based on the proof in Appendix C, we can approximate...
B. Analysis of Performance Metrics

It is noted from Section IV.A that the PDF and CDF expressions of $|\tilde{g}|^2$ in three cases admit the similar forms. This obviously facilitates derivations of unified expressions of the large RIS-THz system for these cases. For notational convenience, let $(C, D)$ denote the coefficients associated with the gamma distribution, i.e., $(C, D) = (C_o, D_o)$ for optimal phase-shifts, $(C, D) = (C_e, D_e)$ for phase estimation errors, and $(C, D) = (C_q, D_q)$ for phase quantization errors.

1) Outage Probability: By performing a similar evaluation as in Section III.B, we obtain the following result.

**Theorem 7**: When $M$ is large, the approximate closed-form expression of the outage probability is given by

$$P_{out,C} = \left\{ \begin{array}{ll}
1 & \gamma_{th} < \frac{1}{k}\frac{D_{th}}{\bar{\gamma}} \\
\frac{1}{\Gamma(C + 1)} \left[ \frac{D_{th}}{\bar{\gamma}^2(1 - \kappa \bar{\gamma}_{th})} \right]^C & \gamma_{th} \geq \frac{1}{k}\frac{D_{th}}{\bar{\gamma}}.
\end{array} \right. \quad (36)$$

**Proof**: The proof is similar to that of Theorem 2, which is omitted for brevity.

**High SNR**: At the high SNR regime, a simpler expression of the OP when $\gamma_{th} < 1/k\sigma^2$ is given by

$$P_{out,C} \approx \frac{1}{\Gamma(C + 1)} \left[ \frac{D_{th}}{\bar{\gamma}^2(1 - \kappa \bar{\gamma}_{th})} \right]^C. \quad (37)$$

2) Ergodic Capacity: With the definition of the ergodic capacity given in (21), we obtain the following result.

**Theorem 8**: When $M$ is large, the approximate closed-form expressions of the capacity are given by

$$C_{L, k^2 \neq 0} = \frac{1}{\Gamma(C)} \ln 2 \left[ G_{3,2}^{1,3} \left( \frac{1 + \kappa \bar{\gamma}}{D_{th}} - 1, \frac{C}{1, 1, 1} \right), \frac{1}{1, 1, 1} \right], \quad (38)$$

and

$$C_{L, k^2 = 0} = \frac{1}{\Gamma(C)} \ln 2 \left[ G_{3,2}^{1,3} \left( \frac{\kappa \bar{\gamma}^2}{D_{th}} - 1, \frac{C}{1, 1, 1} \right), \frac{1}{1, 1, 1} \right], \quad (39)$$

where $G(\cdot ; \cdot)$ is the Meijer $G$-function [38, Eq. (9.301)].

**Proof**: See Appendix D.
**High SNR:** As shown in Appendix D, we can simplify the capacity expressions at the high SNR regime as

\[
C_L^\infty = \begin{cases} 
\log_2 \left( 1 + \frac{1}{\kappa^2} \frac{\bar{\gamma}}{D} \right), & \kappa^2 \neq 0 \\
\log_2(\bar{\gamma}), & \kappa^2 = 0.
\end{cases}
\]

(40)

Similar to Section III.C, we note that for large-size RIS-THz systems, the capacity in the presence of RHI saturates at the high SNR regime, which is determined by the RHI level \( \kappa^2 \). Meanwhile, in case of no RHI, the capacity increases with \( \gamma \). Therefore, the e2e links via different RIS elements are not necessarily co-phased. Consequently, in this section we consider a RIS-THz system with ON/OFF RISs. Since the phase-shifts in the ON/OFF scheme are fixed, issues related to phase-shift estimation and quantization errors are avoided. Moreover, this scheme requires reduced channel overhead compared to that with phase-shift adjustment schemes. Specifically, when \( B \) bits are used to uniformly quantize RIS phase shifts in the discrete phase-shift adjustment scheme, \( M \times B \) bits are required. However, for the ON/OFF scheme, the number of bits is only \( \lceil \log_2 M \rceil \), where \( \lceil \cdot \rceil \) denotes a ceiling function. The instantaneous energy-efficiency in the ON/OFF scheme can be expressed as (cf. (25))

\[
EE(P_0, \mathcal{M}) = \frac{W_B C_L}{P_{tot, \mathcal{M}}},
\]

where \( C_L \) is given by (38)-(40).

**V. Maximize Energy Efficiency of RIS-THz Wireless Systems with ON/OFF RISs**

From a practical perspective, it is noticed that implementing the continuous phase-shift adjustment is challenging due to requirements of sophisticated hardware and signaling overhead, which hinders the original benefits of low-cost, easy-to-deploy of RIS-based solutions. Also, practical implementation remains an issue for the discrete phase-shift adjustment when the number of RIS elements is large. Consequently, in this section, we consider a RIS-THz system with ON/OFF RISs. Since the phase-shifts in the ON/OFF scheme are fixed, issues related to phase-shift estimation and quantization errors are avoided. Moreover, this scheme requires reduced channel overhead compared to that with phase-shift adjustment schemes. Specifically, when \( B \) bits are used to uniformly quantize RIS phase shifts in the discrete phase-shift adjustment scheme, \( M \times B \) bits are required. However, for the ON/OFF scheme, the number of bits is only \( \lceil \log_2 M \rceil \), where \( \lceil \cdot \rceil \) denotes a ceiling function. The instantaneous energy-efficiency in the ON/OFF scheme can be expressed as (cf. (25))

\[
EE(P_0, \mathcal{M}) = \frac{W_B C_L}{P_{tot, \mathcal{M}}},
\]

where \( \mathcal{M} \) is the set of all RIS elements that are in ON state (i.e., on-RISS), and \( |\mathcal{M}| \) denotes the cardinality of \( \mathcal{M} \).

It is worth noting that a larger number of RIS is activated results in higher power consumption. On the other hand, these ON-RISs do not necessarily lead to improved SDNR. This is because the e2e links via different RIS elements are not necessarily co-phased. Therefore, it is important to select which RIS elements are in ON state such that the EE is improved. In addition, we noted that higher transmit power \( P_0 \) will improve the SDNR at the cost of larger power consumption. Motivated by this, a joint optimization of the transmit power \( P_0 \) and the subset of ON-RISs \( \mathcal{M} \) to maximize EE can be formulated as

\[
\max_{(P_0, \mathcal{M})} EE(P_0, \mathcal{M}) \tag{43a}
\]

subject to

\[
\gamma_{\mathcal{M}} \geq \gamma_{th} \tag{43b}
\]

\[
0 < P_0 \leq P_{max}. \tag{43c}
\]

where (43b) and (43c) are the constraints on the performance and the transmit power, respectively. Note that the constraint (43b) is equivalent to \( P_0 \geq \frac{\gamma_{th}^\gamma h}{|\mathcal{M}|(1+\kappa^2)} \). Consequently, in this section, we consider a RIS-THz system with ON/OFF RISs. Since the phase-shifts in the ON/OFF scheme are fixed, issues related to phase-shift estimation and quantization errors are avoided. Moreover, this scheme requires reduced channel overhead compared to that with phase-shift adjustment schemes. Specifically, when \( B \) bits are used to uniformly quantize RIS phase shifts in the discrete phase-shift adjustment scheme, \( M \times B \) bits are required. However, for the ON/OFF scheme, the number of bits is only \( \lceil \log_2 M \rceil \), where \( \lceil \cdot \rceil \) denotes a ceiling function. The instantaneous energy-efficiency in the ON/OFF scheme can be expressed as (cf. (25))

\[
EE(P_0, \mathcal{M}) = \frac{W_B C_L}{P_{tot, \mathcal{M}}},
\]

where \( C_L \) is given by (38)-(40).

**Theorem 9:** Given a subset of ON-RISs \( \mathcal{M} \), the energy efficiency defined in (42) is quasi-concave with respect to (w.r.t.) transmit power \( P_0 \).

Proof: The instantaneous capacity associated with the subset \( \mathcal{M} \) can be expressed as (cf. (4), (21))

\[
C(P_0, \mathcal{M}) = \log_2 \left( 1 + \frac{P_0 |g_M|^2}{\kappa^2 P_0 |g_M|^2 + 2N_0} \right). \tag{44}
\]

Since \( \frac{\partial^2 C(P_0, \mathcal{M})}{\partial P_0^2} = -\frac{|g_M|^4 N_0 (1+\kappa^2) P_0 + 2\kappa^2 N_0 N_0 \log_2 e}{(\kappa^2 |g_M|^2 + 2N_0)^2} < 0 \), \( C(P_0, \mathcal{M}) \) is a concave function w.r.t. \( P_0 \). Thus, (42), which is a ratio between a concave function and an affine function, is quasi-concave w.r.t. \( P_0 \). This completes the proof.

It is obvious from Theorem 9 that given a subset \( \mathcal{M} \), there exists an optimal transmit power \( P_0^{opt} \) such that the EE is maximized. This power can be obtained via search methods, e.g., golden-section search for each subset \( \mathcal{M} \) over a range \([P_{min, \mathcal{M}}, P_{max}]\) [41]. Consequently, the solution to the above problem can be obtained by checking all possible on-RIS subsets. Unfortunately, the number of subsets is \( 2^M - 1 \), which is computational expensive given that the number of RIS elements \( M \) is typically large. Therefore, a suboptimal solution with low-complexity is more preferable from a practical viewpoint. To design an efficient algorithm, we have following observations (cf. (42)):

1) Having more RIS elements in ON state does not necessarily result in improved EE since this depends on whether the benefits from SDNR changes outweigh the disadvantages of additional power consumption requirement;

2) With the same number of ON-RIS elements (i.e., equal value of \( |\mathcal{M}| \)), a subset \( \mathcal{M} \) that introduces higher overall channel gain \( |g_M| = \sum_{m \in \mathcal{M}} g_m \) leads to higher EE.

An issue now is how to efficiently maximize \( |g_M| = \sum_{m \in \mathcal{M}} g_m \), where \( g_m \) is a complex number. To deal with this, we adopt the well-known Kadane’s algorithm that was developed for maximum continuous subarray problem [42]. To realize this, we first need to sort elements of \( G = [g_1, g_2, ..., g_M] \) in an ascending order (or descending order) of the phases, i.e., \( G = \text{sort}(G) \). This step is important since a sum \( |g_m + g_n|, (m \neq n) \) is larger if \( -g_m + g_n \) becomes smaller. Moreover, the Kadane’s algorithm only works on continuous subarrays of an array. Another important point is that the performance will be improved if we consider all \( M \) circularly shifted arrays of an array. A detail about the proposed algorithm is provided in Algorithm 1. Recall that the considered array here is the sorted array \( G \). However, our objective is to maximize the EE, instead of the modulus of the sum of elements of \( G \) as in an original maximum continuous subarray problem. Thus, the Kadane’s algorithm is modified to take this into account (i.e., see Lines 8-25 in Algorithm 1).
Algorithm 1 Joint optimization of $P_0$ and $M$ for maximal EE

1: Initialization: $E_{\text{max}} = 0$; $G = \{g_1, g_2, ..., g_M\}$, where $g_m = h_{1,m}h_{2,m}$
2: Calculate the phases of the elements of $G$, i.e., $\theta_m = \text{mod}(\angle h_{1,m}h_{2,m}, 2\pi)$, where $\text{mod}(x, 2\pi)$ wraps an angle to $[0, 2\pi)$
3: Sort $G$ in an ascending order of the phases, i.e., $G = \text{sort}(G)$
4: Find optimal transmit powers when only one element of $G$ is ON, i.e., $P = \{p_1, p_2, ..., p_M\}$, where $p_m$ is obtained by golden-section search
5: Calculate the EE achieved when only one RIS element is ON, i.e., $\mathcal{E} = \{E_1, E_2, ..., E_M\}$, where $E = \text{EE}(P, G)$ based on (42)
6: for $k = 0 : M - 1$ do
7: Circularly shift elements of $G$, $E$, and $P$ by $k$ positions, i.e., $G_k = \text{circshift}(G, k)$; $E_k = \text{circshift}(E, k)$; $P_k = \text{circshift}(P, k)$
8: Initialize: $E_{\text{sofar}} = E_k(1)$; $\text{start\_index} = 1$; $\text{end\_index} = 1$
9: for $m = 2 : M$ do
10: Obtain $a = E_k(m)$
11: Find optimal power $P_{0,m}$ given $M_{m} = G_k(\text{start\_index} : m)^3$
12: Calculate $b = EE(P_{0,m}, M_{m})$ based on (42)
13: if $a > b$ then
14: $E_{\text{here}} = a$; $P_{\text{here}} = P_k(m)$
15: else
16: $E_{\text{here}} = b$; $P_{\text{here}} = P_{0,m}$
17: end if
18: if $E_{\text{here}} > E_{\text{sofar}}$ then
19: $\text{end\_index} = m$; $E_{\text{sofar}} = E_{\text{here}}$; $P_{\text{sofar}} = P_{\text{here}}$
20: end if
21: if $E_{\text{sofar}} = E_k(m)$ then
22: $\text{start\_index} = m$; $\text{end\_index} = m$; $P_{\text{sofar}} = P_k(m)$
23: end if
24: end for
25: Update selected subset $M_{m} = G_k(\text{start\_index} : \text{end\_index})$
26: if $E_{\text{sofar}} > E_{\text{max}}$ then
27: $P_{0}^{\text{opt}} = P_{\text{sofar}}$; $M^{\text{opt}} = M_{m}$; $E_{\text{max}} = E_{\text{sofar}}$
28: end if
29: end for
30: Outputs: $P_{0}^{\text{opt}}$, $M^{\text{opt}}$, and $E_{\text{max}}$.

With respect to the complexity of the proposed algorithm, we note that the time complexity of the Kadane’s algorithm is $O(M)$. Also, the number of iterations for golden-section search-based power optimization is $I = \lceil\log(e/P_{\text{max}})\rceil/\log(\zeta)$, where $\epsilon$ is the error tolerance, and $\zeta = (\sqrt{5} - 1)/2$. Thus, the complexity of Algorithm 1 is $O(M^2I)$, which is much lower than that with the brute-force approach $O(2^M - 1)$ when $M$ is sufficient large. In addition, on Algorithm 1, we can devise another algorithm that requires a lower complexity of $O(MI)$ at the cost of performance loss. This algorithm, namely Algorithm 2, is accomplished by performing circular shift the elements of $G$ only one time instead of $M$ times as in Algorithm 1, i.e., $G_{k^o} = \text{circshift}(G, k^o)$. Based on the principle of the maximum continuous subarray problem, we chose the shift amount $k^o$ such that the dominant phase, defined as $\omega = \sum_{m=1}^{M} g_m$, is in the middle position with respect to the phases of elements of $G_{k^o}$, i.e., $\angle G_{k^o,M/2} < \omega < \angle G_{k^o,M/2+1}$, where $G_{k^o,i}$ is the $i^{th}$ element of $G_{k^o}$. Performance evaluation of the proposed algorithms compared to their counterparts is provided in the next section.

VI. SIMULATION RESULTS

In this section, we perform numerical evaluations of the system performance based on the approximation approach

Fig. 6 plots the outage probability versus the transmit power $P_0$ for cases of small-size RIS with optimal phase-shifts. It can be seen that the OP performance is better when the number of RIS elements is increased and/or the RHI level is lower. This is because a higher SDNR value can be achieved

devolved in the previous sections. We use similar system parameters as in [25], [28], and [33] for our simulations. In particular, the operating frequency $f = 300$ GHz [25], antenna gains $G_{t} = G_{r} = 40$ dBi [25], molecular absorption coefficient $\varrho = 3.18 \times 10^{-4}$ per meter [25], small-scale fading channels with $\mu_1 = 1$, $\Omega_1 = 1$ and $\mu_2 = 5$, $\Omega_2 = 1$ [25], the SNR threshold $\vartheta = 3$ (dB) [25], and channel bandwidth $W_B = 10$ GHz with noise power density of $-174$ dBm/Hz [25], a number of bits for discrete phase-shift adjustment $B = 3$, PA efficiency $\eta = 0.83$, and $P_{c.r} = P_{c.h} = P_{c.u} = 10$ mW [39]-[40]. For brevity, only the von Mises distribution is considered for phase estimation errors with $\vartheta = 4$ [33]. In cases of small-size RIS, the transmission distances are $d_{BS-RIS} = 2$m and $d_{RIS-User} = 8$m. Meanwhile, $d_{BS-RIS} = 5$m and $d_{RIS-User} = 25$m [25] when a large-size RIS is deployed. Also, the simulation results are averaged over $10^5$ channel realizations.

A. Outage Capacity

Fig. 6 plots the outage probability versus the transmit power $P_0$ for cases of small-size RIS with optimal phase-shifts. It can be seen that the OP performance is better when the number of RIS elements is increased and/or the RHI level is lower. This is because a higher SDNR value can be achieved.
B. Ergodic Capacity

The achieved capacity versus the transmit power $P_0$ is shown in Fig. 8 and Fig. 9 for small-size RIS and large-size RIS scenarios, respectively. In both cases, the capacity is increased with $P_0$ as expected. However, the results reveal that the capacity becomes saturated at the high SNR regime due to the presence of RHI regardless how large the transmit power level and/or the number of RIS elements are deployed. The saturation level disappears where there is no RHI. This behavior agrees with our asymptotic analysis at the high SNR regime presented in Section III.C and Section IV.B.

C. Energy Efficiency

In Fig. 10 and Fig. 11, we plot the energy efficiency versus the transmit power $P_0$ for cases of small-size RIS and large-size RIS, respectively. It can be seen that the presence of the RHI reduces the EE in all scenarios. Moreover, this impact becomes more serious when $P_0$ increases. This is because the capacity is more sensitive to RHI at high-power regime as shown in Fig. 8 and Fig. 9. Note that this behavior was also observed in [28]. In addition, the results show the concavity of the EE w.r.t. $P_0$, which agrees with Theorem 9.

D. Impacts of Hardware Impairment Errors

The impacts of the residual hardware impairment on the system performance are shown in Fig. 12. Here, we plot the outage probability and the capacity versus the RHI level when the number of RIS elements is $M = 50$. It can be seen that a larger value of the RHI errors degrades the system performance in all cases. This is because higher RHI levels reduce the received SNR ratio as shown in Eq. (4). It is also worth noting that these impacts depend on the transmit power $P_0$ as shown in Fig. 6 - Fig. 11.
E. Energy Efficiency with Different ON/OFF RIS Methods

We now consider the EE in the RIS-THz system with different ON/OFF RIS methods when the transmit power $P_0$ is fixed. In Fig. 13, we compare the EE achieved with different ON/OFF strategies, including exhaustive search, the proposed algorithms\(^4\), all RIS elements are always ON (i.e., an All-ON scheme), the maxSNR algorithm [43], and random ON/OFF RIS. The transmission distances are similar to those in the large-size RIS case, i.e., $d_{BS-RIS} = 5\text{m}$ and $d_{RIS-User} = 25\text{m}$. It can be seen that the proposed Algorithm 1 for ON/OFF RIS selection can attain similar EE as the exhaustive method. Also, the EE performance achieved by the Algorithm 1 is higher than those with the remaining methods at the cost of higher complexity. Meanwhile, at the same complexity order, the proposed Algorithm 2 can achieve higher EE than that with the maxSNR algorithm in [43]. Note that the maxSNR algorithm aims to maximize SNR, which does not necessarily lead to maximal EE. In addition, the EE performance is poor if all RISs are in ON state or when RISs are activated randomly. This agrees with our discussion in Section V. These results suggest that adaptive selection approaches for ON/OFF RISs are critical for improved EE.

F. Joint Optimization of $P_0$ and $\mathcal{M}$ for Maximal EE

With respect to the joint optimization of transmit power $P_0$ and RIS activation subset $\mathcal{M}$, we show in Fig. 14 and Fig. 15 the EE versus the number of RIS elements\(^5\). It is obvious that increasing $M$ leads to higher EE for both cases of with and without RHI. This is because the benefits of enhanced capacity thanks to increased $M$ outweigh the disadvantage of additional power consumption requirement (cf. (42)). However, when $M$ becomes very large, the EE of the All-ON and random ON/OFF schemes decrease since the capacity improvement is not significant compared to the impact of additional power consumption. Meanwhile, the EE of the proposed methods tend to be saturated. This implies that very large $M$ does not necessarily result in significant improvement of the EE. In addition, it can be seen that the proposed algorithms outperforms the random ON/OFF and All-ON schemes, especially at the large-size RIS regime.

\(^4\)We note that when the transmit power $P_0$ is fixed, the proposed algorithms for ON/OFF RIS selection are obtained straightforwardly from Algorithm 1 and Algorithm 2 by ignoring steps involving finding optimal power.

\(^5\)Due to the high time-complexity of the exhaustive search method when $M$ is large, an inclusion of the exhaustive search for comparison is feasible for small values of $M$ as demonstrated in Fig. 14.
proposed scheme also attains similar EE performance as the exhaustive search as shown in Fig. 14. This again demonstrates the effectiveness of the proposed approach.

VII. CONCLUSIONS

In this work, we propose an approximate framework to analyze the performance of RIS-THz wireless systems with hardware impairments over α − μ small-scale fading channels. For small-scale RIS, statistical channel characterization was obtained via a novel closed-form approximation of the weighted sum of the cascaded non-identical α − μ fading based on the Gauss-Laguerre quadrature and moment matching. For large-scale RIS, the e2e channel is characterized by the CLT under different cases of RIS phase-shifts, such as optimal phase-shifts, phase estimation errors with the von Mises distribution, and phase quantization errors. Based on the e2e channel characterization, the closed-form expressions of the outage probability, the ergodic capacity and the energy efficiency are derived, which facilitates evaluations of impacts of hardware impairments, a number of the RIS elements, and the THz fading channels on the system performance. In addition, we develop the low-complexity algorithm to jointly optimize the transmit power and ON/OFF RIS state for maximal EE. Our results reveal a good match between the simulations and the analytical approximation method, as well as demonstrate the efficacy of the proposed algorithm.

As for future works, some practical factors, such as point errors and multi-user interference, can be considered for RIS-THz systems. Specifically, the use of highly directive high-gain antennas in THz systems makes the systems prone to the effects of beam misalignment. Therefore, it is essential to develop an analytical framework for the quantification of the pointing error as well as performance evaluation of RIS-THz systems. In case of RIS-THz with multi-antenna base station serving multiple users scenarios, main research problems will be optimizing transmit beamforming/RIS phase-shifts for improved sum-rate or jointly optimizing transmit power, beamforming/RIS phase-shifts to enhance overall energy efficiency. In addition, exploring alternative approximation methods for analyzing RIS-THz systems with these practical factors would be an interesting topic.

APPENDIX A

Proof of Theorem 1

For notational convenience, let us denote \( Z_m = X_mY_m \) and \( U_m = X_mZ_m \). Thus, we can rewrite \( S = \sum_{m=1}^M U_m \). At first, the PDF of the variable \( Z_m \) is computed by (cf. (9))

\[
 f_{Z_m}(z) = \int_0^\infty \frac{1}{x} f_{X_m}(\frac{z}{x}) f_{Y_m}(x) dx
\]

\[
 = \left( \frac{2}{\Omega_1 \Omega_2} \right) \frac{\alpha_1 \beta_1 \mu_1 \alpha_2 \beta_2 \mu_2}{\Gamma(\mu_1) \Gamma(\mu_2)} \int_0^\infty \frac{e^{-\left( \frac{\alpha_2 \beta_2}{\Omega_2} \right) z}}{x^{\mu_2 - \alpha_2 + 1}} dx \int_0^\infty e^{-\left( \frac{\alpha_1 \beta_1}{\Omega_1} \right) x} dx
\]

where the last step is obtained by variable changing. Since the exact result for the integral term in (45) is only expressed via a special function, we adopt the Gauss-Laguerre quadrature for an integral approximation [44]. Specifically, let us define

\[
\xi(x, z) = x^{\mu_2 - \alpha_2 + 1} e^{-\left( \frac{\alpha_2 \beta_2}{\Omega_2} \right) z}
\]

Then, we can express \( f_{Z_m}(z) \) as

\[
\sum_{n=1}^N \omega_n e^{-\left( \frac{\alpha_1 \beta_1}{\Omega_1} \right) x} \int_0^\infty \frac{e^{-\left( \frac{\alpha_2 \beta_2}{\Omega_2} \right) z}}{x^{\mu_2 - \alpha_2 + 1}} dx
\]

where \( \Omega_n = \frac{\Omega_1 \Omega_2}{\mu_2 \alpha_1 \beta_1 \mu_1 \alpha_2 \beta_2} \). Next, by using the transformation method, we can express the PDF of \( U_m = \tau_m Z_m \) as

\[
f_{U_m}(u) \approx f_{Z_m}(\frac{u}{\tau_m}) \frac{1}{\tau_m}
\]

\[
= \sum_{n=1}^N \omega_n e^{-\left( \frac{\alpha_1 \beta_1}{\Omega_1} \right) u} \left( \int_0^\infty \frac{e^{-\left( \frac{\alpha_2 \beta_2}{\Omega_2} \right) z}}{x^{\mu_2 - \alpha_2 + 1}} dx \right)
\]

where \( \Omega_{mn} = \frac{\Omega_1 \Omega_2}{\mu_2 \alpha_1 \beta_1 \mu_1 \alpha_2 \beta_2} \). Since \( S \) is the sum of \( M \) independent random variables, the PDF of \( S \) can be expressed as

\[
f_S(x) \approx f_{U_1}(x) \ast f_{U_2}(x) \ast \cdots \ast f_{U_M}(x)
\]

\[
= \left[ \sum_{n=1}^N \omega_n e^{-\left( \frac{\alpha_1 \beta_1}{\Omega_1} \right) u} \left( \int_0^\infty \frac{e^{-\left( \frac{\alpha_2 \beta_2}{\Omega_2} \right) z}}{x^{\mu_2 - \alpha_2 + 1}} dx \right) \right] \ast \cdots \ast \left[ \sum_{n=1}^N \omega_n e^{-\left( \frac{\alpha_1 \beta_1}{\Omega_1} \right) u} \left( \int_0^\infty \frac{e^{-\left( \frac{\alpha_2 \beta_2}{\Omega_2} \right) z}}{x^{\mu_2 - \alpha_2 + 1}} dx \right) \right]
\]

where * denotes the convolution. An exact closed-form evaluation of the high-order convolution term in (48) is challenging.

7The approximation error of the Gauss-Laguerre quadrature is given by \( E_N(\xi) = \frac{\xi^{(2N)}(0)}{2^N N!} \), for some \( \xi \in (0, \infty) \) [44]. By using the Uspensky theorem [45], we note that \( E_N(\xi) \) is not helpful for further analysis.

6When \( \alpha_1 = \alpha_2 = \alpha \), we have \( \int_0^\infty x^{\mu_2 - \alpha_2 + 1} e^{-x(\frac{\alpha_1 \beta_1}{\Omega_1} + \frac{\alpha_2 \beta_2}{\Omega_2})} dx = 2 \left( \frac{\alpha_2 \beta_2}{\Omega_2} \right) K_{\mu_2 - \alpha_2} \left( 2 \left( \frac{\alpha_1 \beta_1 \Omega_1}{\Omega_2} \right)^2 \right) \), where \( K_{\mu_2 - \alpha_2} \) is the \( \mu_2 \)-th order modified Bessel function of the second kind [38, Eq. (3.784.4)], which is not helpful for further analysis.

7The approximation error of the Gauss-Laguerre quadrature is given by \( E_N(\xi) = \frac{\xi^{(2N)}(0)}{2^N N!} \), for some \( \xi \in (0, \infty) \) [44]. By using the Uspensky theorem [45], we note that \( E_N(\xi) \) is not helpful for further analysis.

8The approximation error of the Gauss-Laguerre quadrature is given by \( E_N(\xi) = \frac{\xi^{(2N)}(0)}{2^N N!} \), for some \( \xi \in (0, \infty) \) [44]. By using the Uspensky theorem [45], we note that \( E_N(\xi) \) is not helpful for further analysis.
us denote $V_n = V_{n_1} + V_{n_2} + \cdots + V_{n_M}$, where $V_{n_m}$ is an $\alpha_n - \mu$ variable with the PDF $f(x; \alpha_n, \mu_n, \Omega_{n,n_m})$. We can express

$$f_{V_n}(x) = f(x; \alpha_1, \mu_1, \Omega_{1,n_1}) \ast f(x; \alpha_1, \mu_1, \Omega_{2,n_2}) \ast \cdots \ast f(x; \alpha_1, \mu_1, \Omega_{M,n_M}) \approx f(x; \alpha_n, \mu_n, \Omega_n),$$

(49)

where $n$ is a subset of indexes $\{n_1, n_2, \ldots, n_M\}$, and $\alpha_n$ and $\mu_n$ are obtained by solving the following equations [24]

$$\begin{align*}
\Gamma^2 \left( \mu_n + \frac{1}{\alpha_n} \right) & \Gamma \left( \mu_n + \frac{2}{\alpha_n} \right), \\
\Gamma(\mu_n) &= \frac{E^2\{V_n\}}{E\{V_n^2\} - E^2\{V_n\}^2},
\end{align*}$$

(50)

\begin{align*}
\Gamma^2 \left( \mu_n + \frac{4}{\alpha_n} \right) & \Gamma \left( \mu_n + \frac{2}{\alpha_n} \right), \\
\Gamma(\mu_n) &= \frac{E^2\{V_n^4\}}{E\{V_n^2\} - E^2\{V_n\}}
\end{align*}

(51)

where $E\{V_n\}$, $E\{V_n^2\}$, and $E\{V_n^4\}$ are the exact moments of $V_n$ that can be evaluated as

$$E\{V_n\} = \sum_{m=1}^{M} E\{V_{n_m}\},$$

(52)

$$E\{V_n^2\} = \sum_{m=1}^{M} E\{V_{n_m}^2\} + \sum_{m_1=1}^{M} \sum_{m_2 \neq m_1} E\{V_{n_{m_1}, V_{n_{m_2}}} V_{n_{m_3}} \},$$

(53)

$$E\{V_n^4\} = \sum_{m=1}^{M} E\{V_{n_m}^4\} + \sum_{m_1=1}^{M} \sum_{m_2 \neq m_1} E\{V_{n_{m_1}, V_{n_{m_2}}}^2 V_{n_{m_3}} \} + \sum_{m_1=1}^{M} \sum_{m_3 \neq m_1} \sum_{m_2 \neq m_3} E\{V_{n_{m_1}, V_{n_{m_2}}, V_{n_{m_3}}} \},$$

(54)

where $k$th moment $E\{V_{n_m}^k\}$ can be computed by using (11). Also, we have $\Omega_n = E\{V_n\}$ and $\beta_n = \Gamma(\mu_n + 1/\alpha_n) / \Gamma(\mu_n)$. This completes the proof.

**APPENDIX B
PROOF OF THEOREM 3**

We start by deriving the PDF expression of the SDNR $\gamma$. In particular, we have (cf. (4))

$$f_{\gamma}(x) = \frac{\partial F_{\gamma}(x)}{\partial x} = \frac{\partial}{\partial x} F_{[g]_2} \left( \frac{x}{1 - K^2 x} \right)$$

$$= \left( \frac{1}{\gamma(1 - K^2 x)} \right) \frac{[g]_2}{(1 - K^2 x)^{3/2}},$$

(55)

where $f_{[g]_2}(\cdot)$ is the PDF of the channel gain that is based on (12). By substituting (55) into (21) and then performing some manipulations including changing variables, we arrive at

$$C \approx \sum_{n=\{n_1, n_2, \ldots, n_M\}} \frac{\Phi_n \alpha_n}{2} \int_{0}^{\infty} e^{-\alpha_n \gamma_{n,M}^2} - e^{-\alpha_n \gamma_{n,n}^2 / 2} \times \log_2 \left( 1 + \frac{\Omega_n^2 \gamma_n^2}{\beta_n^2 + \kappa^2 \Omega_n^2 \gamma_n^2} \right) dz.$$  

(56)

The integral term in (56), labeled as $I$, can be evaluated by using the Laguerre integral approximation, i.e.,

$$I \approx \sum_{q=1}^{Q} \tilde{\omega}_q \tilde{t}_q \frac{\alpha_n^2}{\alpha_n^2 - 1} \log_2 \left( 1 + \tilde{t}_q \gamma_n \right) \sim N \left( \frac{\alpha_n^2}{\alpha_n^2 - 1} \log_2 \left( 1 + \frac{\tilde{t}_q \Omega_n^2 \gamma_n}{\beta_n^2 + \kappa^2 \tilde{t}_q \Omega_n^2 \gamma_n^2} \right) \right).$$

(57)

where $\tilde{\omega}_q$ and $\tilde{t}_q$ ($q = 1, 2, \ldots, Q$) are the weights and abscissas, which can be found in [44]. The result in (22) is thus obtained.

**APPENDIX C
PROOF OF THEOREM 5**

Let us denote $U \triangleq \sum_{m=1}^{M} |\tilde{h}_{1,m}| |\tilde{h}_{2,m}| \cos \varphi_m$ and $V \triangleq \sum_{m=1}^{M} |\tilde{h}_{1,m}| |\tilde{h}_{2,m}| \sin \varphi_m$ for notational convenience. Then, we can express $|\tilde{g}|^2 = U^2 + V^2$. We have

$$E\{|\tilde{h}_{1,m}| |\tilde{h}_{2,m}| \cos \varphi_m \} = E\{|\tilde{h}_{1,m}| |\tilde{h}_{2,m}| \} E\{\cos \varphi_m \}$$

(58)

$$= \Omega_1 \Omega_2 E\{\cos \varphi_m \},$$

where

$$E\{\cos \varphi_m \} = \int_{-\pi}^{\pi} \cos \varphi f_{\varphi_m}(\varphi) d\varphi$$

$$= \int_{-\pi}^{\pi} \cos \varphi e^{j \varphi \varphi_m} d\varphi = I_1(\varphi) \frac{\sin I_0(\varphi)}{I_0(\varphi)}.$$  

(59)

where $I_k(\cdot)$ is the modified Bessel function of the first kind and order $k$, and $f_{\varphi_m}(\cdot)$ is the PDF of the von Mises variable $\varphi_m$ (cf. (7)). Also, we can evaluate

$$E\{|\tilde{h}_{1,m}| |\tilde{h}_{2,m}| \cos \varphi_m \}^2$$

$$= E\{|\tilde{h}_{1,m}|^2 \} E\{|\tilde{h}_{2,m}|^2 \} E\{\cos^2 \varphi_m \}$$

$$= \Omega_1^2 \Omega_2^2 \sum_{i=1}^{2} \Gamma(\mu_i) \Gamma(\mu_i + 2/\alpha_i) \int_{-\pi}^{\pi} \cos^2 \varphi e^{j \varphi \varphi_m} d\varphi$$

$$= \Omega_1^2 \Omega_2^2 \sum_{i=1}^{2} \Gamma(\mu_i) \Gamma(\mu_i + 2/\alpha_i) \frac{I_1(\varphi) \sin I_0(\varphi)}{2 \pi I_0(\varphi)}.$$  

(60)

We note that $U$ is the sum of i.i.d. random variables. Thus, we can approximate it as a Gaussian random variable when $M$ is large based on the CLT. Specifically, we have

$$U \stackrel{M \rightarrow \infty}{\sim} \mathcal{N}(\hat{M} \varphi_{\hat{w}_M}, M \sigma_u^2),$$

(61)

where $\varphi_{\hat{w}_M} = \Omega_1 \Omega_2 I_1(\varphi) I_0(\varphi)$ and $\sigma_u^2 = \Omega_1^2 \Omega_2^2 \frac{I_0(\varphi) + I_2(\varphi)}{\pi \Gamma(\mu_i + 2/\alpha_i)} - \varphi_{\hat{w}_M}$. Similarly, we can approximate $V$ as

$$V \stackrel{M \rightarrow \infty}{\sim} \mathcal{N}(\hat{M} \varphi_{\hat{w}_M}, M \sigma_v^2),$$

(62)

where $\varphi_{\hat{w}_M} = 0$ and $\sigma_v^2 = \Omega_1^2 \Omega_2^2 \left[ 1 - \frac{I_1(\varphi) + I_2(\varphi)}{I_0(\varphi)} \right]$$

$$\times \frac{\Gamma(\mu_i + 2/\alpha_i)}{\Gamma(\mu_i + 1/\alpha_i)}.$$
Following the results in [36] and [47], we have $U$ and $V$ are independent when $M$ becomes large. Then, $|\hat{g}|^2 = U^2 + V^2$ is the sum of two squared Gaussian variables with different mean and variance values. To approximate the distribution of $|\hat{g}|^2$, we first express

$$|\hat{g}|^2 = M\sigma_u^2 \times \frac{U^2}{M\sigma_u^2} + M\sigma_v^2 \times \frac{V^2}{M\sigma_v^2}$$


where $\chi_2^2(b)$ denotes a non-central chi-squared random variable with $a$ degrees of freedom and non-centrality parameter $b$. Note that the second step in (63) is obtained since equal of Gaussian distribution with unit variance follows non-central chi-square distribution with one degree of freedom. Also, the mean and variance of $\chi_2^2(b)$ are given as $a + b$ and $2(a + b)$, respectively. Thus, the mean and the variance of $|\hat{g}|^2$ can be obtained as

$$\mu_{|\hat{g}|^2} = M(\sigma_u^2 + \sigma_v^2)$$

and

$$\sigma_{|\hat{g}|^2}^2 = 2M^2(\sigma_u^4 + \sigma_v^4) + 4M^2\chi_2^2(\sigma_u^2).$$

The value $|\hat{g}|^2$ can be approximated as a gamma random variable, where the shape and the scale parameters are defined as $C_e = \mu_{|\hat{g}|^2}/\sigma_{|\hat{g}|^2}$ and $D_e = \mu_{|\hat{g}|^2}/\sigma_{|\hat{g}|^2}$, respectively.

In case that the phase-shift $\phi_m$ follows the Gaussian distribution with zero-mean and variance of $\sigma_e^2$, the PDF of $\phi_m$, $-\pi \leq \phi_m \leq \pi$ (i.e., a truncated normal distribution), is expressed as

$$f_{\phi_m}(\phi) = \frac{1}{2\sigma \sqrt{\pi}} e^{-\frac{x^2}{2\sigma^2}},$$

where $\Xi(x) = \frac{1}{2} \left[ 1 + erf \left( \frac{x}{\sqrt{2}} \right) \right]$, and $erf(x)$ denotes the error function. To derive closed-form expressions of $E\{\cos \phi_m\}$, $E\{\cos^2 \phi_m\}$, $E\{\sin \phi_m\}$, and $E\{\sin^2 \phi_m\}$, we use the characteristic function instead of working directly with the PDF function. In practical scenarios, we are interested in small standard deviation (e.g., a few degrees). Thus, the characteristic function of $\phi_m$, when $\sigma_e$ is small is given by [34]

$$C_e = \frac{\partial F_e}{\partial x}(x) = \frac{1}{\Gamma(C)(1 - \kappa^2x^2)^{\frac{\gamma}{2}}},$$

where $\gamma = 1 - 2\kappa^2/\kappa^2$. Since $\gamma > 0$, we have

$$E\{\cos \phi_m\} = \frac{1}{2} + \frac{1}{2}E\{\cos 2\phi_m\} = \frac{1}{2} + \frac{1}{2}E\{\cos 2\phi_m\} = \frac{1}{2} + \frac{1}{2}e^{-2\sigma^2}. \quad (69)$$

Similarly, we obtain $E\{\sin \phi_m\} = 0$ and $E\{\sin^2 \phi_m\} = (1 - e^{-2\sigma^2})/2$. The remaining calculations are similar to the case of

\[A\] simple but accurate tool to measure the accuracy of the Gamma approximation was developed based on the Laguerre series expansion in [48]. Accordingly, the approximation error is measured based on the third and the fourth central moments of the random variable. The readers are referred to [48] for more details.

### Appendix D

**Proof of Theorem 8**

Let us first calculate the PDF of the SDNR when $M$ is large. We note that (cf. (4), (26))

$$F_{\gamma}(x) = F_{|\hat{g}|^2} \left( \frac{x}{(1 - \kappa^2x)^{\gamma/2}} \right). \quad (70)$$

Thus, the PDF expression is evaluated by

$$f_{\gamma}(x) = \frac{dF_{\gamma}(x)}{dx} = \frac{1}{(1 - \kappa^2x)^{2\gamma/2}} f_{|\hat{g}|^2} \left( \frac{x}{(1 - \kappa^2x)^{\gamma/2}} \right) = \frac{D^C}{\Gamma(C)(1 - \kappa^2x)^2} \left[ \frac{x}{(1 - \kappa^2x)^{\gamma/2}} \right]^{C-1} e^{-\frac{x}{(1 - \kappa^2x)^{\gamma/2}}}, \quad (x < 1/\kappa^2). \quad (71)$$

Plugging (71) into (21), we can express the capacity as

$$C_L = \frac{D^C}{\Gamma(C)(1 - \kappa^2x)^2} \left[ \frac{x}{(1 - \kappa^2x)^{\gamma/2}} \right]^{C-1} e^{-\frac{x}{(1 - \kappa^2x)^{\gamma/2}}} dx$$

$$= \frac{D^C}{(\gamma/2)^C\Gamma(C)} \int_0^{\infty} u^{C-1} e^{-\frac{u}{\gamma/2\kappa^2}} \log_2(1 + u/(1 + \kappa^2u)) \, du, \quad (72)$$

where a variable changing of $u = x/(1 - \kappa^2x)$ is used for the second step of (72). Noting that $\log(a/b) = \log(a) - \log(b)$, we can rewrite (72) as

$$C_L = \frac{D^C}{(\gamma/2)^C\Gamma(C)} \left[ \int_0^{\infty} u^{C-1} e^{-\frac{u}{\gamma/2\kappa^2}} \log_2(1 + 1 + \kappa^2u) \, du \right] - \left[ \int_0^{\infty} u^{C-1} e^{-\frac{u}{\gamma/2\kappa^2}} \log_2(1 + 1 + \kappa^2u) \, du \right]. \quad (73)$$

By expressing a logarithmic function in terms of the Meijer-G function, i.e., $\ln(1 + x) = G_{2,2}^{1,1}(x|1,1;1,0;0)$ [49], and using the integral result in [38, Eq. (7.813.1)], we obtain (38).

**High SNR regime:** When $\kappa^2 = 0$, (72) can be rewritten as

$$C_L,\kappa^2=0 = \frac{D^C}{(\gamma/2)^C\Gamma(C)} \int_0^{\infty} u^{C-1} e^{-\frac{u}{\gamma/2\kappa^2}} \log_2(1 + u) \, du. \quad (74)$$

By using the approximation of $\ln(1 + x) \approx x/\kappa^2 \ln(x)$ and the integral formula in [38, Eq.(4.352.1)], we obtain

$$C_L,\kappa^2=0 = \frac{1}{\ln 2} \left[ \psi(C) - \ln \left( \frac{D}{\gamma/2} \right) \right]. \quad (75)$$

For case of $\kappa^2 \neq 0$, we note that $\log_2 \left( 1 + \frac{u}{1 + \kappa^2u} \right) \leq \log_2 \left( 1 + \frac{1}{\kappa^2u} \right), \forall u > 0$. Thus, from (72), we can express

$$C_L,\kappa^2\neq0 = \frac{D^C}{(\gamma/2)^C\Gamma(C)} \log_2 \left( 1 + \frac{1}{\kappa^2} \right) \left[ \int_0^{\infty} u^{C-1} e^{-\frac{u}{\gamma/2\kappa^2}} \, du \right] \ln(1 + \frac{1}{\kappa^2})$$

$$= \log_2 \left( 1 + \frac{1}{\kappa^2} \right), \quad (76)$$

where the integral result of $\int_0^{\infty} x^\alpha e^{-bx} dx = b^{-\alpha} \Gamma(a), (a > 0, b > 0)$, was used. This completes the proof.