Artificial Noise and RIS-Aided Physical Layer Security: Optimal RIS Partitioning and Power Control

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Abstract—The synergism of reconfigurable intelligent surfaces (RIS) and artificial noise (AN) shows significant promise in improving physical layer security in wireless networks. Accordingly, this letter proposes the virtual partitioning of RIS elements into two parts such that the phase shifts of the different partitions are configured to improve the intended signal at a legitimate user and enhance the impact of AN on an illegitimate user, respectively. To this aim, two problems are defined to jointly optimize the partitioning ratio, and signal/noise transmit power levels for two main objectives. First, we maximize secrecy capacity by satisfying users’ quality of service (QoS). Second, we optimize transmit power to establish a secure link by satisfying the QoS of the legitimate user. We provide closed-form solutions subject to the rate constraints on both legitimate and illegitimate users. Simulation results validate the closed-from solutions and show that the proposed RIS-partitioning method dramatically improves SC compared to benchmark methods.

Index Terms—Artificial noise, physical layer security, optimization, partitioning, power control, reconfigurable intelligent surface (RIS), secrecy capacity.

I. INTRODUCTION

RECONFIGURABLE intelligent surface (RIS) has been proposed as an emerging technology for next-generation wireless networks to enhance spectral efficiency [1]. The passive beamforming nature of RIS has been integrated into various wireless technologies, such as mmWave [2], full-duplex [3], non-orthogonal multiple access (NOMA) [4], wireless power transfer, wireless security [5], to improve end-to-end performance at low power and hardware cost.

Rapidly evolving attacks on the wireless network have drawn attention to the study of physical layer security (PLS). PLS aims at securing the data link and providing better privacy in combination with existing cryptographic techniques [6]. Recently, RIS-enhanced networks have been introduced to enhance security in wireless communications [7]. The RIS can improve the coverage, the system data rate, and the PLS by accurately directing the signal beam into the desired path. One of the main criteria evaluated in PLS is the secrecy capacity (SC), which characterizes the fundamental limit of secure communications. RIS-assisted secrecy communications were studied in [8]–[10], where active transmit and passive reflect beamforming are jointly designed using various optimization techniques to enhance the achievable SC. Furthermore, the authors in [6], [11] considered the artificial noise (AN) that can be used as a transmit jammer in RIS-assisted secure communication and showed that this architecture improves the network SC.

We propose splitting RIS virtually into two parts such that the phase shifts of different partitions are configured to improve the achievable rate at a legitimate user and enhance the impact of AN on an illegitimate user, respectively. We formulate maximum SC and minimum power consumption problems that jointly optimize the partitioning ratio and signal/noise power levels. We provide closed-form solutions subject to the rate constraints on both legitimate and illegitimate users. Finally, numerical results show that the proposed RIS-partitioning method dramatically improves SC compared to the state-of-the-art benchmarks.

II. SYSTEM MODEL

We consider a secure wireless communication system (Fig. 1), where a transmitter (Alice) aims to convey confidential data to a legitimate user (Bob) with the aid of RIS against an eavesdropper (Eve). We assume that Bob and Eve are equipped with a single antenna and that the RIS has \( K \) reflecting elements that are indexed row-by-row by \( k = 1, \ldots, K \). Moreover, Alice is equipped with two separated antennas to provide a transmit diversity for the desired signal \( s \) and AN \( \{n\} \); both follow complex Normal distribution. The first antenna (Alice_1) transmits message \( s \), while the second one (Alice_2) conveys \( n \) to enhance the jamming effect at Eve. The total transmit power at Alice is \( P_t = P_{t_1} + P_{t_2} \), where \( P_{t_1} \) and \( P_{t_2} \) are the respective power allocations for \( s \) and \( n \).

We assume the presence of direct non-line-of-sight (NLoS) channels from Alice to Bob and Eve, which are respectively denoted as \( h_{1,b} \) and \( h_{1,e} \). Assuming narrow-band and non-

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Fig. 1. Secure communication with the AN-based RIS-partitioning.
dispersive wireless propagation, these channels are given by
\[ h_{i,u} = \sqrt{\rho_{i,u}} e^{-j2\pi f_c \tau_{i,u}} \delta(\tau_{i,u}), \forall \tau_{i,u} \in \{s, n\}, \forall u \in \{b, e\}, \]
where \( \beta_{i,u} \) is the pathloss of the link between Alice, and Bob/Eve, \( f_c \) is the carrier frequency, and \( \tau_{i,u} \) is the propagation delay. Furthermore, assuming a narrow-band channel with only one strong line-of-sight (LoS) path to and from the RIS, the cascaded channel from Alice, to Bob/Eve through the \( K \) RIS elements can be given by [7]
\[ G_{i,u}^K = \frac{\sqrt{\beta_{i,k} \beta_{k,u}}}{\rho_{i,k} \rho_{k,u}} \left| \mathbf{v}(\psi_{i,k}, \theta_{i,k}) \right| \left| \mathbf{v}(\psi_{k,u}, \theta_{k,u}) \right| e^{j2\pi f_c [\mu_{\tau_{i,k}} + \tau_{k,u}]}, \]
where \( \beta_{i,k}, \tau_{i,k}, \psi_{i,k}, \) and \( \theta_{i,k} \) are the pathloss, propagation delay, azimuth and elevation angles between Alice, and the \( k \)th element, respectively. Similarly, \( \beta_{k,u}, \tau_{k,u}, \psi_{k,u}, \) and \( \theta_{k,u} \) denote the pathloss, propagation delay, azimuth and elevation angles of the \( k \)th element - \( u \) link, respectively. The respective transmit and receive steering vectors are denoted by
\[ \mathbf{v}(\psi_{i,k}, \theta_{i,k}) = \mathbf{V}(\psi_{i,k}, \theta_{i,k}) e^{j\mu_{\psi_{i,k}} e^{j\phi_{i,k}}} e^{j\psi_{i,k}}, \]
\[ \mathbf{v}(\psi_{k,u}, \theta_{k,u}) = \mathbf{V}(\psi_{k,u}, \theta_{k,u}) e^{j\mu_{\psi_{k,u}} e^{j\phi_{k,u}}} e^{j\psi_{k,u}}. \]

The channel state information (CSI) of users and RIS can be obtained by applying different channel acquisition methods such as those proposed in [2], [13]. Hence, considering available CSI, Alice can optimize RIS phase shifts and send them back to the RIS controller via separate low-rate wired or wireless links [14]. Moreover, we consider optimal phase shifts, where phases of RIS elements are coherently aligned with phases of cascaded RIS channels. Due to this perfect CSI and phase shift assumptions, this letter provides upper-bound limited results for practical RIS-aided secure communication. We also assume perfect CSI for Eve as it is an active user in the network [15]. However, Eves are not trustworthy when sensitive data is shared with Bob and are not supposed to overhear the private information [6]. This letter proposes a virtual RIS partitioning technique to guarantee security in communication links, where a partitioning factor \( \rho \in [0, 1] \) is used to allocate RIS elements to serve specific users. Thereby, the first group of partitioned elements \( \rho K \) is dedicated to forwarding the message from Alice, to Bob. Moreover, assuming the perfect global CSI and high bit resolution of the RIS elements’ phase shifters [3], the phase shifts \( \Phi_{b} \) of \( \rho K \) elements are coherently aligned with Alice - Bob cascaded channels. The remaining \((1 - \rho)K\) elements create the second group with phases \( \Phi_{e} \) and aligned with Alice - Eve channels to enhance the jamming impact of AN at Eve. We use the function \( \lfloor \rho K \rfloor = Z \) to dedicate integer RIS elements numbers to Bob and Alice. Hence, the signal received at Bob/Eve can be derived as
\[ y_{u} = \sqrt{P_{u}} \left[ h_{s,u} + \rho G_{s,u}^{K} \Phi_{b} + [1 - \rho] G_{s,u}^{K} \Phi_{e} \right] s + \sqrt{P_{n}} . \]

where \( \forall \tau_{u} \in \{b, e\}, w_{u} \) is the additional white Gaussian noise at \( u \) obeying the distribution \( w_{u} \sim \mathcal{CN}(0, \sigma_{u}^2) \). Assuming that Bob knows AN, Bob can subtract it from the received signal, and the signal-to-noise-ratio (SNR) at Bob can be written as
\[ \gamma_{b} = \frac{P_{u} \left[ h_{s,b} + \rho G_{s,b}^{K} \Phi_{b} + [1 - \rho] G_{s,b}^{K} \Phi_{e} \right]^2}{\sigma_{b}^2} \]
where \( \sigma_{b}^2 \) is the thermal noise power at the receiver nodes. On the other hand, Eve is unaware of the AN and treats it as interference. Thus, the signal-to-interference-and-noise-ratio (SINR) at Eve can be expressed as
\[ \gamma_{e} = \frac{P_{u} \left[ h_{s,e} + \rho G_{s,e}^{K} \Phi_{b} + [1 - \rho] G_{s,e}^{K} \Phi_{e} \right]^2}{\sigma_{e}^2} \]
SC for this network, defined as the difference between the achievable rates of Bob and Eve, can be written as
\[ S_{C} = \max \{ R_{b} - R_{e}, 0 \} \]
where \( R_{b} = B \log_{2} [1 + \gamma_{b}], \forall \tau_{u} \in \{b, e\} \), and \( B \) is the channel bandwidth.

### III. Problem Formulation and Solution Methodology

Following from (8), SC can be optimized by maximizing and/or minimizing the achievable rates of Bob and Eve, respectively. To deal with this, PLS literature proposes various methods: one method is to use RIS to improve the achievable rate of Bob by allocating all RIS elements to beamform the signal from Alice to Bob, which ignores the direct link between Alice and Eve. Although this scenario can provide a sufficient SC enhancement by maximizing Bob’s rate, Eve may still achieve a considerable data rate that is enough to corrupt/decode Alice’s data. An alternative method is to allocate some of Alice’s power to transmit AN and suppress Eve. Even if this naturally causes some loss to Bob’s capacity, it may provide desirable SC while minimizing the data decodable by Eve. In this letter, we are interested in reaping the full benefits of both methods to provide a much better secrecy performance, which inherently requires optimization of RIS partitioning and signal/noise power allocation to reach objectives while ensuring QoS constraints on Bob and Eve are satisfied. To this end, this section provides closed-form solutions for two optimization problems to obtain a secure communication link. The former maximizes Bob’s achievable rate and constrains Eve’s rate at a certain threshold. The latter minimizes the total transmit power of Alice subject to QoS constraints on achievable rates of Bob and Eve.

#### A. Secrecy Capacity Maximization

In this problem, we optimize \( P_{s}, P_{n}, \) and \( \rho \) to maximize SC by guaranteeing the QoS of Bob (QoSb) and limiting the

\(^{1}\)AN can be preshared with Bob through a secure connection link before Alice starts transmitting. Alternatively, with the help of the available CSI, Bob can implement a successful interference cancellation technique to cancel AN from the received signal.
achievable rate of Eve (QoS\(_e\)). The SC maximization problem can be formulated using (6) and (7) and written as

\[
P_1 : \max_{\rho, P_s, P_n} SC \quad \text{s.t.} \quad \begin{align*}
C_1^s : & \quad \gamma_b \geq \gamma_b^s \\
C_2^s : & \quad \gamma_c \leq \gamma_c^s \\
C_3^s : & \quad P_s + P_n \leq P_t \\
C_4^s : & \quad 0 \leq P_s \leq P_t, \quad 0 \leq P_n \leq P_t \\
C_5^s : & \quad 0 \leq \rho \leq 1
\end{align*}
\]  

(9)

where \(\gamma_b = 2^{\text{QoS}_b} - 1\), \(\gamma_c = 2^{\text{QoS}_c} - 1\), \(\text{QoS}_b = \zeta \text{QoS}_s\), with \(\zeta\) is the variable that determines the threshold of \(\text{QoS}_s\) with \(0 < \zeta \leq 1\); the constraints \(C_1^s\) and \(C_2^s\) are QoS constraints; \(C_3^s\) requires the power allocated for \(s\) and \(n\) not to exceed maximum transmit power \(P_t\); and \(C_4^s\) specifies the domain of optimization variables while \(C_5^s\) indicates the feasibility values for \(\rho\). In (9), \(P_s^s\) and \(P_n^s\) can be solved using geometric programming (CVX) for fixed values of \(\rho\). Then, optimal \(\rho\) is obtained using a meta-heuristic method, i.e., particle swarm optimization (PSO), that runs a global search of \(\rho\) and evaluates the SC fitness. In addition, \(P_1\) can also be solved analytically, which is shown in Lemma 1.

**Lemma 1:** The optimal power levels providing maximum SC for a given RIS portion, \(\rho\), are given by

\[
P_s(\rho) = \frac{P_t \gamma_c [L_1 + L_2 \rho + L_3 \rho^2] + \gamma_b \sigma_b^2}{S_1 + S_2 \rho + S_3 \rho^2 + \gamma_c [L_1 + L_2 \rho + L_3 \rho^2]},
\]  

\[
P_n(\rho) = P_t - P_s(\rho),
\]  

(10)

(11)

from which the optimal RIS portion of \(P_1\), \(\hat{\rho}_1\), can be derived as in (12). The optimal power levels yielding the maximum SC can be obtained by substituting \(\hat{\rho}_1\) into (10)-(11) and written as \(P_s = P_s(\hat{\rho}_1)\) and \(P_n = P_t - P_s\).

**Proof:** Please see the proofs and notations in Appendix A. ■

**B. Power Consumption Minimization**

Here, the objective is to consume minimal transmit power to establish a secure link that satisfies QoS\(_s\) while limiting the QoS\(_e\) to 1% of QoS\(_s\). We formulate the objective as

\[
P_2 : \min_{\rho, P_s, P_n} P_s + P_n \quad \text{s.t.} \quad \begin{align*}
C_1^s, C_2^s, C_4^s, C_5^s
\end{align*}
\]  

(13)

where the constraints follow (9). Similar to \(P_1\), we solve \(P_2\) using the same numerical approach, while the analytical solution is based on the following lemma:

**Lemma 2:** The optimal power levels providing the minimum total transmit power for a given \(\rho\) are given by

\[
P_s(\rho) = \frac{\Delta_1}{F_1 + F_2 \rho + F_3 \rho^2},
\]  

\[
P_n(\rho) = \frac{\Delta_2 [S_1 + S_2 \rho + S_3 \rho^2] - \sigma_b^2 [F_1 + F_2 \rho + F_3 \rho^2]}{[L_1 + L_2 \rho + L_3 \rho^2] [F_1 + F_2 \rho + F_3 \rho^2]},
\]  

\[\Delta_1 = \sqrt{\Theta + [\Psi + [\Gamma_2 - 2 \Gamma_1 \Gamma_0] - S_3 \Gamma_2 + 2 S_0 \Gamma_1] P_t} \sigma_c + [\Gamma_2 + \chi + S_1 L_2 \Gamma_3 - \Gamma_2 \Gamma_1 + \Gamma_3^2] P_t^2 + [L_3 \gamma_c + S_3] \sigma_c + [\Gamma_1 - \Gamma_4] P_t
\]  

(12)

Fig. 2. Capacity and SC performance versus transmit power \(P_t\) when \(\rho = 0.8\).

from which the optimal RIS portion \(\hat{\rho}_2\) of \(P_2\) can be derived in a closed-form as in (16). The optimal power levels yielding the minimum total power can be obtained by substituting \(\hat{\rho}_2\) into (14)-(15) and written as \(P_s = P_s(\hat{\rho}_2)\) and \(P_n = P_t - P_s(\hat{\rho}_2)\).

**Proof:** We refer readers to Appendix B for the proofs and notations.

**IV. NUMERICAL RESULTS**

This section presents numerical results to validate the analytical expressions and understand the impacts of different system parameters and scenarios on the proposed system model. Unless otherwise stated, the default system parameters are as follows: operating frequency is 4 GHz [16], \(B = 1\) MHz, \(K = 144\), \(\rho = 0.8\), and QoS\(_b\) = 10 Mbit/s, \(\sigma_b^2 = \sigma_c^2 = -105\) dBm [6]. Moreover, 80% of the total power \(P_t\) is dedicated to the signal \(s\). The three-dimensional Cartesian coordinates for RIS and Alice are \((0, 0, 20)\), \((1, 1, 20)\), respectively. We assume the worst-case scenario when Eve is located closer to Alice with coordinates of \((20, 100, 0)\), while Bob is situated farther from Alice with coordinates of \((200, 100, 0)\). Micro-cell pathloss models are used to model LoS and NLoS path-loss [7].

In Fig. 2, we compare the performance of the proposed AN-based RIS-partitioning method with the existing PLS benchmark scenarios regarding the achievable capacity and SC. In the first scenario (AN-only), Alice superimposes AN to jam Eve without the participation of RIS, while the second scenario (RIS-only) assumes no AN, and all RIS elements are allocated to beamform the message \(s\) to Bob. In the third scenario (Tile-based), we use the model proposed in [17], where \(K\) RIS elements are subdivided into \(N\) tiles; hence, each tile contains \(K/N\) RIS elements. In this scenario, 144 RIS elements are divided into 16 tiles.
\[
\hat{\rho}_2 = \frac{\sqrt{\frac{\Delta_b L_2}{\Delta_S^2}} + \frac{\Delta_b L_2^2}{\Delta_S^2} + \frac{\sigma_e^2}{\Delta_S^2}}{2\Delta_b L_2 + 2\Delta_S^2 - 2\sigma_e^2} - \Delta_b L_2 - \Delta_S^2 + \frac{4P_1 P_2 + \Delta_b L_2}{\sigma_e^2} \] 

(16)

From Fig. 2, we can see that when no RIS and no AN are used in the network, the SC is always zero and provides insecure communication for the network by motivating using firm techniques to enhance the security. Furthermore, the capacity of Bob achieves 9 Mbit/s at 30 dBm, while the capacity curve of Eve saturates after 10 dBm by receiving 2.5 Mbit/s in the AN-only scenario. It is noted from the SC curve that this scenario achieves zero SC at low transmit powers and requires higher transmit power to obtain non-zero SC. On the other hand, the RIS-only scenario shows good capacity performance for Bob; however, the capacity of Eve increases as well due to the existence of the direct link. Therefore, using only RIS without AN may not provide excellent secure communication. The SC of this scenario saturates at about 6.5 Mbit/s for all transmit power ranges. Finally, the proposed method shows that Bob’s capacity achieves 18 Mbit/s at 30 dBm, while the capacity of Eve is 1 Mbit/s. Moreover, we can further diminish the capacity of Eve to 0.45 Mbit/s by allocating more RIS elements to Eve, i.e., \(\rho = 0.5\). However, it decreases Bob’s capacity and SC. In addition, the Tile-based scenario also abates Eve’s capacity, but the SC is lower than that of the proposed method. This performance is the lower bound of the proposed one as elements per tile share the common phase shift. The tile-based method is more practical as CSI and phase shift calculations impose less challenge. From the results, it is evident that the proposed method leverages the advantages of RIS and AN, achieving the best SC performance by enhancing the achievable capacity of Bob and aggressively diminishing that of Eve.

Fig. 3 illustrates the numerical and analytical results for \(P_1\) and \(P_2\) to illustrate the impact of different system parameters and scenarios on the proposed system model. The top figure illustrates optimization results for \(P_1\) in (9), where we maximize SC above QoS \(S_0\) by fixing \(P_2 = 30\) dBm. We use CVX and compare it with analytically derived closed-form solutions provided in Lemma 1. Results show a perfect match between CVX and closed-form solutions that validates the analytical derivations. It is observed that the increase of \(K\) improves SC as the number of RIS elements that beamform signal and AN is increased. Moreover, SC improves by increasing \(\rho\) for all RIS sizes. The increase of \(\rho\) means that RIS allocates a higher portion of elements to reflect the signal to Bob, and fewer RIS elements portion is dedicated to beamform AN. Moreover, the optimal \(\hat{\rho}_1\) found using PSO and closed-form in (12) matched to each other. For the considered RIS elements \(K = \{100, 196, 484, 1024\}\), the respective optimal elements’ allocation are found as \(\hat{\rho}_1 = \{0.765, 0.925, 0.985, 0.985\}\). These values prove their correctness by aligning with the simulated CVX results. The bottom figure illustrates the minimum total transmit power for \(P_2\) to achieve QoS \(S_0\) using CVX and analytical closed-forms in Lemma 2. It is observed that the increase of \(K\) decreases the transmit power. Moreover, it is noticed that the transmit power decreases by increasing \(\rho\) until the optimal \(\rho\) is discovered. Similar to the results of the top figure, the closed-form results match with simulated CVX, and the optimal \(\hat{\rho}_2\) values found using PSO and closed-form in (16) for different \(K\) are the same as in the previous plot.

V. CONCLUSION

This letter studied the AN-based RIS-partitioning system model, where RIS was virtually partitioned to reflect the signal to Bob and jam Eve with AN simultaneously. In doing so, we not only maximized Bob’s SC but also limited Eve’s achievable rate. We proposed two optimization problems for this purpose, such as 1) maximization of SC and 2) minimization of the transmit power. The problems were not only solved by the CVX and PSO optimization tools, but we also provided closed-form solutions. Numerical and analytical results showed that the proposed AN-based RIS-partitioning model achieves good SC by improving Bob’s QoS and constraining Eve’s QoS at a low level. Moreover, the proposed model showed its effectiveness by outperforming the traditional AN-only and RIS-only PLS scenarios regarding SC performance. This letter will be extended to the version with multiple antennas at Alice and also considering the scenario with multiple Bobs and randomly located Eves.

APPENDIX A

PROOF OF LEMMA 1

One way of maximizing SC is satisfying Eve’s QoS constraint \(C_1^3\) with equality while pushing \(\gamma_b\) to its maximum subject to the total transmit power constraint in \(C_1^3\). It is worth noting that \(C_1^3\) is also satisfied at the optimal point since
all available transmit power will be consumed to reach the highest SC, which yields a single power control variable, i.e., \( P_a = P_1 - P_s \). Accordingly, substituting \( P_a = P_1 - P_s \) into the equality \( \gamma_e = \gamma_e \) and solving for \( P_s \) yields the optimal \( P_s \) for a given \( \rho \) as follows:

\[
P_s = \frac{h_{s,e} + \rho G_{s,e}^K \Phi_b + [1 - \rho]G_{s,e}^K \Phi_e}{\Omega} \leq \gamma_e. \quad (A.1)
\]

Now, using the notations of \( h_{s,e} = m + jn, \ G_{s,e}^K \Phi_b = o + js, \ G_{s,e}^K \Phi_e = r + jl, \ h_{n,e} = m' + jn', \ G_{n,e}^K \Phi_b = o' + j s, \) and \( G_{n,e}^K \Phi_e = r' \) and after some mathematical manipulations, we can respectively write \( \Omega \) and \( \Lambda \) in (A.1) as:

\[
\Omega = S_1 + S_2\rho + S_3\rho^2, \quad (A.2)
\]

\[
\Lambda = L_1 + L_2\rho + L_3\rho^2, \quad (A.3)
\]

where \( S_1 = [m + r]^2 + [m + l]^2, \ S_2 = 2[m + r][o - r] + 2[m + l][o - l] + S_3 = [o - r]^2 + [o - l]^2, \ L_1 = [m + r]^2 + [m + l]^2, \ L_2 = 2[m + r][o - r] + 2[m + l][o - l] + L_3 = [m + r]^2 - [o - r]^2, \) Then, inserting (A.2) and (A.3) into (A.1) as well as after some mathematical manipulations, the closed-form solution for \( P_s \) can be written as in (10), from which the closed-form for AN power can be derived as in (11). Notice that RIS partitioning and power allocation is decoupled since their respective variables do not complicate each other. Thus, we can further obtain the optimal RIS elements allocation portion \( \rho_1 \) for \( P_1 \), by taking the first-order derivative regarding \( \rho \) and set it equal to zero, i.e., \( \frac{\partial}{\partial \rho} [\Omega(\rho)] = 0. \) Then, after some mathematical manipulations, we can write the optimal RIS allocation factor as in (12), where \( \Theta = \frac{[L_1^2\gamma_c^2 + 2\Gamma\gamma_c + S_3^2]^2}{[\gamma_c^2L_1^2 + 2\Gamma\gamma_cL_1 + S_3^2]} \)

\[
\Psi = \frac{[-2L_3\Gamma_4 + \Gamma_5 + 2\Gamma_1] - L_2 - L_3\Gamma_3}{\beta_c}, \quad \Upsilon = \frac{[S_2^2L_1 - 2S_1\Gamma_1 - S_1\Gamma_2]}{L_2}, \quad \Lambda = L_1S_3, \quad \Gamma_2 = L_2S_2, \quad \Gamma_3 = L_2S_1, \quad \Gamma_4 = L_3S_2, \quad \Gamma_5 = L_3S_1, \quad \Gamma_6 = L_2S_3.
\]

**APPENDIX B PROOF OF LEMMA 2**

The minimal power consumption is achieved when \( C_1^2 \) and \( C_1^2 \) are satisfied with equality since providing a rate higher than QoS increases power consumption. Accordingly, \( P_a \) can be obtained by solving \( \gamma_e = \gamma_e \) for \( P_s \) as follows:

\[
P_a = \frac{\Delta_b h_{s,e} + \rho G_{s,e}^K \Phi_b + [1 - \rho]G_{s,e}^K \Phi_e}{\Omega} \leq \gamma_e. \quad (B.1)
\]

where \( \frac{\Delta_b h_{s,e} + \rho G_{s,e}^K \Phi_b + [1 - \rho]G_{s,e}^K \Phi_e}{\Omega} \leq \gamma_e \). Now, using the following notations for the denominator in (B.1): \( h_{s,e} = a + jb; \ G_{s,e}^K \Phi_b = BX; \ G_{s,e}^K \Phi_e = c + jd \) and applying the equality of \( |x + y|^2 = x^2 + y^2 \) as well as after some mathematical manipulations, \( P_a \) can be rewritten as in (14), where \( F_1 = [a + c]^2 + [b + d]^2, \ F_2 = 2[a + c][B_N - c] - 2[b + d]d + F_3 = d^2[B_N - c]^2. \) In order to find the optimal \( P_{ae} \), we first insert (14) into (7) and rewrite \( \gamma_e \) as:

\[
\gamma_e = \frac{\Delta_b h_{s,e} + \rho G_{s,e}^K \Phi_b + [1 - \rho]G_{s,e}^K \Phi_e}{\Omega} \leq \gamma_e. \quad (B.2)
\]

Next, using (B.2) and \( C_1^2 \) in (13), \( P_a \) is derived as

\[
P_a = \frac{\Delta_b}{\gamma_e \Lambda[1 + F_2 + F_3\rho^2]} - \sigma_e^2 \quad (B.3)
\]

Then, inserting (A.2) and (A.3) into (B.3), the optimal \( P_a \) can be written as in (15), where \( \gamma_e = \frac{\Delta_b}{\gamma_e \Lambda[1 + F_2 + F_3\rho^2]} - \sigma_e^2 \).

Furthermore, summation of (14) and (15) gives the optimal \( P_1 \) as

\[
P_1 = \frac{\Delta_b}{\gamma_e \Lambda[1 + F_2 + F_3\rho^2]} = \frac{\sigma_e^2}{\gamma_e \Lambda[1 + F_2 + F_3\rho^2]} \quad (B.4)
\]

Furthermore, as (B.4) is a function of \( \rho \), we can find the optimal \( \rho_2 \) for \( P_2 \) by taking the first-order derivative regarding \( \rho \) and set it equal to zero. Hence, after some mathematical manipulations, \( \rho_2 \) is solved as in (16).

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