On Secure CDRT with NOMA and Physical-Layer Network Coding

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Abstract—This paper proposes a new scheme to enhance the secrecy performance of non-orthogonal multiple access (NOMA)-based coordinated direct relay transmission (CDRT) systems with an untrusted relay. The physical-layer networking coding (PNC) and the NOMA schemes are combined to improve spectrum efficiency. Furthermore, inter-user interference and friendly jamming signals are utilized to suppress the eavesdropping ability of the untrusted relay without compromising the acceptance quality of legitimate users. Specifically, the far user in the first slot and the near user in the second slot act as jammers that generate jamming signals to ensure secure transmissions of confidential messages. We investigate the secrecy performance of the NOMA-based CDRT systems with the PNC scheme and derive the closed-form expression for the ergodic secrecy sum rate. The asymptotic analysis at a high signal-to-noise ratio is performed to obtain more insights. Finally, simulation results are presented to demonstrate the proposed scheme’s effectiveness and the theoretical analysis’s correctness.

Index Terms—Coordinated direct relay transmission, non-orthogonal multiple access, physical-layer networking coding, physical-layer security, ergodic secrecy sum rate.

I. INTRODUCTION

A. Background and Related Work

Compared with conventional orthogonal multiple access (OMA), non-orthogonal multiple access (NOMA) supports multiple users to simultaneously access the same wireless resources. It resolves massive connectivity requirement and conserves spectrum resources in Internet of Things (IoT) network, which utilizes superimposed coding and successive interference cancellation (SIC) to offer a significant improvement with reliable performance [1], [2]. Cooperative NOMA (C-NOMA) systems have attracted significant attention since they can extend the NOMA users’ coverage and enhance the system performance through diversity technology. A downlink NOMA-based coordinated direct and relay transmission (CDRT) systems was introduced wherein parallel communications between multiple links were allowed [3]. Thus, the higher throughput and spectral efficiency (SE) were achieved. Since the signals for the far user (FU) were decoded on the near user (NU) through the SIC technology in the first time slot, the forward signals for the far user can be easily deleted on the NU. In other works, during the relay forward signals to the FU (the second slot), the transmitter can transmit signals to the NU to improve SE.

There are many outstanding works on the performance analysis of CDRT systems, such as outage probability (OP) [4]-[9], ergodic sum rate [5], [6], [10]-[13], and average bit error rate (ABER) [11], [14]. Liu et al. studied the outage performance of a satellite-assisted CDRT system and derived the closed-form expression for the exact and asymptotic OP in [4]. To enhance SE, Nguyen et al. proposed an IoT-based CDRT scheme in [6] in which the IoT controller node works as a relay to decode and forward the superimposed signals to the FU and IoT user. In [7] and [8], the CDRT scheme was utilized in the underlay cognitive NOMA system with perfect/imperfect SIC, channel state information (CSI), respectively. Yang et al. investigated CDRT systems with multiple NUs and a best-NU scheduling scheme in which the NU with minimum OP was selected to forward the signals for the FU in [9]. To fully utilize the spectrum resources, Zou et al. derived the closed-form expression of the ergodic sum rate for the device-to-device (D2D)-aided CDRT systems in which the relay transmits an additional D2D signal to NU while forwarding FU’s signals in the second slot in [10]. In [11] and [12], the CDRT systems with a full-duplex (FD) relay scheme were considered and the analytical expressions for the OP and ergodic sum rate were derived. The results showed that the performance of FD is worse than HD at larger-SNR regions due to the effect of self-interference. The power allocation (PA) problem of the CDRT system was investigated and the optimal closed-form PA policies under the HD and FD protocols were derived in [13]. Then, an adaptive
relying scheme was designed to maximize the minimum user achievable rate. The authors in [14] derived the approximate expressions for the ABER and the optimal PA was studied to optimize the effective throughput.

A simple physical-layer network coding (PNC)-aided network is a two-way relay channel, in which two user nodes desire to communicate with each other via a relay [15], [16]. There are two slots: the multiple access slot and the broadcast slot. In the multiple access slot, one user transmits message $x_1$ and the other user transmits message $x_2$ to the relay simultaneously. In the second slot, the relay broadcasts $x_1 \oplus x_2$ to the two users. Since each user has its signal then the unknown signals can be easily obtained. By exploiting the characteristic of user interferences, PNC techniques can significantly increase the overall system throughput. The authors in [17] proposed a new SE scheme for the CDRT system in which the uplink and downlink transmissions were held simultaneously via PNC scheme. The analytical expressions for the ergodic sum rate, energy efficiency, and Jain’s fairness index of the system were derived. The decoding condition was considered and an adaptive forward strategy was proposed to implement bidirectional communication in [18]. The closed-form expressions for the OP, outage throughput, and ergodic sum rate were derived. Moreover, the PA coefficient was optimized to maximize the ergodic sum rate.

Relative to C-NOMA, when the relay in CDRT systems is untrusted, the signals transmitted to/from the NU in the second slot are secure since the relay is half-duplex and forwarding signals to FU. Meanwhile, the parallel transmission between multiple links enhances SE and throughput but makes the secrecy performance analysis more complicated and challenging. Lv et al. [19] studied the secrecy performance of a CDRT system with an untrusted relay in both uplink and downlink scenarios. Two novel interference-assisted jamming schemes were proposed in which the inter-user interference and the jamming signals were intelligently designed to suppress the reception quality at the relay. The analytical expressions for the lower bound of the exact and asymptotic ergodic secrecy sum-rate (ESSR) were derived. New adaptive jamming schemes were proposed to enhance the secrecy performance of downlink and uplink CDRT systems in [20]. The problems of maximizing ESSR through the optimization of the jamming power for both downlink and uplink scenarios were studied. The analytical expressions for the lower bound and asymptotic ESSR were derived. The secrecy performance of a CDRT system with multiple NUs was investigated in [21] and the best-user scheduling in which the NU with maximum SNR was selected was proposed to enhance the security. The analytical expressions for the lower bound of the exact and asymptotic ergodic secrecy rate (ESR) were derived. TABLE I outlines recent works in literature related to performance analysis of CDRT systems.

**B. Motivation and Contributions**

The PNC scheme improves system throughput while increasing the risk of information eavesdropping. The CDRT scheme improves system throughput with limited SE. However, the research on the security performance of the CDRT systems with the PNC scheme is still in its infancy. This work considers a joint uplink-downlink CDRT system with two legitimate users and an untrusted relay, utilizing friendly jammer signals and inter-user interference to provide secure transmission. The main contributions of this paper are summarized as follows.

1) A new scheme, termed as CDRT-PNC, is proposed to enhance the secrecy performance of a CDRT with an untrusted relay wherein both PNC and NOMA schemes are utilized to improve the spectrum efficiency. The inter-user interference and friendly jamming signals are utilized to suppress the eavesdropping ability of untrusted relay without affecting the signal-to-interference-noise ratio (SINR) of legitimate signals. Specifically, the FU in the first slot and the NU in the second slot operate as jammers to transmit jamming signals to ensure secure transmission of the confidential messages.

2) We investigate the secrecy performance of the considered CDRT system and derive the closed-form expressions for the lower bound of exact and asymptotic ESSR. Simulation results are presented to prove the accuracy of the derived analytical expressions.

3) Relative to [17] and [18] in which the PNC scheme was utilized to enhance SE and closed-form expressions for the OP and ergodic sum rate were derived, the secrecy performance of CDRT-PNC scheme was investigated in this work and the closed-form expressions for the lower bound of exact and asymptotic ESSR were derived. Technically speaking, it is much more challenging to derive the ergodic secrecy rate than the ergodic rate.

4) Although two jamming schemes were proposed in [19] and [20] for the downlink and uplink CDRT systems, respectively, the prior information has not been utilized to improve the spectral efficiency of CDRT systems. In this work, a new jamming scheme is proposed wherein the PNC scheme is utilized to improve both secrecy performance and SE simultaneously. The joint design of inter-user interference, and uplink and downlink makes the secrecy performance analysis more complicated and challenging.

**C. Organization**

The rest of this paper is organized as follows. Section II describes the system model. In Section III, the analytical expressions for the exact ESSR of CDRT-PNC system are derived and analyzed. Asymptotic ESSR of CDRT-PNC system is derived to gain more insight in Section IV. Section V presents the numerical and simulation results to demonstrate the analysis of the security performance of this system and the paper is concluded in Section VI. TABLE II lists the notation and symbols utilized in this work.

**II. SYSTEM MODEL**

Fig. 1 illustrates the system model consisting of a base station denoted by $S$, a NU denoted by $U_N$, and a FU denoted by $U_F$. There is no direct link between $S$ and $U_F$ due to deep fading and shadowing thereby communication...
TABLE I: Recent Literature Related to Performance Analysis of CDRT Systems.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Forward Scheme</th>
<th>PNC</th>
<th>Optimal PA</th>
<th>Performance Metrics</th>
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<tr>
<td>[3]</td>
<td>DF</td>
<td>without</td>
<td>without</td>
<td>OP, Ergodic sum rate</td>
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<tr>
<td>[4]</td>
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<td>[7]</td>
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<td>[19]</td>
<td>AF</td>
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<td>[20]</td>
<td>AF</td>
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<td>with</td>
<td>ESSR</td>
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<tr>
<td>[21]</td>
<td>AF</td>
<td>without</td>
<td>without</td>
<td>ESSR</td>
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Fig. 1: System model consisting of a base station (S), two users (U_N and U_F), and an untrusted relay (R).

link between S and U_F must be deployed via an intermediate relay R, which is trusted at the service level while untrusted at the data level [20], [22]. In other words, R is a potential eavesdropping node to eavesdrop on the confidential information for U_i (i ∈ {N, F}). The average channel gains and channel coefficients between source i and destination j are denoted by λ_ij and h_ij for i, j ∈ {S, U_N, U_F, R} (i ≠ j), respectively. All the wireless links are assumed to experience quasi-static independent Rayleigh fading and reciprocal. The transmit power at S is denoted as P_S and the transmit powers at U_N, U_F, and R are denoted as P_U. The data transmission in each fading block has three consecutive and equal phases as elaborated below.

In the first time slot ($t_1$), according to downlink NOMA scheme, S broadcasts a superimposed signal of $x_1$ and $x_2$ to $U_N$ and $U_F$. At the same time, $U_F$ transmit a jamming signal.
TABLE II: Notation and Symbols

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tr>
<td>$a_{t1}^d$</td>
<td>PA coefficient of $x_1$ in $t_1$</td>
</tr>
<tr>
<td>$a_{t2}^d$</td>
<td>PA coefficient of $x_2$ in $t_2$</td>
</tr>
<tr>
<td>$a_{t3}^d$</td>
<td>PA coefficient of $x_3$ in $t_3$</td>
</tr>
<tr>
<td>$\lambda_{xy}$</td>
<td>Average channel gain between node $x$ and $y$</td>
</tr>
<tr>
<td>$P_S$</td>
<td>Transmit power at $S$</td>
</tr>
<tr>
<td>$G^*$</td>
<td>Amplifying coefficient in $R$</td>
</tr>
<tr>
<td>$\beta_i(x)$</td>
<td>$i$-order modified Bessel function of the second kind</td>
</tr>
<tr>
<td>$H_{\nu}^{(a,b,c)}(x)$</td>
<td>Fox’s $H$-function</td>
</tr>
<tr>
<td>$\Gamma(x)$</td>
<td>Gamma function</td>
</tr>
<tr>
<td>$\psi^{(k,l)}(x)$</td>
<td>Polygamma function</td>
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</table>

In this work, it is assumed that the jamming signals are able to behave as a Gaussian pseudo-random sequence or utilize deterministic waveforms similar to the structure of the desired signal in [23], [24], [25]. Then, all the jamming signals are known to the legitimate receivers and the transmitter, which means that the jamming signals do not affect the receiving quality of the legitimate users.

$z$ to effectively disrupt the eavesdropping quality of $R$. Then, the received signals at the destination $d$ is expressed as

$$y_{d}^t = h_{sd} \left( \sqrt{a_{t1}^d P_S x_1} + \sqrt{a_{t1}^d P_S x_2} \right) + h_{U_d} \sqrt{P_U z_1} + n_d^t,$$

where $d \in \{U_N, R\}$. $a_{t1}^d$ denotes the PA coefficient for $x_1$, $\bar{a}_{t1}^d = 1 - a_{t1}^d$, and $n_d^t$ signifies the additive Gaussian white noise (AWGN) in slot $t$. After deleting $z_1$ and utilizing perfect SIC detection following the decoding order of $x_2 \rightarrow x_1$ base on transmit power [3], the SNR of $x_1$ at $U_N$ is expressed as

$$\gamma_{x_1}^{R} = \frac{a_{t1}^d \rho_S |h_{SU_N}|^2}{\bar{a}_{t1}^d \rho_S |h_{SR}|^2 + \rho_U |h_{UR_t}|^2 + 1},$$

where $\rho_S = \frac{P_S}{\sigma^2}$ and $\sigma^2$ signifies the noise power.

**Remark 1.** It must be noted there is a premise for the CDRT system that $U_N$ firstly decodes the signal for $U_F$. It is assumed $\lambda_{SR} < \lambda_{SU_N}$ to ensure that $U_N$ firstly decodes $U_F$ signal and then cancels it.

It is assumed that $R$ acts as a potential eavesdropper to decode confidential information based on parallel interference cancellation (PIC) technology [20]. The SNRs of $x_1$ and $x_2$ at $R$ are obtained as

$$\gamma_{x_1}^{R} = \frac{a_{t1}^d \rho_S |h_{SR}|^2}{\bar{a}_{t1}^d \rho_S |h_{SR}|^2 + \rho_U |h_{UR_t}|^2 + 1},$$

and

$$\gamma_{x_2}^{R} = \frac{a_{t2}^d \rho_S |h_{SR}|^2}{\bar{a}_{t2}^d \rho_S |h_{SR}|^2 + \rho_U |h_{UR_t}|^2 + 1},$$

respectively, where $\rho_U = \frac{P_U}{\sigma^2}$.

In the second time slot ($t_2$), $U_N$ broadcasts a superimposed signal of the desired signal $x_3$ and a jamming signal $z_2$ to $S$ and $R$ while $U_F$ synchronously transmit its signal $x_4$ to $R$. Then, the received signals at $S$ and $R$ are expressed as

$$y_{S}^t = h_{SU_N} \left( \sqrt{a_{t3}^d P_S x_3} + \sqrt{a_{t3}^d P_S x_2} \right) + n_{t2}^S,$$

where $a_{t3}^d$ denotes the PA for $x_3$ and $\bar{a}_{t3}^d = 1 - a_{t3}^d$, and $n_{t2}^S$ denotes the AWGN at $S$ and $R$ in slot $t_2$. After canceling $z_2$, the SNR of $x_3$ at $S$ is written as

$$\gamma_{x_3}^{S} = \frac{a_{t3}^d \rho_U |h_{SU_N}|^2}{\bar{a}_{t3}^d \rho_U |h_{SU_N}|^2 + \rho_U |h_{UR_t}|^2 + 1},$$

With PIC method, the SNIRs of $x_3$ and $x_4$ at $R$ are expressed as

$$\gamma_{x_3}^{R} = \frac{\rho_U a_{t3}^d |h_{SU_N}|^2}{\rho_U |h_{SU_N}|^2 + \rho_U |h_{UR_t}|^2 + 1},$$

and

$$\gamma_{x_4}^{R} = \frac{\rho_U |h_{UR_t}|^2}{\rho_U |h_{UR_t}|^2 + 1},$$

respectively.

In the third time slot ($t_3$), $R$ amplifies the received signals and then broadcasts them with power $P_U$. Simultaneously, $U_N$ transmits superimposed signals of new messages $x_5$ and $x_3$ to $S$ and $U_F$. It must be noted that $x_3$ can not be wiretapped by $R$ since $R$ is transmitting at that time to achieve perfect secrecy transmission. Transmitting $x_3$ aims to linearly eliminate the interference of the forwarded signal from $R$ to $U_F$. The received signals at $S$ are expressed as

$$y_{S}^t = h_{SU_N} \left( \sqrt{a_{t5}^d P_U x_5} - \sqrt{a_{t5}^d P_U x_3} \right) + G_{SR} y_{R}^t + n_{t}^S,$$

where $G^2 = \frac{\rho_S \lambda_{SR} + 2 \rho_U \lambda_{SR} + 2 \rho_U \lambda_{SU_N} + 2 \rho_U \lambda_{UR_t} + 2 \rho_U \lambda_{SR} + 2 \rho_U \lambda_{SU_N} + 2 \rho_U \lambda_{UR_t}}{\rho_S \lambda_{SR} + 2 \rho_U \lambda_{SR} + 2 \rho_U \lambda_{SU_N} + 2 \rho_U \lambda_{UR_t} + 2 \rho_U \lambda_{SR} + 2 \rho_U \lambda_{SU_N} + 2 \rho_U \lambda_{UR_t}}$ denotes amplifying coefficient in fixed-gain relay scheme [19], [20], [22]. $n_{S} = G_{SR} n_{R}^t + G_{SR} n_{R}^t + n_{S}^2$, $n_{S}^2$ denotes the AWGN at S in slot $t_3$, $a_{t3}^d$ denotes the PA coefficient for $x_3$, $\bar{a}_{t3}^d = 1 - a_{t3}^d$, and step (a) is obtained since $x_1$, $x_2$, $x_3$, $z_1$, and $z_2$ are known at $S$. Subsequently, $S$ utilizes SIC technology following the decoding order of $x_3 \rightarrow x_4$ and based on average channel gain condition of $\lambda_{SR} < \lambda_{SU_N}$ [20], the SNIRs of $x_5$ and $x_4$ at $S$ are obtained as

$$\gamma_{x_5}^{S} = \frac{a_{t5}^d \rho_U |h_{SU_N}|^2}{G^2 |h_{SR}|^2 \left( \rho_U |h_{UR_t}|^2 + 2 \right) + 1},$$

and

$$\gamma_{x_4}^{S} = \frac{G^2 \rho_U |h_{SR}|^2 |h_{UR_t}|^2}{2G^2 |h_{SR}|^2 + 1},$$

respectively.
Similarly, the received signals at $U_F$ are expressed as
\[
y_{t3}^{U_F} = h_{UNU_F} \left( \sqrt{a_3^3} P_U x_3 - \sqrt{a_3^3} P_{3x3} \right) \\
+ G h_{R_U} y_{t3}^{R} + n_{3}^{U_F} \\
+ G h_{SR} h_{R_U} \left( \sqrt{a_3^3} P_S x_1 + \sqrt{a_3^3} P_{3x2} \right) + \omega_0 x_3 + h_{UNU_F} \sqrt{a_3^3} P_{3x3} + n_{3}^{U_F},
\]
where $\omega_0 = G |h_{R_U}| |h_{R_U}^N| \sqrt{a_3^3} \rho_U - |h_{UNU_F}| \sqrt{a_3^3} \rho_U$, $n_{3}^{U_F} = G h_{R_U} n_{3}^{R} + G h_{R_U} n_{3}^{R} + n_{3}^{U_F}$, $a_3^3$ denotes the AWGN at $U_F$ in slot 3, and step (b) is obtained since $x_4$, $z_1$, and $z_2$ are known at $U_F$. To remove $x_3$ at $U_F$, $a_3^3$ must satisfy $\omega_0 = 0$. Hence, we have
\[
a_3^3 = \frac{G^2 |h_{R_U}|^2 |h_{R_U}^N| a_3^2}{|h_{UNU_F}|^2}.
\]

In this work, it is assumed $\lambda_{USU} \leq \lambda_{USU}$ to ensure $\mathbb{E}[a_3^3] < 1$ [21]. Applying the expectation operation for $a_3^3$, we obtain
\[
\mathbb{E}[a_3^3] = \frac{G^2 a_3^2 \lambda_{USU} \lambda_{R}}{\lambda_{USU} + \lambda_{R}} \\
= \frac{\rho_U a_3^2 \lambda_{USU} \lambda_{R}}{\lambda_{USU} + \lambda_{R}} + \frac{\rho_U a_3^2 \lambda_{USU} \lambda_{R}}{\lambda_{USU} + \lambda_{R}} + \frac{\rho_U a_3^2 \lambda_{USU} \lambda_{R}}{\lambda_{USU} + \lambda_{R}} \\
< \frac{\lambda_{USU} a_3^2 \lambda_{USU} \lambda_{R}}{\lambda_{USU} + \lambda_{R}} \\
< \frac{\lambda_{USU} a_3^2 \lambda_{USU} \lambda_{R}}{\lambda_{USU} + \lambda_{R}} \\
< a_3^3 < 1.
\]

It must be noted that $\mathbb{E}[a_3^3] < 1$ may not guarantee $a_3^3 < 1$. To meet the causality constraint on power coefficient, we set $a_3^3 = \frac{G^2 a_3^2 \lambda_{USU} \lambda_{R}}{\lambda_{USU} + \lambda_{R}}$. Subsequently, $U_F$ utilizes SIC detection following the decoding order of $x_3 \to x_2$, the SINR of $x_2$ is obtained as
\[
\gamma_{t2}^2 = \frac{G^2 a_3^2 \rho_S |h_{SR}|^2 |h_{R_U}|^2}{G^2 |h_{R_U}|^2 (a_3^2 \rho_S |h_{SR}|^2 + 2) + \omega_0^2 + 1} \\
= \frac{G^2 a_3^2 \rho_S |h_{SR}|^2 |h_{R_U}|^2}{G^2 |h_{R_U}|^2 (a_3^2 \rho_S |h_{SR}|^2 + 2) + 1}.
\]

The instantaneous secrecy rate of $x_j$ is expressed as [25]
\[
C_{xtj}^S = \left[ \ln \left( 1 + \gamma_{tj}^2 \right) - \ln \left( 1 + \gamma_{tj}^0 \right) \right]^+, \tag{17}
\]
where $j = 1, \cdots, 5$, $D \in \{S, N, F\}$, and $[x]^+ = \max \{x, 0\}$.

### III. Ergodic Secrecy Sum Rate Analysis

In this section, we analyze the ESSR of the proposed CDRT-PNC scheme, which is expressed as
\[
\bar{C}_{\text{ESSR}} = \frac{1}{3} \sum_{j=1}^{5} C_{xtj}^S \\
= \frac{1}{3} \sum_{j=1}^{4} \mathbb{E} \left[ C_{xtj}^S - C_{xtj}^S \right]^+ + \frac{1}{3} \mathbb{E} \left[ C_{xt5} \right],
\]
where $\bar{C}_{xtj}^S$ denotes the ergodic secrecy rate of $x_j$, $C_{xtj}^S = \ln \left(1 + \gamma_{tj}^2\right)$, $C_{xtj}^S = \ln \left(1 + \gamma_{tj}^0\right)$, a factor of 1/3 is multiplied since the whole communication process is divided into three time slots per unit of time, and $\mathbb{E} \left[ \cdot \right]$ denotes expectation operator.

By utilizing Jensen’s inequality, the lower bound of the ESSR is expressed as
\[
\bar{C}_{\text{ESSR}} = \frac{1}{3} \sum_{j=1}^{5} C_{xtj}^S \\
= \frac{1}{3} \sum_{j=1}^{5} \left[ C_{xtj}^S - C_{xtj}^S \right]^+ + \frac{1}{3} \bar{C}_{xt5}^S,
\]
where $\bar{C}_{xtj}^S = \mathbb{E} \left[ C_{xtj}^S \right], \bar{C}_{xtj}^S = \mathbb{E} \left[ C_{xtj}^S \right],$ and the superscript ‘L’ denotes lower bound. Based on the probability theory, ergodic capacity is expressed as
\[
\bar{C} = \mathbb{E} \left[ \ln \left(1 + \gamma \right) \right] \\
= \int_0^\infty \ln \left(1 + x \right) f_\gamma \left( x \right) dx, \tag{20}
\]
where $f_\gamma(x)$ and $F_\gamma(x)$ are probability density function (PDF) and CDF of $\gamma$, respectively.

The analytical expressions for ergodic rate of all the signals in Section II are derived as follows.

Substituting (22) into (20) and utilizing [26] (4.337.2), \( \bar{C}_{xtj}^S \) is obtained as
\[
\bar{C}_{xtj}^S = \mathbb{E} \left[ \ln \left(1 + a_{tj}^3 \rho_S |h_{SR}|^2 \right) \right] \\
= \frac{1}{\lambda_{USU}} \int_0^\infty \ln \left(1 + a_{tj}^3 \rho_S \right) e^{-\frac{x}{\omega_0}} dx, \tag{21}
\]
where $\phi_1(x) = -\exp \left(\frac{1}{2} t \right) \text{Ei} \left(-\frac{1}{2} t \right)$ and $\text{Ei} \left(x \right) = -\int_0^\infty \frac{t^{-1}}{t} \exp \left(-t \right) dt$ is exponential integral function defined by [26] (8.211.1).

Based on (3), we obtain the CDF of $\gamma_{tj}^2$ as [22], shown at the top of next page, where $\omega_1 = \frac{\rho_S |h_{SR}|^2}{\rho_S |h_{SR}|^2}$.\]

Remark 2. Based on eq. (22), one can easily obtain the OP when $R$ decodes $x_j$ with PIC by substituting $x$ with in the SINR threshold. Thus, it can be observed that the ratio between $a_{t1}$ and $a_{t1}'$ denotes an applicable threshold for the SINR. Specifically, when the target SINR threshold is larger than the ratio, $R$ can not decode $x_1$ successfully.

Substituting (23) into (20) and utilizing [26] (3.352.4), \( \bar{C}_{xtj}^S \) is derived as
\[
\bar{C}_{xtj}^S = \int_0^\infty \frac{1}{x + 1} \left(1 - F_{\omega_{tj}^2} \left( x \right) \right) dx \\
= \frac{\phi_2 \left( \rho_S \lambda_{SR}, a_{tj}^2 \rho_S \lambda_{SR} \right)}{1 - \omega_1} \\
- \omega_1 a_{tj}^2 \phi_2 \left( \omega_1 \rho_S \lambda_{SR}, a_{tj}^2 \rho_S \lambda_{SR} \right), \tag{23}
\]
where $\phi_2 \left( a, b \right) = \phi_1 \left( a \right) - \phi_1 \left( b \right)$. Substituting (21) and (23) into (19), the closed-form expression for lower bound of \( \bar{C}_{\text{ESSR}} \) is obtained.
is faster than the increase of $\gamma_{R,1}$. However, increase of $\gamma_{N,1}$ is faster than the increase of $\gamma_{R,1}$ because $\gamma_{R,1} < \gamma_{N,1}$ and $\gamma_{N,1} = \alpha_1^1 \rho_S |h_{SU,R}|^2$ is proportional to $\alpha_1^1$. Thus, the ESR of $x_1$ increases with increasing $\alpha_1^1$.

The CDF of $x_2$ is obtained as (24), shown at the top of this page, where $\beta_S = \frac{G^2 \rho_S \lambda_{SR} \rho_{|x|^2}}{4}$, $\phi_3 (x) = \sqrt{x} K_1 (\sqrt{x})$, and $K_v (\cdot)$ is the $v^{th}$-order modified Bessel function of the second kind, defined by (26) (8.432.1). Substituting (24) into (20) and utilizing (26) (3.324.1), we obtain

$$
C_{\gamma_{R,2}} = \int_0^\infty \tilde{C}_{\gamma_{R,2}} (x) dx
$$

where $\phi_4 (a,b) = \int_0^\infty \frac{1}{x} e^{-x} \sqrt{x} K_1 (\sqrt{x}) dx$, (9.34.3), eqs.(10) and (11) of (27), and (28) (1.2), $\phi_4 (a,b)$ is denoted as

$$
\phi_4 (a,b) = \int_0^\infty \frac{1}{x} e^{-x} \sqrt{x} K_1 (\sqrt{x}) dx
$$

and $G_{c,d} (\cdot)$ is the Meijer's $G$-function defined by (26) (9.301) and $H_{e,d,p,r,v} (\cdot)$ is the extended generalized bivariate Fox's H-function (EGBFHH) defined by (28) (2.57).

Similar to (23), $C_{\gamma_{R,2}}$ is obtained as

$$
C_{\gamma_{R,2}} = \phi_4 (\rho_S \lambda_{SR}, a_1^1 \rho_S \lambda_{SR})
$$

where $\phi_4 (a,b) = \int_0^\infty \frac{1}{x} e^{-x} \sqrt{x} K_1 (\sqrt{x}) dx$, (9.34.3), eqs.(10) and (11) of (27), and (28) (1.2), $\phi_4 (a,b)$ is denoted as

$$
\phi_4 (a,b) = \int_0^\infty \frac{1}{x} e^{-x} \sqrt{x} K_1 (\sqrt{x}) dx
$$

and $\gamma_{N,1} = \alpha_1^1 \rho_S |h_{SU,R}|^2$ is proportional to $\alpha_1^1$. Thus, the ESR of $x_1$ increases with increasing $\alpha_1^1$.

Remark 3. Based on (2) and (3), one can find that both $\gamma_{R,1}^2$ and $\gamma_{N,1}^2$ increase with increasing $\alpha_1^1$. However, increase of $\gamma_{N,1}^2$ is faster than the increase of $\gamma_{R,1}^2$ because $\gamma_{R,1} < \gamma_{N,1}$.
for $x_2$, which leads to the deterioration of the ESR for $x_2$ as testified in [29].

Similarly to (21) and (23), we obtain $C^{x_3}$ and $\tilde{C}^{x_3}$ as

$$C^{x_3} = \phi_1 \left( a^t_3 \rho_U \lambda_{SU_N} \right),$$  \hspace{1cm} (28)

and

$$\tilde{C}^{x_3} = \frac{a^t_3 \phi_2 \left( a^t_3 \rho_U \lambda_{RU_N} + \rho_U \lambda_{RF} \right) - \phi_2 \left( \rho_U \lambda_{RU_N}, \rho_U \lambda_{RF} \right)}{\omega_2 - 1},$$ \hspace{1cm} (29)

respectively, where $\omega_2 = \frac{\lambda_{RF}}{\lambda_{RU_N}}$. Substituting (28) and (29) into (19), the closed-form expression for lower bound of $C_{ESR}$ is obtained.

Remark 5. Although the equation of the ESR of $x_3$ is similar to that of the ESR of $x_1$, it must be noted that the effect of $a^t_3$ on the ESR of $x_3$ is different from the effect of $a^t_1$ on the ESR of $x_1$ since there is $0 \leq a^t_3 \leq 1$ while $a^t_1 < 0.5$. Based on [1], one can find that increasing $a^t_3$ signifies that $a^t_3$ denotes more power is allocated to $x_3$ and less power is allocated to $x_2$, which leads to increase in both $\gamma^{x_3}_R$ and $\gamma^{x_3}_S$. However, $\gamma^{x_3}_R$ in $0 < a^t_3 < 1$ region increases faster than in $0 < a^t_2 < 0.5$ since the power allocated to $x_2$ tends to zero. Thus, the ESR of $x_3$ increases initially and then decreases as $a^t_3$ increases thereby there exists an optimal $a^t_3$ to maximize the ESR of $x_3$.

Similar to (24), we obtain

$$F_{\gamma^{x_3}_S} (x) = 1 - e^{-\frac{\rho_U x}{\rho_U}} \beta_U \left( \frac{x}{\beta_U} \right),$$ \hspace{1cm} (30)

where $\beta_U = \frac{G^2 \rho_U \lambda_{SU} \lambda_{RF}}{4}$. Similar to (25), we obtain

$$\tilde{C}^{x_3} = \beta_U \phi_4 \left( \beta_U, \frac{G^2 \lambda_{SU}}{2} \right).$$ \hspace{1cm} (31)

The CDF of $\gamma^{x_3}_R$ is expressed as

$$F_{\gamma^{x_3}_R} (x) = \Pr \left\{ |h_{URF}|^2 < \frac{\rho_U x}{\rho_U} |y_{UR}|^2 + z \right\} = \int_0^{\infty} F_{h_{URF}} \left( \frac{\rho_U x}{\rho_U} + y \right) f_{|y_{UR}|^2} \left( y \right) dy = 1 - \frac{\lambda_{RF}}{\lambda_{RU_N} x + \lambda_{RF}} e^{-\frac{x \lambda_{RF}}{\lambda_{RU_N}}}.$$ \hspace{1cm} (32)

Substituting (32) into (20), utilizing (26) (3.352.4)), we obtain

$$\tilde{C}^{x_3} = \int_0^\infty \frac{1}{x + 1} \left( 1 - F_{F_{\gamma^{x_3}_R}} (x) \right) dx = \int_0^\infty \frac{1}{x + 1} \frac{\lambda_{RF}}{\lambda_{RU_N} x + \lambda_{RF}} e^{-\frac{x \lambda_{RF}}{\lambda_{RU_N}}} dx = \frac{\omega_2 \phi_2 \left( \rho_U \lambda_{RU_N}, \rho_U \lambda_{RF} \right)}{\omega_2 - 1}.$$ \hspace{1cm} (33)

Substituting (31) and (33) into (19), the closed-form expression for lower bound of $C_{ESR}$ is obtained.

Based on (11), the CDF of $x_3$ is obtained as (34), showed at the top of next page. Subsequently, the ergodic rate of $x_3$ is expressed as

$$C^{x_3} = \frac{a^t_3}{G^2 \lambda_{SR} \lambda_{RF}} \int_0^\infty \frac{1}{x + 1} e^{-\frac{x \lambda_{RF}}{\lambda_{SU_N}}} \left( \frac{2}{\rho_U \lambda_{RF} + \frac{1}{a^t_3}} + \frac{\tilde{a}^t_3 \lambda_{SU_N}}{G^2 \lambda_{SR} \lambda_{RF}} \right) dx.$$ \hspace{1cm} (35)

The closed-form expression for (35) is difficult to obtain since there is a complicated integral of exponential integral function and so its lower bound is derived based on the method proposed in [21]. We define $X = \tilde{a}^t_3 \rho_U |h_{SU_N}|^2$ and $Y = G^2 |h_{SR}|^2 \left( \rho_U |h_{RF}|^2 + 2 \right)$, and using Jensen’s inequality, we obtain

$$C^{x_3} = \mathbb{E} \left[ \frac{1}{1 + \gamma^{x_3}_S} \right] = \mathbb{E} \left[ \left( 1 + \frac{X}{Y} \right)^{-1} \right] = \mathbb{E} \left[ \left( 1 + e^{\Phi (x)} \right)^{-1} \right] \geq C^{x_3,L} = \mathbb{E} \left[ \left( 1 + e^{\Phi (x)} \right)^{-1} \right] = \mathbb{E} \left[ \left( 1 + e^{\beta_U} \right)^{-1} \right],$$ \hspace{1cm} (36)

where $\Phi = \mathbb{E} \left[ \ln (X) \right] - \mathbb{E} \left[ \ln (1 + Y) \right]$. Utilizing (26) (4.311.1), we derive

$$\mathbb{E} \left[ \ln (X) \right] = \int_0^\infty \ln \left( \tilde{a}^t_3 \rho_U \right) f_{|h_{SU_N}|^2} \left( x \right) dx = \ln \left( \tilde{a}^t_3 \rho_U \lambda_{SU} \lambda_{RF} \right) - C,$$

where $C = 0.577215$ denotes Euler’s constant.\hspace{1cm} (37)

By utilizing

$$\int_{a^t_3}^{\infty} e^{-a^t_3 x^2} dx \approx \sqrt{\frac{\beta_U}{\alpha}} K_1 \left( \sqrt{\frac{\alpha}{\beta_U}} \right),$$ \hspace{1cm} (38)

which is verified in [30], the CDF of $Y$ is obtained as

$$F_Y (y) = \Pr \left\{ |h_{SR}|^2 < \frac{y}{G^2 (\rho_U |h_{RF}|^2 + 2)} \right\} = \int_0^\infty F_{|h_{SR}|^2} \left( \frac{y}{G^2 (\rho_U |h_{RF}|^2 + 2)} \right) f_{|h_{RF}|^2} \left( x \right) dx = 1 - \frac{1}{\lambda_{RF}} \int_0^\infty \frac{e^{-\frac{y}{G^2 \lambda_{RF}}}}{\lambda_{RF}} dx = 1 - \frac{1}{\rho_U \lambda_{RF}} e^{-\frac{y}{\rho_U \lambda_{RF}}} \times \int_0^\infty e^{-\frac{y}{G^2 \lambda_{RF}}} dt \approx 1 - \phi_3 \left( \frac{y}{\beta_U} \right).$$ \hspace{1cm} (39)

Substituting (39) into (20), utilizing (26) (9.31.2) and (27) (21), we obtain

$$\mathbb{E} \left[ \ln (1 + Y) \right] = \int_0^1 \frac{1 - F_Y (x)}{1 + x} dx = \beta_U \int_0^\infty G_{1,1} ^{1,1} \left[ \beta_U t, 1 \right] G_{0,2} ^{2,0} \left[ t \left| 1, 0 \right. \right] dt = G_{1,3} ^{3,1} \left[ \frac{1}{4 \beta_U}, 1 \right]_0^1.$$ \hspace{1cm} (40)
The analytical expression is complicated since many factors affect the ESSR in the high transmit power regime in the following section. To obtain more insights, we derive asymptotic expressions of the ESR of $\rho$. Hence, we have

$$\Phi = \ln \left( \frac{\bar{\alpha}_2}{\rho U \lambda_{SU_N}} \right) - G^{3,1}_{1,3} \left[ \frac{1}{4\bar{\beta}_U} \left| \Gamma \right|_{0,0,0} - C \right], \quad (41)$$

Substituting (41) into $C^{x_5,1}_{\text{ESSR}}$, the closed-form expression for lower bound of $C^{x_5,1}_{\text{ESSR}}$ is obtained.

**Remark 6.** Based on $a_3^{x_5} = \frac{G^{3,2}_{x_5} \lambda_{SU_N} \lambda_{RU_P}}{\lambda_{SU_N} \lambda_{RU_P}}$, one can find that increasing $a_3^{x_5}$ signifies decrease in $a_3^{x_5}$ and $\bar{C}^{x_5}_{\text{ESSR}}$ since the transmit power for $x_5$ leads to the deterioration of the ESR of $x_5$.

Substituting (21), (23), (25), (27), (28), (31), (33), and (36) into (19), the analytical expressions for the lower bound of the ESSR are obtained as (42), shown at the top of this page. The analytical expression is complicated since many factors affect the ESSR of the considered system, specifically, the power coefficients ($a_1^{x_5}, a_2^{x_5}$), the transmit powers ($P_S, P_U$), and the average channel gains ($\lambda_{SU_N}, \lambda_{SU_R}, \lambda_{RU_P}, \lambda_{SU_U}$). To obtain more insights, we derive asymptotic expressions of the ESSR in the high transmit power regime in the following section.

### IV. Asymptotic Analysis for Ergodic Secrecy Sum Rate

To gain more insights on the proposed scheme, we analyze asymptotic ESSR expression in high-SNR region in this section. It is assumed that $\rho_S = \nu \rho_U$ and $\rho_U = \rho \rightarrow \infty$, where $\nu$ denotes the ratio of $\rho_S$ to $\rho_U$.

Based on (21), utilizing $\text{Ei}(x) \sim \frac{e^{-x}}{\nu} C + \ln(-x)$ and $e^{x} \approx x^\nu$, we have $\Phi(x) \approx \ln x - C$. Then, the asymptotic ESR of $x_1$ is obtained as

$$C^{x_1,\rho \rightarrow \infty}_{\text{ESSR}} = C^{x_1,\rho \rightarrow \infty}_{R} - C^{x_1,\rho \rightarrow \infty}_{R}$$

$$= \ln \left( a_3 \rho \lambda_{SU_N} \right) + \frac{\omega_1a_3 \ln(\omega_1)}{(1 - \omega_1)(\omega_1 - a_3)} - \frac{\bar{a}_3 \ln(\bar{a}_3)}{\omega_1 - \bar{a}_3} \approx \ln \left( a_3 \rho \lambda_{SU_N} \right), \quad (43)$$

Similarly, the asymptotic ESR of $x_3$ is obtained as

$$C^{x_3,\rho \rightarrow \infty}_{\text{ESSR}} = C^{x_3,\rho \rightarrow \infty}_{R} - C^{x_3,\rho \rightarrow \infty}_{R}$$

$$= \ln \left( a_3^{x_3} \rho \lambda_{SU_N} \right) + \frac{\omega_2a_3^{x_3} \ln(\omega_2)}{(1 - \omega_2)(\omega_2 - a_3^{x_3})} - \frac{\bar{a}_3^{x_3} \ln(\bar{a}_3^{x_3})}{\omega_2 - \bar{a}_3^{x_3}} \approx \ln \left( a_3^{x_3} \rho \lambda_{SU_N} \right), \quad (44)$$

**Remark 7.** Based on (43) and (44), one can observe that the lower bounds of ESR for $x_1$ and $x_3$ scale as $\ln(\rho)$ at high-SNR region. This is because SIC deletes the inter-user interference on $U_N$ and the jamming signal does not affect $\gamma_{x_1}^{x_1}$ and $\gamma_{x_3}^{x_3}$, which tends to infinity as $\rho \rightarrow \infty$. However, the inter-user interference and the jamming signal on $R$ can not be deleted by PIC thereby $\gamma_{x_1}^{x_1}$ and $\gamma_{x_3}^{x_3}$ tend to be a constant, which is independent of $\rho$. Moreover, the asymptotic ESR of $x_1$ is proportional to $a_1^{x_1}$, which is same as the statement in Remark 3.
where \( \psi^{(k)}(x) \) is the polygamma function defined by (32) (3.63.8)]. The detailed derivation of \( \phi_5 \) \((a, b)\) is provided in Appendix A. Similarly, we obtain

\[
I_2 = \phi_1 \left( \frac{1}{a_1} \nu \rho \lambda_{SR} \right) - C + \ln \left( \frac{2G^2 \lambda_{RU}}{2} \right) - \phi_5 \left( 4a_1^2 \beta S, 2G^2 \lambda_{RU} \right). \tag{51}
\]

When \( \beta_S = \frac{G^2 \rho a_1 \lambda_{SR} \lambda_{RU}}{4} \rightarrow \infty \), \( \phi_5(a, b, S) \) \( \approx 0 \), and \( \phi_1(x) \approx \ln(x) - C \), and \( \phi_2(a, b, S) \approx \ln \left( \frac{1}{\rho} \right) \). Substituting (49) and (51) into (48), we obtain \( C_{x_2, \rho \to \infty} \) as

\[
C_{x_2, \rho \to \infty} = \phi_2 \left( \frac{\nu \rho \lambda_{SR} \lambda_{RU}}{2}, 2 \right) + \phi_5 \left( 4\nu \rho \lambda_{SR} \lambda_{RU}, 2 \right) - \phi_5 \left( 4\nu \rho \lambda_{SR} \lambda_{RU}, 2 \right) \approx \ln \left( \frac{1}{a_1} \right). \tag{52}
\]

Based on (27), utilizing \( \phi_1(x) \approx \ln(x) - C \), \( C_{x_2} \) is asymptotically derived as

\[
C_{x_2, \rho \to \infty} = \frac{a_1^2 \ln \left( \frac{a_1^2}{\omega_1} \right)}{\omega_1 - a_1} - \frac{\omega_1 a_1^2 \ln \left( \frac{\omega_1}{\omega_1 - a_1} \right)}{\omega_1 - a_1}. \tag{53}
\]

Subsequently, the asymptotic ESR of \( x_2 \) is obtained as

\[
C_{x_2, \rho \to \infty} \approx \frac{\omega_1}{\omega_1 - a_1} \left( \ln \left( \frac{1}{a_1^2} \right) + a_1^2 \ln \left( \frac{\omega_1}{\omega_1 - a_1} \right) \right). \tag{54}
\]

Remark 9. Based on (54), one can observe that the asymptotic ESR of \( x_2 \) tends to be a constant independent of \( \rho \) in the high-SNR region.
V. SIMULATION RESULTS AND DISCUSSIONS

In this section, simulation and analysis results are presented to validate the effectiveness of the proposed secure scheme. The effects of system parameters on secure performance of the considered system, such as transmit SNR, average channel gains, power ratio coefficient, and PA coefficients, are investigated. Without loss of generality, the parameters in the simulation and analysis are set as $\lambda_{SU1} = \lambda_{RU_F} = 1$, $\lambda_{SR} = 0.7$, $\lambda_{RU_N} = \lambda_{RU_NU_F} = 0.8$, $\nu = 2$, $a_1 = 0.2$, and $a_2 = 0.5$. ‘Sim’, ‘Ana’, and ‘Asy’ in all the figures denote the simulation, numerical, and asymptotic results, respectively. Since there are five ESRs in the expression of ESSR, we consider system via utilizing the lower bound is feasible. One can observe that simulation and numerical results match perfectly in the second slot $\rho = 1$. 5.

Fig. 2 demonstrates simulation results for the exact ESR and numerical results for the lower bound of ESR versus $\rho$. It is observed that simulation and lower bound results of $x_3 \times x_4$ match perfectly, which justifies approximating the ESR of the considered system via utilizing the lower bound is feasible.

Fig. 3 plots the effect of $\rho$ and $a_1^3$ on the simulation results for the lower bound of ESRs and ESSR. One can observe that simulation and numerical results match perfectly for $x_5$, meanwhile $R$ amplifies received signals $x_4$.

2) A CDRT system based on NOMA and PNC without jamming signal (‘Ben2’): There are three slots in each fading block. $S$ broadcasts superimposed signals $\sqrt{a_1^3} P_S x_1 + \sqrt{a_2^3} P_S x_2$ and $U_F$ emits jamming $\sqrt{P_U} z_1$ in the first slot ($t_1$), $R$ amplifies received signals and forwards to $U_F$ in the second slot ($t_2$), $U_N$ and $U_F$ broadcast uplink NOMA signals $\sqrt{a_3^3} P_U x_3 + \sqrt{a_5^3} P_U z_2$ and $\sqrt{P_U} x_4$ in the third slot ($t_3$), and in final slot ($t_4$) $U_N$ sends a new uplink signal to $SU$.

Fig. 2: ESRs and ESSR versus $\rho$.

Fig. 3: ESRs and ESSR for varying $\rho$ and $a_1^3$. 
to verify the correctness of the analysis, and it is apparent that ESSR increases with increasing $\rho$ and $a_4^1$. This is because the legitimate rates always increase while the eavesdropping rates initially increase and subsequently remain constant with increasing $\rho$. Different from $\rho$, $a_1^4$ only affects the ESR of $x_1$ and $x_2$. The ESR of $x_1$ gets better and continues increasing with increasing $a_1^4$ whereas correspondingly the ESR of $x_2$ gets worse with increasing $a_1^4$. Due to poor channel gain and the presence of inter-user interference, the slope of the decrease in the ESR of $x_2$ is less than the slope of the increase in the ESR of $x_1$, thus the ESSR increases with increasing $a_1^4$.

Fig. 4 demonstrates the lower bound of ESRs and ESSR vs transmit SNR $\rho$ for varying $a_3^2$. It can be observed that the ESSR of the proposed scheme increases with an increase in $\rho$. Different from $a_1^4$, one can observe the ESR increases initially and subsequently decreases with increasing $a_3^2$. This verifies that there is an optimal $a_3^2$ to achieve the optimal performance, which confirms the result in Remark 4.

Fig. 5 presents the impact of $\rho$ for varying $\lambda_{RU_N}$ on the lower bound of ESRs and ESSR. We find that the effect of $\lambda_{RU_N}$ on the ESR in the lower-$\rho$ region is different from that in the higher-$\rho$ region. Within lower $\lambda_{RU_N}$, the larger ESSR is experienced in the lower-$\rho$ region while the lower $\lambda_{RU_N}$, the lower ESSR in the higher-$\rho$ region. This is due the quality of $R-U_N$ link that has a different influence on the ESR of $x_3$ and $x_4$. Lower $\lambda_{RU_N}$ results in a poor eavesdropping rate for $x_3$, which leads to a higher ESR for $x_3$. However, lower $\lambda_{RU_N}$ results in the poor jamming quality of $z_2$, which leads to decrease in ESR of $x_4$, even to zero. In the higher-$\rho$ region, not only the jamming influence of $z_2$ is strong but also amplifying coefficient decreases thereby both $\gamma_{z_4}^z$ and $\gamma_{z_4}^z$ become worse. The ESR of $x_3$ is improved since the eavesdropping and jamming quality at $R$ are enhanced simultaneously.

Fig. 7 demonstrates the lower bound of ESRs and ESSR with varying $\rho$ and $\lambda_{RU_N}$. It is observed that the ESSR with higher $\lambda_{RU_N}$ in the lower-$\rho$ region outperforms that with lower $\lambda_{RU_N}$ in the lower-$\rho$ region, the ESR with lower $\lambda_{RU_N}$ outperforms that with higher $\lambda_{RU_N}$ in the lower-$\rho$ region while the variation of ESR for $x_4$ dominates in the higher-$\rho$ region. ESRs of $x_1$ and $x_3$ increase and ESR of $x_4$ decreases as $\lambda_{RU_N}$ increases.

Fig. 7 provides a comparison of the lower bound of ESRs and ESSR versus $\rho$ with varying $\nu$. A trend is that ESSR increases with an increase in $\rho$ and $\nu$, which represents that the ESSR with higher $\nu$ (higher $\rho_S$) outperforms that with lower $\nu$, which is easy to understand since increasing transmit SNR can enhance the secrecy performance. The ESR of
Due to the time-varying nature of wireless channels, designing $a_{x_1}^3$ based on transient channel state information is bound to achieve perfect cancellation, which poses a substantial analytical challenge due to the superimposed coupling of many random variables. Similar to [21], the average channel gain instead of the instantaneous channel gain is utilized to approximate the linearly canceled signal. In Figs. 2 - 9, it is assumed that $x_3$ can be perfectly deleted at $U_F$. Fig. 10 wherein the practical cancellation is considered is provided to evaluate how good the approximation is. One can observe an error for the ESR of $x_2$ with practical cancellation of $x_3$. However, the influence from the error on the ESSR of considered systems can be omitted since the asymptotic ESR of $x_2$ tends to a constant independent of $\rho$ in the high-SNR region, which is explained in Remark 9.

VI. CONCLUSION

In this paper, we proposed a new scheme called CDRT-PNC to provide reliable and secure communication for the CDRT system with uplink and downlink transmission. Cooperative jamming and inter-user interference prevented information leakage against the untrusted relay, NOMA and PNC schemes are utilized to enhance SE. To characterize the security performance of the proposed schemes, we derived the expressions

$x_1$ predominates in this scenario and ESR of $x_3$ almost is independent on $\nu$. From Figs. 5 - 7, it is easily observed that the scaling law of proposed scheme is obtained via asymptotic results.

Fig. 8 plots the simulation results for the lower bound of ESRs and ESSR for varying $\lambda_{RU,F}$ and $\lambda_{RU,N}$ with $\rho = 20$ dB. It can be found that the ESSR increases with increasing $\lambda_{RU,F}$ while the rate of growth decreases. This is because the ESR of $x_1$ increases with increasing $\lambda_{RU,F}$ and dominates. Moreover, ESSR with lower $\lambda_{RU,F}$ outperforms that with higher $\lambda_{RU,F}$ in lower-$a_1^1$ region. This is because ESRs of $x_1$, $x_2$, and $x_3$ increase and ESRs of $x_4$ and $x_5$ decrease with increasing $\lambda_{RU,F}$. ESR of $x_4$ dominates in lower-$a_1^1$ region while ESR of $x_1$ dominates in higher-$a_1^1$ region.

Fig. 9 demonstrates the effects of $a_{x_3}^2$, $\lambda_{RU,F}$, and $\lambda_{RU,N}$ on the lower bound of ESRs and ESSR. One can observe there is an optimal $a_{x_3}^2$ to maximize the ESSR and the optimal $a_{x_3}^2$ depends on $\lambda_{RU,F}$ and $\lambda_{RU,N}$. This is because ESR of $x_5$ is dominant in lower-$a_3^1$ region and ESR of $x_3$ is dominant in higher-$a_3^1$ region. As $a_{x_3}^2$ increases, ESR of $x_3$ increases to the maximum then decreases. From Figs. 8 - 9 we observe that simulation and numerical results match perfectly to verify the correctness of our analysis. Compared to the benchmarks, the proposed scheme achieves higher secure performance.
for lower bounds of the exact and asymptotic ESSR. The effects of system parameters on the ESSR were analyzed in detail. Simulations were provided to verify the correctness of the analytical results. The results demonstrated that the proposed scheme obtains better security performance than conventional schemes. An interesting work is to investigate the secrecy performance of CDRT systems with imperfect SIC and imperfect CSI, which will be part of our future work.

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APPENDIX A

DERIVATION OF $\phi_5(a, b)$

Utilizing $\ln(y) = \frac{dy^y}{ds}|_{s=0}$, $\phi_5(a, b)$ is expressed as

$$\phi_5(a, b) = \int_0^{\infty} \frac{e^{-by}}{ay+1} \ln(y) dy$$
$$= \int_0^{\infty} \frac{e^{-by}}{ay+1} dy$$
$$= \frac{d}{ds} \int_0^{\infty} \frac{y^s e^{-by}}{ay+1} dy$$

On utilizing [27] (10), (11), (21) and [26] (9.31.2), (9.31.5), we obtain

$$\int_0^{\infty} \frac{y^s e^{-by}}{ay+1} dy = \int_0^{\infty} y^s G_{1, 1}^{1, 1} [ay | 0] G_{0, 1}^{1, 0} [by | 0] dy$$
$$= a^{-s-1} G_{2, 1}^{1, 2} \frac{1}{1+s} \left[ \frac{a}{b} \right].$$

Then, utilizing residue theorem, we have

$$G_{2, 1}^{1, 2} \left[ \frac{1}{1+s} \left[ \frac{a}{b} \right] \right] = \frac{1}{2\pi i} \int_L \left( \Gamma(p) \Gamma(-s+p) \Gamma(s+1-p) \left( \frac{a}{b} \right)^p \right) dp$$
$$= \lim_{p \to -s} \Gamma(p) \Gamma(-s+p) \Gamma(s+1-p) \left( \frac{a}{b} \right)^p$$
$$+ \lim_{p \to 0} \Gamma(p) \Gamma(-s+p) \Gamma(s+1-p) \left( \frac{a}{b} \right)^p + O(a^s)$$

$$= \lim_{p \to -s} \Gamma(p) \Gamma(-s+p) \Gamma(s+1-p) \left( \frac{a}{b} \right)^p$$
$$+ \lim_{p \to 0} \Gamma(p) \Gamma(s+1-p) \left( \frac{a}{b} \right)^p$$
$$= \Gamma(s) \left( \frac{a}{b} \right)^s + \Gamma(-s) \Gamma(s+1),$$

where $O(a^s)$ denotes higher order terms. Substituting [58] into [56], we have

$$\phi_5(a, b) = a^{-1} \frac{d\Theta(s)}{ds}|_{s=0},$$

where $\Theta(s) = b^{-s} \Gamma(s) + a^{-s} \Gamma(-s) \Gamma(s+1).$ By utilizing [26] (8.322), we have

$$\Gamma(s) = \frac{e^{-C s}}{s} \prod_{k=1}^{\infty} \left( 1 + \frac{s}{k} \right)^{-1} e^{\pi i}.$$

By substituting $e^{-C s}$ with its taylor series,

$$e^{-C s} = 1 - C s + \frac{C^2 s^2}{2} + O(s^2),$$

is rewritten as

$$\Gamma(s) = \frac{1 - C s + \frac{C^2 s^2}{2} + O(s^2)}{s}$$

$$= \frac{1}{s} - C + \frac{C^2}{2} s + O(s)$$

$$= \frac{1}{s} - C + \omega_3,$$

where $\omega_3 = \frac{C^2}{2} s + O(s).$ Similarly, we have $\Gamma(-s) = -\frac{1}{s} - C - \omega_3.$ Then, $\Theta(s)$ is rewritten as [62], shown at the top of the next page.
\[ \Theta(s) = b^{-s} \left( \frac{1}{s} - C + \omega_3 \right) + a^{-s} \left( \frac{1}{-s} - C - \omega_3 \right) \Gamma(s+1) \]
\[ = \frac{b^{-s} - \omega_3 b^{-s} - Cb^{-s}}{s} + \frac{a^{-s} \Gamma(s+1)}{s} + \omega_3 a^{-s} \Gamma(s+1) \]

\[ I_4'|_{s=0} = (-C b^{-s} + \omega_3 b^{-s} - Ca^{-s} \Gamma(s+1) - \omega_3 a^{-s} \Gamma(s+1))' \]
\[ = (C - \omega_3) b^{-s} \ln(b) + \frac{C^2}{2} b^{-s} + \left( \ln(a) - \psi(0) \right) (s+1) Ca^{-s} \Gamma(s+1) \]
\[ - \frac{C^2 a^{-s} \Gamma(s+1)}{2} \left( 1 + \left( \psi(0) (s+1) - \ln(a) \right) s \right) \]

Fig. 9: ESRs and ESSR for varying \( a^s \).}

Defining \( f_1(s) = b^{-s} \) and \( f_2(s) = a^{-s} \Gamma(s+1) \), then Taylor series expansions of \( f_1(s) \) and \( f_2(s) \) at \( s = 0 \) are expressed as

\[ f_1(s) = b^{-s} = f_1(0) + f_1'(0) \frac{s}{1!} + f_1''(0) \frac{s^2}{2!} + O(s^2) \]
\[ = 1 - \ln(b) s + \frac{\ln^2(b) s^2}{2} + O(s^2) , \]

and

\[ f_2(s) = a^{-s} \Gamma(s+1) = f_2(0) + f_2'(0) \frac{s}{1!} + f_2''(0) \frac{s^2}{2!} + O(s^2) \]
\[ = 1 + f_2'(0) s + f_2''(0) s^2 + O(s^2) , \]

respectively, where \( f_1''(0) \) and \( f_2''(0) \) are derived as

\[ f_2''(0) = \frac{d}{ds} \left( a^{-s} \Gamma(s+1) \right)|_{s=0} \]
\[ = - \ln(a) a^{-s} \Gamma(s+1) + a^{-s} \Gamma(s+1) \varphi_0(s+1) |_{s=0} \]
\[ = C - \ln(a) , \]
Substituting (65) and (66) into (67), respectively, where \( I' (1) = \Gamma (1) \psi (0) (1) = -C \), and \( I'' (1) = (\psi (1) (1) + (\Gamma (1)^2 = \psi (1) (1) + C^2 \). Subsequently, \( I_3 \) is obtained as

\[
I_3 = \frac{1 - \ln (b) \ s + \ln^2 (b) \ s}{2} - \frac{1 + f_0' (0) \ s + f_0'' (0) \ s^2}{2}.
\]

Substituting (65) and (66) into (67), \( I'_3 \) is obtained as

\[
I'_3 = \frac{\ln^2 (b) - f_0'' (0)}{2!} \frac{2!}{2} = \frac{\ln^2 (a) + 2 C \ln (a) + \psi (1) (1) + C^2}{2}.
\]

Similar to (68), we obtain (69), shown at the top of the previous page. Substituting (68) and (69) into (59), we obtain

\[
\phi_5 (a, b) = \frac{1}{a} \left( I'_3 + I'_4 \right) |_{s=0} = \ln^2 (b) + \frac{C}{a} \ln (ab) + \frac{C^2}{2 a}.
\]

\[
= \ln^2 (a) + 2 C \ln (a) + \psi (1) (1) + C^2.
\]

\[
= \frac{1}{2 a} \ln \left( \frac{b}{a} \right) \ln (ab) - \frac{1}{2 a} \psi (1) (1)
\]

\[
+ \frac{C}{a} \ln (b) + C^2.
\]

REFERENCES


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