Challenges and Prospects to Time Series Burst Overlap Interferometry (BOI): Some Insights from a New BOI Algorithm Test over the Chaman Fault

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Abstract—How to obtain millimeter-scale along-track deformations using phase measurements is still a pending question for InSAR community. Although Burst Overlap Interferometry (BOI) technique makes this question seem tractable, most applications of BOI still focus on extracting centimeter-scale deformations for seismics cases. To further improve measurement accuracy, here we propose a new time series BOI algorithm towards maximizing the performance of BOI and obtaining as high accuracy of the along-track deformations as possible. The algorithm contains three major steps. The first step is to enhance BOI phase signal-to-noise ratio using our newly proposed phase estimator. In the second step, we apply a strain model-based method to further suppress the phase noise and rescue more data points. In the third step, a misregistration correction procedure which considers plate motion is applied to mitigate BOI time series bias. We tested our proposed algorithm over the Chaman fault. Although the derived millimeter-scale deformations demonstrate the effectiveness of our method, experimental results show that decorrelation and ionospheric disturbance are still two great challenges of BOI techniques.

Index Terms—Burst Overlap Interferometry, Along-track Deformation, Strain Model, Chaman Fault, Decorrelation, Ionosphere.

I. INTRODUCTION

INTERFEROMETRIC Synthetic Aperture Radar (InSAR) has been a rising geodetic tool to monitor the crustal deformations associated with earthquake cycles [1-9]. As SAR satellites operate in near-polar orbit (along-track) and sense ground targets along orbit-perpendicular (cross-track) direction, InSAR technique is more sensitive to surface deformations in East-West (EW) and vertical directions than to North-South (NS) direction. In contrast, other InSAR techniques are more sensitive to the NS deformation. The dual-beam interferometer [10], which has two constantly symmetric antennas along the radar zero-doppler time, is such a representative technique. Through simply differentiating interferograms from two antennas, along-track deformations, which are more sensitive to the NS direction, can be directly captured. However, this method is inapplicable for one-antenna platforms that usually operate in Scanning Synthetic Aperture Radar (ScanSAR) and Terrain Observation with Progressive Scans (TOPS) imaging mode. To obtain along-track deformations based on single antenna platforms, three representative techniques come to play.

The first is the Multi-Aperture Interferometry technique [11] which splits azimuth spectrum into two different bands. Limited by the narrow azimuth bandwidth of only ~50 Hz (ScanSAR) [12] and ~320 Hz (TOPS) [13], this technique can only be used to obtain the large-magnitude along-track deformations such as seismo-deformations of large earthquakes [14].

The second is the offset-tracking technique [15] which obtains along-track signals by cross correlating SAR intensity or amplitude maps. The ever-present multiplicative speckle noise and the spatial resolution of SAR images limit its achievable accuracy to few centimeters (1/30-1/10 of pixel resolution) [16].

The third is the Burst Overlap Interferometry (BOI) for TOPS mode which can derive along-track deformations through differentiating two consecutive burst overlap interferograms [17-21]. The rationale behind is that TOPS mode can capture one target in the burst overlap with two complete beam patterns in two different spectral frequencies (the spectral difference is ~5000 Hz) (scanning process is given in Fig.S1). Assuming that there is no decorrelation or ionospheric delay difference during the acquisition time interval for two overlaps [22], this Doppler centroid difference presents a constant phase proportional to the misregistration...
and along-track deformations. Once the systematic misregistration is estimated and removed from the double difference phase, along-track deformations can be extracted. However, the phase decorrelation and ionospheric delay still exists as main error sources in BOI [23].

While ionospheric forecasting maps can partly mitigate the ionospheric delay [24], the decorrelation noise issue is more challenging than the ionospheric one, especially in cases towards extracting mm-level interseismic deformations. A hybrid procedure using multi-looking and Goldstein filtering [25] proves to be efficient in suppressing noise and extracting interseismic velocity in Southern Dead Sea Fault [18]. Yet, regions of high coherence, such as the South Dead Sea fault, are rare. For regions with lower coherence, BOI accuracy loses dramatically. Therefore, to further improve the performance of BOI, a denoising procedure is required.

Some pioneering efforts towards improving phase signal-to-noise ratio (SNR) have been made over the past decades. The likelihood estimation of deformation rate and Digital Elevation Model (DEM) error is derived under the Complex Circular Gaussian (CCG) assumption [26]. On this basis, incoherent distributed scatterers (DS) are further considered in phase estimation and the quality of time series phase observation is optimized by taking coherence matrix as noise weight. This idea is further extended to the Phase Linking (PL) technique in which the likelihood estimation of phase time series is proposed [27]. In PL, the neighboring points within a certain distance are used to improve the phase SNR. Similarly, the inverse of the coherence matrix (precision matrix), which is the inverse of the modulus of the sample covariance matrix (SCM), is treated as the weight matrix in PL. Since adjacent points do not conform to the same scattering characteristic in multi-scattering mechanism scenes, the homogeneous pixel selection (HPS) algorithm emerges to handle this heterogeneity [28]. SqueeSAR integrated HPS into SCM estimation and further proposed the phase triangularity analysis (PTA) algorithm, which derived the maximum likelihood estimation of phase time series [28]. In PTA, the optimal phase series are usually obtained by unconstrained nonlinear optimization routines, e.g., Broyden–Fletcher–Goldfarb–Shanno (BFGS). Eigen decomposition based Maximum likelihood estimator of Interferometric phase (EMI) is a more efficient maximum likelihood estimator [29], which saves the computational burden using eigen decomposition [30] without accuracy loss. Through introducing Sequential Estimator (Seq) [31], the accuracy of EMI can be improved to a high level, which is close to the Cramér–Rao lower bound (CRLB) [32].

This hybrid method of Seq+EMI has been extended to BOI phase estimation, and its accuracy also has a further improvement. The rationale behind is that, a double sample coherence matrix estimation (2k) mitigates the coherence matrix estimation error [22]. Assuming that there is no scattering difference for two overlaps covering the same region, 2k directly includes samples from two burst overlaps in estimation instead of only using samples from one burst overlap. Therefore, doubling the sample number can mitigate the coherence estimation error to some extent.

Doubling sample numbers also increases the possibility that coherence matrix is positive definite. When the sample number is less than the image number, the probability density function of coherence matrix is characterized by a degenerate distribution and its determinant is nearly zero [33]. If we directly use the inverse matrix of this singular-type coherence matrix as the weight in Seq+EMI, estimation error of the coherence matrix will be magnified, and 2k method just reduces its occurrence. There is a left clue here for us to further improve the phase estimation accuracy of Seq+EMI+2k. That is how to mitigate the precision matrix (the inverse of coherence matrix) error when the double sample number is still less than the image number.

In this article, we focus on this left clue and further propose an error mitigation procedure based on Rao-Blackwell Ledoit-Wolf shrunk estimator [34]. We integrate this procedure into Seq+EMI+2k that we originally proposed to estimate the azimuth misregistration [22]. Moreover, to rescue more data points, we also introduce the strain model [35, 36] which considers the spatial correlation of neighboring pixels to further reconstruct all burst overlap interferograms. After reconstruction, a misregistration correction method that considers long-term plate motion is integrated into the new BOI method which is aimed to mitigate the bias on BOI time series. This paper is organized as follows. Section II outlines the principle of BOI and rationale of the proposed method. In Section III, synthetic data tests are presented. Real data results are described in Section IV. In Section V, we give some discussions and prospects. Finally, conclusions are outlined.

II. METHODOLOGY

To better illustrate our proposed method, we present the workflow diagram in Fig.1. We start with two burst overlaps covering the same region (Fig.1a). Based on these two stacks, HPS samples for the following BOI phase estimator can be obtained. In the first step (Fig.1b), we perform the newly proposed BOI phase estimator to each stack. In the second step (Fig.1c), we reconstruct all burst overlap interferograms using the strain model. In the third step (Fig.1d), we calculate the misregistration series and remove them from all interferograms. Finally, the following time series analysis can be applied. Deformation velocity and time series can be extracted based on the final burst overlap interferograms. The overall process is given in Fig.1e.

In the following subsections, we address five detailed questions in this workflow: 1) what is BOI? 2) how to improve BOI phase SNR? 3) how to rescue more data points using the strain model? 4) how to accurately estimate misregistration in the context of plate motion? 5) what accuracy can the proposed method achieve? The answers to these questions are foundations of our time series BOI method.

A. Burst Overlap Interferometry (BOI)

Under TOPS mode, radar antenna scans the same region for two times in different viewing angles. These overlap regions locate at consecutive radar image bursts in one swath or adjacent bursts of neighboring swaths (Fig.S1). Owing to the
different velocity vectors related to the different viewing geometries, there is a high spectral separation $\Delta f_{\text{vel}}$ of $\sim5000$ Hz between the consecutive burst overlaps. This separation can further cause a large variation of interferometric phase difference $\phi_{\text{BOI}}$ [37]

$$\phi_{\text{BOI}} = 2\pi \Delta f_{\text{vel}} \Delta t$$

(1)

when out-of-sync azimuth time error $\Delta t$ exists. This time error possibly results from orbital error, along-track deformation, or ionospheric delay as mentioned in earlier. For each burst overlap region, we can estimate this phase difference through directly differentiating two burst overlap interferograms from two radar echoes [37, 38]:

$$\phi_{\text{BOI}} = \angle (r_i \cdot s_i^*) \cdot (r_{i+1} \cdot s_{i+1}^*)$$

(2)

where $r$ and $s$ respectively represent the reference and secondary burst overlap image, $i$ and $i+1$ each denote the first and second radar scan (upper and lower one), $*$ represents the complex conjugate, and $\angle$ is to take the phase of a complex number. $r_i \cdot s_i^*$, for example, is thus the burst overlap interferogram of the $i$th echo.

(2) can be further extended into four phase components: misregistration error $\phi_{\text{mis}}$, along-track deformation phase $\phi_{\text{def}}$, ionospheric phase delay $\phi_{\text{tec}}$, and phase noise $\phi_{\text{noi}}$ shown as

$$\phi_{\text{BOI}} = \phi_{\text{mis}} + \phi_{\text{def}} + \phi_{\text{tec}} + \phi_{\text{noi}}$$

(3)

A deterministic orbit control with an accuracy of 5 cm within a 95% confidence bound can ensure $\phi_{\text{mis}}$ is topographically independent and can be considered as a fixed value for the whole burst overlap [23, 39, 40]. In (3), along-track deformation component is mainly related to the phase contribution from NS deformations. Given that a phase cycle ranging from $-\pi$ to $\pi$ corresponds to approximately -64.3 cm to 64.3 cm displacements (the derivation is described as Text.S1), we need to reach a phase estimation accuracy down to 0.3 degree if we want to detect a displacement of 1 mm. When considering a strong ionospheric gradient, an azimuth gradient variation of $10^{-3}$ TECU/m will present about an azimuth shift of 37 cm on along-track deformations, thus this effect can introduce a nonlinear phase bias along the azimuth direction. Such a strong ionospheric delay does not exist all the time and can be partly mitigated by the third-party data (e.g. TEC map) or time-domain low-pass filtering. The remaining noise component is the trickiest limit to a so-called 0.3-degree accuracy. Assuming one pixel in burst overlap with a high coherence of 0.75, the related phase noise standard deviation (std) will reach 1 rad which corresponds to $\sim20$ cm. Double difference in (2) increases the noise to $\sim28$ cm. In the following subsections, we introduce the theoretical foundation of our proposed denoising procedure.

B. Theoretical Foundation of the New BOI Phase Estimator

In the past decade, phase triangularity is proved to be a good constraint for phase SNR enhancement [28]. EMI, one of the most efficient phase estimators based on triangularity, can obtain the maximum likelihood estimation (MLE) of phase solutions through squeezing the minimal eigenvector of the full covariance matrix. Its mathematical formulation can be described as [29]

$$\hat{\gamma} = \arg \min_x \frac{1}{N} \sum_{i=1}^{N} (\xi_i - \gamma_i)^2$$

(4)

where $\hat{\gamma}$ is the estimated phase series and $\xi$ is the

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Fig. 1. Schematic diagram of the proposed workflow. (a) input data; (b) the proposed BOI phase estimator described in Section II-B; (c) strain model described in Section II-D; (d) estimate misregistration; (e) summary of the workflow.
minimum eigen vector of the Hadamard product $\gamma^{-1} \circ \Sigma$, in which $\gamma^{-1}$ is a key factor to weight the noise level. The superscript $H$ represents Hermitian transposition. $\Sigma$ is the estimated full covariance matrix formed by $m$ Single Look Complex (SLC) images $x \in \mathbb{C}_{m \times n}$ and $n$ pixels in the corresponding HPS region $\Omega$ [41]. $\hat{\gamma}$ is the coherence matrix (the modulus of $\Sigma$) in which each element is coherence $\hat{\gamma}_{i,j}$ between two time nodes $i$ and $j$,

$$
\hat{\gamma}_{i,j} = \sum_{p \in \mathbb{Z}} e^{\gamma \tau \left( \xi(x_{i}^{p}x_{j}^{p}) - \phi_{p}^{\text{local}} \right)}
$$

(5)

where $\phi_{p}^{\text{local}}$ represents a combination of the local topographical phase and flat earth phase to mitigate the non-stationarity of the coherence estimation window.

Given the theoretical probability density function ($pdf$) of $\Sigma$ is formed by [33]

$$
p(\Sigma|\theta) = \frac{\Gamma(\frac{m+n}{2})}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})}\left|\Sigma\right|^{-\frac{m+n}{2}}e^{-\frac{\text{Tr}(\Sigma^{-1})}{2}}
$$

(6)

where $\text{Tr}(\cdot)$ means the matrix trace and $\Gamma$ is the hypergeometric function. When $n < m$, covariance matrix $\Sigma$ is singular as its determinant is nearly zero. For this case, (6) is considered as a degenerate distribution. This means that the coherence matrix $\hat{\gamma}$ is not full ranked, and its inverse $\gamma^{-1}$ will magnify the estimation error of $\hat{\gamma}$, further amplifying to the estimation error of $\phi$.

To better integrate EMI into BOI to improve the phase SNR, we proposed a double-sample coherence estimator named as 2$k$ estimator [22, 42]. In 2$k$ estimator, we assumed that two burst overlap echoes undergo nearly the same decorrelation and tropospheric effect, thus the original coherence estimator (5) can be replaced by

$$
\hat{\gamma}_{2k}^{\text{loc}} = \sum_{p \in \mathbb{Z}} e^{\gamma \tau \left( \xi(x_{i}^{p}x_{j}^{p}) - \phi_{p}^{\text{local},u} + \xi(x_{i}^{p}x_{j}^{p}) - \phi_{p}^{\text{local},l} \right)}
$$

(7)

where $u$ and $l$ respectively denote the upper and lower burst overlap.

2$k$ estimator improves EMI performance in two main ways. The first is to mitigate the coherence estimation error by introducing twice as many samples. The other is that doubling the sample numbers can reduce the occurrence of the singular case of the coherence matrix because while $n$ is sometimes less than $m$, $2n$ is very likely to be more than $m$. Although 2$k$ estimator reduces the occurrence of singularity, it is sometimes difficult for $2n$ to be greater than $m$, because $m$ is usually a large number for long time series.

Seq [31] can help to further reduce the occurrence of singularity. Seq divides the full covariance matrix into several block diagonals and sequentially compresses them into several 1-rank subspace clusters. Based on a subspace clustering-like connection, block diagonals are connected to a unique phase datum and the estimated phase can reach a higher SNR than the original EMI. The block diagonal size of Seq is manually selected by users. In this study, we choose 20 as the diagonal matrix size, corresponding to a 120-day temporal baseline. In this case, 2$k$ only needs to be greater than 20 and the occurrence of singularity can be reduced. The left clue here we did not notice in our previous efforts is that how to mitigate the estimation of the precision matrix $\gamma^{2k-1}$ when 2$k$ is even less than diagonal matrix size.

To suppress the singularity of the matrix, regularization of the matrix is usually used. Typically, a matrix can be forced to be positive definite by heuristically adding a small quantity to its diagonal or truncating those small eigenvalues. However, in the process of forcing matrix to be positive definite, we need to specify an eigenvalue threshold to be truncated or a value added to the diagonal. This value is difficult to determine towards minimizing the estimation error of $\gamma^{2k-1}$ and simultaneously maximizing the estimation accuracy of $\hat{\phi}$.

In this context, we proposed to apply a non-thresholding shrunk method to Seq+EMI+2$k$ to further improve the BOI phase estimation accuracy. We generally review two commonly used matrix regularization methods in Text S2-S3, which are ridge regression and minimum eigenvalue replacement. In the next section, we further introduce the new non-thresholding precision matrix estimator in the new BOI phase estimator.

C. Non-thresholding Precision Matrix Estimator Based on Minimum Mean-Squared Error (MMSE) Criterion

When the sample number is much greater than the SLC number, (5) can perform well, and the estimated sample coherence matrix is invertible. It coincides with the MLE estimate. However, when the sample number is less than the SLC number, substituting the sample coherence matrix as a MLE estimate results in a significant SNR loss because it does not minimize the mean-squared error (MSE) criterion which minimizes the Frobenius-norm distance of $\|\hat{\gamma} - \gamma\|_F$ [43]. To cope with it, a simple but always positive definite estimate of coherence matrix can be given by

$$
\hat{\gamma} = \frac{\text{Tr}(\hat{\gamma})}{n} I
$$

(8)

Although this estimation can ensure the coherence matrix to be always well conditioned, it increases the estimation bias. A reasonable trade-off between well condition and low bias can be achieved through shrinking $\gamma$ towards $I$. It is given by [44]

$$
\hat{\gamma}_{\beta} = (1 - \beta) \hat{\gamma} + \beta I
$$

(9)

where $\beta \in [0, 1]$ and $\hat{\gamma}_{\beta}$ is the shrunk coherence matrix which is meant to regularize $\gamma$. Particularly $\hat{\gamma} = I$ is proven to be effective in the case of gaussian noise. Therefore, accurate selection of $\beta$ is the key point. Towards the optimization of

$$
\min_{\beta} E[\|\hat{\gamma}_{\beta} - \gamma\|_F^2]
$$

(10)

an initial solution has been derived out [45]

$$
\beta = \frac{\left(1 - \frac{2}{m+1}\right) \text{Tr}(\hat{\gamma}) + \text{Tr}(\gamma)}{\left(\frac{m+1}{m+2}\right) \text{Tr}(\gamma^2) + (1 - \frac{m}{n}) \text{Tr}(\gamma^2)}
$$

(11)

It is proved that the initial solution is the optimal solution, but it cannot be implemented, since it depends on the real coherence matrix which is unknown. Rao-Blackwell Ledoit-Wolf (RBLW) proposed an approximation of (11) [34]. The RBLW solution is formed by

$$
\beta_{\text{RBLW}} = \min \left\{ \frac{\left(1 - \frac{2}{m+1}\right) \text{Tr}(\gamma) + \text{Tr}(\gamma)}{\left(\frac{m+2}{m+1}\right) \text{Tr}(\gamma^2) - \text{Tr}(\gamma^2)} \right\}^{-1}
$$

(12)
One can see that (12) has no empirical parameter required to be set. It can automatically adjust the shrunk ratio to fit the MMSE criteria of (10) according to different scenarios. In the new BOI phase estimator, we integrate this shrunk estimation into Seq+EMI+2k to well condition the coherence matrix. In Section III-A, we give a performance comparison of this non-thresholding estimator to two traditional regularization methods described in Text.S2-S3.

D. Rescuing More Data Points Using the Strain Model

The overlapping area only accounts for about 8% of one burst, so there are few available pixels to begin with. Even though phase denoising can improve SNR to some extent, those regions with severe decorrelation would still maintain a poor SNR, leading to fewer pixels available for further time series analysis.

To rescue more pixels, we apply the Strain-model based InSAR for Geo-hazards Monitoring Approach (SIGMA) to reconstruct the BOI interferograms after denoising [36]. Here we make some changes to the original SIGMA method. We remove the relevant part of the elevation direction from the solution process because BOI contains only horizontal deformation. The relationship between the reconstructed BOI phase \( \hat{\phi}_{BOI} \) and the input phase \( \phi_{BOI} \) can be formulated as a simple linear equation:

\[
\hat{\phi}_{BOI,k+1} = B_{k3} X_{3,1}
\]

\[
\phi_{BOI,k+1} = [\phi_{BOI,k}, \hat{\phi}_{BOI,k}, ..., \phi_{BOI,k}]^T 
\]

\[
B_{k3} = \begin{bmatrix} \Delta e_i, \Delta n_i \end{bmatrix}^T, i = 1,2, ..., k
\]

\[
X_{3,1} = [\hat{\phi}_{BOI,k} \partial X/\partial e, \partial X/\partial n]^T
\]

where \( B \) is the designed matrix, \( X \) is the solution, and \( k \) means \( k \) nearest neighboring pixels (we set to 100 in this study); \( \Delta e_i \) and \( \Delta n_i \) are respectively the distance in meters between the \( i \)th neighboring pixel and the center pixel in the east and north direction; \( \partial X/\partial e \) and \( \partial X/\partial n \) each denote the elasticity of these two directions. To consider the different phase ambiguities of neighboring pixels, it is necessary to subtract the average value of all adjacent pixels and add it back after performing (13).

We further solve (13) using an iteratively reweighted least square method. In the initial iteration, the initial weight is set as temporal coherence of each edge connected by the center pixel and \( k \) neighboring pixels [46]. In the next iteration, the weight matrix is updated by the residuals from the previous estimates. Once the subsequent solution is less than \( 10^{-4} \) or the number of iterations reaches 100, the iteration stops.

E. Mitigating Azimuth Misregistration in the Context of Plate Motion

In the case provided by (3), misregistration is required to be accurately estimated and further mitigated from SAR images. To accurately estimate the misregistration from \( \phi_{BOI} \), a periodogram method was proposed and it can be formulated as [38]

\[
\hat{\phi}_{mis} = \arg \max_{\phi_{mis}} \left\{ \mathcal{R} \left( \sum_{p=1}^{P} e^{-i(\phi_{BOI,p} - \phi_{mis})} \right) \right\} \tag{14}
\]

where \( \mathcal{R} \) means the whole burst overlap, and \( \mathcal{R} \) is to take the real part of a complex number.

This periodogram function can accurately detect the peak phase which means the constant phase value related to the orbital error. Particularly, short wavelength along-track deformations can hardly affect this peak phase, whereas long wavelength components due to plate motion, for example, can lead an estimation bias (Fig.S2).

In this context, we apply an azimuth misregistration correction strategy to mitigate the plate motion effect. We summarize it into six steps as shown in Fig.S3. In the first step, the newly proposed BOI estimator and the strain model are applied to each burst overlap, and the BOI phase SNR can thus be improved. Then in the second step, periodogram is performed on all burst overlaps to detect peak phase. All pixels in one burst are used to estimate one peak phase for one time node. In the third step, we fit a long-term velocity base on these detected peak phases. This long-term velocity can represent the effect of long-term plate motion. In the fourth step, the fitted long-term velocity is removed from each peak phase time series. Note that velocity fitting should be performed on each burst overlap alone. In the fifth step, those detrended peak phase values of all bursts are averaged. The final averaged misregistration phase values can be a good misregistration estimation. In the last step, misregistration estimation is subtracted from all pixels of all bursts.

III. RESULTS AND ANALYSIS: SYNTHETIC DATA

A. Capability of RBLW to Suppress Error of InSAR Precision Matrix

This subsection, we carry out a simple comparison of the proposed InSAR precision matrix estimation against the commonly used regularization methods, including Ridge

![Fig.2](image-url) Coherence and its inverse matrix (homogeneous pixel number is set to 50). (a) true coherence; (b) inverse matrix of (a), precision matrix; (c) difference between the inverse matrix of a sample coherence matrix and (a); (d-f) sample coherence matrix after ridge regression, mini eigenvalue replacement and RBLW; (g-i) difference between estimation in (d-f) and truth in (e).
Regression and Minimum Eigenvalue Replacement. For these two thresholding regularization methods, the maximum condition number is set to an empirical value of 10000.

Before starting to address the significance of the proposed InSAR precision matrix estimation, we need to clarify when the regularization is required. That is to introduce when the singularity of coherence matrix will occur. We did a synthetic experiment to illustrate this problem.

Referred to synthetic coherence matrix simulation described in [29, 31], we employ an exponential decorrelation model to simulate coherence matrix with the constant time of 27 days, a short-term coherence of 0.6 and a long-term coherence of 0.1. We set the revisit cycle of images to 6 days. The SLC number is set to 60 and 90, respectively. The used HPS sample number for coherence matrix estimation is set to a range from 1 to 150.

We repeat the coherence matrix simulation for each sample number 8000 times. We record the number of occurrences of singularity. Fig.S4 shows the results with different SLC numbers. When the sample number is smaller than the SLC number, the probability of occurrence of the singularity is 100%, in which case the estimated coherence matrix is definitely singular. When the sample number starts to be greater than the SLC number, the probability of singular matrix occurrence drops rapidly to 0%. When the sample number is not significantly greater than the SLC number, the coherence matrix cannot always be guaranteed to be positive definite. Therefore, the key is to compare the performance of our proposed non-thresholding shrunk estimator with other regularization methods when the sample number for coherence matrix estimation is insufficient.

For such comparison, we present a statistical comparison for the three methods. Fig.2a-b presents a case sample coherence matrix and its inverse. In this case, sample coherence matrix is estimated by only 30 samples. We repeat the sample coherence matrix estimation for 1000 times. In each simulation, we directly pseudo-inverse the estimated coherence matrix and record their difference between the true precision matrix. Fig.2d-f and Fig.2g-i give the averaged regularized coherence matrixes and the averaged difference of their corresponding inverse matrixes between the true precision matrix, respectively. Ideally, the closer the estimation error is to 0, the better the regularization method works. In contrast to Fig.2c, Fig.2g demonstrates the bias mitigation after the ridge regression. After the minimum eigenvalue replacement, the precision matrix estimation error also has a sharp decrease. It is clear that the precision matrix after the newly proposed estimation method outperforms minimum eigenvalue replacement.

B. Performance of the New BOI Estimator

To compare the phase estimation performance, we test different methods in Fig.3. These methods include EMI, Seq+EMI, Seq+EMI+2k and Seq+EMI+2k+Shrunk. We test these four methods in two scenarios. In both scenarios, the temporal baselines are all set to 600 days (100 SLCs). The HPS sample numbers are set to 9 and 81, respectively. The parameters of the simulated coherence matrix are consistent with the previous experiments, as the coherence matrix shown in the subplot of Fig.3b.

As introduced in Section II, Seq+EMI+2k using 2k coherence estimator achieve the higher estimation accuracy than the original EMI and Seq+EMI, but Seq+EMI+2k+Shrunk phase estimator, which integrates Seq+EMI+2k and shrunk estimator, achieve the best performance. Note that in Fig.3a, the performance of Seq+EMI+2k and Seq+EMI+2k+Shrunk are identical, whereas in Fig.3b new estimator has an improvement.

This performance improvement is attributed to the precision matrix estimation using shrunk estimation. In Fig.3a, we use 81 samples for estimation, and the 2k estimator increases the sample number to 162, which is much larger than the mini stack size used by Seq of 20 (time baseline 120 days), so the coherence matrix is always positive definite in Seq+2k (similar to the simulation in Fig.S4). Therefore, there is no need for regularization in this case. However, a regularization is required when the estimated samples are reduced to 9 (Fig.3b). Even though 2k samples can increase the sample number to 18, it is still less than 20. As shown in the previous experiment, the performance of conventional regularization methods is poorer than the shrunk estimation. Therefore, the performance improvement here indicates that shrunk estimation is conducive to improving phase SNR. This improvement of estimation accuracy is contributed to the sufficiency of precision matrix close to truth.

![Fig.3. Statistical results of the phase estimation accuracy. (a) and (b) represent experimental results when sample number is equal to 81 and 9, respectively; Coherence matrix in (b) denotes the simulated coherence matrix for (a-b).](image-url)
generated BOI deformation phase rate and the related profile are presented in Fig. S6. Referenced to synthetic coherence matrix in Fig. 3, we simulate 100 SAR images in SAR coordinate reference for each image. Each image size is set to 114 by 500 (azimuth by range). The noise phases are simulated according to the coherence matrix (Hanssen, 2001). Topographical phase and flat earth phase are simulated using precise orbit and external DEM. To simulate azimuth Doppler centroid histories of two overlaps, an azimuth Doppler Centroid separation of 5000 Hz is applied to two overlaps. To highlight the performance of shrunk estimation, we simulate a scenario with two different HPS sample numbers as shown in Fig. S7. Homogeneous pixel number in the yellow region is set to 81 and those in blue are set to 9.

Next, we use the above-mentioned methods in Fig. 4 to recover all burst overlap interferograms. Original phase and phase after Goldstein (in this paper, the square window size of Goldstein filter is set to 64, the alpha is set to 1, and the zero padding times is set to 32), EMI, Seq+EMI, Seq+EMI+2k, Seq+EMI+2k+Shrunk are set as benchmarks. The new method is to recover phase through applying strain model on the results of Seq+EMI+2k+Shrunk.

When there are fewer HPS samples (the bar area in the middle of the interferogram), the phase SNR is less improved. However, it can be seen from the coherence that the SNR of the regions with fewer HPS samples increased after the introduction of the shrunk estimation, which also reflects the positive effect of the better precision matrix estimation on the phase SNR improvement. There is no difference between the two regions with the HPS number of 81, which also indicates that these regions do not produce singular matrix, and the BOI estimator integrated with shrunk estimation is no different from Seq+EMI+2k in this case. In general, results of the new method exhibit obvious improvement, especially after introducing the strain model (last column of Fig. 4); more points are rescued.

C. Theoretical Accuracy of the Proposed BOI Estimator

To give a statistical evaluation on the theoretical accuracy of our proposed BOI method, a Monte-Carlo simulation test is carried out. In the simulation, we focus on deriving the achievable accuracy of the BOI velocity. As in the simulation in Fig. 4, we simulate an interseismic case, but set HPS samples in BOI phase estimator and neighboring pixels in the strain model to different ensembles.

The number of HPS samples may not always be sufficient, which is common in vegetated areas. Phase recovery in these areas is always unsatisfactory. The strain model can be a further plan to rescue points in these areas through considering the spatial relations of adjacent points.

Fig. 5 shows the accuracy of the estimated rate under different HPS samples and adjacent pixel numbers. The increase of HPS number improves the velocity accuracy. However, when HPS number is large and the number of neighboring pixels still
increases, the velocity accuracy exhibits little improvement. In such cases, HPS and interferogram reconstruction can be very time consuming. Therefore, as a compromise between accuracy and efficiency, we set the selection window size in HPS to 15 \times 15 and the neighboring pixel number for strain model to 100 in the test with real data.

To judge quantitatively how small velocity BOI can reveal, we define a concept of maximum detectable along-track deformation rate. When the velocity uncertainty is less than the slip rate, BOI results are reliable, but not when the uncertainty exceeds the velocity limit.

Fig.5. Theoretical velocity uncertainty of the proposed BOI method. The X-axis means the different sample number when estimating the covariance matrix, and the color of markers means different neighboring pixel numbers we used in the strain model.

Fig.5 shows that under the simulated coherence condition (the long-term coherence of 0.6 in our simulation is relatively ideal), the theoretical accuracy of our method can only approach 1 cm/yr no matter how large the window and the number of adjacent pixels are used.

Currently, one feasible way to lower this bottom line is to average more pixels to reduce this limit uncertainty after velocity inversion, but it directly sacrifices resolution to a large extent. To illustrate, the average velocity of 100 pixels can reduce the uncertainty to 1 mm/yr. Therefore, this experiment reminds us that, towards millimeter-level deformations, BOI still faces great challenges from noise. Note that this experiment only considers the influence of noise on BOI technique and does not consider the influence of other error sources such as ionosphere. Detailed discussions on challenges from more error sources are given in Section V-A and B.

IV. RESULTS AND ANALYSIS: REAL DATA

A. Study Region and Data Preparation

We select the Chaman fault as the test ground for the new BOI method. Chaman fault is sinistral fault bounded the northwestern Indian plate and the Eurasian plates [47] (Fig.6). The relative motion between the two plates is \( \sim 29 \) mm/yr [47-50], mostly accommodated by the ~1200 km long Chaman fault, one of the longest continental strike-slip faults in the world.

Note that, no large earthquakes (Mw>7) have been reported instrumentally along the Chaman fault, suggesting that the fault is either releasing strain in aseismic slip or it has relatively long earthquake cycle and the instrumental era has not yet captured large earthquakes [50].

Studies using historical GNSS and InSAR observations have revealed a heterogeneous distribution of fault creep and interseismic coupling [47-51], but great discrepancies exist between these studies. The inset of Fig.6 shows that the slip rate derived from InSAR [49] increases significantly from A and B to C. Therefore, we select this region to test the capability of the new BOI method to map the interseismic deformation and slip rate across the fault. The northern end of the study area is densely populated with mountains and forests, with poor coherence, while the southern end is gentle and less vegetation (see Fig.S8), with high coherence.

We totally process 224 time epochs of SAR images covering the North Chaman Fault system acquired by Sentinel-1.
acquisitions of a descending track 151 between Nov 2017 and Aug 2021 are used to derive the interseismic deformation for the orange region in Fig.6. 115 acquisitions of a descending track 78 covering the same period are used for the blue region in Fig.6. We accomplish the coregistration task of all images using the Coregistration Toolbox for Sentinel-1 (CtSent) software (https://zenodo.org/record/4774694#.YhC9FJNBzgF). CtSent performs burst-wise geometrical coregistration on all images by means of Shuttle Radar Topography Mission (SRTM) 30 DEM [52] and precise orbit [53]. After coregistration, we extracted 96 burst overlaps from two tracks. We then removed flat-earth and topographical phases from all burst overlaps using SRTM and precise orbit before performing the new BOI estimator. That is to mitigate local fringes for covariance matrix estimation in the next phase recovery step.

The subsections below present the performance comparison of three selected methods: Goldstein, Seq+EMI+2k and the new method proposed in this study. Unlike the synthetic data tests, we only compare our method with the Goldstein and Seq+EMI+2k methods, which has a known good phase optimization performance, for simplicity in this case.

B. Visual Inspection and Quantitative Assessment on Phase SNR Improvement

In this subsection, we compare phase optimization performance of the three methods. The first row of Fig.7 represents the results of the northern part of Chaman Fault. The Goldstein method does not give fringes in the upper and lower burst overlap interferograms (Fig.7b1-b2). Seq+EMI+2k clearly generate fringes (Fig.7b3-b4), and our methods give even clearer fringes (Fig.7b5-b6). The SNR of all burst overlap interferograms is calculated and compared to further validate this performance gain.

We first evaluate the accuracy of different phase estimators using SNR defined as:

\[ \text{SNR} = \frac{\text{Power of signal}}{\text{Power of noise}} \]

Fig.7. Example burst overlap interferograms after phase estimation by means of Goldstein, Seq+EMI+2k and the new method. Phase SNR of Seq+EMI+2k and the new method are respectively show in (a) and (c). The left quarter of the semicircle is Seq+EMI+2k’s SNR, and the right quarter is the SNR of the new method. The transition from blue to yellow color indicates an increase in time. (b1-6) are the upper and lower burst overlap interferograms (20171112-20190211) of three methods. These six interferograms are from one overlap in the northern study region. (d1-6) are the upper and lower burst overlap interferograms covering the southern region. The square window size of Goldstein filter is set to 64, the alpha is set to 1, and the zero padding times is set to 32.
\[ SNR = 10 \log_{10} \frac{\sigma_{\phi_{BOI}}}{\sigma_{\phi_{BOI}^*}} \]  

where \( \sigma_{\phi_{BOI}} \) and \( \sigma_{\phi_{BOI}^*} \) respectively represent sample phase standard deviation of all pixels in all burst overlap interferograms before and after phase optimization. A larger SNR denotes a better phase optimization performance.

We treat the results with the Goldstein method as a benchmark. \( \sigma_{\phi_{BOI}} \) is set to its phase std. Fig.7a depicts the results of Seq+EMI+2k and our method in radial bar charts. The color of the radial bar chart from blue to yellow shows the progression of time. The upper and lower bounds of SNR are set to 0 and 8. The greater the angle of rotation of the bar, the higher the SNR. As expected, our method significantly improves the BOI phase SNR for the whole time-series. This result agrees well with the root-mean squares error (RMSE) of the simulation in Fig.3 and confirms the effectiveness of proposed method.

Additionally, Fig.7c-d denote the results of the southern part with a higher coherence. Although the fringes of the burst overlap interferograms do not improve as significantly as that of the northern region, a significant difference in the upper right corner of the burst overlap interferograms is observed. That is from mostly noises (Fig.7d1-d2) to obvious signals (Fig.7d3-d6). Its related radial bar chart (Fig.7c) also indicates a significant SNR improvement. The new method achieves a SNR of 7.2 in both the southern and northern regions, which is about an 80% improvement over the SNR of 4.0 from Seq+EMI+2k.

Fig.8. Misregistration estimated for two regions before and after plate motion correction. (a) and (d) each represent the misregistration values for the northern and southern region. (b) and (e) are modeled plate motion from (a) and (d). (c) and (f) are final misregistration values after subtracting (b) and (e) from (a) and (d). (g) and (h) shows the mean misregistration values and their related standard deviations of all burst overlaps presented in (a)-(c) and (d)-(f). Uncorrected ones show in gray color and corrected ones show in green.
C. Misregistration Correction and the Related Effects on BOI Time Series

There are two things left to resolve before we estimate the along-track deformation time series.

The first, which is rarely discussed in estimating azimuth misregistration, is that tectonic movements can also lead to BOI peak phase offset, as described in Fig.S2 (also provided in (2)), and these consequent misregistration have different magnitudes on faults. In the case of the Chaman Fault, the subzone on the western side of the fault has less misregistration values than on the other side. A slip rate of 29 mm/yr along this plate boundary will result in approximately 2 cm/yr motion along the flight direction, which is reflected as 0.1 rad/yr at the magnitude of misregistration. If we only estimate misregistration using periodogram to remove an average misregistration value of all burst overlaps, an unknown average misregistration will be removed because of the plate motion and may bring subzones on either side of the fault to an unknown movement reference. Thus, we should model this plate motion when estimating the misregistration values.

Here, we model the plate motion (Fig.8b and Fig.8e) of the Chaman fault from the estimated misregistration (Fig.8a and Fig.8d) using the method described in Section II-C. The mitigated misregistration values are presented in Fig.8c and 8f. Gray squares show the mean misregistration values of all burst overlaps before correction and green dots denote the detrended ones. We also present their related stds from all burst overlaps on either side of the fault to an unknown movement reference. Thus, we should model this plate motion when estimating the misregistration values.

To further address its effect on BOI time series, we also compare time series before and after the correction. We select ten points for each test region and compare their time series in Fig.S9-10. Before the correction, the trend of plate motion (linear trend) is gradually prominent with the increase of time, but misregistration is always disturbing the time series. After the correction, the time series become much smoother with a clearer linear trend. This is also the direct cause of the increase of correlation coefficient in velocity fitting. It is worth noting that time series in the northern test region has an outlier in February 2020 (Fig.S10). This may be due to the influence of...
We discuss this potential error source in Section V-B.

D. Comparison of Posterior RMSE

The region we study lacks GNSS data to verify the BOI absolute accuracy. Therefore, here we propose a posterior RMSE indicator to illustrate the accuracy performance of the three methods. The posterior RMSE is calculated from the inversion process in which the small baseline BOI sets are transformed into single time reference. It can be defined as

\[
\tilde{\sigma}_{\phi_{\text{BOI,SR}}} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (\phi_{\text{BOI,SB}}^i - \tilde{\phi}_{\text{BOI,SR}})^2}
\]  

(16)

where \( N \) means the interferogram number in the small baseline subsets, \( \phi_{\text{BOI,SB}} \) is the \( i^{th} \) differenced burst overlap interferogram after phase SNR improvement, \( \tilde{\phi}_{\text{BOI,SR}} \) is the modeled BOI phase transformed from the inverted single time reference interferogram \( \phi_{\text{BOI,SR}} \) and incidence matrix \( G \) in this

Fig.10. Posterior root mean square error (RMSE) in single reference conversion for three methods. (a-c) RMSE maps of Goldstein, Seq+EMI+2k and new method for the northern region. (d-f) the corresponding results for the southern region. (g) Histograms of (a-c). (h) Histograms of (d-f).

Fig.11. Comparison between the BOI velocity map estimated applying the (a) Goldstein, the (b) Seq+EMI+2k and (c) New method. Negative velocity values mean a movement of the target toward northeast. Insert maps are results of the northern study region. Dashed circles represent the locations of two example points in Fig.12.

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article, we set temporal baseline limit to 600 days for generating small baseline subsets).

The first and second rows of Fig.10 respectively show the posterior RMSE of the northern and southern test region. The first column compares histograms of all burst overlaps, indicating that RMSE of the transformation from the phase of the small baseline set to the single reference phase of the new method is significantly smaller. The discussion above demonstrates that the improvement of BOI phase SNR of the new method is beneficial to the solution of BOI time series.

E. BOI Velocity Maps and Displacement Time Series Comparison

Fig.11 shows a comparison of along-track velocity maps estimated by the Goldstein method, the Seq+EMI+2k method, and our method. In the velocity estimation, we only select points with RMSE in Fig.10 less than 0.05 rad. In the northern region (insert maps in Fig.11), the Goldstein Seq+EMI+2k, and our methods obtain 3049, 17359, and 66399 points, respectively. In the southern region with better coherence and less vegetation, the Goldstein, Seq+EMI+2k, and our methods reserve 15382, 87114, and 236195 points, respectively.

Overall, the number of measurement points obtained by the new method is about 15 times and 3 times larger than that obtained by the Goldstein and Seq+EMI+2k methods, respectively. The traditional methods miss most of mountainous areas (northern part). In contrast, our method provides relatively dense points over these vegetated areas. Overall, there is a huge increase in the number of data points in both northern and southern regions. In addition, the relative motion on both sides of the fault is delineated more clearly (Fig.11c). Section IV-F presents a detailed discussion and analysis of this result.

Fig.12. BOI displacement time series of two exampled points. (a) time series of point A. (b) time series of point B.

Fig.13. Slip rate derivation based on the derived BOI velocity map. (a) the derived BOI velocity and four profiles. The white dashed line represents the fault trace of the Chaman Fault. The translucent buffer zone along the fault is treated as the area likely to be affected by creep. (b-e) are four velocity profiles projected to the fault-parallel direction and their corresponding long-term slip rates.
In addition to the velocity maps comparison, this subsection also presents a BOI displacement time series comparison in Fig. 12. Due to the few available points in the northern test region, the time series comparison only gives two common locations in the southern test region. These two locations (dashed circles in Fig. 11) are present in the results of all three methods. The time series obtained by the Goldstein method are noisier than the results of the other two methods. The results of Seq^1-EM+2k are improved compared with Goldstein’s method. This improvement is attributed to the better phase denoising capability. This is also consistent with the results obtained in Fig. 7. The better noise suppression of BOI interferograms significantly reduces the noise of time series. The time series obtained by the new method are the smoothest, showing the superiority of the new method in denoising. It is worth noting that noise reduction is not the only reason for the smoothness of the time series. Fig. 5.9 and Fig. 5.10 directly illustrate that the correct correction of registration errors is also another reason for the smoothness of the time series. This indicates that phase noise suppression and coregistration error correction are indispensable steps for BOI time series analysis.

F. Long-term Slip Rate Revealed by the New BOI method versus the Derived Slip Rate by Other Space Geodetic Techniques

Although our method has obtained relatively dense points, only a few observations can be obtained due to the dense vegetation on the western side of the fault in the northern study area. Sparse observations are insufficient for deriving the slip rate there. In the southern region, observations are denser and four profiles across the fault can be utilized to obtain the slip rate. Savage et al. [54] pointed out that the fault parallel velocity of far field profiles is controlled by the motion of the free slip of deep displacements, and the velocity difference between two sides of the fault is ideally equal to the long-term slip rate, as shown in the simulated interseismic slip in Fig. 5.6. Since the striking direction of the fault is very close to the heading angle, the along-track velocity is considered as a relatively reliable fault parallel velocity estimation. Given the effects of possible creep on the fault, we perform buffer analysis on the fault, masking out observation points within ±10 km to the fault trace. We perform a simple fitting of the fault-parallel velocities on both sides of four profiles. The fitting process is a simple weighted average operation, where the weight of each point is the reciprocal of the posterior root mean square errors calculated in Fig. 10f. The related std values are calculated using a 100 times 80% Jackknife test [55].

The results of our fitting analysis are shown in salmon lines of Fig. 13b-e. Our estimates for the fault long-term slip rate of $6.8^{+1.2}_{-1.2}$ mm/yr (profile 1), $8.7^{+1.4}_{-1.4}$ mm/yr (profile 2), $7.7^{+1.7}_{-1.7}$ mm/yr (profile 3) and $8.4^{+1.2}_{-1.2}$ mm/yr (profile 4) appear to increase southward from profile 1 to profile 2, then decrease from profile 2 to profile 3, and rise again from profile 3 to profile 4, implying that there is a heterogeneous distribution of slip rate.

We compare our derived slip rate to other historical slip rate reported by InSAR and GPS in Fig. 14. A detailed summary of geolocations, slip rate and the used techniques is given in Table. S1. Our derived slip rate shows that from 30°N to 32°N shear from the relative plate motion is accommodated by the Chaman Fault with estimated slip rates of $8.4^{+1.1}_{-1.1}$ mm/yr, $7.7^{+1.7}_{-1.7}$ mm/yr, $8.7^{+1.4}_{-1.4}$ mm/yr and $6.8^{+1.2}_{-1.2}$ mm/yr. These results are consistent with a slip rate of $8.5^{+1.5}_{-1.5}$ mm/yr derived from 5 years GPS measurements, which indicates the Chaman Fault accommodate 25% relative Indian-Eurasia motion [51]. Our results also well coincide with slip rates of $6.6^{+1.9}_{-1.9}$ mm/yr (32°N) and $9.1^{+1.5}_{-1.5}$ mm/yr (30.43°N) revealed by InSAR [48, 49].

These slip rate results indicate that there may be a slip rate decrease northward between 30.4°N and 32°N, but the rate decrease stops near 32°N and rises north of 32°N. The latest InSAR results indicate a slip rate of $10.5^{+6.5}_{-6.5}$ mm/yr at 32.06°N. Due to the lack of BOI observations with high SNR there, a further analysis of this rate discrepancy is still not feasible. However, compared with the results of other space geodetic observations, the slip rate derived from our new BOI method can be considered reliable in areas with high SNR observations. The average slip rate of 7.9 mm/yr calculated by our results indicates that our studied fault segment accommodates ~27% of the convergence of 29 mm/yr of the Indian-Eurasia plate.

It should be noted that the fluctuation amplitude of the four profiles is about 1 cm/yr, which is consistent with the derived uncertainty of our method obtained in Section III-C. The slip rate we derived here is obtained after averaging tens of thousands of pixels, and its uncertainty is reduced, so it can be considered reliable. However, such a large uncertainty is still an obstacle to analyze one specified point. To describe more details about slip rates and/or even creep dynamics in near fault zone, higher SNR BOI observations are needed, which requires us to develop better phase denoising estimators for BOI.

![Fig.14. Historical long-term slip rate revealed by different space geodetic observations. Cyan and purple markers respectively reported by [49] and [48] are derived from InSAR results. Orange marker reported by [51] is from GPS measurements. Green dots reported by us in this paper are from BOI results.](image)

V. DISCUSSIONS

A. Some Insights into Azimuth Misregistration Correction

The conventional Sentinel-1 TOPS data coregistration scheme always follow a two-step procedure: geometrical coregistration and Enhanced Spectral Diversity [37, 38]. That
is a combination of rough coregistration and precise coregistration. Currently, the mainstream InSAR data processing software [23, 56-59] follows almost the same procedure: after rough registration, all overlaps are used to estimate azimuth misregistration and further mitigate the roughly registered data.

For along-track deformation derivation using BOI, this coregistration scheme needs to further consider the influence of tectonic movements on accurate misregistration estimation. The linear rate compensation scheme proposed in this paper is a very simple solution, but there are still some limitations, therefore future efforts are needed for further improvements.

To illustrate, when extracting along-track deformations over an area experienced a large earthquake, post seismic deformation caused by afterslip and viscoelastic relaxations governed by exponential or power law may be included in the data [60, 61]. A pure linear rate mitigation for this case is not appropriate. As Sentinel-1 will operate more years in orbit, more sophisticated empirical models should be considered during misregistration estimation, aiming to obtain more accurate estimates of misregistration values. If we simply ignore these tectonic elements, derived BOI time series will bear misregistration errors due to orbital uncertainty.

Moreover, if the relative plate motion is not considered for a large study area, not only the results of BOI will be biased, but also that of InSAR. Tectonic deformations over a wide range are usually variable, and the estimated misregistration will also change the reference of the interferograms over the whole range. Nontectonic factors such as earth tides and ocean tides, should also be considered during the extraction of a wide range of along-track deformations. Removal of these nontectonic deformations depends on the established models instead of a simple linear fitting [62-64]. Additionally, given that the GPS antenna reference point (ARP) of Copernicus Sentinel-1 Precise Orbit Determination has changed at the end of July 2020 (S1A:29/07/2020, S1B:30/07/2020) [65], this ARP correction directly introduces the orbital position change. 3D position change can reach almost 6 cm for some regions [24]. It can result in an azimuth misregistration shift of almost 0.2 rad. In our test case covering Chaman Fault, we do not find obvious misregistration jump at the end of July 2020 (see Fig.8c). But for a case covering Central San Andreas Fault (CASF) creeping section (Fig.S11a), we can observe obvious shifts of misregistration resulted by ARP correction after July 2020 (Fig.S11b). To exclude this jump introduced by ARP correction, \textit{Heaviside} function can be a good choice. That is, we must consider a more complex nonlinear fitting model to exclude the aliasing effects resulted by a combination of orbit position correction and other mentioned tectonic/nontectonic contributions.

In summary, a generic misregistration correction method which is applicable to a variety of scenarios is needed.

\textbf{B. Limits of Ionospheric Delay to BOI}

Ionosphere is a dispersive medium, mainly formed by ionized free electrons of atmospheric molecules and neutral particles, and the degree of ionization is closely related to solar radiation intensity and geomagnetic field changes, which determines that the ionosphere changes with the latitude of the earth in space. Disturbance tends to be severe near the equator (within 30°N/S) and in the polar regions (above 60°N/S) [66, 67].

For SAR satellites operating in polar orbit, ionospheric effects in LOS direction are generally considered to be related to the carrier frequency, whereas those in the along-track direction are closely related to more elements: ionospheric gradient variation along azimuth direction, instrument carrier frequency, and the slant range between antenna and ground target [68, 69].

BOI uses Doppler frequency separation and azimuth out-of-sync time errors of adjacent burst overlap region to estimate the along-track deformation of consecutive burst overlap regions. However, the phase delay caused by the azimuth gradient change of the ionosphere can cause the change provided by (2). When the ionospheric gradient is large, such as high-order ionospheric disturbance (ionospheric scintillation), it is enough to cause significant “fake” along-track deformation signals, up to tens of centimeters, even exceeding the monitoring accuracy of BOI [68], which is much larger than the 1 cm/yr theoretical accuracy as shown in the synthetic test.

Currently, the correction of azimuth ionospheric errors is mainly based on split spectrum methods and filtering methods (e.g., directional filtering and frequency domain filtering), which are commonly used for along-track monitoring of ALOS-1/2 SAR operating in the low-frequency band of ~300 Hz [70-74], but the ionospheric errors of BOI along-track monitoring based on Sentinel-1A/B SAR data which operates in high-frequency up to ~5000 Hz is hard to handle with.

In this context, Lazecky and Hooper proposed to use IR12016 global model to mitigate ionospheric errors for each pixel in burst overlap regions [24]. Although this can partially remove ionospheric errors, the accuracy of the IR12016 model itself cannot still meet the requirements for high-precision monitoring in local regions [75]. A fine-scale ionospheric correction method for BOI technology is highly expected.

Although the ionospheric error in BOI measurement cannot be removed well at present due to the high sensitivity of this technique to ionospheric changes, it can better detect local ionospheric disturbances. Through interpreting the BOI measurements, the ionospheric signals may contribute to a more detailed ionospheric map. Going deeply, its added value of high-resolution ionospheric maps of local ionospheric disturbances, computed from burst overlap interferograms, in short-range ionospheric predictability is now rarely discussed and can be further assessed.

\textbf{VI. CONCLUSIONS}

Towards deriving millimeter-scale along-track deformations, we have developed a time series BOI algorithm which combines Seq+EMI+2k, non-thresholding shrunk estimation and strain model. We firstly tested our method on synthetic data, and the results show that it outperforms other state-of-the-art techniques in phase recovery. We also applied the proposed method to the Northern Chaman Fault. Experimental results...
validate the effectiveness of our algorithm. In the new BOI method, we also applied a misregistration correction procedure which considers plate motion. Experimental results covering Chaman Fault illustrate the significance of plate motion correction for misregistration estimation and BOI time series analysis. Moreover, our BOI results revealed that the Chaman Fault we studied accommodates about 27% of the 29 mm/yr convergence of the Indian-Eurasian plate. It well coincides with the results of other space geodetic techniques. It is proved that the new BOI method can accurately depict the roughly NS striking plate motion.

Although the proposed method can achieve good results over the Chaman Fault, synthetic data test shows that millimeter-scale BOI deformation derivation still confronts a significant challenge. A better phase denoiser is urgently needed to overcome this limit. Beyond that, another prospect in this article is a high-resolution BOI ionospheric delay mitigation method is needed to better remove the ionospheric effect for deriving more accurate BOI time series in the future.

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