Robust optimization of geothermal recovery based on a generalized thermal decline model and deep learning

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A B S T R A C T

Geothermal reservoir simulation often considers the coupled thermo-hydro-mechanical physics, so the computational cost is remarkably expensive, which brings challenges for rapid reservoir optimization for geothermal management. In this work, we developed a parsimonious thermal decline model with only 3 parameters, namely HyperReLU model. It can accurately predict the produced fluid temperature behavior in geothermal recovery, which captures both the early thermal breakthrough and the later decline behavior. Further, a forward surrogate model based on deep neural network is developed to map the reservoir parameters to the HyperReLU model parameters and the ultimate total net energy. The forward model is integrated with a multi-objective optimizer (MOO) based on Non-dominated Sorting-based Genetic Algorithm II (NSGA-II), which considers reservoir uncertainties of rock properties and subjects to nonlinear engineering constraints for robust reservoir optimization. The HyperReLU model is validated through processes including enhanced geothermal recovery (EGS) and geothermal recovery from hot sedimentary aquifers (HSA) without fracturing. The mean relative error of the HyperReLU model is less than 1%. We also examined the deep neural network to predict 4 parameters including the total energy and 3 HyperReLU model parameters in EGS, with decent $R^2$ scores 0.998, 0.998, 1.000 and 0.946, respectively. The MOO converges well to achieve the optimum total energy, and solutions with different (low, median, high) risk levels are consistent with the results based on reservoir simulation. The decision variables including injection temperature and rate, extraction well pressure and well distance are provided based on the MOO framework. The number of forward model evaluations during optimization is 20000, and the average CPU time of MOO based on the forward surrogate model is 28.32 s, while the optimization based simulation is estimated to be around 600 min. Therefore, the newly proposed workflow is highly scalable and ready for field or regional scale geothermal optimization.

1. Introduction

Geothermal energy is a type of renewable and clean energy, and it widely exists beneath the earth’s crust with capacity equivalent to $4 \times 10^{13}$ Watts of energy [1,2]. Hence, it has remarkable potential to be an alternative to the fossil fuels. Hot dry rock (HDR) is located at 3 to 10 kilometers beneath the earth surface and can be a viable geothermal energy source. HDR is almost dry and impermeable in nature and requires the enhancement of permeability. Hydraulic fracturing is the most commonly used method to enhance its permeability, and the generated hydraulic fracture system acts as fluid pathways between the injection and production wells. Therefore, cold water is injected through injection well, flows through the fractures, extracts the heat from the surrounding rock matrix, and is ultimately produced at the production well. The fracture system also acts as heat exchange medium between the host rock matrix and injected fluid. This type of heat extraction from the geothermal HDR is also known as enhanced geothermal system (EGS) [3–6]. On the other hand, hot sedimentary aquifer (HSA) is available between 0.4 to 4 kilometers, and its energy...
The geothermal recovery from HDR by EGS or HSA is a multi-physics process that couples heat transfer, fluid flow, and mechanical deformation. To simulate the geothermal recovery process, it is necessary for HSA and its primary heat transfer mechanism is heat conduction.

As the porosity and permeability of HSA is relatively high, hydraulic fracturing is not necessary for HSA and its primary heat transfer mechanism is heat conduction.
important to consider the variation of fluid properties, rock mechanical and thermal properties at different operational conditions [4,11,12]. Existing numerical simulation models help geothermal engineers to predict the thermal front movement and production temperature behavior both in EGS and HAS. Thermo-hydro (TH), thermo-hydro-mechanical (THM), thermo-hydro-mechanical–chemical (THMCC) simulation models were developed for geothermal reservoir simulations [13–21]. However, TH models may underestimate the production temperature at lower injection rates [19]. They fail to capture the fracture aperture increase with injection during EGS [20,22], as produced fluid temperature decreases. Therefore, geothermal thermal recovery is highly impacted by the coupled thermo-hydro-mechanical physics [23]. Nevertheless, in a long time span geochemical reaction is important in specific rock types, such as sandstone or carbonate, but it is negligible in the relative short life span (30 to 40 years) of geothermal reservoirs [19,23]. Therefore, it becomes necessary to consider THM model for both EGS and HAS systems.

Operational parameters including perforation depth, well distance, well pressure, injection temperature and geo-fluid type, are the crucial for the design of EGS or HAS system [17,21,24,25]. Therefore, optimization coupled with numerical simulation has been usually utilized to optimize these operational parameters, which can effectively influence the geothermal reservoir performance. However, as the coupled simulations are computationally intensive, it becomes difficult and costly to perform optimization with coupled simulation models [21,25,26] for geothermal reservoir management, such as well placement and well control optimization [21,25,27,28]. Multi-objective Optimization (MOO) techniques have often been employed to the reservoir management application in the oil and gas industry, and have also been applied to maximize the economic value or net energy recovery [11,21,29]. Song et al. [26] proposed a Multi-objective Optimization (MOO) method that considers the injection fluid rate and temperature, and production well pressure as the decision parameters to optimize net energy recovery with TH model. Schulte et al. [24] developed a Multi-Objective Particle Swarm Optimization and the Design of Experimentation (DOE) based proxy models to optimize injection fluid temperature and well distance. Samin et al. [21] developed a hybrid optimization methodology to improve the EGS performance, and they integrated the finite element simulation model and genetic algorithm for optimizing well distance, reservoir depth and injection pressure. Martinez-Gomez et al. [30] proposed a MOO to choose suitable geo-fluid for the downstream process (i.e., organic Rankine cycle) of geothermal field with the consideration of economic, environmental and safety factors. However, most of the proposed MOO methods are based on the TH modeling [21,24,26], which makes the simulation capability very limited. Since TH models tend to underestimate the production temperature [19], it is also very necessary to consider the THM model during geothermal reservoir optimization.

On the other hand, production decline models were introduced in the oil and gas industry to forecast the fluid production rate, and it is more an empirical approach compared to full-physics reservoir simulation discussed before. These decline models provide robust prediction of flow rate with sufficient history rate data, and often were used in production forecast and reserve estimation. Further, it was adopted to the thermal recovery because of its simple principle and generalization. In literature most decline models related to geothermal recovery predict flow rate. Ripperda and Bodvarsson [31] reported that at least 5-years history rate data is necessary in order to perform a robust decline curve analysis for geothermal reservoirs. Different decline models can be appropriate to fit the hot fluid production rate with descent accuracy, including harmonic decline model [31], exponential decline model [32,33]. However, in modern geothermal recovery, it is important to keep constant production rate with high production temperature in order to maintain stable energy generation. Unless significant leakage occurs, the injection and production rates in most scenarios of EGS or HAS systems stay constant, and thus the prediction of produced fluid temperature becomes more interesting and important for geothermal reservoir management.

Based on previous research, TH simulation models were widely used in geothermal reservoir optimizations, and the lack of consideration in geomechanics resulted in inaccurate prediction of production temperature and total net energy recovery. Therefore, the use of coupled THM models for EGS and HAS is demanded. Further, the development of a decline curve model to forecast the production temperature trends in both EGS and HAS is of the utmost significance on behalf of its simplicity, efficiency and generalization. As a result, this research aims to perform the study of thermal (temperature) decline curve analysis (TDCA) based on THM simulation of both EGS and HAS systems. Further, a deep-learning based forward surrogate model is constructed to take the reservoir parameters and estimate the TDCA model parameters and total net energy recovery. We then establish the integrated optimization methodology for geothermal reservoir management by integrating the TDCA, the forward surrogate model and a multi-objective optimizer based on Non-dominated Sorting-based Genetic Algorithm II (NSGA-II) proposed by [34]. The methodologies is fully validated for different geothermal systems based on synthetic simulation results.

The rest of the manuscript is structured as following. In Section 2, we briefly introduce the mathematical formulation used in the THM modeling for both EGS and HAS scenarios. In Section 3, we present the reservoir model description, and illustrate the full details about the TDCA model (Section 3.1), the forward surrogate model (Section 3.2) and robust optimizations based on MOO (Section 3.3). In Section 4, we present the performance of the developed TDCA model (Section 4.2), feature correlation (Section 4.3). In Section 5, the results of forward surrogate model (Section 5.1) and MOO (Section 5.2) are discussed. In Section 6, we summarize the advantages and highlights of the research work.

2. Mathematical formulation for Thermal-Hydro-Mechanical process

The geothermal recovery process in EGS or HAS can be described by the heat transfer, fluid flow in the rock matrix with mechanical variations during cold fluid injection and hot fluid production. The governing equation which describes the heat transport in the rock matrix is given in Eq. (1) [17,35–37].

\[
\frac{\partial \rho \alpha T}{\partial t} + \nabla \cdot (\rho \beta \mbox{C}_p (\rho \alpha T - \rho \alpha B)) = \nabla \cdot \left( \frac{K}{\mu_e} \nabla T \right)
\]

where \(\rho \beta \mbox{C}_p\) is the effective heat capacity within the matrix; \(\lambda_{m}\) is the effective thermal conductivity within the matrix; \(\rho \alpha\) is the coupled source/sink term for matrix and fracture.

\[
(\rho \mbox{C}_p)_{v,m} \frac{\partial \lambda_{m}}{\partial t} + \nabla \cdot (\rho \alpha \lambda_{m} (\rho \alpha T - \rho \alpha B)) = -\rho \alpha \lambda_{m} \frac{\partial \lambda_{m}}{\partial t} + \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial t}
\]

\[
\frac{\partial \rho \alpha \phi}{\partial t} + \nabla \cdot (\rho \beta \mbox{C}_p (\rho \alpha T - \rho \alpha B)) = \rho \alpha \lambda_{m} \frac{\partial \lambda_{m}}{\partial t} + \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial t}
\]

The governing equation which represents the fluid flow in the rock matrix and poro-elastic storage are described in Eq. (4), and Eq. (5), respectively [17],

\[
\frac{\partial \phi}{\partial t} + \nabla \cdot (\rho \beta \mbox{C}_p (\rho \alpha T - \rho \alpha B)) = -\rho \alpha \lambda_{m} \frac{\partial \lambda_{m}}{\partial t} + \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial t}
\]

\[
\phi = \frac{\lambda_{m}}{\rho \alpha \lambda_{m} \frac{\partial \lambda_{m}}{\partial t} + \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial t}}
\]

where the first term in the right hand side of Eq. (4) represents the coupled geomechanics and fluid flow.

The Biot–Willis coefficient (\(\alpha_0\)) in Eq. (4) is presented as [38,39],

\[
\alpha_0 = 1 - \frac{K_e}{K_{fj}}
\]
The Biot’s modulus \((M)\) in Eq. (5) is presented as \([38,39]\),
\[
\frac{1}{M} = \frac{\phi_m}{K_{fj}} + \left(a_b - \frac{\phi_m}{K_d}\right)\frac{1}{K_d}
\]

The combined mathematical equation which represents the fluid flow and poroelastic behavior in rock matrix is given in Eq. (8).
\[
\rho_u \left[ \frac{\partial u_j}{\partial t} \right] - \frac{1}{K_{fj}} \frac{\partial \phi_m}{\partial t} - \nabla \cdot \left( \rho_u \left( K_m \frac{\partial \phi_m}{\partial u} \nabla p_m - \rho_u \phi \right) \right) = - \left( 1 - \frac{K_d}{K_{fj}} \right) \frac{\partial \varepsilon_{vol}}{\partial t} + k_f \frac{\partial \varepsilon_{m,f}}{\partial u} + \frac{\partial \varepsilon_{f}}{\partial u}
\]

The total strain is calculated from the thermo-poroelasticity is calculated by using Eq. (9) \([17,40,41]\).
\[
\varepsilon_{ij} = \frac{\sigma_{ij}}{2G} - \frac{v}{1-v} \frac{\partial \phi_m}{\partial u} \partial \phi_m \delta_{ij} + \frac{1}{2} \rho_j \partial \phi_m \, (T - T_{ref}) \delta_{ij}
\]

In Eq. (9), the first three terms on the right hand side represent the strain due to the flow and geomechanics, and the fourth term on the right hand side denotes the thermal strain due to temperature variation in the rock matrix and fracture.

The equilibrium of forces acting on the computational domain and strain–displacement relations are presented in Eq. (12) and Eq. (13), respectively.
\[
\nabla \cdot [\sigma_{ij}] + \phi_m \rho_u \partial u + \rho_i = 0
\]

\[
\varepsilon_{ij} = \frac{1}{2} \left( a_{ij} + a_{ji} \right)
\]

The heat transport in the fractures is represented in Eq. (14) \([40]\).
\[
d_j \left( \rho C_P \lambda_{f,c} \frac{\partial T_f}{\partial t} + d_j \left( \rho_u C_{P,u} \right) \frac{\partial T_u}{\partial t} \right) \left( \frac{\partial a_{ij}^2}{\partial R_f \mu_u} \nabla T_u \psi_f \right) \cdot \nabla T_u \psi_f
\]

\[- V_{T_u} \cdot \left( d_j \left( \lambda_{f,c} \right) \nabla V_{T_u} \psi_f \right) + d_j V \cdot \left( \lambda_{f,u} \nabla V_{T,u} \psi_f \right) = 0
\]

During geothermal recovery, cold geo-fluid is injected into the injection well, and extracts heat from the surrounding rock matrix and migrates towards the extraction well in terms of hot fluid. Due to interwell distance, flow and thermal connectivity between injection and extraction wells, the thermal front moves more slowly than pressure front. At early period the temperature of extracted geo-fluids \(T_{raw}(t)\) equals to the initial reservoir temperature \(T_{res,i}\) and it stays constant for a period until the thermal front break through in the extraction well and the temperature declines. The duration reaching to thermal breakthrough is denoted as \(t_b\).

\[Fig. 1. Temperature of extracted water from EGS versus time.\]

The governing equation to describe the fluid flow in the EGS is given in Eq. (15) \([41]\).
\[
d_j \frac{\partial \phi_m}{\partial t} - V_{T_u} \cdot \left( \frac{\rho_u d_j}{\partial R_f \mu_u} \nabla T_u \psi_f \right) = d_j \rho_u \frac{\partial a_{ij}^2}{\partial \psi_f} + d_j \frac{K_f}{\mu_u} \frac{\partial \varepsilon_{m,f}}{\partial \varepsilon_{f}}
\]

The fractures in the present work are considered as thin elastic layer. Force per unit area acting on the fracture is represented mathematically as a function of spring constant \(k_{A,f}\), damping constant per unit area \(d_{A,f}\) and fracture thickness \(t_{A,f}\) of fracture aperture \(t_{A,f}\) is given in the following Eq. (16) \([42]\).
\[
F_{A,f} = -k_{A,f} \left( u_{A,f} - u_{d,f} - u_0 \right) - d_{A,f} \frac{\partial (u_{A,f} - u_{d,f} - u_0)}{\partial t}
\]

Spring constant for unit area acting on the thin elastic layer is described in Eq. (17) \([42]\).
\[
k_A = k_{A,n} \otimes n + k_{A,f} \left( I_n - n \otimes n \right)
\]

The fracture stiffness \(k_{A,n}\) in the normal direction, and fracture shear stiffness \(k_{A,f}\) are defined as a function of both elastic modulus and Poisson’s ratios of fractures are given in Eqs. (18) and (19), respectively \([42]\).
\[
k_{n,f} = \frac{E_f (1 - v_f)}{d_f (1 + v_f) (1 - 2v_f)}
\]

\[k_{s,f} = \frac{E_f}{2d_f (1 + v_f)}\]

We used COMSOL Multiphysics to implement numerical simulations for EGS and HSA scenarios using THM (i.e., presented in 2). The implementation is presented in Appendix. The mathematical formulation and validation of the dynamic properties for rock, fracture, and fluid are presented in the Appendix Section.

3. Methodology

3.1. HyperReLU: Thermal decline model to predict extracted fluid temperature

During geothermal recovery, cold geo-fluid is injected into the injection well, and extracts heat from the surrounding rock matrix and migrates towards the extraction well in terms of hot fluid. Due to interwell distance, flow and thermal connectivity between injection and extraction wells, the thermal front moves more slowly than pressure front. At early period the temperature of extracted geo-fluids \(T_{raw}(t)\) equals to the initial reservoir temperature \(T_{res,i}\) and it stays constant for a period until the thermal front break through in the extraction well and the temperature declines. The duration reaching to thermal breakthrough is denoted as \(t_b\).

\[Fig. 1. Temperature of extracted water from EGS versus time.\]

*Case 0
Case 8
Case 15*

The initial temperature \(T_{res,i}\), thermal breakthrough time \(t_b\) and temperature declining trend of each case varies. For example, even though the well distance \(L_{w,i}\) of Case 0 (red curve) is short, the low fracture aperture \(d_f\) and low water injection rate \(q_{inj}\) delays the thermal breakthrough and leads to a constant temperature profile for 30 years. On the contrary, Case 8 (green curve) has breakthrough at only 0.78 years and declines much faster than Case 0, which likely results from the largest fracture aperture \(d_f\) and the highest water injection rate \(q_{inj}\). Compared to Case 8, Case 15 (black curve) has quite shallow decline in temperature, which can be caused by its relatively low injection rate \(q_{inj}\). Therefore, in geothermal recovery, the intricate behavior of thermal front movement is a function of variables including well design, the properties of flow, thermal and geo-mechanics of reservoir rocks and geo-fluid (e.g., water).

In the subsurface porous media flow community, engineers \([42–47]\) often leverages analytical data-driven decline models to curve-fit production history of fluids and forecast the future production from the subsurface reservoirs. This is straightforward to implement and versatile to process large volume of field production data. These decline
curvature models usually performs prediction of the declining period since the peak rate, including hyperbolic decline, exponential decline, and harmonic decline. The hyperbolic decline behavior of fluid rates [43] has been often witnessed when reservoir is supported by gas cap, water drive or injection of water and gas, and it can be expressed as Eq. (20),

\[ \frac{q(t)}{q_i} = \frac{1}{1 + b \cdot d_i \cdot t} \]  

(20)

where \( q \) is the fluid extraction rate at time \( t \); \( q_i \) is the initial (peak) fluid rate; \( d_i \) denotes the initial nominal decline rate at \( t = 0 \), which ranges between 0 and 1; \( b \) denotes the hyperbolic decline constant. Overall, \( d_i \) determines the steepness of the hyperbolic decline curve \( (q(t) \) versus \( t) \), while \( b \) factor controls the amount of its curvature or bend. The larger \( b \) factor, the more the curve bends. Notice that Eq. (20) can incorporate other scenarios such as exponential decline \((b = 0)\) and harmonic decline \((b = 1)\).

The hyperbolic decline model has only three parameters to regress during curve-fitting. Since it always calculates production rate with a decline from time zero, it is still limited to predict the dynamic behavior of thermal breakthrough, as shown in Fig. 1. This makes Eq. (20) not appropriate to predict produced fluid temperature from most of the geothermal reservoirs. To improve the capability of the hyperbolic decline model, we introduce the rectified linear unit function (ReLU) as shown in Eq. (21).

\[ \sigma_{relu}(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \]  

(21)

ReLU is a function that returns the value provided as input directly, or the value 0.0 if the input is negative. It is often used in the deep learning community as an activation function between neural network layers [48,49]. During geothermal recovery, it is critical to capture the thermal breakthrough time \( \beta_i \), at which the extracted fluid temperature \( T_{prod} \) starts declining in extracted geo-fluids. Besides, \( \beta_i \) is an important metric of geothermal performance, and thus it is beneficial to incorporate it into the decline model.

With the aids of ReLU, we replace the term \( t \) in Eq. (20) with \( \sigma_{relu}(t - \beta_i) \) in order to transform the time origin from 0 to \( \beta_i \), at which the extracted fluid temperature starts declining. As a result, the time-dependent term in Eq. (20) becomes \((b \cdot d_i \cdot \sigma_{relu}(t - \beta_i))\). This term is 0 at \( t < \beta_i \) to hold the constant temperature, while it becomes \((b \cdot d_i \cdot t)\) with a hyperbolic decline at \( t \geq \beta_i \). Moreover, since there is a widely varying steepness observed in the temperature profiles as geothermal reservoir parameters change, as shown in Fig. 1, the initial nominal decline rate \( d_i \) in Eq. (20) is re-written in the form of \( 10^{\delta_i} \). As a result, the new \( d_i \) becomes negative but with a much smaller range, which eases the training processes of the neural network models in Section 3.2. Therefore, the extended decline curve model for prediction of production temperature can be written as a \( H \) function as shown in Eq. (22) and the flow chart for the development of the HyperReLU model is depicted in Fig. 2.

\[ H(t; b, d_i, \beta_i) = \frac{T_{prod}(t)}{T_{res,i}} = \frac{1}{1 + b \cdot 10^{\delta_i} \cdot \sigma_{relu}(t - \beta_i)^T} \]  

(22)

where \( T_{prod}(t) \) is the extracted fluid temperature at time \( t \); \( T_{res,i} \) is the initial reservoir temperature, which is usually provided from measurements; \( \beta_i \) inside the ReLU function is the thermal breakthrough time; \( b \) denotes the hyperbolic decline constant; \( d_i \) is a control factor of the initial nominal decline rate at \( t = \beta_i \). The variables of \( b, d_i, \beta_i \) are the fitting parameters for the time series of the extracted fluid temperature. To simplify the parameterization, we group these fitting parameters in \( H \) as a curve-fitting parameter vector \( \tilde{\theta} \), shown as, \n
\[ \tilde{\theta} = [b, d_i, \beta_i]^T \]  

(23)

To favor the parameterization, we also define an intermediate variable called the extracted fluid temperature drop \( \eta(t) \), shown as Eq. (24).

\[ \eta(t) = 1 - \frac{T_{prod}(t)}{T_{res,i}} \]  

(24)
Based on the characteristics of the extended decline model, we name it as Hyperbolic-ReLU Decline or \textit{HyperReLU} for the sake of a simple nomenclature. Compared to traditional decline model, we highlight here \textit{HyperReLU} brings four folds of benefits: (1) \( \beta_t \) here represents the thermal breakthrough time, and is coupled in the new model through the \textit{ReLU} function, which directly shows the timing the cold fluid breakthrough. However, traditional decline model such as Arps model Eq. (20) cannot achieve this; (2) \( \beta_t \) term helps us to directly evaluate the impact of reservoir model parameters on cold fluid breakthrough; (3) \( \beta_t \) term benefits the robust optimization for geothermal, for instance, we directly include \( \beta_t \) as a nonlinear constraint; (4) the \textit{ReLU} term itself is also applicable in other breakthrough and depletion processes, where the decline stage may not necessarily follow Arps model.

With geothermal simulation or field data for produced fluid temperature, the parameters of \textit{HyperReLU} in Eq. (23) is based on the non-linear least squares method to perform the curve-fitting. The solver \textit{HyperlReLU} directly include term benefits the robust optimization for geothermal, for instance, we consider it as Hyperbolic-ReLU Decline or \textit{HyperReLU} in Eq. (23). This procedure brings two advantages: (1) it reduces the dimensionality of the output \( \tilde{y} \) significantly; (2) it predicts the essential physical parameter of \( \beta_t \), which eases the sensitivity analysis of this parameter if we need. Therefore, the low-fidelity forward surrogate model can be represented as Eq. (27).

\[ \tilde{\phi}, W(t_{i,\bar{a}}) = f_i(\tilde{m}) \]  
\[ Eq. (28) \text{ can be used to further obtain the time series of } \frac{\tau_{\text{prod}}(t)}{\tau_{\text{res}}}, \frac{\tau_{\text{prod}}(t)}{\tau_{\text{res}}} = H(t; b, d_i, \beta_t) = H(t; \tilde{\phi}) \tag{28} \]

In Eq. (27), the number of input variables (the length of \( \tilde{m} \)) is small, for example, 9 parameters for EGS or 13 parameters for HSA, and the number of output variables (the length of vector [\( \tilde{\phi}, W(t_{i,\bar{a}}) \]) is 4. Therefore, \( f_i \) here is a multi-variable regression problem and can be approached through the fully connected neural network (FCN), \( \mathcal{N} \mathcal{N} \). We clarify that for more complex heterogeneous problems, image-based neural networks such as CNN are recommended to encode heterogeneous property maps, which can be referred to our previous work [52,53].

Mathematically, \( \mathcal{N} \mathcal{N} \) is a layered structure with \( d \) layers, and the \( i \)-layer \( l_i \) is a function of its previous layer \( l_{i-1} \) through an activation function \( \sigma \), as shown in Eq. (29).

\[ [\tilde{\phi}, W(t_{i,\bar{a}})] = \mathcal{N} \mathcal{N}(\tilde{m}; \bar{\sigma}) = \sigma(l_{i,\sigma}(\ldots \sigma(l_1(\sigma(\tilde{m})(\ldots))) \ldots)) \tag{29} \]

\[ \sigma_{\text{tanh}}(a, x) = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}} \tag{30} \]

where \( \theta \) are the hyper-parameters of \( \mathcal{N} \mathcal{N} \), which consists of the bias and weight matrices; \( \sigma \) is a nonlinear activation function, and the adaptive hyperbolic tangent \( \sigma_{\text{tanh}} \) function in Eq. (30) is used for \( \sigma \) [52,54], which is beneficial to avoid gradient exploding or vanishing in the training backpropagation. Hereafter we use \( \mathcal{N} \mathcal{N} \) to replace \( f_i \), since in this work our \( f_i \) is developed based on deep neural network.

For both EGS and HSA modeling, we set 9 hidden layers and there are 20 neurons per hidden layer, and the workflow of the forward surrogate model in EGS is shown in Fig. 3. At first, \( \mathcal{N} \mathcal{N} \) takes \( \tilde{m} \) and predict \( [\tilde{\phi}, W(t_{i,\bar{a}})] \). Further, the extracted fluid temperature \( T_{\text{prod}}(t) \) is predicted via the \textit{HyperReLU} function \( H \). The workflow of the forward surrogate model of HSA only differs in the number of input neurons in \( \mathcal{N} \mathcal{N} \), which is not presented for conciseness.

Since the ranges of the input and output variables in \( \mathcal{N} \mathcal{N} \) are significantly different, they are normalized to be between 0 and 1, by Eq. (31). This helps to ensure that different parameters are scaled into a similar scale, and thus ease the training process based on gradient descent method.

\[ x' = \frac{x - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} \tag{31} \]

The loss function \( \mathcal{L} \) during training \( \mathcal{N} \mathcal{N} \) is defined as,

\[ \mathcal{L}(\theta) = ||\tilde{\phi} - \bar{\phi}|| + ||W(t_{i,\bar{a}}) - W(t_{i,\bar{a}})|| + \lambda \cdot ||H(t_{i,\bar{a}}; \tilde{\phi}) - H(t_{i,\bar{a}}; \bar{\phi})|| \tag{32} \]

where \( || \cdot || \) is the root-mean-square-operator; \( y (\tilde{\phi} \text{ and } W(t_{i,\bar{a}})) \) denotes the ground truth from reservoir simulation, and \( \tilde{y} \) denotes the prediction of \( y \) by \( \mathcal{N} \mathcal{N} \); the first and second terms are data-driven loss for mismatch; the third term is to regularize \( \mathcal{N} \mathcal{N} \) by the loss of predicted temperature drop based on \textit{HyperReLU} function, and \( \lambda \) is the regularization coefficient.

Training of the \( \mathcal{N} \mathcal{N} \) with an architecture shown in Fig. 3 is based on the Adam optimizer [49], and the goal is to find the learnable parameters \( \theta^* \) by minimizing \( \mathcal{L}(\theta) \). The implementation of \( \mathcal{N} \mathcal{N} \) and its training procedures is based on the deep learning library PyTorch [49].
parameters are considered as decision parameter vector \( \vec{m} \), which is ultimately required to be determined by engineers, and such type of reservoir parameters we label them as given model parameter vector \( \vec{g} \), injection fluid mass rate \( \vec{m}_i \), initial reservoir pressure \( p_{r0} \), and temperature \( \vec{T}_{\text{res}} \). For such type of reservoir model parameters, we parameterize them as uncertainty parameter \( \vec{u} \). Therefore, the original reservoir model parameter vector \( \vec{m} \) is represented as a concatenation or combination of 3 vectors, including \( \vec{m}_u, \vec{m}_i, \vec{m}_d \),

\[
\vec{m} = [\vec{m}_u, \vec{m}_i, \vec{m}_d]^T
\]

where,

\[
\vec{m}_u = [\vec{A}_t, \vec{d}_f, \vec{c}_pr]^T
\]

\[
\vec{m}_i = [ \vec{T}_{\text{res}}, \vec{p}_{\text{res}}]^T
\]

\[
\vec{m}_d = [\vec{L}_{\text{inj}}, \vec{T}_{\text{inj}}, \vec{q}_{\text{inj}}, \vec{p}_{\text{prod}}]^T
\]

The goal of geothermal optimization is to determine \( \vec{m}_u \) when achieving the maximum reservoir performance metric or objective function \( J \) with considering the uncertainty in \( \vec{m}_u \), so it is a robust optimization [40]. According to Eq. (26), the total net power \( W(t_f) \) is an effective metrics to quantify the total energy harvested from a geothermal reservoir until well abandonment, and selected as the reservoir performance objective \( J \). Given the uncertainty in \( \vec{m}_u \), a ensemble of stochastic forward models is evaluated and thus we obtain an ensemble of \( J \) correspondingly. Therefore, both the expected value, \( E(J) \), and its corresponding risk or standard deviation \( \sigma(J) \), are considered as the objective functions during the reservoir optimization,

\[
E(J) = \frac{1}{N} \sum_{i=1}^{N} J(\vec{m}_i), \quad \sigma(J) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (E(J) - J_i)^2},
\]

where \( N \) is the total number of realizations for uncertainty parameter \( \vec{m}_u \).

The nonlinear constraints are listed in Eqs. (38) and (39) as below,

\[
\beta_i^* = \min_{1 \leq i \leq N} (\beta_i) \leq 0, \quad \text{max}_{1 \leq i \leq N} (\eta(t_a)) - \eta^* \leq 0,
\]

where \( t_a \) is the abandonment time of geothermal reservoir; Eq. (38) denotes that the thermal breakthrough time \( \beta_i \) of all the \( N \) realizations is higher than a lower limit \( \beta_i^* \), so we can produce hot-fluid at stable temperature before \( \beta_i^* \); Eq. (39) stands that the abandonment extracted fluid temperature drop \( (\eta(t_a)) \) of all the \( N \) realizations is lower than an upper limit \( \eta^* \).

Both Eqs. (38) (39) are nonlinear constraints, as \( \beta_i \) and \( \eta(t_a) \) are calculated from the forward simulators \( f_j \) or surrogate models \( f_j, \lambda \). Here we strictly impose these constraints on all the realizations of \( \vec{m}_u \), since we have only 2 constraints here. However, when the number of nonlinear constraints is high, alternatively these constraints can be formulated as a penalty term for joint optimization [40]. The ultimate goal of the reservoir optimization is to maximize the reservoir performance objective \( E(J) \) (or minimize objective \( -E(J) \)) and minimize the risk objective \( \sigma(J) \) simultaneously, which becomes a MOO problem. Therefore, this can be solved by the aforementioned NSGA-II algorithm. The overall flow structure of the \( \mathcal{N} \mathcal{N} \) model and the reservoir optimization is presented in the Fig. 5.

Fig. 3. Forward surrogate model workflow in EGS.

Fig. 4. Mapping from decision space into objective space. Modified after [56].
4. Thermal decline curve analysis

4.1. Data preparation

The size of reservoir domain is 2000 × 2000 m², for the EGS scenario. The injection well and production well are connected with fracture and it will act as the primary fluid flow conduit. The matrix is considered as homogeneous initially with a porosity of 0.01 and a permeability of 1 × 10⁻¹⁸ m². Variation of rock porosity, permeability, elastic modulus and other properties employed in the present work and mathematical formulation is presented in Appendix A. The length of the fracture and fracture aperture are considered as model parameters and ranges are presented in Table 2. Quad mesh was employed with mapped structure to all the EGS scenarios. We run total 5000 simulation with different combinations of the parameters mentioned in Table 2 for deep learning purpose.

For the HSA scenarios, the reservoir domain is 3000 × 30000 × 200 m³ located at 2800 m (i.e., top of the reservoir) from the earth surface. Vertical wells (50 m) are considered for both injection and production operation for HSA scenario. The distance between the injection and production wells \( L_w \) and the ranges are presented in Table 3 and located 75 m distance from the reservoir top. The ranges of matrix porosity, permeability, elastic modulus and other properties employed for the HSA simulations are presented in Table 3.

In order to generate a decent physics simulation database for thermal decline curve analysis (TDCA), deep learning model training and optimization, we run in total 5000 THM simulation for EGS and 3000 simulations for HSA system using Latin Hypercube sampling.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Model parameters for EGS simulation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_s )</td>
<td>W/m/K</td>
</tr>
<tr>
<td>( d_f )</td>
<td>mm</td>
</tr>
<tr>
<td>( c_p )</td>
<td>J/kg/K</td>
</tr>
<tr>
<td>( T_{inj} )</td>
<td>°C</td>
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<td>( L_w )</td>
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<td>( \phi_{inj} )</td>
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<td>( p_{prod} )</td>
<td>MPa</td>
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<table>
<thead>
<tr>
<th>Table 3</th>
<th>Model parameters for HSA simulation.</th>
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</thead>
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<td>( T_{inj} )</td>
<td>°C or K</td>
</tr>
<tr>
<td>( \phi_{inj} )</td>
<td>kg/s</td>
</tr>
<tr>
<td>( L_w )</td>
<td>m</td>
</tr>
<tr>
<td>( p_{inj} )</td>
<td>MPa</td>
</tr>
<tr>
<td>( \alpha_p )</td>
<td>–</td>
</tr>
<tr>
<td>( C_p )</td>
<td>J/kg/K</td>
</tr>
<tr>
<td>( \phi_{rock} )</td>
<td>–</td>
</tr>
<tr>
<td>( \rho_{rock} )</td>
<td>mD</td>
</tr>
<tr>
<td>( \mu_{rock} )</td>
<td>GPa</td>
</tr>
<tr>
<td>( \nu )</td>
<td>–</td>
</tr>
<tr>
<td>( L_w = \frac{w}{\nu} )</td>
<td>–</td>
</tr>
<tr>
<td>( F_i )</td>
<td>MPa</td>
</tr>
</tbody>
</table>

Table 4

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>EGS: ( \mu \pm \sigma )</th>
<th>HSA: ( \mu \pm \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>%</td>
<td>0.39 ± 0.89</td>
<td>0.035 ± 0.074</td>
</tr>
<tr>
<td>( \eta(t_i) )</td>
<td>%</td>
<td>15.00 ± 19.00</td>
<td>0.00 ± 1.00</td>
</tr>
<tr>
<td>( W(t_i) )</td>
<td>10⁶ kWh</td>
<td>0.43 ± 0.20</td>
<td>0.93 ± 0.29</td>
</tr>
<tr>
<td>( \beta )</td>
<td>years</td>
<td>17.01 ± 0.26</td>
<td>47.04 ± 7.86</td>
</tr>
<tr>
<td>( b )</td>
<td>–</td>
<td>3.44 ± 0.88</td>
<td>2.73 ± 0.97</td>
</tr>
<tr>
<td>( \Delta t )</td>
<td>–</td>
<td>-1.76 ± 0.48</td>
<td>-2.05 ± 0.14</td>
</tr>
</tbody>
</table>

4.2. Thermal decline curve analysis results of EGS and HSA

Based on the physics-based simulation dataset, we perform thermal decline curve analysis (TDCA) based on the HyperReLU model. We use the non-linear least squares to fit the HyperReLU function in order to determine the three fitting parameters in Eq. (22). For EGS simulation, it takes 15.08 s to fit 2638 simulation cases, with average CPU cost 3.66 × 10⁻³ seconds/case. For HSA simulation, it takes 76.08 s to fit 2638 simulation cases, with average CPU cost 2.88 × 10⁻² seconds/case. Therefore, the curve-fitting process is very efficient such that it is not necessary to parallel the fitting process. Following Eq. (26), we also calculate the total energy \( W(t_i) \) of each case through numerical integration based on the composite trapezoidal rule.

Table 4 presents the mean (\( \mu \)) and standard deviation (\( \sigma \)) of fitting error, HyperReLU parameters (\( b, \Delta t, \beta \)), \( \eta(t_i) \) and total energy \( W(t_i) \).
HyperReLU presents extremely low curve-fitting error, with EGS error 0.39 ± 0.89% and HSA error 0.053 ± 0.074%. The error distributions are shown in Fig. 6(a). This basically guarantees the high fidelity to use HyperReLU to encode the time-series data of extracted fluid temperature. b factor in HyperReLU (Fig. 6(b)) ranges from 0 to 7. Based on the original decline type classification in [43], it basically covers exponential, hyperbolic and harmonic declines. \(d_i\) in Fig. 6(c) is a negative value between −3.0 and 0.0, which indicates the temperature profiles include different levels steepness of temperature decline. In HSA scenario, \(d_i = −2.05 ± 0.14\) explains that HSA has relatively shallow temperature decline, since both heat conduction and advection contributes to geo-fluid temperature dynamics. \(β\) in Fig. 6(d) is the time of thermal breakthrough and decides the duration of constant extracted temperature. In both EGS and HSA simulations, the peak values of \(b = 3.0\) and \(d_i = −2.0\) are because these cases with constant temperature converges to the initial guess of \(\alpha = 0\) during curve-fitting.

Further, we present the histograms of \(\eta(t_u)\) and total energy \(W(t_u)\) when the geothermal wells are abandoned at \(t_u = 30\) years for EGS and \(t_u = 50\) years for HSA. \(\eta(t_u)\) in HSA (red) is relatively low (Fig. 6(e)), since in the simulations HSA exhibits a long duration of constant temperature (Fig. 6(d)). On the other hand, \(\eta(t_u)\) in EGS (green) changes from 0% to 70%, which is caused by its relatively early thermal breakthrough (Fig. 6(d)). Finally, the predicted final total energy \(W(t_u)\) is shown in Fig. 6(f), and the difference of \(W(t_u)\) between EGS and HSA is mainly caused by their difference in \(t_u\).

In Fig. 7, we present the comparison of extracted fluid temperature profile between COMSOL and HyperReLU. To demonstrate the capability of HyperReLU, we select 3 representative cases of EGS and HSA with different thermal breakthrough time and also extracted fluid temperature decline trends. HyperReLU can capture the thermal breakthrough time very well with the aids of the \(σ_{relu}\) in Eq. (21). Compared to COMSOL reservoir simulation, HyperReLU has a sharp transition at the thermal breakthrough, which is also caused by \(σ_{relu}\). The hyperbolic form of Eq. (22) makes HyperReLU excel in capturing either shallow or steep declining with early stable temperature occurring in both EGS and HSA. The maximum relative error in both scenarios with three cases is less than 0.4%, which successfully demonstrates the power of HyperReLU.

### 4.3. Feature correlation and importance analysis

As TDCA can extract performance variables such as \(β\), \(W(t_u)\), and \(\eta(t_u)\), we further evaluate the impact of different reservoir parameters on these metrics with different methods in two different ways. Firstly, we train 3 different machine learning models based on the gradient boost model (GBM) with 50 estimators [57]. In EGS scenario, the prediction accuracy in terms of \(R^2\) score is 0.943 for \(β\), 0.965 for \(\eta(t_u)\), and 0.936 for \(W(t_u)\). In HSA scenario, the prediction accuracy in terms of \(R^2\) score is 0.976 for \(β\), 0.913 for \(\eta(t_u)\), and 0.992 for \(W(t_u)\). Based on these accurate machine learning models for specific labels, we perform a feature importance analysis to the individual labels. Secondly, we compute pairwise correlation between the performance variables and the input reservoir model parameters based on Pearson correlation coefficient. Pearson correlation coefficient \(ρ_{X,Y}\) defines as the ratio between their covariance \(cov(X,Y)\) and the product of their standard deviations \((σ_X \cdot σ_Y)\), shown as Eq. (40). We expect that the ranking of the absolute value of feature correlation by Pearson correlation is basically consistent with the ranking of the feature importance by GBM.

\[
ρ_{X,Y} = \frac{cov(X,Y)}{σ_X \cdot σ_Y} \quad (40)
\]

The feature importance and correlation of \(β\) are shown in Figs. 8(a) and 8(d). \(d_i\), the fracture aperture, is strongly negatively correlated with \(β\) as the most important factor, since it determines fracture...
permeability based on the cubic law, and higher fracture permeability will accelerate the thermal breakthrough. $L_{in}$, the inter-well distance, is positively correlated to $\beta_i$ as the second most important variable, since higher $L_{in}$ means longer the migration distance for cold fluid front before breakthrough. $q_{inj}$, the injection mass rate, determines the fluid flow velocity in the fracture in EGS, and higher $q_{inj}$ can accelerate the thermal breakthrough with lower $\beta_i$, which makes it the third most important factor to negatively correlate to $\beta_i$.

Figs. 8(b) and 8(c) present the feature importance and correlation of $\eta(t_a)$, respectively. $L_{in}$ is negatively correlated with $\eta(t_a)$ as the most important feature to $\eta(t_a)$, since larger $L_{in}$ leads longer $\beta_i$ (Fig. 8(d)) and thus shallower temperature decline with lower $\eta(t_a)$. $d_f$ is the second most important feature to $\eta(t_a)$ and positively correlates with it, since higher $d_f$ increases fracture permeability and accelerate the movement of cold fluid front, and thus leads to more rapid extracted fluid temperature decrease with high $\eta(t_a)$. $q_{inj}$ is the third most important factor and positively correlates with $\eta(t_a)$. Similar to $d_f$, higher $q_{inj}$ can also accelerate the movement of cold fluid front, and thus the extracted fluid temperature drops faster, which results in higher $\eta(t_a)$.

The feature importance and correlation of $W(t_a)$ are shown in Figs. 8(c) and 8(f). The most important factor for $W(t_a)$ in EGS becomes $q_{inj}$, because in EGS $q_{inj}$ basically equals to fluid production rate $q_{prod}$, which is positively correlated to $W(t_a)$ based on Eq. (26). $T_{res}$ is the second most important feature to $W(t_a)$, since it determines the upper limit of $T_{prod}$, which positively impacts $W(t_a)$ in Eq. (26). Longer $L_{in}$ leads to longer $\beta_i$ with $T_{prod}(t) = T_{res}$, which makes it also positively correlates to $W(t_a)$. Higher $d_f$ represents higher fracture conductivity, which shortens the thermal breakthrough time $\beta_i$ with $T_{prod}(t) = T_{res}$ (Fig. 8(d)), which makes it negatively correlate to $W(t_a)$.

Other EGS model parameters contributes much less to $\beta_i$ and $W(t_a)$ than above parameters, so we will not go through the details. Overall, the dominant features to $\beta_i$, $W(t_a)$, and $\eta(t_a)$ are $L_{in}$, $q_{inj}$, $d_f$, and $T_{res}$, and during forward model construction it is essential to consider these parameters as a result. Besides, $L_{in}$ and $q_{inj}$ are engineering decision parameters and it is essential to perform optimization and find the optimized design of these parameters.

In HSA, since there is not fracture in the geothermal reservoir, the top two features contributing to $\beta_i$ becomes $L_{in}$ and $q_{inj}$ (Figs. 8(g) and 8(j)), which is quite consistent with that in EGS (Figs. 8(a) and 8(d)). In Figs. 8(b) and 8(k), the rock heat capacity $c_p$ is negatively correlated to $\eta(t_a)$ as the third most important feature in addition to $L_{in}$ and $q_{inj}$, because high $c_p$ helps stabilize the extracted fluid temperature due to the advection and conduction in the rock matrix. This is different from that in EGS we mainly consider heat conduction in the rock matrix. Finally, $W(t_a)$ in HSA is positively correlated to $\frac{d}{dz}$ similar to $T_{res}$ in EGS, because with a fixed reservoir depth $Z$, we have the reservoir temperature $T_{res} = T_{surf} + Z \cdot \frac{d}{dz}$. Besides, $T_{inj}$ becomes an important feature to negatively correlate with $W(t_a)$ since it is involved in the total energy calculation based on Eq. (26).

5. Forward surrogate model and reservoir optimization of EGS

5.1. Forward surrogate model

After the feature importance, we train the forward surrogate model based on the input reservoir model parameters and the output state variables. The forward surrogate model is generalized to predict the temperature profile and total energy from geothermal reservoirs. In this work, we take the EGS as an example, and build the forward surrogate model ($\mathbb{N}^F \mathbb{V}$) following Eq. (29). Therefore, the reservoir model parameter vector $\bar{m}$ in EGS include 9 parameters,

$$\bar{m} = [L_{in}, d_f, c_p, T_{res}, p_{res}, L_{in}, T_{inj}, q_{inj}, p_{prod}]$$

(41)

The corresponding loss function follows Eq. (32), and we set the regularization coefficient $\lambda = 1$. When training $\mathbb{N}^F \mathbb{V}$ with Adam optimizer [49], we use a learning rate of $10^{-3}$ and weight decay of $10^{-4}$, and total number of training epochs is 20000. The training process is quite efficient in the GPU environment (NVIDIA RTX A6000), with total training CPU time 1233.75 s. The change of loss function with training epochs is plotted in Fig. 9. Overall we see a good convergence of both training and validation loss functions in $RMSE$, and both of them follow the same trends without over-fitting.

The prediction of $\mathbb{N}^F \mathbb{V}$ is very efficient, since we can perform prediction in a batch mode with 10000 samples/batch. The total prediction time for 412 testing cases is 0.962 s, with average CPU cost $2.3 \times 10^{-3}$ seconds/case. Based on the 412 testing cases, we further compare the predictions by $\mathbb{N}^F \mathbb{V}$ with the ground truth from the curve-fitting based on $HyperReLU$ function, and the results are presented in Fig. 10. In Fig. 10(a), the accuracy of the total energy $W(t_a)$ is high with $R^2 = 0.998$ and $RMSE = 9.23 \times 10^3$ kWh. This ensures the optimization quality when we optimize engineering decision parameters to maximize $W(t_a)$. Overall, the fitting parameters for $HyperReLU$ are also accurately predicted, especially the $R^2$ scores of $d_i$ and $\beta_i$ (Figs. 10(b) and 10(c)) reach up to 0.998. A factor in $HyperReLU$ (Figs. 10(d)) is slightly less accurate with $R^2$ score 0.946 than the previous two parameters in $HyperReLU$. We consider $\mathbb{N}^F \mathbb{V}$ predicts $b$ less accurately at the low ($b < 3.0$) and high ($b > 6.0$) value ranges, but this will not impact too much in our optimization accuracy. A potential strategy to further improve this is to increase the training samples with $b$ values falling into the two ranges.
Fig. 8. Feature importance and correlation of $\beta_t$, $\eta(t_a)$, $W(t_a)$ in EGS and HSA.
To gauge the accuracy of $\mathcal{N}\mathcal{N}$ in temperature prediction, we use the reservoir parameter model vector $\vec{m}$ to feed into $\mathcal{N}\mathcal{N}$ and predict the $\text{HyperReLU}$ parameters, and a predicted temperature profile based on $\mathcal{N}\mathcal{N}$ can be obtained, which is the $\mathcal{N}\mathcal{N}$ scenario. We also present the extracted fluid temperature from COMSOL simulation, namely COMSOL scenario, and the direct $\text{HyperReLU}$ curve-fitted temperature based on COMSOL simulation results, namely $\text{HyperReLU}$ scenario. Using the three typical EGS cases in Fig. 7(a), we present the temperature profiles by the above three scenarios in Fig. 11(a). The prediction by $\mathcal{N}\mathcal{N}$ is very close to the ground truth of COMSOL in the three different decline trends, and sometimes $\mathcal{N}\mathcal{N}$ is even more stable than $\text{HyperReLU}$; for instance, Case 47. The relative errors of the three cases and all the 419 testing cases are presented in Table 5. It shows that overall the error of $\mathcal{N}\mathcal{N}$ is slightly higher than that of $\text{HyperReLU}$, and their error distributions are also quite similar to each other (Fig. 11(b)).

### 5.2. Robust optimization

As we developed an accurate forward surrogate model $\mathcal{N}\mathcal{N}$ in Section 5.1, we further apply it for robust reservoir optimization. In the EGS optimization, we consider three different sets of parameters, as mentioned in Eq. (35). The given model parameter vector $\vec{m}_g = [T_{res}, P_{res}]^T$ is given and varies for each optimization task. Further, the uncertainty parameter vector is $\vec{m}_u = [\lambda_1, d_1, c_p]^T$. The MOO framework in this work is generalized to handle different distributions of the uncertainty parameters. We consider that these three uncertainty parameters above ($\lambda_1$, $d_1$, and $c_p$) can be estimated based on reservoir history matching or lab experiments, and assume that they follow Gaussian distribution. The mean and standard deviation of each parameter is specified in Table 6. We use Latin-Hypercube design to perform the sampling of these three parameters and in total generate $N = 50$ realizations, and their distributions are presented in Fig. 12. Finally, the decision parameter vector $\vec{m}_d = [T_{res}, T_{ini}, q_{ini}, P_{prod}]^T$ are about the well design and control parameters. The searching ranges of each parameters during MOO are specified in Table 7.

For each optimization (MOO) task, $\vec{m}_u$ is given and the uncertainty of $\vec{m}_u$ is considered, the optimization goal is to optimize $\vec{m}_g$ with maximizing the expectation $E(W(t_j))$ and minimizing the standard deviation $\sigma(W(t_j))$, where $W(t_j)$ is the total energy recovered from the EGS after 30 years. Hereafter we use $W$ to stand for $W(t_j)$ for conciseness. The optimization is subjected to two non-linear constraints described in Eqs. (38) and (39), and we set $\beta^* = 1$ year, and $\eta^* = 60\%$.

When searching for the Pareto Front, MOO requires a large number of simulation evaluations based on the forward models. We set the population size to be 200 and the maximum number of iterations to be 200, and thus in total 20000 simulation runs are required for MOO. The computational cost is prohibitively high for physics-based reservoir simulator such as COMSOL, mainly coming from grid discretization, linear/nonlinear solvers etc. Based on our numerical experiments, the average CPU time per a single COMSOL simulation takes 3.0 min. If we optimistically assume the 200 COMSOL simulations at each MOO iteration can run in parallel and be completed in 3 min, 200 iterations will take 600 minutes/task to converge MOO. This is an optimistic CPU cost estimation for COMSOL, since in reality the parallel computing for these coupled simulation models will also be greatly constrained by memory limitation. On the other hand, since $\mathcal{N}\mathcal{N}$ can be evaluated in a scalable way (batch-mode) in the GPU environment, the average CPU time for a single prediction based on $\mathcal{N}\mathcal{N}$ is $2.3 \times 10^{-3}$ s, which is remarkably faster than COMSOL. If optimizing by $\mathcal{N}\mathcal{N}$, the actual average CPU time per optimization task (200 iterations) based on $\mathcal{N}\mathcal{N}$ is 28.32 seconds/task or 0.47 minutes/task. Therefore, MOO based on $\mathcal{N}\mathcal{N}$ can be as high as 1277 times faster for an optimization task than MOO based on physics-based reservoir simulator such as COMSOL. Therefore, the developed forward surrogate model $\mathcal{N}\mathcal{N}$ becomes a great alternative option for such intensive optimization tasks. We adopt MOO based on $\mathcal{N}\mathcal{N}$ for EGS reservoir performance optimization, and benchmark its ultimate optimized results with COMSOL with the same setup of $\vec{m}_g, \vec{m}_u, \vec{m}_d$.

We present two optimization examples with different initial reservoir temperature ($T_{res}$) and pressure ($p_{res}$) conditions. The Pareto Front (maroon dots) after optimization are shown in Fig. 13. Every single data point of $E(W)$ and $\sigma(W)$ in Fig. 13 is based on the aggregation of $N = 50$ uncertainty realizations, and on the Pareto front there is a trade-off between $E(W)$ and $\sigma(W)$. Therefore, MOO can provide us with multiple solutions on the Pareto Front, we choose low (blue), median (green) and high (red) risk levels of solutions based on the minimum, median and maximum of $\sigma(W)$. The high risk solution (red) has the highest $E(W)$ and $\sigma(W)$, which is an optimistic solution with more uncertainty to achieve. The low risk solution (blue) has the lowest $E(W)$ and $\sigma(W)$, which is often a very conservative solution with least uncertainty to achieve. The median solution strikes well between $E(W)$ and $\sigma(W)$, and thus it is our recommended solution.

Table 6 shows the mean and standard deviation to generate $N = 50$ samples of $\vec{m}_u$ following Gaussian distribution.

### Table 6

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.1</td>
</tr>
<tr>
<td>$d_1$</td>
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</tr>
<tr>
<td>$c_p$</td>
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<td>10.0</td>
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Table 7 shows the ranges of decision parameter vector $\vec{m}_d$ during MOO.

### Table 7

<table>
<thead>
<tr>
<th>Variable</th>
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<th>Minimum value</th>
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<td>50.0</td>
</tr>
<tr>
<td>$q_{ini}$</td>
<td>kg/s</td>
<td>10.0</td>
<td>50.0</td>
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<tr>
<td>$P_{prod}$</td>
<td>MPa</td>
<td>2.5</td>
<td>10.0</td>
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</table>
standard deviation $\sigma(W)$. In both cases, their low-risk solution has the lowest value of $\sigma(W) = 1.0 \times 10^5$ Wh, yet it yields also lowest $E(W)$, e.g., $408.35 \times 10^6$ Wh in Case 30. With risk level rising up, we see gradually increasing $E(W)$ and $\sigma(W)$. As we observed in Figs. 8(c) and 8(f), the decision parameters $q_{\text{inj}}$ and $L_w$ are two of the most important and positively correlated features to impact $W(t_a)$ in EGS, their values basically increases as risk level or $E(W)$ increases until reaching to their upper limit in the high-risk scenario, with $L_w \approx 550$.

Fig. 10. Parity plot of parameters predicted by forward surrogate model $\mathcal{X}_N$.

(a) Parity plot of total energy $W(t_a)$.  
(b) Parity plot of $d_i$.  
(c) Parity plot of $\beta_i$.  
(d) Parity plot of $b$.

Fig. 11. Accuracy of $\mathcal{X}_N$.

(a) Temperature profiles based on COMSOL, $\text{HyperReLU}$, and $\mathcal{X}_N$.  
(b) Relative error comparison between $\text{HyperReLU}$ and $\mathcal{X}_N$.  

<table>
<thead>
<tr>
<th>Error (%)</th>
<th>HyperReLU</th>
<th>COMSOL: Case 47</th>
<th>HyperReLU: Case 47</th>
<th>NN: Case 47</th>
<th>COMSOL: Case 4090</th>
<th>HyperReLU: Case 4090</th>
<th>NN: Case 4090</th>
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<tbody>
<tr>
<td>0</td>
<td>0.39 ± 0.89%</td>
<td>0.39 ± 0.89%</td>
<td>0.49 ± 1.12%</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td>0.39 ± 0.89%</td>
<td>0.39 ± 0.89%</td>
<td>0.49 ± 1.12%</td>
<td></td>
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<tr>
<td>10</td>
<td>0.39 ± 0.89%</td>
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<td>0.49 ± 1.12%</td>
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<tr>
<td>15</td>
<td>0.39 ± 0.89%</td>
<td>0.39 ± 0.89%</td>
<td>0.49 ± 1.12%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.39 ± 0.89%</td>
<td>0.39 ± 0.89%</td>
<td>0.49 ± 1.12%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0.39 ± 0.89%</td>
<td>0.39 ± 0.89%</td>
<td>0.49 ± 1.12%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Optimization of EGS. \( \bar{m}_p \) in Case 30: \( T_{res} = 219.12 \, ^\circ C, P_{res} = 15.79 \, MPa; \bar{m}_p \) in Case 150: \( T_{res} = 186.80 \, ^\circ C, P_{res} = 18.03 \, MPa. 

<table>
<thead>
<tr>
<th>Case</th>
<th>Risk</th>
<th>( L_m )</th>
<th>( T_{ini} )</th>
<th>( q_{ini} )</th>
<th>( P_{prod} )</th>
<th>( E(W) )</th>
<th>( \sigma(W) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Low</td>
<td>491.58</td>
<td>32.79</td>
<td>18.54</td>
<td>2.50</td>
<td>408.35</td>
<td>0.10</td>
</tr>
<tr>
<td>30</td>
<td>Median</td>
<td>549.93</td>
<td>36.68</td>
<td>43.42</td>
<td>9.90</td>
<td>856.27</td>
<td>3.89</td>
</tr>
<tr>
<td>30</td>
<td>High</td>
<td>549.93</td>
<td>25.01</td>
<td>50.00</td>
<td>2.50</td>
<td>1056.57</td>
<td>12.55</td>
</tr>
<tr>
<td>150</td>
<td>Low</td>
<td>296.10</td>
<td>44.41</td>
<td>18.63</td>
<td>4.25</td>
<td>303.36</td>
<td>0.10</td>
</tr>
<tr>
<td>150</td>
<td>Median</td>
<td>549.92</td>
<td>27.62</td>
<td>43.09</td>
<td>9.76</td>
<td>769.59</td>
<td>4.47</td>
</tr>
<tr>
<td>150</td>
<td>High</td>
<td>549.93</td>
<td>25.01</td>
<td>50.00</td>
<td>2.50</td>
<td>925.89</td>
<td>13.03</td>
</tr>
</tbody>
</table>

\( m \) and \( q_{ini} \) = 50 kg/s, \( T_{ini} \) is minor important to \( W(t) \) based on the feature important analysis, and it hits to its lower limit in the high-risk scenario due to its negative correlation with \( W(t) \) (Fig. 8(f)). As for \( P_{prod} \), it is the least important and least correlated feature to \( W(t) \) among the reservoir model parameters, so its change in different risk levels does not exhibit a monotonic trend. In a nutshell, if following the recommended median-risk decision parameters to operate EGS, Case 30 can ultimately deliver \( W(t) = (856.27 \pm 3.89) \times 10^6 \, Wh \) after 30 years, and Case 150 can produce \( W(t) = (769.59 \pm 4.47) \times 10^6 \, Wh \).

To improve the trust of decision-making, we also run COMSOL simulation with decision parameters at different risk levels along with the 50 uncertainty parameter realizations, and use it to gauge the ultimate accuracy of MOO based on \( \mathcal{N}' \). Fig. 14 presents the comparison of the total energy for both Cases 30 and 150. At each risk level of both cases, \( \mathcal{N}' \) delivers very close values of \( E(W) \) and \( \sigma(W) \) compared to COMSOL, which demonstrates the robustness of \( \mathcal{N}' \) when predicting \( W(t) \) during MOO.

In Fig. 15, we present the produced fluid temperature \( T_{prod} \) of the 50 uncertain realizations predicted by \( \mathcal{N}' \) (gray) and COMSOL (cyan) based on the \( \bar{m}_p \) at different risk levels, and also aggregate the P10 (red), P50 (blue) and P90 (black) of both \( \mathcal{N}' \) (dashed lines) and COMSOL (solid lines). In both cases, we clearly see that the thermal breakthrough time \( \beta_t \) is always greater than the target \( \eta^* = 1 \, \text{year} \), and the extracted fluid temperature drop \( \eta(t) \) is less than the target \( \eta^* = 60\% \), which is validated by \( T_{prod} \geq (1 - \eta^* \cdot T_{res} = 74.72 \, ^\circ C \) in Case 30 and \( T_{prod} \geq (1 - \eta^* \cdot T_{res} = 74.72 \, ^\circ C \) in Case 150. Therefore, the non-linear constraints described in Eqs. (38) and (39) hold very well during MOO. Besides, as risk level increases, the trend of \( T_{prod} \) declines much steeper. This is mainly caused by the aggressive or higher value of \( q_{ini} \), since higher \( q_{ini} \) can accelerate thermal breakthrough (shorter \( \beta_t \)) and increase \( \eta(t) \). On the other hand, in the decision parameter space the increasing \( L_m \) with rising risk cannot compete with \( q_{ini} \), even though \( L_m \) impacts \( \beta_t \) and \( \eta(t) \) in an opposite way compared to \( q_{ini} \). As shown in Fig. 15, \( T_{prod} \) by \( \mathcal{N}' \) matches very well with \( T_{prod} \) by COMSOL among different uncertainty realizations (ensemble) and aggregated levels (P10, P50, P90). This demonstrates that the time series of \( T_{prod} \) can be accurately predicted based on \( \mathcal{N}' \) and \textit{HyperReLU}.
6. Summary

In this work, we successfully developed a novel thermal decline model in geothermal reservoirs. It is parsimonious with only three curve-fitting parameters and couples a Hyperbolic decline model with the rectified linear unit (ReLU) function, namely HyperReLU model. The ReLU function directly parameterizes the thermal breakthrough time $\beta_t$ and the hyperbolic function captures the followed thermal decline behavior. HyperReLU is treated as an encoder to compress the extracted fluid temperature $T_{prod}(t)$. Taking EGS as an example, a deep neural network $\mathcal{N}$ is developed to directly map the reservoir model parameters $\tilde{\mathbf{m}}$ to the curve-fitting parameters in HyperReLU and the total energy $W(t_a)$ recovered from geothermal. As a result, the combination of HyperReLU and $\mathcal{N}$ becomes the forward model for geothermal simulation. We further integrated the forward model with the Multi-Objective Optimization (MOO) Framework based on the Non-dominated Sorting-based Genetic Algorithm II (NSGA-II) to maximize the expectation of $W(t_a)$ while minimizing its standard deviation by considering the uncertainty of rock properties. We highlight the following remarks from this work.

Firstly, we built two Thermal-Hydro-Mechanical (THM) models to simulate EGS and HSA separately, and use the simulation data to examine HyperReLU in these two common geothermal systems. The accuracy of HyperReLU in both systems is decently high, with errors...
The median-risk solution stretches well between the expected to different non-linear engineering constraints. We ultimately choose with considering the uncertainty in rock properties and subjecting curve-fitting error \( \sigma_{\text{fit}} \) to accelerate thermal breakthrough. The top three parameters to accelerate thermal breakthrough. The top three parameters to achieve decent efficiency with CPU time 28.32 s, which can be further accelerated by paralleling \( T_{\text{RTS}} \) to around 0.39 \pm 0.89\% and 0.053 \pm 0.074 separately. The curve-fitting results show that the thermal breakthrough time \( t_b \) changes from 0 to the total simulation time, the hyperbolic decline constant \( b \) in HyperReLU varies from 0 to around 7, and the initial nominal decline \( d_i \) is always negative in the range from -2.5 to 0. These parameter ranges provide good guidance for geothermal engineers to use HyperReLU for thermal decline prediction.

Secondly, we also performed feature correlation and importance analysis between reservoir model parameters and typical geothermal performance metric parameters based on Pearson correlation and Gradient Boost method (GBM). In EGS, the three dominant parameters to \( \beta \) consist of fracture aperture \( d_f \) (negatively correlated), inter-well distance \( L_w \) (positively correlated) and fluid injection mass rate \( q_{\text{inj}} \) (negatively correlated), while higher \( d_f \) and \( q_{\text{inj}} \), and lower \( L_w \) can accelerate thermal breakthrough. The curve-fitting results successfully searched the Pareto Front in HSA are

\[
\phi_m = \frac{\phi_{\text{fit}} + \Delta \phi_{\text{fit}} - \Delta \phi_F}{1 + \Delta \phi_{\text{fit}} - [\sigma_{\text{fit}}(1 - \phi_{\text{fit}})]dT]}
\]

(A.1)

The elastic modulus of the rock matrix and fractures is a function of porosity \( \phi \) and temperature gradient \( \frac{dT}{dx} \) (proxy of \( T_{\text{res}} \), where both are positively correlated with \( W(t_b) \). Similar to EGS, the two dominant factors to \( \beta \) in HSA are \( L_w \) (positively correlated) and \( q_{\text{inj}} \) (negatively correlated), while the two dominant factors to \( W(t_b) \) in HTS become \( q_{\text{inj}} \) and temperature gradient \( \frac{dT}{dx} \). The prediction accuracy of \( x \) is high with \( R^2 \) score at least 0.946 among the 4 predicted parameters, and \( T_{\text{pred}}(t) \) obtained, which further validate the robustness of MOO based on HyperReLU curve-fitting \( \phi_{\text{fit}} \) reaches to error as low as 0.49 \pm 1.12\%, which is quite close to the purely HyperReLU curve-fitting error 0.39 \pm 0.89\%. The average prediction CPU time per case is as low as 2.3 \times 10^{-3} \text{ s}.

Finally, MOO based on \( x \) successfully searched the Pareto Front with considering the uncertainty in rock properties and subjecting to different non-linear engineering constraints. We ultimately choose high, median and low risk levels for solutions for operation, and median-risk solution stretches well between the expected \( W(t_b) \) and its standard deviation (risk) \( \sigma(W(t_b)) \). With the reservoir model parameters from different risk-level solutions, we also performed physics simulation with COMSOL and compared its results with MOO based on \( x \). Highly consistent results in terms of \( W(t_b) \) and \( T_{\text{pred}}(t) \) are obtained, which further validate the robustness of MOO based on \( x \).

The high accuracy and efficiency of MOO based on \( x \) makes it ready to lift up to cloud computing infrastructure with minimum effort of customization and makes it ready for field or regional scale geothermal performance optimization.

Acknowledgments

Bicheng Yan, and Manojkumar Gudala thanks King Abdullah University of Science and Technology (KAUST), Saudi Arabia for the Research Funding through the grants FCC/1/4491-22-01 and BAS/1/1423-01-01; Shuyu Sun thanks for the Research Funding from King Abdullah University of Science and Technology (KAUST), Saudi Arabia through the grants BAS/1/1351-01-01 and URF/1/4074-01-01.

Appendix A. Mathematical formulation for rock, fracture and fluid properties in thm modeling for EGS and HSA

The porosity of rock matrix used in the present work depends on the strain variations generated by mechanical and thermal loads, shown as Eq. (A.1) [9],

\[
\phi_m = \frac{\phi_{\text{fit}} + \Delta \phi_{\text{fit}} - \Delta \phi_F}{1 + \Delta \phi_{\text{fit}} - [\sigma_{\text{fit}}(1 - \phi_{\text{fit}})]dT]}
\]

(A.1)

The elastic modulus of the rock matrix and fractures is a function of porosity variation (i.e., Eq. (A.1)) and is represented in Eq. (A.2) [9].

\[
\ln \left( \frac{E}{E_{\text{fit}}} \right) = -d(\phi_m - \phi_{\text{fit}})
\]

(A.2)

The permeability variations of rock matrix and fractures are represented in Eqs. (A.3) and (A.4), respectively [42,58,59],

\[
\ln \left( \frac{k}{k_{\text{fit}}} \right) = \left( \frac{1 - \phi_{\text{fit}}}{h_1} + b_1 \phi_{\text{fit}} \right) \epsilon_{\text{vol}} = C_a \epsilon_{\text{vol}}
\]

(A.3)

\[
k_f = k_{f,0} \exp \left( -\frac{\sigma_a}{\alpha} \right)
\]

(A.4)

The heat capacity and thermal conductivity of rock matrix and fracture are calculated by Eq. (A.5), and Eq. (A.6) respectively [4,9],

\[
C_p,m(t) = \begin{cases} 
2.6 \times 10^{-2} \text{ J kg}^{-1} \text{ K}^{-1} & \text{if } -20 \leq \log(k) \leq -11 \\
13 \log(k) + 699 & \text{if } -11 \leq \log(k) \leq -2
\end{cases}
\]

(A.5)

\[
\lambda_m(T) = 2.6 \times 0.0025(T - 293.15)
\]

(A.6)

The effect of temperature variation on water viscosity, water density, water specific heat and water thermal conductivity are calculated by Eqs. (A.7), (A.8), (A.9) and (A.10), [3],

\[
\eta(T) = \begin{cases} 
1.38 - 0.0287 T + 1.36 \times 10^{-4} T^2 & \text{if } 273.15 \leq T \leq 413.15 \\
-4.61 \times 10^{-3} T^3 + 8.9 \times 10^{-10} T^4 & \text{if } 413.15 \leq T \leq 553.15
\end{cases}
\]

(A.7)

\[
\rho_w(T) = 838.47 + 1.4 T - 0.003 T^2 + 3.72 \times 10^{-7} T^3
\]

(A.8)

\[
C_p,w(T) = 12010.15 - 80.41 T + 0.31 T^2 - 5.38 \times 10^{-4} T^3 + 3.62 \times 10^{-7} T^4
\]

(A.9)

\[
\lambda_w(T) = -0.869 + 0.009 T - 1.58 \times 10^{-3} T^2 + 7.98 \times 10^{-9} T^3
\]

(A.10)

Appendix B. Implementation in COMSOL multi-physics

The developed mathematical model is implemented in the COMSOL Multiphysics for both EGS and HSA systems. Several researchers used COMSOL multiphysics to investigate the dynamics occurring in the geothermal reservoirs [4,60–68]. The COMSOL built-in modules including heat transfer, poro-elastic, Darcy flow, solid mechanics, thermal expansion modules are utilized, and the dynamic rock/fracture/fluid
properties are implemented as local variables in the COMSOL Multiphysics. In the solid-mechanics module, natural and hydraulic fractures are treated as a thin elastic layer, and mathematical formulations are available at Section 2 and Appendix A. Stationary and transient solvers are sequentially coupled in the present work. Backward difference formula (BDF) with Euler backward initializing technique is adopted to linearize the governing partial differential equations. The multifrontal massively parallel sparse (MUMPS) direct linear solver is used to solve pressure and displacement, while the parallel direct sparse solver (PARDISO) is used to solve temperature.

Appendix C. Validation of the mathematical model

C.1. TH model for flow and heat transport in single fracture system

As we simulate the fluid flow and heat transport in a single fracture in EGS, it is critical to validate the accuracy. A conceptual geothermal reservoir model with a pair of injection (blue)-production (red) wells connected by a fracture (green) is depicted in Fig. C.16, and it is further solved by COMSOL.

To validate the COMSOL solution, we further adopt the following the analytical solution that describes the temperature changes as a function of distance \(x\) and time \(t\), as proposed by [69].

\[
T(x, t) = T_0 + (T_{\text{inj}} - T_0) \operatorname{erfc} \left( \frac{x}{\lambda_m} \sqrt{\frac{t}{\rho_m C_p}} \right) \frac{U}{u_w} \left( t - \frac{x}{u_w} \right) \quad \text{(C.1)}
\]

Where \(T_{\text{inj}}\) is the injected fluid temperature, \(T_0\) is the rock temperature, \(U\) is the unit step function and \(u_w\) is the flow velocity of water.

\[\begin{align*}
\text{Fig. C.16. Schematic of single fracture conceptual model.} \\
\text{Fig. C.17. Validation of the simulation model for flow and heat transport in a single-fracture geothermal reservoir.}
\end{align*}\]

\[\begin{align*}
\text{Fig. C.18. Schematic of heat flow in matrix conceptual model.} \\
\text{Fig. C.19. Validation of the simulation model for flow and heat transport in a non-fracture geothermal reservoir.}
\end{align*}\]

\[\begin{align*}
\text{Fig. C.17 shows the comparison the simulation results and analytical results. The temperature inside the fracture increases from the} \\
\text{injection (}x = 0\text{) to the producer (}x = 100\text{), and the producer well temperature further decreases with time (e.g., 75 days) after} \\
\text{the cold injected fluid breakthrough in the fracture channel. This is well captured by both the} \\
\text{numerical simulation (marker lines) and the analytical solutions (solid lines). Therefore, the} \\
\text{numerical simulation model used in this research can accurately predict the heat flow in the} \\
\text{fracture.}
\end{align*}\]

C.2. THM model for flow and heat transport in a non-fractured geothermal reservoir

Here we further develop the THM model to simulate flow and heat transport in a non-fractured geothermal reservoir. A two-dimensional THM model was constructed with COMSOL, and the schematic plot is shown in Fig. C.18. There are fixed boundary conditions (no displacement) on the left and lower sides, and constant stress condition on the upper and right sides in the domain. Constant pore pressure boundaries are set on the four sides of the domain. In the center of the domain, there is a cold fluid injector with constant injection temperature.
The analytical solution of this model was proposed by Ghassemi and Zhang [70], shown as Eq. (C.2),

\[ T(x, t) = \frac{1}{T_0} \sum_{N=1}^{J} T_i^h(x, t - \tau) \Delta \phi 

(C.2)

Where \( J \) is the total number of elements; \( \phi \) is the heat flux discontinuity; \( \tau \) is the occurring time; the super-script \( ch \) represents the continuous heat source.

Han et al. [22] provided the validation parameters for petrophysical and hydrodynamic properties of the model. The transient heat transport at near-well locations were compared between the numerical simulation model and the analytical solution (Eq. (C.2)), and results are shown in Fig. C.19. The comparison revealed that the temperature dynamics in both the numerical and analytical solutions were consistent. This implies that the modified mathematical model is appropriate for predicting thermal, hydro, and geomechanical variations in the rock matrix. Therefore, the developed mathematical simulation model can be used to forecast heat transport in non-fractured porous media.

Appendix D. Simulation results for EGS and HSA

Fig. D.20 illustrates the variation in temperature over time and space for different EGS cases. During the EGS operation, heat is exchanged between the injected fluid and the rock matrix, and it takes time for the injected fluid to extract heat from the surrounding rock matrix. This causes the temperature of the injected fluid to increase and the temperature of the rock matrix in the vicinity of the fracture to decrease. At the early time, the production temperature equals to the initial temperature of the rock matrix, but then it decreases due to the temperature decrease in the matrix nearby the fracture. This zone controls the heat exchange between the rock matrix and the fluid in the fracture, and its shape during the EGS operation is conical, as shown in Fig. D.20.

![Spatial and temporal variation of cold thermal (°C) front in the matrix in EGS for different cases.](image-url)
in Fig. D.20. As the operation progresses, the low temperature zone expands and affects the heat transfer from the rock matrix to the fluid in the fracture, which is reflected in the production temperature. The geometry of the cold temperature zone near the fracture depends on the fluid mass injection rate, the fracture length, and fracture aperture. This is evident from Figs. D.20a to D.20i.

Fig. D.21 illustrates the spatial and temporal evolution of temperature for different HSA cases. The cold front moves from the injection well to the production well and is mainly distributed in the vicinity of the injection well for all the cases, as shown in Fig. D.21. Thermal breakthrough occurs faster in case 734 compared to case 1265 and case 1878, as seen in Fig. 7b. After thermal breakthrough, the production temperature gradually decreases over the period of HSA operation, similar to EGS. However, the expansion of the cold temperature zone in HSA is relatively higher compared to EGS, and the shape of the cold front changes from a circle to a conical shape as it moves from the injection well to the production well, as shown in Fig. D.21.

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