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\textbf{Abstract}—Viscosity measurement has wide ranging applications from oil industry to pharmaceutical industry. However, measuring viscosity in real-time is not a facile process. This paper provides an elaborate mathematical model and study of viscosity measurement in real-time using pressure sensors. For a given flowrate, a change in liquid viscosity gives rise to a change in pressure difference across a particular section of the pipe. Hence, by recording the pressure change, viscosity can be calculated dynamically. A mathematical model as well as a finite element analysis model has been presented to determine viscosity from flowrate and pressure difference. A set of pressure sensors can be placed at a fixed distance from each other to get the real-time pressure change, while flowrate can be obtained using a flowmeter. For the finite element analysis, the pressure sensors were placed 60 mm away from each other. The radius of the pipe was 19 mm. A mixture of water and glycerol, with different ratios, was used to provide variable viscosity.

\textbf{Index Terms}—Viscosity, Flowrate, Microfluidic channel, Pressure sensor, Real-time measurement

I. INTRODUCTION

Viscosity can be defined as the resistance or friction of a fluid when it is in motion. It is one of the most important and unique properties of chemical and biological fluids [1]–[4]. The measurement of viscosity has applications in industries like fossil fuel, biomedical, chemical and so on [1]. In the past few decades, there have been several developments to measure viscosity to test and monitor liquids like blood, mucus, drug delivery, lubricants and fuels [5]–[8]. Further, by measuring viscosity of a mixture of fluid, the ratio of individual components in the mixture can be determined, for example, amount of medicine in blood, amount of water in oil [9]–[13].

To measure viscosity, many techniques have been reported such as falling ball [14]–[16], moving paddle [17], capillary force [18]–[19], resonating microtube [20], tuning fork [21]. In recent years, several MEMS based viscometers have been developed using microfabrication and microfluidic technology [22]–[24]. Even though measuring viscosity is a very important task, its measurement in real-time is not trivial [25]. However, there are many applications where a continuous real-time measurement of viscosity can prove invaluable.

Recently, a flexible viscometer is presented in which the velocity is being monitored to measure viscosity in real-time and without disturbing the flow of the liquid [26], [27]. The conventional measuring instrument installation has different drawbacks such that flow disturbance, tube damage, and is more suitable for laminar flow only. The simplified illustration of the proposed viscosity measurement system is shown in the Fig. 1. An affordable system is presented which is a fully flexible system and which is connected to the inner surface of the pipe with different diameters and curvatures. The fully flexible system consists of a Poly-(dimethyl siloxane) (PDMS) microfluidic channel with a microfluidic flowmeter [26], [27]. The flowmeter consists of three square-shaped capacitive pressure sensors to monitor pressure at three different points. Although the capacitive pressure sensors require large die area and influenced with by parasitic behaviour; however, it is less affected by temperature drift and offers high sensitivity which is useful for precise measurement of ultra-low pressure [28]–[30]. The analysis is performed to define the relationship between pressure and viscosity at different diameters of pipes. The capacitive pressure sensors consist of two square-shaped parallel plates which sense the change in capacitance for different viscosity of fluids with the pressure range of 20 to 120 Pa which also depends on the curvature and diameter of the pipe [26], [27].

Fig. 1. Simplified illustration of the proposed viscosity measurement system.

This article presents the analytical foundation to measure viscosity using this technique. By measuring pressure changes in the fluidic channel, the dynamic viscosity can be determined by placing a channel inside a pipe. The channel material used is poly-(methyl methacrylate) (PMMA), and the substrate is PDMS. From the difference in pressure between inlet and outlet, we can determine the viscosity of the fluid. Because the measurement of pressure and flowrate is in real-time, any change in viscosity will be reflected in these measurements. Thus, with continuous measurement of flowrate and pressure difference, we can calculate viscosity in real-time. We have performed mathematical modelling as proof of concept and a finite element analysis has been done for verification of the model.
II. SYSTEM STRUCTURE

The structure of the proposed system is presented in Fig. 1. The system consists of a narrow PMMA channel with capacitive pressure sensors fabricated using copper bilayers on a PDMS substrate. The PMMA channel has height of 250 μm from the inside of the pipe, while the length of the channel is 60 mm. Such a narrow channel provides a laminar flow, thus allowing for stable measurement of pressure difference. The pressure sensors are placed at the ends of the channel to read the pressure continuously. When fluid passes through the pipe, it also enters the channel at the velocity dependent on the total flowrate in the pipe. The fluid exerts pressure on the walls of the channel, which can be measured using the two pressure sensors. The difference in the pressure measurement is proportional to the flowrate (as in case of Ohm’s Law for flow of electrons), with the constant of proportionality being the resistance of the channel. This resistance is a function of fluid viscosity, thus, for a given flowrate, the pressure difference in the channel can be used to measure the viscosity of the fluid.

III. MATHEMATICAL MODELLING

To obtain the relationship between the pressure difference and fluid viscosity, we assume an incompressible, steady, laminar fluid of viscosity \( \mu \) in cylindrical pipe of radius \( R \). In the circular pipe, the fluid flowrate counters from concentric circles \([31]\). Thus, according to the Hagen-Poiseuille equation, derived from Navier-Stokes equation, the volumetric flowrate of a thin circular fluid layer of thickness \( dr \), at the distance \( r \) from the center, is given by:

\[
Q(r)dr = \frac{1}{4\mu} \frac{\Delta P}{L} (R^2 - r^2) 2\pi r dr
\]

where, \( \Delta P \) is the pressure difference between two points at a distance \( L \) from each other. For the large circular pipe, the total flowrate in the pipe is given by:

\[
Q_R = \int_0^R Q(r)dr
\]

Thus, from equation (1), we obtain the in the pipe flowrate as:

\[
Q_R = -\frac{1}{4\mu} \frac{\Delta P}{L} 2\pi \int_0^R (R^2 - r^2) dr
\]

\[
Q_R = -\frac{1}{4\mu} \frac{\Delta P \pi R^2}{2}
\]

For a channel placed at the inside boundary of the big circular pipe (Fig. 1), the flow rate in the channel is given by:

\[
Q_{R-h} = \frac{\theta}{2\pi} \int_{R-h}^R Q(r)dr
\]

where \( h \) is the height of the channel above the surface of the pipe and is the sectoral angle of the channel. From eq. (1), we have:

\[
Q_R = -\frac{1}{4\mu} \frac{\Delta P \pi R^2}{2} (R^2 - (R-h)^2) \left[1 - \frac{1}{R^2} (R-h)^2\right]
\]

In terms of the total flowrate inside the pipe, the flow in the channel is given by:

\[
Q_{R-h} = \frac{Q_R}{R^2} \left[R^2 - (R-h)^2\right] \left[1 - \frac{1}{R^2} (R-h)^2\right]
\]

The average velocity in the channel of height can be obtained from the total flowrate using the as relationship \( v = Q/A \) as:

\[
\bar{v}_{R-h} = \frac{Q_R}{R^2} \left[1 - \frac{1}{R^2} (R-h)^2\right]
\]

From [32], we know that the average velocity in a tubular channel of height \( h \) is given by:

\[
\bar{v}_{R-h} = \frac{\Delta P R^2}{8\mu L} \left[1 + \frac{(R-h)^2}{R^2} + \frac{(R-h)^2}{\ln \left(\frac{R}{R-h}\right)} - 1\right]
\]

Eq. (8) and (9) represent two different ways of expressing the same quantity. Hence, we can equate these two equations, and solve for the viscosity of the fluid in terms of the other parameters:

\[
\mu = \frac{\Delta P \pi R^4}{8LQ_R \left[1 - \frac{1}{R^2} (R-h)^2\right]}
\]

Thus, if pressure and flowrate through a pipe are measured continuously, we can obtain the viscosity of the fluid in real-time.

IV. SIMULATION AND ANALYSIS

Finite element analysis is performed using COMSOL Multiphysics to validate the mathematical model. The geometry used for simulation consisted of a circular pipe of radius 19 mm and length 60 mm. A channel with 0.25 mm height and 3 mm width was inserted at the inner radius of the pipe (Fig. 2). We measured the pressure at the two ends of the channel (where the pressure sensors are assumed to be present) for liquids of different viscosities using water and glycerol at different mix ratios. We also recorded the values of average velocity inside the channel to compare the results with the mathematical model.

For the simulation, we fixed the flowrate in the pipe to be 1500 ml/min. The velocity distribution in the pipe and the channel is found to be consistent with the expected value from the mathematical model (Eq. 8). The key result is the distribution of pressure inside the channel as a function of the distance. As seen from Fig. 4, the pressure is higher close to the inlet of the fluid and reduces to zero (relatively) at the outlet. In this case, the maximum pressure in the channel was
observed to be 895 Pa. While the pressure varies from higher to lower when going from inlet to outlet, the velocity at each point in the channel remains constant in steady state.

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Fig. 2. Geometry of the simulated system with the channel inserted in the pipe. The meshing is done such that the detailing of the tiny channel is captured in the simulation.

Fig. 3. Distribution of fluidic velocity in the cross section of the pipe and the channel for flowrate of 1500 ml/min and liquid viscosity of 0.13 Pa·s. The colour scales are in m/s.

V. RESULTS AND DISCUSSION

In our mathematical analysis, we found that the pressure difference along the channel depends on the geometry of the channel and pipe (length and height of the channel, and the radius of the pipe), the flow rate and the viscosity of the fluid. Given that geometry of a system is fixed for a particular deployment, we compared the results of the simulation by varying the viscosity of the fluid to observe the pressure drop across the channel. If the predicted pressure drop is achieved, the same equation can be used to estimate viscosity given a known flow rate and pressure-difference. The viscosity of the simulated fluid was varied between that of water and glycerol by increasing the percentage of glycerol in water in steps of 5% (to eventually reach 100% glycerol). The variation of the observed pressure-difference with fluids of various viscosities is plotted in Fig. 5. The symbols represent results from the simulation, while the line represents the analytical results obtained from eq. (10). These results compare favourably.

The experimental analysis which is presented to monitor viscosity and pressure of fluid flowing inside the pipe, based on its diameter, utilized mechanically flexible square-shaped capacitive pressure sensors [26], [27]. The sensors electrodes are designed using copper sheet/foil and PDMS acts as dielectric material. These sensors measure the pressure at three different point of channel [26], [27]. For enhancement in the sensitivity of the sensor, the elliptical or circular shape capacitive pressure sensors are better choice in terms of sensitivity, given that the pressure range is in almost 20 to 120 Pa range, however, the sensors response will be more non-linear [28], [29], [33]–[34]. The combination of cantilevers and diaphragms could also be used to design the highly sensitive pressure sensors based on the desired pressure range. This approach is suitable to measure the high range of pressure applications.

VI. CONCLUSION

This paper presents the mathematical background for the measurement of fluid viscosity in real-time. A flow meter and a pair of pressure sensors is used to create the setup for viscosity measurement. A mathematical model was developed to determine the viscosity of a liquid, with given flow rate and the pressure-difference in the channel. We have analysed by keeping the flow rate fixed at 1500 ml/min and changing the viscosity step by step. We recorded pressure difference at two points who are 60 mm away from each other in our finite element analysis model and matched with mathematical calculations to prove the concept. As we can see the analytical and simulation are almost perfectly matched, thus, we can conclude that viscosity can be calculated if we can monitor the change in pressure which is caused by the fluid inside the pipe along with the flow rate. We have showed the pressure distribution inside the pipe to support our concept and also velocity distribution with fixed flow rate inside pipe was
shown. To the industries where we need to monitor change in viscosity continuously such as petroleum industry, chemical industry, medical industry, etc. this model will be very useful. This system can be applied to help determine continuously the amount of oil in water in petroleum industry, ratio of liquids in a solution in chemical industry, amount of medicine in blood in medical industry, and so on.

**REFERENCES**


