Symbiotic Ambient Backscatter Systems: Outage Behavior and Ergodic Capacity

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Abstract—This paper investigates a symbiotic ambient backscatter communication (AmBC) system, where for the primary system, a source node T1 transmits information to a destination node T2. Whereas for the backscatter system, by riding on T1’s signal, backscatter device passively conveys its own information c(n) to T1 and T2 via backscattering. For such, the coexistence outage probability (COP) and ergodic capacity (EC) of the AmBC system are characterized for three cases of coexistence constraints, i.e., (i) both T1 and T2 decode c(n) (Case I), (ii) only T2 decodes c(n) (Case II), and (iii) only T1 decodes c(n) (Case III). It is analytically shown that for sufficiently high transmit signal-to-noise ratio (SNR), the COP obeys the scaling law of \( \frac{p}{P} \) as well as \( \frac{1}{s} \) for Case I. In addition, it is shown that the restriction condition of decoding c(n) at T1 results in a dominating term \( \frac{1}{P_s} \) for the COP at high SNR, whereas the restriction condition of decoding c(n) at T2 results in an infinitesimal relative to \( \frac{1}{P_s} \). It is also shown that for different cases, the effects of the T1-T2 channel statistics on the COP are significantly different. However, unlike the metric of COP, for the EC, the impacts of decoding constraints of c(n) gradually disappear at high SNR and the ECs of the backscatter channels for Cases II and III approach respectively toward the counterpart for Case I.

Index Terms—Ambient backscatter communications, symbiotic communications, outage behavior, ergodic capacity.

I. INTRODUCTION

SPECTRUM scarcity and energy efficiency are envisioned as two primary issues for the large-scale deployment of a future Internet of Things (IoT), which is supposed to connect billions of small computing devices embedded in objects and environments such as books, furniture, home appliances, and even implantable medical devices [1]–[4]. To address this, a promising solution, namely, ambient backscatter communications\(^1\), is attracting significant attentions from both academic and industrial communities. For such, by modulating and reflecting the environmental RF signals from ambient TV towers, cellular base stations, and Wi-Fi routers, the information belonging to the backscatter device (BD) is conveyed in a passive and low-power manner, which improves the spectrum efficiency and lowers energy consumption at the BD node.

This work was supported in part by the National Key R&D Program of China under Grant 2018YFED0100500, in part by the National Natural Science Foundation of China under Grant 61871387, in part by the NUDT Research Fund under Grant ZK20-21, and in part by the Natural Science Basic Research Program of Shaanxi under Grant 2021JQ-378. H. Ding, K. Xin, H. Li, and S. Xu are with the Youth Innovation Team of Shaanxi Universities. H. Ding, K. Xin, H. Li, and S. Xu are with the School of Information and Communications, National University of Defense Technology, Xi’an, 710106, China. (email: dhingy2003@hotmail.com)

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\(^1\)Unlike the monostatic and bistatic backscatter communications that employ dedicated RF sources in the BD, in this work we focus on ambient backscatter communication which utilizes the ambient RF sources, such as TV towers, cellular base stations, and Wi-Fi APs, as carrier signal.

A. Related Works

1) Ambient Backscatter Communications (AmBC): In [5], a seminal AmBC prototype was introduced, which enables connectivity among small computing devices with the aid of ambient RF signals. Then, Parks et al. presented the first multi-antenna interference cancellation design for AmBC in addition to a novel multiple-access enabled coding method [6]. After that, [7] proposed a high-throughput, sufficient range and low-power AmBC prototype, a.k.a. BackFi, by using ambient WiFi signals, where the range and throughput of the prototype can be further boosted by utilizing multiple antennas at the WiFi AP. The aforementioned works focused on the enabling techniques for the prototype design of AmBC systems although their achievable performance is not fully understood, especially for large-scale deployment. In view of this, [1] employed stochastic geometry to model the large-scale AmBC deployment, and then quantified the coverage and capacity of the considered network. To guarantee non-overlapping sub-channels for different backscatter users, the authors of [8] considered shifting the frequencies of backscatter users to avoid co-channel interference (CCI). Then, the authors of [9] studied capacity scaling law of backscatter communication systems with the number of backscatter tags. In addition, [10] and [11] analyzed the capacity and outage performance of an AmBC system and no cooperation was considered between the primary and backscatter systems.

2) Symbiotic AmBC: Unlike [10] and [11], knowing the symbiotic nature of the AmBC systems, a cooperative backscatter mechanism was recently given to tackle the CCI problem from the primary system, where a successive interference cancellation (SIC) algorithm was used to remove the primary signal at the backscatter receiver [12], [13]. Recently, the authors of [14] carried out a resource allocation for a cooperative AmBC system and proposed several symbiotic transmission schemes. Further, [15] characterized the transmission robustness of the cooperative AmBC system proposed by [14] and unveiled the achievable outage performance of several transmission schemes at high signal-to-noise ratio (SNR). In [16], the authors studied the performance of an AmBC system which rides over a non-orthogonal multiple access (NOMA) downlink, and proposed the optimal setup to maximize the ergodic capacity of the AmBC system. [17] further proposed the backscatter cooperation (BC) scheme for NOMA downlinks, where the two end-users cooperate with each other by backscattering surplus power of the received downlink signals. In addition, [18] considered a NOMA uplink scenario, where a delay-sensitive non-energy-constrained IoT device and multiple delay-tolerant energy-constrained devices communicate with the same access point. The authors of [19] studied a symbiotic AmBC system, where along with the ambient primary transmission from a MU to an AP, a BD node passively conveys its own information to the MU via the ambient carrier emitted by MU. For such, a MAC-layer analysis of the false-alarm/detection probability at AP and a physical-layer outage analysis of the backscatter link \( BD \rightarrow MU \) and the primary link \( MU \rightarrow AP \) were respectively carried out. In [20], the
authors analyzed the effect of the tag sensitivity on the capacity of a symbiotic AmBC system, where an energy harvesting (EH) and then backscattering mechanism was adopted at the BD node. In [21], the authors considered a three-node symbiotic radio (SR) system, where a secondary transmitter (STx) transmits messages by modulating its information over the radio frequency (RF) signals received from a primary transmitter (PTx). Based on this system model, a maximum-likelihood (ML) detector was used to perform a joint detection of the primary/secondary signals at the destination and the BER performance for the primary and secondary transmissions was then developed, respectively. In [22], the base station (BS) transmits information to two cellular users based on NOMA protocol, while a BD backscatters its own signal by riding on the BS’s signal to the two cellular users. In particular, when the BD backscatters its information only to the near cellular user, the backscatter-NOMA system degenerates into a symbiotic radio (SR) system. For such, the authors derived the outage probability, ergodic rate, diversity order and the scaling slope of the ergodic rate for primary link as well as for the backscatter link within the backscatter-NOMA and SR systems, respectively. In [23], the authors developed the ergodic rates of the primary link as well as the secondary/backscatter link of a cooperative ambient backscatter system which is composed of a multi-antenna RF source, a single-antenna backscatter transmitter and a multi-antenna cooperative receiver that aims to decode the signals from the RF source and the backscatter transmitter. Particularly, transmit beamforming is deployed at the RF source and multi-antenna combining is utilized at the cooperative receiver.

B. Motivations and Contributions

It is noteworthy that in the forgoing works, the transmission robustness of the symbiotic AmBC system has been studied either for the backscatter system that rides on the environmental RF carrier [10], [15], [22], [23] or for the primary system that emitting the RF carrier [15], [22], [23]. Although the primary system and the backscatter system co-exist as a whole, the symbiotic system’s co-existing capability for the whole symbiotic AmBC system has not been well understood yet. In particular, the coexistence outage probability and the ergodic capacity of the backscatter link is not clear under the symbiotic constraints. Therefore, the motivations of this work are twofold. On one hand, for the considered symbiotic AmBC systems, along with the primary transmission from T1 to T2, the backscatter (secondary) transmitter BD employs the primary signal $s(n)$ from T1 as a carrier to passively convey its own signal $c(n)$ to T2 and/or to T1. For such a symbiotic AmBC system, we first need to characterize its coexistence capability, where we use the metric of coexistence outage probability, i.e., COP. Specifically, a coexistence outage event occurs when either the primary transmission from T1 to T2, or the backscatter transmission from BD to T1 and/or from BD to T2 is not successful. Further, we need to depict the scaling law of the coexistence capability with respect to the key system parameters. It is noteworthy that whether the primary transmission and the backscatter transmission can coexist as a whole, the backscatter transmitter BD can successfully decode $c(n)$. Therefore, we are curious to know the impacts of decoding constraints at T2 and/or at T1 on the coexistence capability of the symbiotic AmBC systems. According to the specific decoding requirements/possibilities, we have to consider three possible cases, which include Case I: both T1 and T2 are required to decode $c(n)$, Case II: only T2 is required to decode $c(n)$, and Case III: only T1 is required to decode $c(n)$. By comparing the COP difference between Case I and the other two cases, we are able to determine the specific impact of the individual decoding operation of $c(n)$ at T2 or at T1 on the COP of the whole symbiotic AmBC system.

On the other hand, the decoding requirements of the three cases mentioned above establish the coexistence constraints for one specified backscatter link. For example, in order to satisfy the coexistence constraints of Case I, the ergodic capacity (EC) of the backscatter link from BD to T2 would be affected by the backscatter link from BD to T1 (i.e., the decoding operation of $c(n)$ at T1), and vice versa. As thus, we need to characterize this impact by analyzing the EC differences of the backscatter links for the aforementioned three cases. In particular, we attempt to determine the impact of the individual decoding operation of $c(n)$ at T2 or at T1 on the EC of the specified backscatter links.

The main contributions of our work can be summarized as follows:

(i) A symbiotic AmBC framework is firstly established, where a primary transceiver T1 conveys information $s(n)$ to a primary receiver T2 via a direct link. Meanwhile, a backscatter device (BD) passively modulates its own information $c(n)$ on the incidence signal from T1 and then backscatters the resulting signal toward T1 and T2. At T2, the primary signal $s(n)$ and the backscattered signal $c(n)$ are recovered via the SIC procedure. At T1, the backscattered signal $c(n)$ is decoded by cancelling its own self-interference signal $s(n)$. In particular, we consider three cases for the system model. For Case I, both T1 and T2 are required to decode $c(n)$. For Case II, only T2 is required to decode $c(n)$, whereas for Case III, only T1 is required to decode $c(n).$ This model depicts a primary transmission from T1 to T2 along with an AmBC transmission from BD to T1 and from BD to T2, where the impacts of decoding operation of $c(n)$ are examined for the above three cases.

(ii) The coexistence capability of the symbiotic AmBC system is measured in terms of coexistence outage probability and our analysis shows that at high SNR, the coexistence outage behavior scales as $\frac{1}{n^{a_2}}$ for Cases I and III, with $P_2$ denoting the transmit power at source. In contrast, for Case II, the scaling law of outage behavior is jointly dominated by $\frac{1}{n^{a_1}}$ and $\frac{\log(n)}{n}$ at high SNR. In addition, the impacts of the restriction conditions of decoding $c(n)$ at T1 and T2 on the coexistence outage probability are characterized, which shows that the restriction condition of decoding $c(n)$ at T1 leads to a dominating term $\frac{1}{n^{a_1}}$ for the coexistence outage probability at high SNR, whereas the restriction condition of decoding $c(n)$ at T2 gradually loses impacts on the coexistence outage probability at high SNR in comparison with that of decoding $c(n)$ at T1.

(iii) Our analytical results show that for the above three cases, the slope of the ergodic capacity of the backscatter channels with respect to the transmit SNR preserves $\frac{1}{n^{a_2}}$ at high SNR. Meanwhile, unlike the metric of coexistence outage probability, the impacts of decoding operation of $c(n)$ at T1 or T2 on the ergodic capacity of the backscatter channels gradually disappears in the high SNR regions. In addition, with the improvement of the channel quality within the symbiotic AmBC system, the ergodic capacity gap between the backscatter links BD-T1 and BD-T2 gradually narrows.

Before proceeding to the next section, Table I shows a table which lists the symbols and their definitions used throughout this paper.
Herein, the channel reciprocity is assumed and we have $f_p$ power gains conform to exponential distribution with probability density functions (PDF) given by $f_{h_{12}}(x) = e^{-x/\lambda_1}$, where $h_1 (l \in \{1, 2, B\})$ denotes the channel coefficient (of the links $T1 \leftrightarrow T2$, $T1 \leftrightarrow BD$, and $T2 \leftrightarrow BD$, respectively). Herein, the channel reciprocity is assumed and we have $h_{12} = h_{21}$, $h_{1B} = h_{B1}$, and $h_{2B} = h_{B2}$.

### II. System Model

#### A. System and Channel Model

As shown by Fig. 1, we consider a symbiotic ambient backscatter communication system, which is composed of a primary system and a backscatter system. For the primary system, the end-source $T1$ communicates with $T2$. For the backscatter system, in addition to acquiring the information from $T1$, $T2$ also intends to attain information backscattered by BD. Whereas for $T1$, it also tries to decode the signal backscattered by BD. Without loss of generality, we assume a quasi-static flat Rayleigh fading channel such that the channel power gains conform to exponential distribution with probability density functions (PDF) given by $f_{h_{12}}(x) = e^{-x/\lambda_1}$, where $h_1 (l \in \{1, 2, B\})$ denotes the channel coefficient (of the links $T1 \leftrightarrow T2$, $T1 \leftrightarrow BD$, and $T2 \leftrightarrow BD$, respectively). Herein, the channel reciprocity is assumed and we have $h_{12} = h_{21}$, $h_{1B} = h_{B1}$, and $h_{2B} = h_{B2}$.

#### B. Signal Transmission Model

In what follows, the signal transmission process is introduced. First of all, the received signal at $T2$ can be expressed as

$$y_{T2}(n) = \sqrt{P_s} h_{12} s(n) + \sqrt{\alpha \eta} P_s h_{1B} h_{2B} c(n) s(n) + u_2(n),$$  

(1)

where $s(n)$ is the signal transmitted by $T1$, and $P_s$ denotes the transmit power at $T1$. $\alpha$ and $\eta$ represent the normalized reflection coefficient and the backscatter efficiency at BD. $c(n)$ denotes the $n$-th symbol transmitted by BD, and $u_2(n)$ is the normalized additive white Gaussian noise (AWGN) at $T2$. In particular, $s(n)$ and $c(n)$ are zero mean circular symmetric complex symbols with normalized variance. Note that the second term in (1) indicates the backscattered signal from BD. On the other hand, the backscattered signal from BD is also received by $T1$, which can be given by

$$y_{T1}(n) = \sqrt{\alpha \eta} P_s h_{1B} h_{2B} s(n) c(n) + u_1(n),$$  

(2)

where $u_1(n)$ denotes the normalized AWGN at $T1$. Based on the above results, the received signal-to-interference-plus-noise ratio (SINR) at $T2$ to decode $s(n)$ is given by

$$\gamma_{T2,s}(n) = \frac{P_s |h_{12}|^2}{\alpha \eta P_s |h_{1B}|^2 |h_{2B}|^2 + 1}.$$  

(3)

Denoting the SINR threshold to decode $s(n)$ at $T2$ as $\gamma_{T2,s}$, we assume that $T2$ decodes $s(n)$ and $c(n)$ by utilizing SIC. Therefore, if $\gamma_{T2,s} \geq \gamma_{T2,c}$, $s(n)$ can be recovered such that the received SNR to decode $c(n)$ at $T2$ can be expressed as

$$\gamma_{T2,c} = \frac{\alpha \eta P_s |h_{1B}|^2 |h_{2B}|^2}{\alpha \eta P_s |h_{1B}|^2 |h_{2B}|^2 + 1}.$$  

(4)

At $T1$, the backscattered signal $c(n)$ is decoded by cancelling its own self-interference signal $s(n)$ such that the received SNR at $T1$ is given as

$$\gamma_{T2,c} = \frac{\alpha \eta P_s |h_{1B}|^4}{\alpha \eta P_s |h_{1B}|^4 + 1}.$$  

(5)

As shown by Figure 2, along with the ambient primary transmission from $T1$ to $T2$, there are two possible backscatter transmission streams which respectively complete the passive backscatter transmission from BD to $T2$ and from BD to $T1$. Specifically, each backscatter frame (either from BD to $T2$ or from BD to $T1$) is composed of two parts, i.e., the preamble part and the data transmission part. Similar to [5] and [29], the preamble part consists of the 0/1 wakeup signals, timing signals and the training signals, whereas the data transmission part consists of the data bits. Without loss of generality, we assume the time duration of the data transmission part is much longer than that of the preamble part such that the time

2The channel state information (CSI) of loop-interference channel is supposed to be available at $T1$ to cancel its self-interference signal $s(n)$ such that $c(n)$ can be decoded at $T1$ for Cases I and III. This can be performed by transmitting pilot signaling by $T1$ and then estimating the loop-interference channel as in [24], [25], [26]. In this case, a self-interference-free assumption can be made as in [27] and [28].
duration of the latter can be safely omitted in comparison with that of the former one. In particular, our theoretical analysis of COP/EC will focus on the data transmission part, as in [14] and [15]. By default, the BD node is in a sleep mode to save energy if it has no data to transmit. Once it has enough data to transmit, the BD’s backscatter transmission is activated [5], [29]. In addition, the length of the AmBC frame from BD to T2 and from BD to T1 is exactly the same since the backscattered signal \( c(n) \) is emitted by the same BD node.

C. System Setup and Applications

For the considered system model, along with the ambient primary transmission from T1 to T2, we focus on three possible cases for the backscatter transmission from BD to T1 and/or to T2, which include Case I: both T1 and T2 decode \( c(n) \), Case II: only T2 decodes \( c(n) \), and Case III: only T1 decodes \( c(n) \).

The proposed three scenarios may find applications in smart home. For instance, both BS (i.e., T1) and MS (i.e., T2) may need to acquire the status data of a home appliance such as the temperature setup of a refrigerator (i.e., BD), and the data to transmit, the BD’s backscatter transmission is activated [14] and [15]. By default, the BD node is in a sleep mode to save energy if it has no data to transmit. Once it has enough data to transmit, the BD’s backscatter transmission is activated [5], [29]. In addition, the length of the AmBC frame from BD to T2 and from BD to T1 is exactly the same since the backscattered signal \( c(n) \) is emitted by the same BD node.

III. COEXISTENCE PROBABILITY AND OUTAGE ANALYSIS

In this section, we characterize the coexistence capability of the primary system and the backscatter systems in terms of coexistence outage probability (COP) for three possible cases, i.e., Case I: both T1 and T2 decode \( c(n) \), Case II: only T2 decodes \( c(n) \), and Case III: only T1 decodes \( c(n) \).

A. Both T1 and T2 Decode \( c(n) \)

We define \( \tau_0 \) as the SNR threshold to recover \( c(n) \). In other words, to ensure that T1 and T2 decode BD’s signal \( c(n) \) successfully, inequalities \( \gamma_{T_{1,c}} \geq \tau_0 \) and \( \gamma_{T_{2,c}} \geq \tau_0 \) have to be satisfied. Therefore, the condition to successfully decode \( s(n) \) and \( c(n) \) at T2 can be expressed as

\[
\left\{ \frac{P_s|h_{12}|^2}{\alpha n P_s|h_{1B}|^2|h_{2B}|^2} + 1 \geq \tau_p, \alpha n P_s|h_{1B}|^2|h_{2B}|^2 \geq \tau_0 \right\},
\]

which can be equivalently written as

\[
\left\{ \frac{\tau_0}{\eta P_s|h_{1B}|^2|h_{2B}|^2} \leq \alpha \leq \frac{P_s|h_{12}|^2}{\eta P_s|h_{1B}|^2|h_{2B}|^2} - 1 \right\}.
\]

In other words, if the normalized reflection coefficient \( \alpha \) is adjusted according to (6), T2 would be able to decode \( s(n) \) and \( c(n) \). To make this happen, inequalities \( \tau_0 \leq \frac{P_s|h_{12}|^2}{\eta P_s|h_{1B}|^2|h_{2B}|^2} - 1 \) and \( \frac{P_s|h_{12}|^2}{\eta P_s|h_{1B}|^2|h_{2B}|^2} \leq \tau_0 \) have to hold. On the other hand, for T1, the reflection coefficient \( \alpha \) has to be adjusted based on the following condition in order to decode \( c(n) \) successfully:

\[
\alpha \geq \frac{\tau_0}{\eta P_s|h_{1B}|^2},
\]

in which the condition \( \frac{P_s|h_{12}|^2}{\eta P_s|h_{1B}|^2|h_{2B}|^2} \leq 1 \) has to hold to keep a feasible setup of \( \alpha \). To summarize, to make the primary transmission T1 → T2 as well as the backscatter transmission BD → T2 and BD → T1 suffer from no information outage, the following conditions have to be satisfied:

\[
\begin{align*}
\frac{\tau_0}{\eta P_s|h_{1B}|^2|h_{2B}|^2} &\leq 1, \quad \tau_0 \leq \frac{P_s|h_{12}|^2}{\eta P_s|h_{1B}|^2|h_{2B}|^2}, \\
\tau_0 &\leq \frac{P_s|h_{12}|^2}{\eta P_s|h_{1B}|^2|h_{2B}|^2} - 1.
\end{align*}
\]

Accordingly, the probability of successful coexistence\(^3\) can be written as (9), shown at the top of the next page.

In what follows, we first focus on \( I_1 \), which can be reformulated as

\[
I_1 = \frac{1}{\lambda_{1B}} e^{-\frac{\tau_0}{\lambda_{1B} P_s}} \times \left( \int_{\frac{\tau_0}{\lambda_{2B} P_s}}^{\infty} e^{-\frac{\tau_0}{\lambda_{1B} P_s} - \frac{\tau_0}{\lambda_{2B} P_s} z} dz - \int_{\frac{\tau_0}{\lambda_{1B} P_s}}^{\frac{1}{\lambda_{1B} P_s} + \frac{1}{\lambda_{2B} P_s}} e^{-\frac{\tau_0}{\lambda_{1B} P_s} - \frac{\tau_0}{\lambda_{2B} P_s} z} dz \right).
\]

Unfortunately, so far as the authors are concerned, there is no closed-form expression for \( \xi \) in (10). To proceed, for sufficiently large \( P_s \), \( \xi \) can be asymptotically written as

\[
\xi \approx \sum_{i=0}^{\infty} \left( \frac{\tau_0}{\lambda_{2B} P_s} \right)^i \left( \frac{1}{\lambda_{1B} P_s} \right)^{1-i} e^{-\frac{\tau_0}{\lambda_{1B} P_s} z} dz.
\]

3Instead of defining the transmission robustness for the primary and backscatter systems separately [15], [4], [10] and [11], in this work we define the probability of successful coexistence (or equivalently the coexistence outage probability) to evaluate the overall transmission robustness of the whole symbiotic AmBC system.
\[ P_{\text{coex}}^{T_1\rightarrow T_2} = \Pr \left( |h_{2B}|^2 \leq |h_{1B}|^2, \frac{\tau_0}{\eta P_s |h_{1B}|^2} \leq \frac{\lambda_{2B}}{\lambda_{1B} + \lambda_{2B}} \leq \frac{P_s |h_{12}|^2}{\eta P_s |h_{1B}|^2}, \tau_0 \leq \frac{\lambda_{12}}{\eta P_s} \right) \]

\[ + \Pr \left( |h_{2B}|^2 > |h_{1B}|^2, \frac{\tau_0}{\eta P_s |h_{1B}|^2} \leq \frac{\lambda_{12}}{\eta P_s} \leq \frac{P_s |h_{12}|^2}{\eta P_s |h_{1B}|^2}, \tau_0 \leq \frac{\lambda_{12}}{\eta P_s} \right). \]

(9)

\[ \sqrt{\frac{\gamma P_s}{\tau_0}} e^{-\frac{\lambda_{12}}{\gamma P_s} \sqrt{\frac{\gamma P_s}{\tau_0}}} \]. By inserting (11) into (10) and making use of [30, Eq. (8.214.1)], we have

\[ I_1 \simeq \frac{\lambda_{1B}}{\lambda_{1B} + \lambda_{2B}} + \frac{1}{P_s} \left( \frac{\tau_0}{\eta \lambda_{1B} \lambda_{2B}} \right)^2 \left( \frac{\lambda_{1B} + \lambda_{2B}}{\lambda_{1B}} \right)^2 \left( C_{\text{Euler}} + \ln \left( \frac{1}{\lambda_{1B}} \sqrt{\frac{\lambda_{1B}}{\eta P_s}} \right) \right) \]

\[ - \frac{\tau_0 \lambda_{1B} \lambda_{2B}}{2 \eta \lambda_{1B} \lambda_{2B}} - \frac{1}{\lambda_{1B}} + \frac{1}{\lambda_{2B}} \right)^2, \]

(12)

where \( C_{\text{Euler}} = 0.5772156649 \) denotes the Euler’s constant.

On the other hand, we define \( X \triangleq |h_{1B}|^2 \), \( Y \triangleq |h_{2B}|^2 \), and \( Z \triangleq |h_{12}|^2 \), and rewrite \( I_2 \) as

\[ I_2 = \lambda_{2B} e^{-\frac{\tau_0 (\tau_0 + 1)}{\eta P_s \lambda_{1B} \lambda_{2B}}} \sqrt{\frac{\gamma P_s}{\tau_0}} \left( \frac{\lambda_{1B}}{\lambda_{1B} + \lambda_{2B}} \right)^2 \]

\[ - e^{-\frac{\lambda_{1B}}{\gamma P_s \lambda_{1B} \lambda_{2B}}} \frac{\lambda_{1B} \lambda_{12}}{P_s} \int_{I_{21}} e^{-\frac{\lambda_{1B}}{\gamma P_s \lambda_{1B} \lambda_{2B}}} \frac{P_s}{\tau_0 \lambda_{1B} \lambda_{12}} \frac{1}{\lambda_{1B} - 1} \frac{1}{\lambda_{1B}} dz. \]

(13)

With the aid of [30, Eq. (3.352.2)], \( I_2 \) can be asymptotically written as

\[ I_{21} \simeq -\frac{\tau_0 \lambda_{12}}{P_s} \]

\[ \times \left\{ C_{\text{Euler}} + \ln \left( \frac{1}{\lambda_{12}} + \frac{1}{\tau_0 \lambda_{12}} \right) \left( \frac{\tau_0 P_s}{\eta} \right) \right\} \]

\[ + \frac{\tau_0 \lambda_{12}}{P_s} \left( \frac{1}{\lambda_{12}} - \frac{1}{\tau_0 \lambda_{12}} \right) \left( \frac{\tau_0 P_s}{\eta} \right) \]

(14)

By inserting (14) into (13), \( I_2 \) can be asymptotically expressed as

\[ I_2 \simeq \lambda_{2B} \frac{\lambda_{1B} + \lambda_{2B}}{\lambda_{1B} + \lambda_{2B}} \]

\[ \times \left\{ 1 + \frac{\lambda_{1B} + \lambda_{2B}}{\lambda_{1B} + \lambda_{2B}} \right\} \left( \frac{\tau_0}{\eta P_s} \right) \]

\[ - \frac{\tau_0 \lambda_{12} \lambda_{2B}}{2 \eta \lambda_{1B} \lambda_{2B}} - \frac{1}{\lambda_{1B}} + \frac{1}{\lambda_{2B}} \right)^2, \]

(12)

where \( C_{\text{Euler}} = 0.5772156649 \) denotes the Euler’s constant.

By summarizing (12) and (15), we can arrive at the following conclusion.

**Proposition 1:** When both T1 and T2 are required to decode \( c(n) \) (Case I), the asymptotic coexistence behavior of the symbiotic AmBC system can be written as

\[ P_{\text{out}}^{T_1\rightarrow T_2} = 1 - P_{\text{coex}}^{T_1\rightarrow T_2} \simeq P_1 + P_2 + P_3 \rightarrow P_1, \]

in which we have \( P_1 = \frac{1}{\lambda_{1B} + \lambda_{2B}} \sqrt{\frac{\gamma P_s}{\tau_0}} \),

\[ P_2 \]

\[ = \frac{\tau_0 \lambda_{12}}{\lambda_{12}} \ln \left( \frac{1}{\lambda_{12}} + \frac{1}{\lambda_{2B}} \right) \sqrt{\frac{\gamma P_s}{\tau_0}} - \frac{1}{\lambda_{1B} + \lambda_{2B}} \ln \left( \frac{1}{\lambda_{1B} + \lambda_{2B}} \right) \sqrt{\frac{\gamma P_s}{\tau_0}} \],

and

\[ P_3 \]

\[ = \frac{1}{\lambda_{1B} + \lambda_{2B}} \ln \left( \frac{1}{\lambda_{1B} + \lambda_{2B}} \right) \sqrt{\frac{\gamma P_s}{\tau_0}}. \]

**Remark 1:** It follows from (16) that the asymptotic scaling law of the symbiotic backscatter system is dominated by three kinds of ingredients, i.e., \( \sqrt{\frac{\gamma P_s}{\tau_0}} \), \( \frac{1}{\lambda_{1B} + \lambda_{2B}} \), and \( \frac{1}{\lambda_{1B} + \lambda_{2B}} \ln \left( \frac{1}{\lambda_{1B} + \lambda_{2B}} \right) \sqrt{\frac{\gamma P_s}{\tau_0}} \). It will be shown in the numerical results section that all of the three kinds of ingredients are indispensable to accurately depict the high-SNR outage behavior of the symbiotic backscatter systems. However, for sufficiently high SNR, the asymptotic scaling law of the symbiotic backscatter system is dominated by \( \sqrt{\frac{\gamma P_s}{\tau_0}} \).

**B. Only T2 Decodes \( c(n) \)**

In this case, to guarantee a successful SIC decoding of \( s(n) \) and \( c(n) \) at T2, the normalized reflection coefficient has to satisfy

\[ \frac{\tau_0}{\eta P_s |h_{1B}|^2 |h_{2B}|^2} \leq \alpha \leq \frac{P_s |h_{12}|^2 - 1}{\eta P_s |h_{1B}|^2 |h_{2B}|^2}. \]

Correspondingly, to ensure a feasible setup of \( \alpha \), the following condition has to hold:

\[ \tau_0 \leq \frac{P_s |h_{12}|^2 - 1}{\eta P_s |h_{1B}|^2 |h_{2B}|^2}. \]

That is to say: the probability of successful coexistence between the backscatter
transmission and primary transmission can be expressed as
\[
P_{\text{coex}}^{\text{ND-T1}} = \Pr\left(\tau_0 \leq \frac{P_s |h_{12}|^2}{\tau_p} - 1, \frac{\tau_0}{\eta P_s |h_{1B}|^2 |h_{2B}|^2} \leq 1\right) = e^{-\frac{\tau_0 (\tau_0 + 1)}{2\tau_p}} \left[1 - F_V\left(\frac{\tau_0}{\eta P_s}\right)\right],
\]
where \(F_V(v) = G_{2,1}^1\left(\frac{v}{\lambda_{1B} \lambda_{2B}}, 1\right)\) denotes the CDF of \(V \sim |h_{1B}|^2 |h_{2B}|^2\), with \(G_{2,1}^1(-,-)\) being the Meijer’s G function. To characterize the asymptotic outage behavior of the considered symbiotic backscatter system, we establish the scaling behavior of \(F_V\left(\frac{\tau_0}{\eta P_s}\right)\) at high SNR, given in the following Lemma.

**Lemma 1:** For sufficiently high SNR, \(F_V\left(\frac{\tau_0}{\eta P_s}\right)\) can be asymptotically expressed as
\[
F_V\left(\frac{\tau_0}{\eta P_s}\right) \simeq \frac{\tau_0}{\eta P_s \lambda_{1B} \lambda_{2B}} \ln\left(\frac{\eta P_s \lambda_{1B} \lambda_{2B}}{4\tau_0}\right) + \frac{\tau_0}{\eta P_s \lambda_{1B} \lambda_{2B}} \ln\left(P_s\right) \propto \frac{\tau_0}{\eta P_s}.
\]
**Proof:** According to \([30, \text{Eq. (9.34.3)}]\) and the definition of \(F_V(v)\), we have
\[
F_V\left(\frac{\tau_0}{\eta P_s}\right) = \frac{2}{\lambda_{12} \lambda_{22}} \int_0^{\frac{\tau_0}{\eta P_s}} \frac{t}{\lambda_{22}} K_0\left(2 \sqrt{\frac{t}{\lambda_{12} \lambda_{22}}}\right) dt.
\]
With the aid of \([31, \text{Eq. 9.6.8}]\) and the change of variables, it follows that
\[
F_V\left(\frac{\tau_0}{\eta P_s}\right) \simeq -\int_0^{\frac{\tau_0}{\eta P_s}} \frac{2\sqrt{\pi \tau_0 \lambda_{12} \lambda_{22}}}{\lambda_{12} \lambda_{22}} x \ln(x) dx.
\]
Next, invoking the integration by parts and L’Hospital’s rule, we complete the proof. \(\square\)

**Proposition 2:** When only T2 is required to decode \(c(n)\) (Case II), the coexistence outage probability for the symbiotic AmBC system can be asymptotically expressed as
\[
P_{\text{out}}^{\text{ND-T2}} = 1 - P_{\text{coex}}^{\text{ND-T1}} \simeq \frac{\tau_0}{\eta P_s \lambda_{1B} \lambda_{2B}} \ln\left(\frac{\eta P_s \lambda_{1B} \lambda_{2B}}{4\tau_0}\right) + \frac{\tau_0}{\eta P_s \lambda_{1B} \lambda_{2B}} \ln\left(P_s\right) \propto \frac{\tau_0}{\eta P_s}.
\]
**Remark 2:** It follows from (21) that the coexistence outage probability scales as \(\frac{\tau_0}{\eta P_s}\) at high SNR. In addition, by comparing Proposition 2 with Proposition 1, we notice that the impacts of restriction condition of decoding \(c(n)\) at T1 results in a dominating scaling law of \(\frac{\tau_0}{\eta P_s}\) for the coexistence outage probability of the whole symbiotic system.

**C. Only T1 Decodes \(c(n)\)**

In this case, T2 is only required to decode the signal from T1, i.e., \(s(n)\), whereas T1 is required to decode \(c(n)\). As a result, to make the primary and the backscatter transmissions coexist with each other, the normalized reflection coefficient \(\alpha\) has to satisfy \(\frac{\tau_0}{\eta P_s |h_{1B}|^2} \leq \alpha \leq \frac{P_s |h_{12}|^2 - 1}{\eta P_s |h_{1B}|^2 |h_{2B}|^2}\). Correspondingly, to make the set of \(\alpha\) feasible, inequalities \(\frac{\tau_0}{\eta P_s |h_{1B}|^2} \leq 1\) and \(\frac{\tau_0}{\eta P_s |h_{1B}|} \leq \frac{P_s |h_{12}|^2 - 1}{\eta P_s |h_{2B}|^2}\) have to hold.

Therefore, the probability of coexistence can be written as
\[
P_{\text{coex}}^{\text{ND-T2}} = \Pr\left(\frac{\tau_0}{\eta P_s |h_{1B}|^2} \leq 1, \frac{\tau_0}{\eta P_s |h_{1B}|^2} \leq \frac{P_s |h_{12}|^2 - 1}{\eta P_s |h_{2B}|^2}\right).
\]
By defining \(X \sim |h_{1B}|^2, Y \sim |h_{2B}|^2\), and \(Z \sim |h_{12}|^2\), it follows from \([30, \text{Eq. (3.352.2)}]\) that the coexistence outage probability of the symbiotic AmBC system can be characterized in the following proposition.

**Proposition 3:** When only T1 is required to decode \(c(n)\) (Case III), the coexistence outage probability of the symbiotic AmBC system can be asymptotically expressed as
\[
P_{\text{out}}^{\text{ND-T2}} \simeq \frac{1}{P_s} \left[\frac{\tau_0}{\lambda_{12}} - 2\left(\frac{\tau_0}{\lambda_{1B}}\right)^2 - \frac{\tau_0}{\lambda_{1B}} G_{\text{Euler}}\right] + 1 \frac{\tau_0}{\lambda_{1B}} \frac{\tau_0}{\lambda_{12} \lambda_{2B}} P_0 \ln\left(\frac{\tau_0}{\lambda_{1B}} + \frac{\tau_0}{\lambda_{12} \lambda_{2B}}\right) \rightarrow \frac{1}{\lambda_{1B}} \frac{\tau_0}{\lambda_{12} \lambda_{2B}}.
\]

**Remark 3:** It follows from Propositions 1-3 that unlike Case II, the asymptotic coexistence outage behavior of the symbiotic AmBC system for Case III is exactly the same with the counterpart for Case I. In other words, the impacts of restriction condition of decoding \(c(n)\) at T2 on the coexistence outage probability gradually disappears at high SNR, which is different from the restrictions of the restriction condition of decoding \(c(n)\) at T1.

**IV. ERODIC CAPACITY**

**A. Decoding \(c(n)\) at T2 and T1**

To maximize the rate of the backscatter system, it follows from (6) that the optimal normalized reflection coefficient is
\[\alpha^* = \min\left[1 - \frac{|P_s |h_{1B}|^2 - 1}{\eta P_s |h_{1B}|^2 |h_{2B}|^2}, \frac{P_s |h_{12}|^2 - 1}{\eta P_s |h_{1B}|^2 |h_{2B}|^2}\right].\]
Accordingly, the ergodic capacity of the backscatter transmission at T2 can be written as
\[
C_{\text{T2}} = E\left\{\log_2\left(1 + \frac{|P_s |h_{1B}|^2 |h_{2B}|^2}{\tau_p}\right)\right\} = E\left\{\log_2\left(1 + \min\left[\eta P_s |h_{1B}|^2 |h_{2B}|^2, \frac{P_s |h_{12}|^2 - 1}{\tau_p}\right]\right\},
\]
where the expectation operation is taken over (8). As before, we define \(X \sim |h_{1B}|^2, Y \sim |h_{2B}|^2\), and \(Z \sim |h_{12}|^2\), and then reformulate (24) as
\[
C_{\text{T2}} = E_{\eta P_s X Y} \min\left(P_s |h_{1B}|^2 |h_{2B}|^2, \frac{P_s |h_{12}|^2 - 1}{\tau_p}\right) = \frac{J_1}{J_2},
\]
where \(J_1 = \int_{\eta P_s X Y} \log_2\left(\frac{P_s Z}{\tau_p}\right) dP_s X Y\) and \(J_2 = \int_{\eta P_s X Y} \log_2\left(\frac{P_s Z}{\tau_p}\right) dP_s X Y\).

\[4\text{To set the optimal reflection coefficient } \alpha \text{ at BD, global CSI is supposed to be available at BD. This can be achieved by estimating the local CSI } h_{1B} \text{ and } h_{2B} \text{ at BD from the pilot signaling sent by T1 and T2, respectively. In addition, the non-local CSI } h_{12} \text{ can be estimated at T2 from the pilot signaling sent by T1 and then } h_{12} \text{ will be forwarded from T2 to BD, similar to the approach adopted by } [32, 33], \text{ and } [34].\]
Next, we first focus on $J_1$, which can be written as

$$J_1 = \int \int f_X(x) f_Y(y) f_Z(z) \left( \begin{array}{c} P_{s,z} \tau_p - 1, z \leq \frac{P_{s,z}}{\tau_p} \\ \frac{\tau_0}{\eta P_{s,x} z^2} \leq 1, \tau_0 \leq \frac{P_{s,z}}{\tau_p} - 1, \tau_0 \leq \frac{P_{s,z}}{\tau_p} - 1, \tau_0 \leq \frac{P_{s,z}}{\tau_p} - 1 \end{array} \right) \text{d}x \text{d}y \text{d}z,$$

(26)

where the integration region is given by

$$\frac{\tau_0}{\eta P_{s,x} z^2} \leq 1, \tau_0 \leq \frac{P_{s,z}}{\tau_p} - 1, \tau_0 \leq \frac{P_{s,z}}{\tau_p} - 1, \tau_0 \leq \frac{P_{s,z}}{\tau_p} - 1.$$  (27)

Based on Appendix A-1, the asymptotic behavior of $J_1$ can be determined.

On the other hand, $J_2$ can be reformulated as (28), given at the top of the next page.

Then, it follows from [30, Eq. (3.324.1)] that

$$J_2 \simeq \int_{\tau_0 (\tau_0 + 1)}^{\infty} \text{d}z \log_2 \left( \frac{P_{s,z}}{\tau_p} \right) J_{211} J_{212} \tau_p.$$

(29)

Utilizing the change of variables $u = \frac{P_{s,z}}{\tau_p} - 1$ and $v = \frac{u}{P_{s,z}}$, we rewrite $\phi_1$ as

$$\phi_1 \simeq \frac{1}{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \left[ \frac{4 \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) + \frac{1}{2} \frac{\tau_0}{\lambda_2 \lambda_3} \ln \left( \frac{P_{s,z}}{\tau_p} \right) \right],$$

(30)

where $\phi_3$ and $\phi_4$ are defined as $\phi_3 = \lim_{t \to \infty} \frac{1}{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \left( \frac{4 \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right)$ and $\phi_4 = \lim_{t \to \infty} \frac{1}{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \left( \frac{4 \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right)$. For $\phi_2$, with the aid of the change of variables $u = \frac{P_{s,z}}{\tau_p} - 1$ and $t = \frac{u}{P_{s,z}}$, it follows from [30, Eqs. (3.381.3), (8.351.9), and (8.214.1)] that

$$\phi_2 \simeq \frac{1}{\lambda_2 \lambda_3 \lambda_4} \left( \frac{1}{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) \right)$$

where $\phi_5$ and $\phi_6$ are defined as $\phi_5 = \lim_{t \to \infty} \frac{1}{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \left( \frac{4 \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right)$ and $\phi_6 = \lim_{t \to \infty} \frac{1}{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \left( \frac{4 \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right)$. Which scales as

$$\phi_5 \simeq \frac{1}{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \left( \frac{4 \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) \text{d}t.$$  (31)

Summarizing the foregoing results, we establish the following proposition.

**Proposition 4:** When both T1 and T2 are required to decode $c(u)$ (Case 1), the asymptotic behavior of the ergodic capacity of the backscatter link BDC-T2 can be expressed as

$$C_{BC,T2,c} \simeq \psi_0 \log (P_s) + \psi_1,$$

(32)

in which $\psi_0 \triangleq \frac{1}{\lambda_1 \lambda_2} \left( \frac{1}{\lambda_1 \lambda_2} \right) \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right)$ and $\psi_1 \triangleq \left( \frac{1}{\lambda_1 \lambda_2} \right) \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) - \left( \frac{1}{\lambda_1 \lambda_2} \right) \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right)$.

$W_{a,b}(-)$ denotes the Whittaker W function and, $\varpi_0$, $\varpi_1$, $\varpi_3$, and $\varpi_4$ are defined as before.

**Remark 4:** It follows from Proposition 4 that the ergodic capacity scales linearly with respect to $\log (P_s)$ in the high SNR regime, and the scaling coefficient is $\psi_0$. On the other hand, interestingly, as $P_s \to \infty$, it can be checked that $\tau_p \varpi_3 - \varpi_4 \to 0$ such that the scaling coefficient $\psi_0$ will not be affected by the channel statistics $\lambda_{12} \lambda_{1B}$, and $\lambda_{2B}$ for sufficiently high SNR. Note that at high SNR, the scaling rate of the ergodic capacity of the backscatter link BD-T2 in Case I is the same with the counterparts of the backscatter-NOMA and symbiotic radio systems in [22, Sect.IV.C]. Similar phenomenon can also be observed in Cases II and III, as will be shown by Propositions 5 and 6.

Now we turn to the ergodic capacity of the backscatter link BD-T1, which can be given by $C_{BC,T1,c} = E \left[ \log_2 \left( 1 + \gamma_{T1,c} \right) \right]$, where the expectation is taken over (8). Thus, we have

$$C_{BC,T1,c} = E_{0} \left[ \log_2 \left( 1 + \min \left( \eta P_s X^2, \frac{P_{s,z}}{\tau_p} - 1 \right) \right) \right]$$

$$= E_{0, \eta P_s X^2 + \left( \frac{P_{s,z}}{\tau_p} - 1 \right)} \left\{ \log_2 \left( 1 + \eta P_s X^2 \right) \right\}$$

$$+ E_{0, \eta P_s X^2 - \left( \frac{P_{s,z}}{\tau_p} - 1 \right)} \left\{ \log_2 \left( 1 + \frac{X P_s Z}{\tau_p} \right) \right\}.$$  (33)

By exchanging the integration order, we have

$$S_1 \simeq \frac{1}{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \left( \frac{4 \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) \log \left( \frac{\eta \lambda_1 \lambda_2}{\lambda_3 \lambda_4} \right) \int_{\frac{P_{s,z}}{\tau_p}}^{\infty} e^{-\frac{t}{\lambda_1 \lambda_2 \lambda_3 \lambda_4}} \text{d}t$$

$$+ \frac{2}{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \int_{\frac{P_{s,z}}{\tau_p}}^{\infty} e^{-\frac{t}{\lambda_1 \lambda_2 \lambda_3 \lambda_4}} \text{d}t.$$

(31)

By invoking Taylor's series expansion of $e^{-\frac{t}{\lambda_1 \lambda_2 \lambda_3 \lambda_4}}$ and with the aid of [30, Eq. (3.351.5)], we can arrive at $S_{11} \simeq \lambda_{1B} e^{-\frac{1}{\lambda_1 \lambda_2 \lambda_3 \lambda_4}} + \frac{\lambda_{3B} \eta P_s X^2}{\tau_p} E_{1} \left( \frac{1}{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \right)$.
\[ J_2 = \int_{\tau_p (\varphi_0+1)}^{\infty} f_Z(z) \log_2 \left( \frac{P_x z}{\tau_p} \right) \left( \int_{x \geq \sqrt{\frac{\tau_p}{\eta P_s} \gamma} \leq \frac{1}{\eta} \left( \frac{P_x z}{\tau_p} - 1 \right) x, y \geq \frac{P_x z}{\eta \tau_p} \right) dy \]
\[ S_2 \simeq \ln (P_s) \left[ \frac{1}{\ln (2)} + \frac{\lambda_{12}}{\eta_{\tau_p} \lambda_{1B} \lambda_{2B} \ln (2)} e^{\frac{\tau_p \lambda_{12}}{\eta_{\tau_p} \lambda_{1B} \lambda_{2B}}} Ei \left( -\frac{\lambda_{12}}{\eta_{\tau_p} \lambda_{1B} \lambda_{2B}} \right) \right] + \ln \left( \frac{\lambda_{1B}}{\lambda_{2B}^2} \right) \frac{\lambda_{12}}{\eta_{\tau_p} \lambda_{1B} \lambda_{2B}} - \frac{\lambda_{12}}{\eta_{\tau_p} \lambda_{1B}^2 \lambda_{2B}^2} \frac{\tau_p}{\lambda_{12} \ln (2)} \int_0^\infty \frac{1}{\sqrt{\eta_{\tau_p}}} f_X (x) \ln \left( \frac{\tau_p}{\lambda_{12}^2} + \frac{1}{\lambda_{1B} \lambda_{2B}} \frac{\tau_p}{\lambda_{12}} \right) dx. \] (38)

**Proof:** Please refer to Appendix A-3. \[ \Box \]

Summarizing the foregoing results, an asymptotic expression can be derived for \( K_1 \). For \( K_2 \), by using [30, Eq. (3.471.9)], the change of variables \( u = \frac{\rho_{e2}}{\tau_p} - 1, v = \frac{\eta_{\tau_p}}{\lambda_{1B} \lambda_{2B}} \), and after several algebraic manipulations, it follows from (41) that

\[ \begin{align*}
K_2 & \sim \sqrt{\frac{4}{\eta_{\lambda_{1B} \lambda_{2B}}} \frac{\tau_p}{\lambda_{12}}} \\
& \times \lim_{P_s \to \infty} \int_0^\infty e^{-\frac{\tau_p v}{\lambda_{12}}} \log_2 (P_s v) \sqrt{\eta_{\lambda_{1B} \lambda_{2B}}} \left[ \sqrt{\frac{4v}{\eta_{\lambda_{1B} \lambda_{2B}}} \ln (P_s)} \right] dv
\end{align*} \]

where \( \omega_3 \) and \( \omega_4 \) are the same as before. It is noteworthy that (45) is exactly the same with (30). Summarizing the foregoing results, one can establish the following proposition.

**Proposition 6:** At high SNR, when only \( T_2 \) decodes \( c(n) \) (Case II), the ergodic capacity of the backscatter link BD-T2 is exactly the same with (32).

**Remark 5:** One can observe that at high SNR, the ergodic capacity of the backscatter link BD-T2 remains unchanged regardless of whether the terminal \( T_1 \) is required to decode \( c(n) \). In other words, the effects of decoding operation at \( T_1 \) to decode \( c(n) \) on the ergodic capacity of the backscatter link BD-T2 disappears in the high SNR regions.

### C. Only \( T_1 \) Decodes \( c(n) \)

In this case, we evaluate the ergodic capacity of the backscatter link BD-T1. To guarantee that \( T_1 \) can successfully decode \( c(n) \) while \( T_2 \) can successfully decode \( s(n) \), the reflection coefficient \( \alpha \) has to satisfy \( \eta_{\tau_p} / |h_{1B}|^2 \leq \alpha \leq \frac{P_{s|h_{1B}|^2}}{\eta_{\tau_p} / |h_{1B}|^2} - 1 \). To boost the rate of the backscatter link, \( \alpha \) is set to \( \alpha = \hat{\alpha} = \min \left\{ 1, \frac{P_{s|h_{1B}|^2}}{\eta_{\tau_p} / |h_{1B}|^2} - 1 \right\} \). Accordingly, the ergodic capacity can be written as \( C_{BC,T_1} = E \{ \log_2 (1 + \eta_{\tau_p} c_{1B}) \} \), where the expectation is taken over the following domain:

\[ \frac{\tau_0}{\eta_{P_s} |h_{1B}|^4} \leq 1, \quad \tau_0 \leq \frac{P_{s|h_{1B}|^2}}{\eta_{\tau_p} / |h_{2B}|^2} - 1, \quad \frac{P_{s|h_{2B}|^2}}{\tau_p} - 1 \geq 0. \] (46)

Furthermore, the ergodic capacity can be expressed as

\[ C_{BC,T_1} = E_{\eta_{P_s} x^2 \leq \hat{\alpha} \left( \frac{P_{e2}}{\tau_p} - 1 \right)} \left\{ \log_2 \left( 1 + \frac{P_{e2} x^2}{\hat{\alpha} \left( \frac{P_{e2}}{\tau_p} - 1 \right)} \right) \right\} + E_{\eta_{P_s} x^2 > \hat{\alpha} \left( \frac{P_{e2}}{\tau_p} - 1 \right)} \left\{ \log_2 \left( 1 + \frac{Y}{L_2} \right) \right\}. \] (47)

By exchanging the integration order and invoking [30, Eqs. (3.352.2) and (4.331.2)], we have

\[ L_1 \simeq -\frac{\lambda_{12}}{\lambda_{1B} \lambda_{2B} \eta_{\tau_p} \ln (2)} \times \left( \ln (\eta) + \ln (P_s) \right) e^{\frac{\tau_p \lambda_{12}}{\eta_{\tau_p} \lambda_{1B} \lambda_{2B}}} \ln (\eta) \ln (2) \right) \left( \frac{\lambda_{12}}{\lambda_{1B} \lambda_{2B} \eta_{\tau_p}} \right) - 2 \omega_7. \] (48)

Next, we focus on \( L_2 \). According to Appendix A-4, an asymptotic expression of \( L_2 \) can be achieved. Therefore, one can attain an asymptotic expression of \( C_{BC,T_1} \) \( \propto \log (P_s) \), which is summarized as follows.

**Proposition 7:** In the high SNR regions, when only \( T_1 \) decodes \( c(n) \) (Case III), the ergodic capacity of the backscatter link BD-T1 can be asymptotically written as

\[ C_{BC,T_1} \propto \frac{\ln (P_s)}{\ln (2)} \left( 2 \ln \left( \frac{\lambda_{1B}}{\lambda_{2B}} \right) - \frac{\lambda_{12}}{\eta_{\tau_p} \lambda_{1B} \lambda_{2B}} \right) \] (49)

We are curious to know the capacity difference between (49) and (39), in order to understand the effects of decoding operation of \( c(n) \) at \( T_2 \) on the backscatter reception at \( T_1 \), which is given as below.

**Proposition 8:** For sufficiently high SNR, the ergodic capacity of the backscatter link BD-T1 is not affected by the decoding operation of \( c(n) \) at \( T_2 \).

**Proof:** Please refer to Appendix A-9.

**Remark 6:** For the metric of ergodic capacity, it follows from Propositions 6 and 8 that the impacts of decoding operation of \( c(n) \) at \( T_1 \) or \( T_2 \) gradually disappears at high SNR, which is different from the impacts of these decoding constraints on the coexistence outage probability as shown in Propositions 1-3. This means that the impacts of decoding constraints on the different performance metrics are dramatically different.

### V. NUMERICAL RESULTS AND DISCUSSION

In this section, we first validate the analytical results presented in the foregoing sections, and then examine the observations made from the theoretical analysis via Monte-Carlo simulations.

#### A. Outage Behavior

Fig. 3 shows the coexistence outage probability where primary and backscatter transmissions coexist. It is first observed that the asymptotic outage curves match well with simulations at least in the medium and high SNR regions. Also, it can be seen that the outage curve for the scenario where both \( T_1 \) and \( T_2 \) decode \( c(n) \) approaches toward that of the scenario where only \( T_1 \) decodes \( c(n) \), as predicted in the foregoing remarks. This is because the restriction condition of decoding \( c(n) \) at \( T_1 \) results in a dominating term \( \frac{1}{\sqrt{\eta_{\tau_p}}} \) for the coexistence outage.
probability at high SNR, whereas the impacts of the restriction condition of decoding \( c(n) \) at T2 on the coexistence outage probability tend to be marginal relative to \( \frac{1}{\lambda_{1B}} \) at high SNR. For small values of transmit SNR, it follows from (8), (17), and (22) that the coexistence outage probability of Cases I, II, and III tends to the same limit, i.e., unity, as shown by Fig. 3.

Fig. 4 illustrates the coexistence outage probability when only T2 decodes \( c(n) \) while T1 is not required to do so. It is shown that the outage performance improves dramatically with an increase in the average channel gain \( \lambda_{12} \) especially in the low and medium SNR regions, which is in accord with (21). However, in the high SNR regions, the outage curves with different values of \( \lambda_{12} \) tend to converge but always with a noticeable gap among them. This is due to the fact that in the asymptotic expression given by (21), the term with \( \lambda_{12} \) acts as a dominant ingredient in the high SNR regions. Note that this phenomenon is different from the scenario where T1 is required to decode \( c(n) \). Unlike Fig. 4, it follows from Fig. 5 that when both T1 and T2 are required to decode \( c(n) \), the coexistence outage probability converges in the high SNR regions regardless of the value of \( \lambda_{12} \), which is in accord with (16). As shown in Fig. 6, when only T1 is required to decode \( c(n) \), the high-SNR outage behavior is again not affected by the average channel gain \( \lambda_{12} \), as in the scenario where both T1 and T2 are required to decode \( c(n) \). This accords with (23). On the other hand, the coexistence outage performance improves with an increase in the average channel gain \( \lambda_{1B} \), as predicted by (23).

B. Ergodic Capacity

Fig. 7 shows the ergodic capacity of the backscatter links when both T1 and T2 decode \( c(n) \). It can be first seen that the asymptotic curves match well with simulations in the medium and high SNR regions, which confirms our theoretical analysis. In addition, it is shown that the ergodic capacity of the backscatter link T1-BD-T2 is always larger than that of the backscatter link T1-BD-T1 and the capacity gap gradually narrows with the improvement of the channel conditions. This means that under good channel conditions, the ergodic capacity of the two backscatter links turns out to be comparable to each other.

Fig. 8 demonstrates the ergodic capacity of the backscatter link when only T1 or T2 decodes \( c(n) \). Once again, the accuracy of our asymptotic analytical results is first confirmed. As in the scenario where both T1 and T2 decode \( c(n) \), the ergodic capacity of the backscatter link when only T2 decodes \( c(n) \) outperforms the counterpart when only T1 decodes \( c(n) \), and the performance gap shrinks as the channel conditions become better as before.

Fig. 3: Coexistence outage probability of the symbiotic AmBC systems.

Fig. 4: Coexistence outage probability when only T2 decodes \( c(n) \).

Fig. 5: Coexistence outage probability when both T1 and T2 decode \( c(n) \).
In what follows, we try to discover the effects of decoding operation of $c(n)$ on the ergodic capacity of the backscatter links. Fig. 9 makes a comparison of the ergodic capacity of the backscatter link BD-T2 between the case of decoding $c(n)$ at T1 and the case of no decoding of $c(n)$ at T1. As shown by the figure, when the decoding requirement of $c(n)$ is removed at T1, the ergodic capacity of the backscatter link T1-BD-T2 increases, especially in the low and medium SNR regions. Nonetheless, in the high SNR regions, the capacity improvement owing to the removal of decoding operation of $c(n)$ at T1 gradually disappears, which complies with Remark 5. In another word, the effects of decoding operation of $c(n)$ at T1 on the ergodic capacity of the backscatter link BD-T2 becomes barely noticeable in the high SNR regions. Similarly, it can be observed from Fig. 10 that the removal of decoding operation of $c(n)$ at T2 can indeed boost the ergodic capacity of the backscatter link T1-BD-T1 at least in the low and medium SNR regions. However, this performance improvement lessens with an increase in transmit SNR and finally disappears at high SNR, which is predicted by Proposition 8.

To make a pertinent comparison with [22], Figure 11 considers the scenario of Case II where no decoding operation...
omitted here due to space limit.

C. Advantages of Different Scenarios

Based on the forgoing results, it is ready to confirm that for Case I, the advantage is that both T1 and T2 could attain the information \( c(n) \) at BD, whereas the disadvantage is that the CSI of the loop-interference channel has to be attained at T1 to cancel its self-interference signal \( s(n) \) such that \( c(n) \) can be decoded at T1. Relative to Case I, the advantage of Case II is that the COP of the symbiotic AmBC system can be improved by removing the constraint of decoding operation of \( c(n) \) at T1. Also, by removing the decoding operation of \( c(n) \) at T1, the round-trip fading effects of the BD-T1 channel to decode \( c(n) \) are eliminated, resulting in a steeper decreasing rate of the COP (i.e., a scaling law of \( \frac{\ln(COP)}{\eta P_s} \)). In comparing with Case I, the COP of Case III can also be improved by removing the decoding operation of \( c(n) \) at T2 but the COP still suffers from the round-trip fading effects of the BD \( \rightarrow \) T1 channel (i.e., a scaling law of \( \frac{COP}{\eta P_s} \)). In addition, in comparison with Case I, both Cases II and III lead to a higher ergodic capacity of the backscatter channels as shown by Figures 9 and 10.

VI. CONCLUDING REMARKS

In this paper, we characterized the scaling behavior of coexistence outage probability and ergodic capacity for a symbiotic AmBC system, in which a source T1 communicates with a destination T2, whereas a backscatter device BD also passively conveys its own information \( c(n) \) to T1 and/or T2 via backscattering links. Particularly, we considered three cases for the symbiotic AmBC system. It was shown that the restriction condition of decoding \( c(n) \) at T1 leads to a dominating term \( \sqrt{\frac{\lambda_{1B}}{\eta P_s}} \) for the coexistence outage probability at high SNR, whereas the impacts of the restriction condition of decoding \( c(n) \) at T2 on the coexistence outage probability were shown to be radically different. Finally, the ergodic capacity of the backscatter link was analyzed at high SNR for the three cases and their relationship was disclosed as well.

APPENDIX A

A-1: Asymptotic Behavior of \( \hat{J}_1 \)

Firstly, (26) can be expressed as (A.1), shown at the top of the next page. With the aid of [30, Eq. (4.331.2)] and the change of variables \( u = 1 + \eta P_s xy \), \( \hat{J}_{11} \) in (A.1) can be rewritten as (A.2), shown at the top of the next page. In what follows, we first concentrate on the analysis of \( \hat{J}_1 \). Utilizing [30, Eqs. (3.351.4) and (8.214.1)] and after some algebraic manipulations, \( \hat{J}_1 \) can be asymptotically written as

\[
\hat{J}_1 \simeq \lambda_{1B} \left[ 1 - \frac{1}{\lambda_{1B} \sqrt{\frac{\tau_0}{\eta P_s}}} + \frac{1}{2} \left( \frac{1}{\lambda_{1B}} \right)^2 \frac{\tau_0}{\eta P_s} \right] \\
+ \frac{\tau_0}{\lambda_{1B} \eta P_s} \left[ C_{\text{Euler}} + \ln \left( \frac{1}{\lambda_{1B}} \sqrt{\frac{\tau_0}{\eta P_s}} \right) \right] \triangleq \hat{J}_{11}^{\infty}. \quad (A.3)
\]

Invoking the change of variables \( t = \frac{\tau_0}{\sqrt{\eta P_s}} \) and utilizing [30, Eqs. (4.331.2) and (3.351.4)], \( \hat{J}_2 \) can be asymptotically written
\[ J_1 = \int_{\mathbb{R}^n} f_{X}(z) \sum_{(27a),(27c),(27d),(27e)} f_{Y}(y) \log_2 \left(1 + \eta P_{xy} \right) dxdydz. \] (A.1)

\[ J_{11} = \frac{\ln(1 + \tau_0)}{\lambda_1 \ln(2)} \int_{\mathbb{R}^n} e^{-z_{12} - \eta \tau_0 z_{12}} dz - \frac{1}{\lambda_1 \ln(2)} \int_{\mathbb{R}^n} e^{-z_{12} + \frac{1}{z_{12}} \eta \tau_0 z_{12}} Ei \left(-1 + \frac{\tau_0}{\lambda_2 \eta P_{xy}}\right) dx, \]

\[ J_{12} = -\ln \left(\frac{\tau_0}{\lambda_1 \lambda_2 \eta P_{xy}}\right) \int_{\mathbb{R}^n} e^{-z_{12} - \eta \tau_0 z_{12}} dz - \frac{1}{\lambda_1 \ln(2)} \int_{\mathbb{R}^n} e^{-z_{12} + \frac{1}{z_{12}} \eta \tau_0 z_{12}} Ei \left(-\frac{z}{\eta \tau_0 \lambda_2 B} \right) dx. \] (A.2)

As can be observed by (A.8), as \( P_s \rightarrow \infty \), the scaling law of \( J_1 \left( j_2^\infty \right) \) is log \((P_s)\). By its turn, we have

\[ J_1 \left( j_3^\infty \right) \simeq -\sqrt{\frac{4 \lambda_1 B}{\eta \tau_0 \lambda_2 B}} \int_0^\infty e^{-\frac{4 z}{\eta \tau_0 \lambda_2 B}} K_0 \left(2 \sqrt{\frac{4 z}{\eta \tau_0 \lambda_2 B}}\right) dz \simeq \varpi_1, \]

where \( \varpi_0 \) and \( \varpi_1 \) can be calculated as \( \varpi_0 = \int_0^\infty \sqrt{2 \tau} e^{-\frac{z^2}{2 \tau}} K_0 \left(2 \sqrt{\frac{2 \tau}{\eta \tau_0 \lambda_2 B}}\right) dz \). With the aid of [30, Eqs. (6.614.4)], \( J_1 \left( j_3^\infty \right) \) can be further asymptotically expressed as

\[ J_1 \left( j_3^\infty \right) \simeq -\sqrt{\frac{4 \lambda_1 B}{\eta \tau_0 \lambda_2 B}} \int_0^\infty e^{-\frac{4 z}{\eta \tau_0 \lambda_2 B}} K_0 \left(\sqrt{\frac{4 z}{\eta \tau_0 \lambda_2 B}}\right) dz \times W_{-\frac{1}{2},0} \left(\frac{\lambda_1 B}{\eta \tau_0 \lambda_2 B}\right), \] (A.10)

which is a constant irrelevant to \( P_s \). Summarizing the foregoing results, we can arrive at the scaling behavior of \( J_1 \), which completes the proof.

**A-2: Asymptotic Behavior of \( S_2 \)**

Firstly, we rewrite \( S_2 \) (A.11), shown at the top of the next page. To proceed, with the aid of [30, Eq. (4.331.2)], one can attain (A.12), shown at the top of the next page.

By utilizing the change of variables \( t = \eta \tau_0 \lambda_2 B \) and \( u = \eta \tau_0 \lambda_2 B \), it follows from [30, Eqs. (6.224.1), (4.331.1), and (4.337.2)] that a reformulated expression of \( S_{221} \) is given as (A.13) shown in the next page, where \( \varpi_8 \) is

\[ \varpi_8 \triangleq \int_{\mathbb{R}^n} e^{-\frac{4 z}{\eta \tau_0 \lambda_2 B}} \left[ K_0 \left(2 \sqrt{\frac{4 z}{\eta \tau_0 \lambda_2 B}}\right) dz \right. \]

and

\[ \varpi_9 \triangleq \int_{\mathbb{R}^n} e^{-\frac{4 z}{\eta \tau_0 \lambda_2 B}} \left[ K_0 \left(\sqrt{\frac{4 z}{\eta \tau_0 \lambda_2 B}}\right) dz \right. \].

Utilizing [30, Eqs. (3.532.2), (3.531.4), (3.214.1), and (4.331.1)], we have \( S_{221} \simeq -\frac{\lambda_1 B}{\eta \tau_0 \lambda_2 B} K_0 \left(\sqrt{\frac{4 z}{\eta \tau_0 \lambda_2 B}}\right) - \lambda_1 B \left( C_{Euler} + \ln \left(\lambda_2 B \right) \right) - \lambda_1 B \left( C_{Euler} + \ln \left(\lambda_2 B \right) \right) + \lambda_1 B \left( C_{Euler} + \ln \left(\lambda_2 B \right) \right) \]
\[
S_2 = \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} f_Z(z) \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} f_X(x) \log_2 \left( x \left( \frac{P_z z}{\tau_p} - 1 \right) \right) \left[ e^{-\frac{\tau_0}{\tau_2 B \eta P_s x}} - e^{-\frac{\tau_0}{\tau_2 B \eta P_s}} \right] dxdz
\]

\[
S_2 = \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} f_Z(z) \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} f_X(x) \frac{\lambda_p}{\lambda_p \ln(2)} \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} \left( \frac{P_z z}{\tau_p} - 1 \right) e^{-\frac{\tau_0}{\tau_2 B \eta P_s x}} dxdz.
\]

\[
S_{22} = \frac{1}{(\beta_2^2)^{\frac{1}{2}}} \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} f_Z(z) \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} f_X(x) \lambda_p \ln(2) \left[ \lambda_2 B Ei \left( -x \lambda_2 B \eta P_s x \right) - \lambda_2 B \ln \left( \frac{x}{\tau_0} \left( \frac{P_z z}{\tau_p} - 1 \right) \right) e^{-\frac{\tau_0}{\gamma B \eta P_s x}} \right] dxdz.
\]

\[
S_{22} = \frac{1}{\lambda_1 B \ln(2)} \left[ \lambda_2 B \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} f_Z(z) \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} f_X(x) \frac{\lambda_p}{\lambda_p \ln(2)} \left( \frac{P_z z}{\tau_p} - 1 \right) e^{-\frac{\tau_0}{\gamma B \eta P_s x}} dxdz \right.
\]

\[
- \frac{1}{\lambda_1 B \ln(2)} \left( C_{Euler} + \ln \left( \frac{1}{\eta \lambda_2 B} \right) \right) \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} f_Z(z) \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} f_X(x) \frac{\lambda_p}{\lambda_p \ln(2)} \left( \frac{P_z z}{\tau_p} - 1 \right) e^{-\frac{\tau_0}{\gamma B \eta P_s x}} dxdz.
\]

\[
S_{22} \approx -\frac{\tau_0}{\lambda_1 B \ln(2)} \left[ \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} e^{-\frac{\tau_0}{\tau_2^{\gamma B \eta P_s x}} \ln \left( t + \frac{\lambda_2 B \eta P_s}{\lambda_1 B} \right) dt - \int_{\frac{\tau_0 (\gamma + 1)}{\tau_p}}^{\infty} e^{-\frac{\tau_0}{\tau_2^{\gamma B \eta P_s x}} \ln(t) dt} \right] dxdz \]
...where it follows from [30, Eq. (4.311.2)] that \( \sigma \) is given by

\[
\sigma = \frac{1}{\ln(2) \ln(2)} \int_{0}^{\frac{z}{\tau p}} e^{-\frac{z}{\tau p} 2\beta x} \left[ \ln \left( \frac{z}{\tau p} \right) + \ln \left( \frac{x}{\pi} \right) \right] dx.
\]

(A.16)

where it follows from [30, Eq. (4.311.2)] that \( \sigma \) is given by

\[
\sigma = \frac{1}{\ln(2) \ln(2)} \int_{0}^{\frac{z}{\tau p}} e^{-\frac{z}{\tau p} 2\beta x} \left[ \ln \left( \frac{z}{\tau p} \right) + \ln \left( \frac{x}{\pi} \right) \right] dx.
\]

(A.16)

As a result, at high SNR, by inserting (A.17) into (43) and then by using [30, Eq. (6.6.1.4)], we complete the proof.

A-4: Asymptotic Behavior of \( L_{2} \)

To begin with, \( L_{2} \) can be rewritten as (A.18), shown at the top of the next page. After some algebraic manipulations and

invoking [30, Eq. (4.311.2)], we have

\[
L_{2} = \ln \left( \frac{\pi \tau}{\tau p} \right) \int_{0}^{\infty} \left\{ e^{-\frac{z}{\tau p} 2\beta x} - e^{-\frac{\pi}{\tau p} x} \right\} dx.
\]

(A.19)

By utilizing the change of variables \( u = \frac{z}{\tau p} \) and [30, Eq. (4.311.2)], we have

\[
L_{2} = \lambda_{2} B \ln \left( \frac{\pi \tau}{\tau p} \right) \left\{ e^{-\frac{\pi}{\tau p} x} + e^{-\frac{\pi}{\tau p} x} \right\} dx.
\]

(A.19)

Recall that \( S_{22} = S_{21} + S_{22} \sim S_{22} \) at high SNR. As a result, an asymptotic expression of \( S_{2} \) can be attained.

A-3: Proof of Lemma 3

At high SNR, by utilizing the change of variables \( u = \eta x \) and \( u = \eta x \), it follows from [30, Eq. (4.311.1)] that

\[

e_{\eta} \simeq -C_{Euler} + \ln \left( \lambda_{1} \eta x \right) + \ln \left( \ln(2) \right) - \tau_{p} \lambda_{2} B \ln(2) \eta_{x} + \int_{0}^{\infty} e^{-\frac{z}{\tau p} 2\beta x} \left[ \ln \left( \frac{z}{\tau p} \right) + \ln \left( \frac{x}{\pi} \right) \right] dx.
\]

(A.16)

where it follows from [30, Eq. (4.311.2)] that \( \eta \) is given by

\[
\eta = \frac{1}{\ln(2) \ln(2)} \int_{0}^{\frac{z}{\tau p}} e^{-\frac{z}{\tau p} 2\beta x} \left[ \ln \left( \frac{z}{\tau p} \right) + \ln \left( \frac{x}{\pi} \right) \right] dx.
\]

(A.16)

As a result, at high SNR, by inserting (A.17) into (43) and then by using [30, Eq. (6.6.1.4)], we complete the proof.

Proof: Please refer to Appendix A-5.

Similarly, for \( M_{14} \), by exchanging the integration order and utilizing the change of variables \( u = \frac{x}{\pi \tau p} - \frac{1}{\pi \tau p} \), \( u = \frac{x}{\pi \tau p} \), we have

\[
\varpi_{10} \propto \log P_{s}, \varpi_{11} \propto \log P_{s}.
\]

(A.21)
\[ L_2 \overset{\approx}{=} \int_0^\infty f_Z(z) \int_0^\infty f_X(x) \left\{ \frac{\pi_y}{\pi_p} \int_{\pi_p}^{\pi_y} \frac{e^{-\frac{t}{\pi_p}}}{\sqrt{\pi_p}} \right\} f_Y(y) \log_2 \left( \frac{P_s x z}{\tau_p y} \right) dy dx dz. \]  

(A.18)

\[
L_2 (M_{11}) = \int_0^\infty f_Z(z) M_{11} dz \\
\overset{\approx}{=} \frac{1}{\ln(2)} \left( \sqrt{\frac{\tau_0}{\eta P_s}} \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) \right) - \frac{1}{\ln(2)} \lambda_1^2 \ln(2) \int_0^\infty e^{-\frac{\tau_p}{\eta P_s}} \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) dx \\
+ \frac{\lambda_1^2}{\ln(2)} \int_0^\infty e^{-\frac{\tau_p}{\eta P_s}} \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) dx, 
\]

(A.20)

and [30, Eqs. (4.331.1) and (3.352.2)], we have

\[
L_2 (M_{14}) \overset{\approx}{=} \frac{C_{\text{Euler}}}{\ln(2)} + \frac{\lambda_1^2 C_{\text{Euler}} e^{\frac{\tau_p}{\lambda_1^2}}} {\lambda_1^2 \ln(2)} + \frac{\lambda_1^2}{\lambda_1^2 \ln(2)} - \frac{1}{\lambda_1^2 \ln(2)} \lambda_1^2 \ln(2) \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) \\
+ \frac{\lambda_1^2}{\ln(2)} \int_0^\infty e^{-\frac{\tau_p}{\eta P_s}} \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) dx, 
\]

(A.22)

where \( \omega_{12} = \int_0^\infty e^{-\frac{\tau_p}{\eta P_s}} \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) dx \), \( \omega_{13} = \int_0^\infty e^{-\frac{\tau_p}{\eta P_s}} \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) dx \), and \( \Theta \) is given by

\[
\Theta = \frac{1}{\ln(2)} \left( C_{\text{Euler}} + \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) \right) \propto \log P_s. 
\]

For \( R_1 \), we have

\[
R_1 \overset{\approx}{=} -\frac{\lambda_1^2 \lambda_2 B \eta \tau_p}{\ln(2)} - \frac{\lambda_1^2 \lambda_2 B \eta \tau_p}{\ln(2)} \propto \log(P_s) 
\]

(A.25)

For \( R_2 \), we have

\[
R_2 \overset{\approx}{=} \frac{\lambda_2 B \eta \tau_p}{\ln(2)} \left[ C_{\text{Euler}} + \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) \right] 
\]

(A.26)

For \( R_3 \), we have

\[
R_3 \overset{\approx}{=} \frac{\lambda_2 B \eta \tau_p}{\ln(2)} \left[ C_{\text{Euler}} + \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) \right] 
\]

(A.27)

where we have \( \omega_{16} \propto \log(P_s) \). The proof of the scaling law of \( \omega_{16} \) is provided in Appendix A-7. By its turn, we have

\[
\omega_{16} \propto \log(P_s) \propto \log(P_s) \]

(A.26)

For \( M_{13} \), utilizing the change of variables \( u = x \lambda_2 \eta \tau_p \), one can attain

\[
M_{13} \overset{\approx}{=} \int_0^\infty e^{-\frac{\tau_p}{\eta P_s}} \ln \left( \frac{\eta P_s}{\lambda_1^2} + \frac{1}{x \lambda_2 \eta \tau_p} \right) dx 
\]

(A.27)

\[
-2 K_0 \left( \frac{\eta P_s}{\lambda_1^2 \lambda_2 \eta} \right) \right) 
\]

Next, for \( L_2 (M_{13}) \), making use of the change of variables \( u = x \lambda_2 \eta \tau_p \) and [30, Eq. (6.14.4)],
\[ L_2 (M_{12}) \simeq \]
\[- \frac{1}{\lambda_1 B \ln (2)} \int_{\tau_p}^{\infty} f_{\tau_p} (z) \ln \left( \frac{\eta P_{\tau_p}}{\tau_p} \right) e^{-\sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \left( \frac{\tau_p}{\lambda_2 B \tau_0} - 1 \right)} \, dz - \frac{\sqrt{\frac{\eta P_{\tau_p}}{\tau_p}}}{\lambda_1 B \ln (2)} \int_{\tau_p}^{\infty} f_{\tau_p} (z) \ln \left( \sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \right) e^{-\sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \left( \frac{\eta P_{\tau_p}}{\tau_p} + \frac{1}{\lambda_2 B \tau_0} \left( \frac{\tau_p}{\tau_p} - 1 \right) \right)} \, dz \]
\[ + \frac{\sqrt{\frac{\eta P_{\tau_p}}{\tau_p}}}{\lambda_1 B \ln (2)} \int_{\tau_p}^{\infty} f_{\tau_p} (z) \left( - \sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \left( \frac{1}{\lambda_1 B} + \frac{1}{\lambda_2 B \tau_0} \left( \frac{\tau_p}{\tau_p} - 1 \right) \right) \right) \, dz, \]  
(A.23)

we have
\[ L_2 (M_{13}) \simeq - \frac{e^{\frac{\tau_p}{\lambda_2 B \tau_0}} \lambda_2 B \tau_0}{\ln (2)} \times \sqrt{\frac{\eta P_{\tau_p} \lambda_1 B \lambda_2 B}{\lambda_1 \lambda_2}} W_{-0.5,0} \left( \frac{\lambda_2 B \tau_0}{\eta P_{\tau_p} \lambda_1 B \lambda_2 B} \right). \] (A.28)

For \( M_{15}, \) by utilizing the change of variables \( u = \frac{x}{\sqrt{\eta P_{\tau_p}}} \) and using [30, Eq. (4.331.2)], we have \( M_{15} \simeq \)
\[ \frac{1}{1 + \frac{1}{\lambda_2 B \tau_0} \left( \frac{\tau_p}{\tau_p} - 1 \right)} \text{Ei} \left( - \sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \frac{1}{\lambda_1 B} + \frac{1}{\lambda_2 B \tau_0} \left( \frac{\tau_p}{\tau_p} - 1 \right) \right) + \]
\[ \ln \left( \frac{\tau_p}{\lambda_2 B \tau_0} \right) e^{-\sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \left( \frac{\tau_p}{\lambda_1 B} \right)} - \sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \lambda_2 B \tau_0 \]
\[ \times \text{Ei} \left( - \sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \frac{1}{\lambda_1 B} + \frac{1}{\lambda_2 B \tau_0} \left( \frac{\tau_p}{\tau_p} - 1 \right) \right) \]  
\[ + \frac{1}{\lambda_2 B \tau_0} \tau_p \ln \left( \frac{\tau_p}{\lambda_1 B} \right) e^{-\sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \left( \frac{\tau_p}{\lambda_2 B \tau_0} \right)} Q_1\]
\[ + \frac{\tau_p}{\lambda_2 B \tau_0} \ln \left( \frac{\tau_p}{\lambda_1 B} \right) e^{-\sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \left( \frac{\tau_p}{\lambda_2 B \tau_0} \right)} Q_2, \] (A.29)

where we have \( Q_1 = - \frac{\lambda_2 B \tau_0}{\lambda_1 B} e^{-\sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \left( \frac{\tau_p}{\lambda_2 B \tau_0} - 1 \right)} \)
\[ \times \text{Ei} \left( - \sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \frac{1}{\lambda_1 B} + \frac{1}{\lambda_2 B \tau_0} \left( \frac{\tau_p}{\tau_p} - 1 \right) \right), \] (A.30)
\[ Q_2 = \frac{\lambda_2 B \tau_0}{\lambda_1 B} \text{Ei} \left( - \sqrt{\frac{\eta P_{\tau_p}}{\tau_p}} \frac{1}{\lambda_1 B} + \frac{1}{\lambda_2 B \tau_0} \left( \frac{\tau_p}{\tau_p} - 1 \right) \right), \] (A.31)

in which we have defined \( \text{Ei} = \int_{\infty}^{\tau_p} e^{-x} \left( \frac{1}{\lambda_1 B} + \frac{1}{\lambda_2 B \tau_0} \left( \frac{\tau_p}{\tau_p} - 1 \right) \right) \, dx \).

Next, we focus on the asymptotic behavior of \( Q_1 \) and \( Q_2 \). Firstly, \( Q_1 \) can be expressed as
\[ Q_1 \simeq - \frac{\lambda_2 B \tau_0}{\lambda_1 B} \left( C_{\text{Euler}} + \ln \left( \frac{\tau_p}{\lambda_1 B} \right) + \frac{\tau_p}{\lambda_2 B \tau_0} \right) \]
\[ \text{Log}_P. \] Next, based on the derivation given in Appendix A-8, at high SNR, we have \( \text{Ei} \simeq \text{Log}_P \). As a result, we can arrive at \( L_2 (M_{15}) \simeq \text{Log}_P \).

Finally, let us focus on \( M_{16} \). At high SNR, by making use of [30, Eq. (6.224.1)], \( M_{16} \) can be asymptotically written as \( M_{16} \simeq - \ln \left( 1 + \frac{1}{\lambda_1 B} \frac{\lambda_2 B \tau_0}{\tau_p} \right) \). By its turn, we can attain \( L_2 (M_{16}) \simeq \lim_{u \to 0} \ln \left( 1 + \frac{\lambda_2 B \tau_0}{\lambda_1 B} \right) / \ln (2) + \lim_{u \to 0} \left( C_{\text{Euler}} + \ln \left( \frac{\tau_p}{\lambda_1 B} \right) \right) / \ln (2) \).

To summarize, \( L_2 \) can be asymptotically written as \( L_2 \simeq L_2 (M_{11}) + L_2 (M_{13}) + L_2 (M_{14}) \simeq \text{Log}_P \). This completes the derivations.

**A-5: Asymptotic Behavior of \( \varpi_{10} \) and \( \varpi_{11} \)**

With the aid of [30, Eq. (8.214.1)], \( \varpi_{10} \) can be asymptotically expressed as
\[ \varpi_{10} \simeq C_{\text{Euler}} \lambda_1 B + \ln \left( \frac{\tau_p}{P_{\tau_p}} \right) \lambda_1 B \]
\[ + \int_{\infty}^{\tau_p} e^{-\frac{\tau_p}{\lambda_1 B}} \left[ \frac{1}{\lambda_1 B} + \frac{1}{\lambda_2 B \eta P_{\tau_p}} \right] \, dx \simeq \text{Log}_P, \] (A.32)

where \( \varpi_{19} \) approaches to a constant in the high SNR regions. For \( \varpi_{11} \), it follows from [30, Eqs. (8.214.1) and (3.352.2)] that \( \varpi_{11} \simeq \int_{\infty}^{\tau_p} e^{-\frac{\tau_p}{\lambda_1 B}} \left( C_{\text{Euler}} + \ln \left( \frac{\tau_p}{P_{\tau_p}} \right) + \frac{1}{\lambda_1 B} \frac{\lambda_2 B \tau_0}{\tau_p} \right) \, dx \simeq \text{Log}_P \).

**A-6: Asymptotic Analysis of \( \varpi_{14} \)**

Firstly, \( \varpi_{14} \) can be asymptotically expressed as \( \varpi_{14} \simeq \int_{\infty}^{\tau_p} e^{-\frac{x}{\lambda_1 B}} \left( \frac{1}{\lambda_1 B} + \frac{1}{x \lambda_2 B \eta P_{\tau_p}} \right) \, dx \).

By defining \( t = \frac{x}{\lambda_1 B} \), we
have \( \omega_{14} \simeq \int_{0}^{\infty} \left\lfloor \frac{\ln(t)+1}{\omega_{p}} \right\rfloor e^{-\frac{t}{\omega_{p}}} t^{\lambda_{12}\ln(2)} \int_{t}^{\infty} e^{-\frac{x}{\omega_{p}}} \left( \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}t^{\tau_{p}}}x} \right) dx \right\rfloor dt. \)

The aid of [30, Eqs. (3.381.6) and (8.211.1)], we have \( \omega_{14} \simeq \int_{0}^{\infty} \left\lfloor \frac{\ln(t)+1}{\omega_{p}} \right\rfloor e^{-\frac{t}{\omega_{p}}} t^{\lambda_{12}\ln(2)} \int_{t}^{\infty} e^{-\frac{x}{\omega_{p}}} \left( \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}t^{\tau_{p}}}x} \right) dx \right\rfloor dt. \)

Next, by invoking [30, Eq. (8.214.1)] and after some algebraic manipulations, we can complete the derivation.

\section*{A.7: Asymptotic Behavior of \( \omega_{16} \)}

To begin with, one can arrive at

\[ \omega_{16} = \int_{0}^{\infty} e^{-\frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}}}x} \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}}x} t^{\lambda_{12}\ln(2)} \left( \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}}x} \right) dx \simeq \int_{0}^{\infty} e^{-\frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}}x}x} \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}}x} \left( \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}}x} \right) dx \simeq \omega_{18} \]

by invoking the change of variables \( x = u + \frac{\lambda_{12}0}{\lambda_{12}B^{\tau}} \).

Equivalently, \( \omega_{18} \) can be rewritten as

\[ \omega_{18} = -\int_{0}^{\infty} e^{-\frac{\lambda_{12}}{\lambda_{12}B^{\tau}}x} \left( \frac{1}{\lambda_{12}B^{\tau}} \right) dx \]

where \( \omega_{18} \) is a tight lower bound and will be used to characterize the scaling behavior of \( \omega_{18} \) (or equivalently, \( \omega_{16} \)). Making use of the change of variables \( x = \frac{\lambda_{12}0}{\lambda_{12}B^{\tau}} \), we have \( \omega_{18} = \omega_{18}^{LB} \), which completes the proof.

\section*{A.8: Asymptotic Behavior of \( \omega_{17} \)}

For sufficiently high SNR, \( \omega_{17} \) can be asymptotically written as \( \omega_{17} \simeq \int_{0}^{\infty} e^{-\frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}}}x} \left( \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}}} \right) dx \). By using the change of variables \( x = u + \frac{\lambda_{12}0}{\lambda_{12}B^{\tau}} \), it follows that

\[ \omega_{17} \simeq \int_{0}^{\infty} e^{-\frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}}}x} \left( \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}}} \right) dx \simeq \omega_{18} \]

\vspace{1cm}

\section*{A.9: Proof of Proposition 8}

To prove the equivalence of the two asymptotic expressions, we rewrite the last term of (39) as

\[ \phi \simeq -\frac{1}{\lambda_{12}B^{\tau}} \int_{0}^{\infty} e^{-\frac{t}{\omega_{p}}} t^{\lambda_{12}\ln(2)} \left( \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}t^{\tau_{p}}}x} \right) dx + \frac{\lambda_{12}}{\lambda_{12}B^{\tau}} \int_{0}^{\infty} e^{-\frac{t}{\omega_{p}}} t^{\lambda_{12}\ln(2)} \left( \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}t^{\tau_{p}}}x} \right) dx \]

Invoking the change of variables \( t = 1 + \frac{\eta_{p}t^{\lambda_{12}}}{\lambda_{12}B^{\tau}} \) and [30, Eq. (4.331.1)], one can show that

\[ \phi \simeq \frac{\lambda_{12}}{\lambda_{12}B^{\tau}} \int_{0}^{\infty} e^{-\frac{t}{\omega_{p}}} t^{\lambda_{12}\ln(2)} \left( \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}t^{\tau_{p}}}x} \right) dx \]

Making use of [30, Eqs. (4.331.2) and (3.352.2)], we have

\[ \psi \simeq \frac{\lambda_{12}(\lambda_{12}B^{\tau})}{\eta_{p}t^{\lambda_{12}B^{\tau}}} \int_{0}^{\infty} e^{-\frac{t}{\omega_{p}}} t^{\lambda_{12}\ln(2)} \left( \frac{\lambda_{12}}{\lambda_{12}+t^{\lambda_{12}+\tau_{p}t^{\tau_{p}}}x} \right) dx \]

Finally, utilizing the definitions of \( \psi \) and \( \omega_{a,b} \) [30, Eqs. (9.222.1) and (8.211.1)], one can attain

\[ \omega_{17} \simeq \omega_{17}^{LB} \]

which leads to the fact that (A.35) is equal to (A.36). This completes the proof.

\section*{References}


[21] Q. Zhang, Y.-C. Liang, H.-C. Yang, and H. V. Poor, “Mutualistic mechanism in symbiotic radios: When can the primary and secondary transmissions be mutually beneficial?” IEEE Trans. Wireless Commun., Accepted for publication in early access issues.


