A Discontinuous Galerkin Time-Domain Method to Simulate Metasurfaces using Generalized Sheet Transition Conditions

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Abstract—The generalized sheet transition conditions (GSTCs) are incorporated into a discontinuous Galerkin time-domain (DGTD) method to efficiently simulate metasurfaces. The numerical flux for GSTCs is derived for the first time using the Rankine-Hugoniot jump conditions. This numerical flux includes time derivatives of fields and therefore explicit time integration schemes that are traditionally used with DGTD do not yield a stable time marching method. To alleviate this bottleneck, a new time marching scheme, which solves a local matrix system for the unknowns of the elements touching the same GSTC face, is developed. Numerical results, which validate the accuracy of the proposed method against analytical solutions and demonstrate its applicability to the simulation of curved and space/time-varying metasurfaces, are presented.

I. INTRODUCTION

In recent years, metasurfaces have attracted significant attention due to their unprecedented ability to manipulate electromagnetic fields and found a diverse range of applications, such as anomalous reflection and refraction, beam steering and focusing, and cloaking, etc. [1]. More recently, curved, space/time-varying, tunable and active, and nonlinear metasurfaces have also been explored and shown to have many benefits in various real-life applications.

Numerical modeling has played a vital role in the development of metasurfaces and their widespread use. The generalized sheet transition conditions (GSTCs), which model a metasurface as a zero-thickness sheet and relate the fields on its two sides using surface susceptibility functions, provide an efficient way to efficiently account for metasurfaces in various electromagnetic solvers. Indeed, GSTCs have been successfully incorporated into finite element method (FEM) [2], finite-difference time-domain (FDTD) method [3][4], and a surface integral equation (SIE) solver [5].

Having said that, electromagnetic solvers with GSTCs retain the strengths and the weaknesses of their original versions without GSTCs. The discontinuous Galerkin time-domain (DGTD) method combines advantages of FDTD and FEM [6]: Like FDTD, it can account for nonlinearity and has a very high parallel efficiency, meanwhile, like FEM, it can model arbitrarily shaped geometries using unstructured meshes. These properties render DGTD an ideal candidate for simulating space/time-varying and nonlinear metasurfaces with complex shapes.

In this work, GSTCs are incorporated into a DGTD scheme to efficiently simulate metasurfaces. The numerical flux for GSTCs is derived using the Rankine-Hugoniot jump conditions. The resulting expression includes time derivatives of the electromagnetic fields averaged across the discontinuity introduced by the metasurface. This makes the explicit time integration schemes that are often used with the traditional DGTD scheme unstable. To overcome this problem, a locally-implicit time marching scheme is developed.

II. FORMULATION

GSTCs describe the “jumps” in the electromagnetic fields across the two sides of a metasurface as

\[
\hat{n} \times \begin{bmatrix} H \end{bmatrix} = -\hat{\epsilon}_0 \hat{P}_|| + \hat{n} \times \nabla \begin{bmatrix} M_\perp \end{bmatrix}
\]

\[
\hat{n} \times \begin{bmatrix} E \end{bmatrix} = \mu_0 \hat{H}_\| + \frac{1}{\epsilon_0} \hat{n} \times \nabla \begin{bmatrix} P_\perp \end{bmatrix}.
\]

(1)

Here, \( E \) and \( H \) are the electric and the magnetic field intensities, \( \epsilon_0 \) and \( \mu_0 \) are the permittivity and the permeability in free space, the jump operator is defined as \([f] = f^- - f^+\), superscript “+” or “−” means that the associated variable is on the exterior (+) or the interior (−) side of the metasurface, \( \hat{n} \) is the normal unit vector pointing from the interior side to the exterior side, subscript “\|” or “\perp” attached to a variable means that only the normal [along \( \hat{n} \)] or the tangential [transverse to \( \hat{n} \)] component of that variable is considered, and finally \( P \) and \( M \) are the electric and the magnetic polarization densities.

For the sake of simplicity and without loss of generality, it is assumed that the metasurface is mono-anisotropic and uniaxial, and \( M_\|=0 \) and \( P_\|=0 \) . It is further assumed that the susceptibilities \( \chi_{ee} \) and \( \chi_{hh} \), which relate the polarization densities with electric and magnetic fields, are time-varying but not dispersive [1]. Therefore, (1) is simplified to [1]

\[
\hat{n} \times \begin{bmatrix} H \end{bmatrix} = \epsilon_0 \hat{\epsilon} \partial_t \left( \chi_{ee} \begin{bmatrix} E \end{bmatrix} \right)_h
\]

\[
\hat{n} \times \begin{bmatrix} E \end{bmatrix} = -\mu_0 \hat{\mu} \partial_t \left( \chi_{hh} \begin{bmatrix} H \end{bmatrix} \right)_h
\]

(2)

where \( \{f\} = (f^- + f^+) / 2 \) is the average operator. Using the Rankine-Hugoniot jump condition, the upwind numerical flux incorporating (2) is expressed as

\[
F^H = F^{H\text{up}} + \partial_t F^{H\text{GS}}
\]

\[
F^E = F^{E\text{up}} + \partial_t F^{E\text{GS}}
\]

(3)

where \( F^{H\text{up}} \) and \( F^{E\text{up}} \) are the normal upwind flux [6] without the GSTCs, and \( \partial_t F^{H\text{GS}} \) and \( \partial_t F^{E\text{GS}} \) are the new numerical flux terms due to the GSTCs and are given by
The proposed method is validated against analytical solutions. Consider a mono-anisotropic, uniaxial, and time-independent planar metasurface located on the $z = 0$ plane in free space with $\chi_{xx} = \chi_{yy} = \chi_{zz}$. A plane wave with $E^{\text{inc}}(r,t) = \hat{z}E_0 \cos(\omega t - k_0(x-z_0))$ is normally incident on the metasurface. Here, $z_0 = -6$ m, $E_0 = 1$ V/m, $k_0 = \omega/c_0$, $\omega = 2\pi f$, $f = 100$ MHz, and $c_0$ is the speed of light in free space. The dimensions of the computation domain are $L_x = 6$ m, $L_y = 1.5$ m, and $L_z = 18$ m along the $x$, $y$, and $z$ directions, respectively. Periodic boundary conditions are used along the $x$ and $y$ directions and perfectly matched layers [8] are used along the $z$ directions. The average edge length of the elements in the mesh is 0.25 m.

Fig. 1 shows $E_x$ (the $\hat{x}$ component of the electric field) computed at $t = 80$ ns in the whole computation domain for four cases (from top to bottom): (a) $\chi = 1$; (b) $\chi = 5$; (c) $\chi = 2c_0/(3\omega)$; and (d) $\chi = 2c_0/(\omega)$. For cases (a) and (b), the metasurfaces are reflectionless but with different phase jumps at the metasurface. For case (c) the metasurface is fully-absorbing, while for case (d) the metasurface is reflectionless and half-absorbing. All these numerical results match perfectly with analytical results.

ACKNOWLEDGMENT

This work is supported by the King Abdullah University of Science and Technology (KAUST) Office of Sponsored Research (OSR) under Award No. 2019-CRG8-4056. The authors thank the KAUST Supercomputing Laboratory (KSL) for providing the required computational resources.

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