Coexistence of Terrestrial and Satellite Networks in the 28 GHz band

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ABSTRACT

As we move towards the sixth generation (6G) of connectivity, satellites have been identified as an indispensable solution to bridge the digital divide. The satellites offer an extensive coverage footprint and can reach the most remote regions with high throughput, fueled by the large bandwidth available in higher frequency bands. As the low earth orbit (LEO) satellites are closer to the earth and therefore have lower latency, we could use a mega-constellation of LEO satellites to complement the terrestrial networks in 6G.

However, the satellite and terrestrial networks may compete for the same spectrum band, thereby being a source of interference for each other. The mmWave bands have attracted the attention of LEO satellite networks and terrestrial mobile operators alike. Specifically, the 28-GHz mmWave band (27.5-29.5 GHz) is licensed to Fixed Satellite Services (FSS) for earth-to-satellite uplink transmissions, while the terrestrial networks will use it for downlink operation. The satellite networks are the primary users of the 28 GHz band, while it is also available for licensing to International Mobile Telecommunication (IMT) networks. In some countries, the 28 GHz band is also used for point-to-multipoint (PMP) wireless backhaul links.

Therefore, in this work, we aim to understand the impact of the earth station uplink transmissions on the terrestrial users, viz., the cellular users, and the backhaul points, and suggest methods to facilitate the coexistence of these networks in the 28 GHz band through exclusion zones.

The average data rate of the networks is derived through stochastic geometry, which results in expressions that are not closed-form. To optimize the data rates of the coexisting networks jointly, we first approximate the coverage probability expressions as closed-form sigmoid curves. This enables us to use gradient descent methods to determine the optimal radii of the exclusion zones.
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Chapter 1

Introduction

As we move towards the sixth generation (6G) of connectivity, satellites have been identified as an indispensable solution to bridge the digital divide [1, 2]. The satellites offer a large coverage footprint [3] and can reach the most remote regions with high throughput, fueled by the large bandwidth available in higher frequency bands.

Among the satellites, the geostationary (GEO) satellites have the largest footprint; however, since they are furthest from the earth, they have long delays and significant Doppler shifts [3], making them unsuitable for low-latency applications of 5G and beyond networks. To circumvent this problem, we could make use of a mega-constellation of low earth orbit (LEO) satellites [4, 5] which have recently gained traction, as evident from the projects by Starlink, OneWeb, and Kepler, to name a few [6].

The push for integrating satellites into the future network architecture to complement the terrestrial networks is reasonable [6], given that connecting the global population through terrestrial networks alone is cost-prohibitive. However, there may be cases where the satellite and terrestrial networks compete for the same spectrum band, thereby being a source of interference for each other. This is an exciting problem, referred to as spectrum coexistence in the literature.

The mmWave bands have attracted the attention of LEO satellite networks and terrestrial mobile operators alike. Specifically, the 28-GHz mmWave band is licensed to Fixed Satellite Services (FSS) for earth-to-satellite uplink transmissions [7], while the terrestrial networks will use it for downlink operation [8]. The satellite networks are the primary users of the 28 GHz band (27.5-29.5 GHz). This
band is vital for commercial satellite connectivity solutions and 5G non-terrestrial networks (NTN) [9]. The 28 GHz band is also available for licensing to International Mobile Telecommunication (IMT) networks in countries like US, Canada, Norway, Australia, Japan, and Singapore [8]. In Latin America, Africa, and some countries in the Middle East, the 28 GHz band is used for point-to-multipoint (PMP) wireless backhaul links [10].

Therefore, in this work, we aim to understand the impact of the earth station (ES) uplink transmissions on the terrestrial users (TUs), viz., the cellular users and the backhaul points (BPs), and suggest methods to facilitate the coexistence of these networks in the 28 GHz band. In Fig. 1.1 we depict the signal and interference among the different networks elements involved in our coexistence study.

![Figure 1.1: Coexistence scenario of interest.](image)

Coexistence of different technologies utilizing the same frequency band has always been a subject of interest for wireless researchers due to the ever increasing demand for spectrum resources. In cognitive radio (CR) [11] research, the focus is on managing the spectrum among various users that have different access rights to a given frequency band. CR networks rely heavily on spectrum sensing
techniques [12] to assess the level of spectrum usage by primary users, which then allows to dynamically allocate [13] the available spectrum among the secondary users, while ensuring that the collective interference from the secondary users does not exceed a certain threshold [14]. In the recent literature on cognitive radio, the spectrum sensing and dynamic spectrum allocation tasks are being solved through machine learning [15] tools such as deep learning [16, 17, 18] and reinforcement learning [19, 20, 21]. An alternative approach to model the interaction of different networks in a CR setup is through game theory [22, 23, 24].

Coexistence can be achieved either by temporal bandwidth slicing or by spatial. In spatial coexistence [25], the goal is to maximize the separation between the users that use the same frequency bands. One way of achieving spatial coexistence is through exclusion zones [22]; the interference sources can be viewed as having a circular exclusion zone around them, the radius of which is referred to as the protection distance. The secondary users within the exclusion zones should not use the common spectrum, as the interference is high and will deteriorate performance. Authors in [26] have studied the coexistence of 5G network with the incumbents in the 28 GHz and 70 GHz bands. They characterized the minimum separation between the ESs and cellular users in the order of a few kilometres, in order to protect the cellular users from the ES interference. In [27], authors find the minimum protection distance between the terrestrial user and earth station by varying the elevation angle of the FSS. They report that for elevation angle below 37°, require a minimum protection distance greater than 100 m, while for elevation angle above 37°, protection distance lower than 100 m is sufficient to maintain a minimum SINR of −8.8 dB. However both these works rely on deterministic setups, not taking into account the spatial randomness of network elements.

In our work, we adopt a stochastic geometry based approach and derive the distribution of the interference caused by the earth stations based on the spatial density of the satellites in the orbit. We then scatter the terrestrial network ele-
ments, such as cellular base stations and backhaul points as independent Poisson point processes, and characterize their signal-to-interference ratio using stochastic geometry which enables us to assess the performance of the networks as we vary the protection distances. Finally, the goal is to find the optimal values of the protection distances that maximizes the data rates of the cellular and backhaul networks jointly.
Chapter 2

System Model

2.1 Satellite Network

There are $N$ satellites in the constellation, at an orbit height of $h$ above the earth’s surface. The distance of the nearest satellite from the Earth Station (ES) transmitter is $r_0$. The elevation angle $\theta_0$ is the angle that the tangent passing through the surface of the earth at the ES transmitter makes with the direction of the nearest satellite (see Fig. 2.1).

![Figure 2.1: Illustration of the geometric model.](image)

The antenna pattern of 28 GHz transmitters [28], as seen in Fig. 2.2, is directional, and the angle that the boresight of the ES antenna makes with the terrestrial users while tracking the satellite changes with time. Therefore, we obtain the distribution of this interference angle which helps us in capturing the interference from the side lobes of the ESs accurately.

We know from [29, Eq. 13] that the contact angle $\varphi_0$ measured from the
centre of the earth follows the distribution:

\[ f_{\varphi_0}(\varphi) = \frac{N}{2} \sin \varphi \exp \left( -\frac{N}{2} (1 - \cos \varphi) \right). \]  (2.1)

Moreover, we can express the elevation angle \( \theta_0 \) as a function of the contact angle \( \varphi_0 \) as follows:

\[ \theta_0 = g(\varphi_0) = \tan^{-1} \left( \cot \varphi_0 - \gamma_1 \csc \varphi_0 \right); \quad \gamma_1 = \frac{r_\oplus}{r_\oplus + h}. \]  (2.2)

Conversely, we can express the contact angle \( \varphi_0 \) in terms of the elevation angle \( \theta_0 \) as:

\[ \varphi_0 = g^{-1}(\theta_0) = \cot^{-1} \left( \gamma_1^2 \tan \theta_0 + \frac{\gamma_1^2 \sec^2 \theta_0 - 1}{\gamma_1^2 - 1} \right). \]  (2.3)

Therefore, the distribution of the elevation angle can be evaluated through transformation of random variable \( \varphi_0 \rightarrow \theta_0 \):

\[ f_{\theta_0}(\theta) = f_{\varphi_0}(g^{-1}(\theta)) \left| \frac{dg^{-1}(\theta)}{d\theta} \right|. \]  (2.4)
We verify the analytical result with Monte-Carlo simulations in Fig. 2.3 establishing that they are in agreement.

![Figure 2.3: Distribution of the elevation angle $\theta_0$, $h = 1000$ km.](image)

![Figure 2.4: Illustration of interference angle $\alpha_0$.](image)

To characterize the interference that the ES transmitters cause the terrestrial users, we need to find the distribution of interference angle $\alpha_0$ (see Fig. 2.4).

**Definition 1.** The angle that the terrestrial user $u$ makes with the boresight of the ES transmitter $s$ is defined as the interference angle $\alpha_0$.

$$\alpha_0 \triangleq \cos^{-1} (\cos \beta \cos \theta_0),$$ (2.5)
where $\beta \sim U(0, 2\pi)$, and $\theta_0$ is the elevation angle of the satellite wrt the ES transmitter.

Lemma 1. The distribution of the interference angle is:

$$f_{\alpha_0}(\alpha) = |\sin \alpha| \int_{|\cos \alpha|}^{1} \frac{f_{\theta_0}(\cos^{-1}(v))}{\pi \sqrt{(1 - v^2)(v^2 - \cos^2 \alpha)}} dv.$$  (2.6)

The interference angle distribution is shown in Fig. 2.5. As the satellite density in the orbit increases, the boresight of the ES transmitter is pointed upwards towards the sky, i.e., $\alpha_0$ is close to $\pi/2$, which means that the TUs are affected primarily by the side lobes of the ES transmitter.

Figure 2.5: Interference angle $\alpha_0$ distribution, $h = 1000$ km.

2.2 Terrestrial Network

The space in $R^2$ is populated by ESs as a Poisson point process (PPP) with density $\lambda_s$, and each ES transmits at a power $p_s$. Similarly, the cellular base stations (BSs), and BPs are distributed as PPP with densities $\lambda_c$ and $\lambda_b$ respectively. We denote the PPPs describing the ESs, cellular BSs, and BPs with $\Phi_s$, $\Phi_c$ and $\Phi_b$ respectively. Moreover, the BSs and BPs have transmit powers $p_c$ and $p_b$ respectively. For wireless propagation of signal, we use Nakagami-$m$ fading
model, where \( m \) is the shape factor and \( \zeta > 2 \) is the path loss exponent. Since the ES transmitter antenna pattern is directional \([28]\), the gain at an angle \( \alpha \) measured from the main lobe is \( G(\alpha) \). For the BP antenna, the main lobe has a beam width of \( 2\Theta \), and gain \( G_1 \), while the side lobe has gain \( G_0 \) (see Fig. 2.6).

![Figure 2.6: Simplified antenna pattern for backhaul point transmitters.](image)

In Fig. 2.7 we depict the different sources of interference for each terrestrial user, and formalize the same in the definitions that follow.

![Figure 2.7: Signal and Interference for TUs.](image)

**Definition 2.** The signal-to-interference ratio (SIR) experienced by a typical cellular user at the origin, connected to the nearest cellular BS at \( x_c^0 \in \Phi_c \) is:

\[
\text{SIR}_c \triangleq \frac{p_c H_{x_c} ||x_c^0||^{-\zeta}}{I_s + I_b},
\]  

(2.7)
where $I_c^0$, $I_s$, and $I_b$ denote the interference from the cellular BSs, ESs and BPs respectively.

**Definition 3.** The signal-to-interference ratio (SIR) experienced by a typical backhaul point at the origin, connected to the nearest BP at $x_b^0 \in \Phi_b$ is:

$$\text{SIR}_b \triangleq \frac{p_b G_1 H \|x_b^0\|^{-\zeta}}{I_c + I_s + I_b^0},$$

(2.8)

where $I_c$, $I_s$, and $I_b^0$ denote the interference from the cellular BSs, ESs and BPs respectively.
Chapter 3

Coverage Analysis

Stochastic Geometry is a valuable framework for analyzing wireless networks to derive mathematical performance insights [30]. It can capture the randomness in the network in terms of the spatial distribution of network elements as well as communication parameters such as path loss, fading, and power control [31, 32].

In order to determine coverage probability, we first need to characterize the interference from various sources, described by a random point process Φ. One way to do this, is through Laplace transform of the Interference $I$, defined as:

$$\mathcal{L}_I(s) \triangleq \mathbb{E}_{\Phi} [e^{-sI}], \quad (3.1)$$

which proves to be a useful tool in the mathematical analysis of networks. For example, to find the expectation of the interference $\mathbb{E}[I]$ we can write:

$$\mathbb{E}_{\Phi}[I] = -\frac{\partial}{\partial s} \mathcal{L}_I(s) \bigg|_{s=0}. \quad (3.2)$$

3.1 Laplace Transform of the Interference

The Laplace transform of the interference from various sources is presented in the Lemmas that follow, with the proofs deferred to Appendix A.

Lemma 2. The Laplace transform of the interference from the earth station transmitters described by a PPP of density $\lambda_s$ is expressed as:

$$\mathcal{L}_{I_s}(s, v, \lambda_s) = \int_0^\pi \exp \left( -2\pi \lambda_s \int_v^\infty \left\{ 1 - \left( 1 + \frac{G(\alpha)p_s x^{-\xi}}{m s} \right)^{-m} \right\} x \, dx \right) f_{\alpha_0}(\alpha) \, d\alpha. \quad (3.3)$$
**Lemma 3.** The Laplace transform of the interference from the backhaul points described by a PPP of density $\lambda_b$ is expressed as:

$$
\mathcal{L}_b(s, v, \lambda_b) = \exp \left( \mathbb{E}_{G \sim f_G} \left[ -2\pi \lambda_b \int_{v}^{\infty} \left\{ 1 - \left( 1 + \frac{Gp_b x^{-\zeta}}{m} s \right)^{-m} \right\} x \, dx \right] \right),
$$

(3.4)

where $f_G(G_1) = \frac{\Theta}{\pi}$ and $f_G(G_0) = 1 - \frac{\Theta}{\pi}$.

**Lemma 4.** The Laplace transform of the interference from the cellular BSs described by a PPP of density $\lambda_c$ is expressed as:

$$
\mathcal{L}_c(s, v, \lambda_c) = \exp \left( -2\pi \lambda_c \int_{v}^{\infty} \left\{ 1 - \left( 1 + \frac{p_c x^{-\zeta}}{m} s \right)^{-m} \right\} x \, dx \right).
$$

(3.5)

### 3.2 Coverage Probability

Consequently, the coverage probability of the cellular users and backhaul points is expressed in Theorems 1 and 2 respectively. The proof is sketched on the same lines as [33, Appendix F], and is therefore skipped.

**Theorem 1.** The coverage probability of the cellular users $\mathbb{P}(\text{SIR}_c > \tau)$ is:

$$
P_c(\tau) = 2\pi \lambda_c \sum_{k=1}^{m} \binom{m}{k} (-1)^{k+1} \int_{0}^{\infty} \exp \left( -\pi \lambda_c v^2 \right) \times \mathcal{L}_c(\Omega_c, 0, 0) \mathcal{L}_c(\Omega_c, v, \lambda_c) \mathcal{L}_b(\Omega_c, 0, \lambda_b) \, dv,
$$

(3.6)

where $\beta \triangleq (m!)^{-\frac{1}{m}}$ and $\Omega_c \triangleq \frac{\beta \mu_{\text{pp}} \tau^\zeta}{\mu_{\text{pp}}}$.

**Theorem 2.** The coverage probability of the backhaul points $\mathbb{P}(\text{SIR}_b > \tau)$ is:

$$
P_b(\tau) = 2\pi \lambda_b \sum_{k=1}^{m} \binom{m}{k} (-1)^{k+1} \int_{0}^{\infty} \exp \left( -\pi \lambda_b v^2 \right) \times \mathcal{L}_c(\Omega_b, 0, 0) \mathcal{L}_c(\Omega_b, v, \lambda_c) \mathcal{L}_b(\Omega_b, v, \lambda_b) \, dv,
$$

(3.7)

where $\beta \triangleq (m!)^{-\frac{1}{m}}$ and $\Omega_b \triangleq \frac{\beta \mu_{\text{pp}} \tau^\zeta}{\mu_{\text{pp}} G_1}$.
It can be seen from equations (3.6) and (3.7) that the performance of the terrestrial networks is interdependent on the transmission parameters and deployment density of the terrestrial as well as satellite uplink network. In the following section we introduce the concept of exclusion zones and restate the coverage probabilities as a result of the introduction of such zones, and see how controlling the radius of these exclusion zones can result in performance gains for each terrestrial network involved.
Chapter 4

Exclusion Zone

To protect the TUs from ES transmissions, we create an exclusion zone of radius $R_s$ around ESs within which the TUs do not operate in 28-GHz. Moreover, we encircle the BPs with an exclusion zone of their own, of radius $R_b$ within which the cellular users are not allowed to operate in the 28-GHz band. This is graphically depicted in Fig. 4.1, where the exclusion zones divide the space in $\mathbb{R}^2$ into four regions denoted by Roman numerals I, II, III and IV. The frequency usage by the TUs in these regions is tabulated in Table 4.1.

![Diagram](image)

Figure 4.1: Different regions created as a result of ESs and BPs having exclusion zones.

<table>
<thead>
<tr>
<th>Region</th>
<th>Cellular users</th>
<th>Backhaul points</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>28 GHz + sub-6 GHz</td>
<td>28 GHz + 6 GHz</td>
</tr>
<tr>
<td>II</td>
<td>sub-6 GHz</td>
<td>6 GHz</td>
</tr>
<tr>
<td>III</td>
<td>sub-6 GHz</td>
<td>28 GHz + 6 GHz</td>
</tr>
<tr>
<td>IV</td>
<td>sub-6 GHz</td>
<td>6 GHz</td>
</tr>
</tbody>
</table>

Table 4.1: Frequency usage by the TUs based on the region.

Based on the new frequency usage rules presented in Table 4.1, the density of the PPP describing the cellular BS transmitting in sub-6GHz band is $\lambda_c$, while the cellular BS transmitting in 28-GHz band is described by a thinner PPP of
Similarly, the BPs operating in the 6-GHz is described by a PPP of density $\lambda_b$, however those operating in 28-GHz are described by a thinner PPP of density $\tilde{\lambda}_b$. The thinner PPPs are approximations of Poisson Hole Process (PHP), which is defined formally in the following approximation.

**Approximation 1.** If circular holes of radius $\rho$ are cut out of a PPP of density $\lambda$, at the points that follow a PPP of density $\lambda_0$, then the remaining points result in a Poisson Hole Process (PHP), which can be approximated as a PPP of density $\tilde{\lambda}$, defined as:

$$\tilde{\lambda} = \lambda \exp\left(-\pi \lambda_0 \rho^2\right). \quad (4.1)$$

In Fig. 4.2, we show that the BPs and cellular BSs within the exclusion zones can still transmit in the 28-GHz band. As an approximation, we define $r_c$ and $r_b$ as the average distance of the nearest neighbor in PPPs of density $\lambda_c$ and $\lambda_b$ respectively, which means that on average, a cellular BS can connect to a cellular user $r_c$ distance away. Similarly, a BP connects to a BP which is, on average, $r_b$ distance away. In the figure, this is depicted through gray areas.

![Figure 4.2](image_url)

**Figure 4.2:** BPs and Cellular BSs within exclusion zones which can still transmit in the 28-GHz band.

**Lemma 5.** The average nearest neighbor distance of a PPP of density $\lambda$ is $\frac{1}{2\sqrt{\lambda}}$. 


Proof. The distribution of the nearest neighbor distance of a homogeneous Poisson point process of parameter $\lambda$ is $f_{nn}(r) \triangleq 2\pi \lambda r \exp(-\pi \lambda r^2)$. To determine the average nearest neighbor distance, we take the expectation over $r$ as:

$$\mathbb{E}_{r \sim f_{nn}(r)}[r] = \int_{r \geq 0} r \cdot f_{nn}(r) \, dr = 2\pi \lambda \int_0^\infty r^2 \exp(-\pi \lambda r^2) \, dr = \frac{1}{2\sqrt{\lambda}}, \quad (4.2)$$

which concludes the proof. \qed

The PHP describing the BPs transmitting in 28-GHz band can be seen as what remains after removing the BPs which fall inside circles of radius $R_s - r_b$ centred around each ES, i.e., each point in $\Phi_s$. Through Approximation 1, we can define the PHP as a PPP of density $\tilde{\lambda}_b$ as:

$$\tilde{\lambda}_b \triangleq \lambda_b \exp \left( -\pi \lambda_s (R_s - r_b)^2 \mathbb{1}\{R_s > r_b\} \right); \quad r_b \triangleq \frac{1}{2\sqrt{\lambda_b}} \quad (4.3)$$

Similarly, the PHP describing the cellular BSs transmitting in 28-GHz frequencies can be visualized as the points remaining after removing points from circles of radius $R_s - r_c$ centred at each ES in $\Phi_s$, and circles of radius $R_b - r_c$ centred around each BP in $\Phi_b$. We approximate this PHP as a PPP of density $\tilde{\lambda}_c$, defined as:

$$\tilde{\lambda}_c \triangleq \lambda_c \exp \left( -\pi \lambda_c (R_s - r_c)^2 \mathbb{1}\{R_s > r_c\} - \pi \lambda_b (R_b - r_c)^2 \mathbb{1}\{R_b > r_c\} \right), \quad (4.4)$$

where $r_c \triangleq \frac{1}{2\sqrt{\lambda_c}}$.

### 4.1 Coverage Probability

In the following Lemmas we present the coverage probability of the cellular BSs and the BPs transmitting in different frequency bands. The proof of these Lemmas are similar to those of Theorems 1 and 2, and is therefore skipped.
Lemma 6. The coverage probability of cellular users in the 28-GHz band is:

\[ P_c(\tau) = 2\pi \tilde{\lambda}_c \sum_{k=1}^{m} \left( \frac{m}{k} \right) (-1)^{k+1} \int_{0}^{\infty} \exp \left(-\pi \tilde{\lambda}_c v^2\right) \times L_s(\Omega_c, R_s, \lambda_s) L_c(\Omega_c, v, \tilde{\lambda}_c) L_b(\Omega_b, R_b, \tilde{\lambda}_b) v \, dv. \]  

(4.5)

Lemma 7. The coverage probability of cellular users in the sub 6-GHz band is:

\[ P_{cn}(\tau) = 2\pi \lambda_c \sum_{k=1}^{m} \left( \frac{m}{k} \right) (-1)^{k+1} \int_{0}^{\infty} e^{-\pi \lambda_c v^2} L_c(\Omega_c, v, \lambda_c) v \, dv. \]  

(4.6)

Lemma 8. The coverage probability of backhaul points in the 28-GHz band is:

\[ P_b(\tau) = 2\pi \tilde{\lambda}_b \sum_{k=1}^{m} \left( \frac{m}{k} \right) (-1)^{k+1} \int_{0}^{\infty} e^{-\pi \tilde{\lambda}_b v^2} \times L_s(\Omega_b, R_s, \lambda_s) L_c(\Omega_b, 0, \tilde{\lambda}_c) L_b(\Omega_b, v, \tilde{\lambda}_b) v \, dv. \]  

(4.7)

Lemma 9. The coverage probability of backhaul points in the 6-GHz band is:

\[ P_{bn}(\tau) = 2\pi \lambda_b \sum_{k=1}^{m} \left( \frac{m}{k} \right) (-1)^{k+1} \int_{0}^{\infty} e^{-\pi \lambda_b v^2} L_b(\Omega_b, v, \lambda_b) v \, dv. \]  

(4.8)

Figure 4.3: Coverage Probability, simulation vs. theory.
4.2 Average Data Rate

Since the users, viz. cellular and backhaul points, are scattered uniformly in the $\mathbb{R}^2$ plane, we need to characterize their data rate as the average of the data rates experienced by the users operating in different frequency bands. We denote the amount of bandwidth offered in 28-GHz, 6-GHz and sub 6-GHz bands with $B$, $B_{bn}$ and $B_{cn}$ respectively.

**Theorem 3.** The average data rate of cellular users is given as:

$$D_c(R_s, R_b, \tau) = e^{-\pi \lambda_s R_s^2 - \pi \tilde{\lambda}_b R_b^2} \left( BP_c(\tau) - B_{cn} P_{cn}(\tau) \right) + B_{cn} P_{cn}(\tau) \log_2 (1 + \tau).$$

(4.9)

**Proof.** The average data rate of the cellular users operating in 28-GHz band is $P_c(\tau)B \log_2 (1 + \tau)$ and of those operating in sub 6-GHz is $P_{cn}(\tau)B_{cn} \log_2 (1 + \tau)$. Moreover, the fraction of cellular users operating in 28-GHz band is $e^{-\pi \lambda_s R_s^2 - \pi \tilde{\lambda}_b R_b^2}$. On taking the average over the fraction of cellular users operating in the 28-GHz and sub 6-GHz frequency we get the expression in (4.9), which concludes the proof.

**Theorem 4.** The average data rate of backhaul points is given as:

$$D_b(R_s, R_b, \tau) = \left( e^{-\pi \lambda_s R_s^2} (BP_b(\tau) - B_{bn} P_{bn}(\tau)) + B_{bn} P_{bn}(\tau) \right) \log_2 (1 + \tau).$$

(4.10)

**Proof.** The proof is similar to that of Theorem 3 and is therefore skipped.
Chapter 5

Data Rate Optimization

5.1 Problem Formulation

The goal of this paper is to improve the performance of the terrestrial users in terms of their data rate, by managing the interference through the adoption of exclusion zones of certain radii. In other words, we aim to find the radii of the exclusion zones which improves the data rate of the cellular users and backhaul points. For this, we define two objective functions. The first one is the sum of data rates \( f_{\Sigma} \triangleq D_c + D_b \), and the other is the product of data rates \( f_{\Pi} \triangleq D_c \cdot D_b \).

**Problem 1.** \( \max_{R_s, R_b} f_{\Sigma}(R_s, R_b, \tau) = D_c + D_b, \quad s.t. \quad R_s \geq 0, R_b \geq 0. \)

**Problem 2.** \( \max_{R_s, R_b} f_{\Pi}(R_s, R_b, \tau) = D_c \cdot D_b, \quad s.t. \quad R_s \geq 0, R_b \geq 0. \)

In Problem 1, we try to maximize the sum of the data rates in which the data rate of one network can be sacrificed to improve the data rate of another network. However, in Problem 2, this is prevented as the product of two opposing terms is maximum only when the two terms are equal. So, the data rate of one network will not be sacrificed to increase the data rate of another, thereby ensuring fairness in data rate.

These problems appear simple. However the only way to solve them directly is through exhaustive search, as the coverage probability expressions \( P_c, P_b, P_{cn}, \) and \( P_{bn} \), are not available in closed-form. To expedite the optimization process, we approximate the coverage probability expressions as sigmoid curves that are function of \( R_s, R_b \) for a given \( \tau \). The approximation technique is described in detail, in the next section.
5.2 Sigmoid Approximation

Consider that we have a coverage probability expression \( p(\tau) \triangleq P(\text{SIR} > \tau) \), where \( \tau \) is the SIR (signal-to-interference ratio) threshold. Often times deriving coverage expressions using stochastic geometry results in complicated expressions involving several integrals which makes it computation-intensive and difficult to analyze the trends wrt the variables of the system model.

This motivates us to come up with a closed-form expression for \( p(\tau, x) \) which depends on several variables denoted by the vector \( x \in \mathcal{V} \subseteq \mathbb{R}^d \). The goal is to be able to use the coverage expression in objective functions which can be optimized through gradient descent methods, in contrast to exhaustive search, thereby cutting down on computation time.

We start with the assumption that the SIR has a logistic distribution \([34]\), which allows us to express the cumulative distribution function (CDF) of SIR, i.e., \( P(\text{SIR} \leq \tau) \) as a sigmoid function. We are interested in determining the coverage probability, which is the complementary CDF of the SIR, i.e., \( p(\tau) \triangleq P(\text{SIR} > \tau) \), which is also a sigmoid function\(^1\). The sigmoid curve is a natural choice to approximate the coverage probability expression as the domain \( \in \mathbb{R} \), and range \( \in [0, 1] \). The function is closed-form and differentiable, which makes calculations easier. For example, authors in \([35]\), use sigmoid curve to approximate the line-of-sight probability CDF as a function of the elevation angle. The CDF2PDF method in \([36]\) employs a shallow neural network with one hidden layer having sigmoid activation functions and one linear output node, for CDF estimation.

Therefore, we approximate \( p(\tau, x) \) as a sigmoid function of \( \tau \), as follows:

\[
p_{1}(\tau, x) = \sigma\left(-w_{1}(x)\tau - w_{0}(x)\right) = \frac{1}{1 + \exp\left(w_{1}(x)\tau + w_{0}(x)\right)}, \tag{5.1}
\]

where \( \tau \) is in dB scale, \( \sigma(x) \) is the sigmoid function, while the weight \( w_{1}(x) \in \mathbb{R} \), and the bias \( w_{0}(x) \in \mathbb{R} \) are functions of \( x \).

\(^{1}\) \( 1 - \sigma(ax + b) = \sigma(-ax - b) \).
First, we find the values of the weight \( w_1(x) \) and bias \( w_0(x) \) through linear regression \([37]\) of \( \log \left( \frac{1}{p_1(\tau, x)} - 1 \right) \) constrained to the dataset such that \( \epsilon < p(\tau, x) < 1 - \epsilon \), where \( \epsilon > 0 \), to avoid extremely large values. The next step involves approximating the weight \( w_1(x) \) and bias \( w_0(x) \) through regression. The resulting regression surface can be described as a function of \( x \), i.e.,

\[
f_w(x, a_k) = a_k + \sum_{i=1}^{d} \sum_{j=1}^{m} a_{ijk} \phi_j(x_i), \tag{5.2}\]

where \( \phi_j(\cdot) \) is the \( j \)th basis function. Therefore, we can express the coverage probability as:

\[
p_2(\tau, x) = \sigma \left( -f_w(x, a_1)\tau - f_w(x, a_0) \right), \tag{5.3}\]

where the parameter \( a_k \) is learnt by feeding the data points \( w_k(x) \). We refer to \( p_1(\tau, x) \) as sigmoid-I fit, and \( p_2(\tau, x) \) as sigmoid-II fit.

In this work, we define the vector of variables as \( x \triangleq [R_s R_b]^T \), and the basis functions as \( \phi_1(x) = x \), and \( \phi_2(x) = x^2 \).

In Fig. 5.1, we show the surfaces \( f_w(x, a_1) \) and \( f_w(x, a_0) \) that fit the data points of weight \( w_1 \) and bias \( w_0 \) of sigmoid-I fit respectively.

![Figure 5.1: Regression surface \( f_w \).](image)

(a) \( w_1(R_s, R_b) \) regression.  
(b) \( w_0(R_s, R_b) \) regression.

In Fig. 5.2, we show how the sigmoid fit curves compare with the simulated
coverage probability curve. We quantify the RMSE between the simulated coverage probability curve with the sigmoid fit curves in Fig. 5.3 establishing the utility of our approach.

Figure 5.2: Comparing sigmoid fit with simulated coverage probability. $R_s = 110 \text{ m}, R_b = 10 \text{ m}$.

Figure 5.3: Cumulative distribution of the RMSE between the simulated coverage probability curve and the sigmoid fits.
5.3 Optimization Results

We first obtain the closed-form expressions of the coverage probability $P(·)$ through sigmoid-II fit denoted by $\hat{P}(·)$. Then the approximate average data rates $\hat{D}(·)$ can be expressed as follows. The average data rate of the cellular users is given by:

$$\hat{D}_c(R_s, R_b, \tau) = \left( e^{-\pi \lambda_s R_s^2 - \pi \lambda_b R_b^2} \left( B\hat{P}_c(\tau) - B_{cn}\hat{P}_{cn}(\tau) \right) + B_{cn}\hat{P}_{cn}(\tau) \right) \log_2(1 + \tau).$$

(5.4)

Similarly, the average data rate of the backhaul points is given by:

$$\hat{D}_b(R_s, R_b, \tau) = \left( e^{-\pi \lambda_s R_s^2} \left( B\hat{P}_b(\tau) - B_{bn}\hat{P}_{bn}(\tau) \right) + B_{bn}\hat{P}_{bn}(\tau) \right) \log_2(1 + \tau).$$

(5.5)

In Fig. 5.4, we compare the data rate curves obtained through simulations with those obtained from the closed-form expressions in equations (5.4) and (5.5), and we observe that the surfaces overlap.

In Fig. 5.5, we show the position of the maxima for objectives $f_\Sigma$ and $f_\Pi$ for different values of bandwidth ratio $B/B_{cn}$. The bandwidth ratio is the ratio between the bandwidth offered by 28-GHz band, and that offered by sub-6 GHz or 6 GHz. We perform the optimization through gradient descent methods.
In Fig. 5.6, we show the improvements in the data rate as the bandwidth ratio increases. We measure the improvement wrt the absence of exclusion zones, i.e., $R_s = 0$ m, and $R_b = 0$ m. We see that the improvement saturates at
∼ 5% for cellular users, as we increase the bandwidth ratio, and for backhaul points, instead of improvement, the data rate worsens by ∼ 1%. When we have similar bandwidths, we see up to 30% improvement for cellular users, and 5% improvement for backhaul points.

Figure 5.6: Data rate improvement vs. bandwidth ratio.
Chapter 6

Concluding Remarks

6.1 Summary

In this work, we minimized the interference caused by the earth-to-satellite transmissions on the terrestrial users through exclusion zones. In practice, this can be deployed using a spectrum access database, where the location of the earth stations, and backhaul points is updated regularly, and based on the current location of the terrestrial users, they are allowed to receive signals in a particular frequency, as per the rules. We quantified the performance of terrestrial users in terms of average data rate by employing tools from stochastic geometry. The theoretical expressions were verified through Monte Carlo simulations, and converted into closed-form expressions through two-step sigmoid approximation. The approximation method presented in this work can be useful for the research community, as any complicated coverage probability curve can be approximated as a sigmoid curve by the use of appropriate basis functions.

6.2 Future directions

The work can be further generalized by considering users that are capable of operating in several frequency bands, and based on the current interference from the surroundings, can choose a frequency band that offers the highest QoS. This would require modelling each user as an agent in a multi-agent game setting.
REFERENCES


A Proof: Laplace Transforms

A.1 Proof of Lemma 2

The interference from the earth stations can be expressed as:

\[ I_s = \sum_{x \in \Phi_s} HG(\alpha)p_s ||x||^{-\zeta}, \]  

(A.1)

where \( H \sim \Gamma(m, \frac{1}{m}) \) because of Nakagami-m fading channel, \( \alpha \sim f_{\alpha_0} \), and \( \Phi_s \) is a PPP of intensity \( \lambda_s \).

Proof. The Laplace transform of \( I_s \) is:

\[
\mathcal{L}_{I_s}(s) = \mathbb{E}_{H, \Phi_s, \alpha} \left[ \exp \left( -s I_s \right) \right],
\]

(A.2)

\[
= \mathbb{E}_{H, \Phi_s, \alpha} \left[ \exp \left( -s \sum_{x \in \Phi_s} HG(\alpha)p_s ||x||^{-\zeta} \right) \right], \quad \text{(A.3)}
\]

\[
= \mathbb{E}_{H, \Phi_s, \alpha} \left[ \prod_{x \in \Phi_s} \exp \left( -s HG(\alpha)p_s ||x||^{-\zeta} \right) \right], \quad \text{(A.4)}
\]

\[
= \mathbb{E}_\alpha \left[ \exp \left( -2\pi \lambda_s \int_{v}^{\infty} \left( 1 - \mathbb{E}_H \left[ \exp \left( -s HG(\alpha)p_s x^{-\zeta} \right) \right] \right) x \, dx \right) \right], \quad \text{(A.5)}
\]

\[
= \int_{0}^{\pi} \exp \left( -2\pi \lambda_s \int_{v}^{\infty} \left\{ 1 - \left( 1 + \frac{G(\alpha)p_s x^{-\zeta}}{m} \right)^{-m} \right\} x \, dx \right) f_{\alpha_0}(\alpha) \, d\alpha,
\]

(A.6)

where step (a) follows from definition of a PGFL [30], and step (b) follows from the MGF of a Gamma distributed random variable [38]. \( \square \)
A.2 Proof of Lemma 3

The interference from the backhaul points can be expressed as:

\[ I_b \triangleq \sum_{x \in \Phi_b} H G p_b \| x \|^{-\zeta}, \tag{A.7} \]

where \( H \sim \Gamma(m, \frac{1}{m}) \) because of Nakagami-m fading channel, \( \Phi_b \) is a PPP of intensity \( \lambda_b \), and \( G \) is a discrete random variable with \( f_G(G_1) = \frac{\Theta}{\pi} \), and \( f_G(G_0) = 1 - \frac{\Theta}{\pi} \).

**Proof.** The Laplace transform of \( I_b \) is:

\[
L_{I_b}(s) = \mathbb{E}_{H, \Phi_b, G} [\exp (-s I_b)] , \tag{A.8} \\
= \mathbb{E}_{H, \Phi_b, G} \left[ \exp \left( -s \sum_{x \in \Phi_b} H G p_b \| x \|^{-\zeta} \right) \right] , \tag{A.9} \\
= \mathbb{E}_{H, \Phi_b, G} \left[ \prod_{x \in \Phi_b} \exp \left( -s H G p_b \| x \|^{-\zeta} \right) \right] , \tag{A.10}
\]

\[
\overset{(a)}{=} \exp \left( \mathbb{E}_G \left[ -2\pi \lambda_b \int_v^\infty \left( 1 - \mathbb{E}_H \left[ \exp \left( -s H G p_b x^{-\zeta} \right) \right] \right) x \, dx \right] \right) , \tag{A.11}
\]

\[
\overset{(b)}{=} \exp \left( \mathbb{E}_G \left[ -2\pi \lambda_s \int_v^\infty \left( 1 - \left( 1 + \frac{G p_b x^{-\zeta}}{m} \right)^{-m} \right) x \, dx \right] \right) , \tag{A.12}
\]

\[
= \exp \left( -2\pi \lambda_s \frac{\Theta}{\pi} \int_v^\infty \left( 1 - \left( 1 + \frac{G_1 p_b x^{-\zeta}}{m} \right)^{-m} \right) x \, dx \right) \times \exp \left( -2\pi \lambda_s \left( 1 - \frac{\Theta}{\pi} \right) \int_v^\infty \left( 1 - \left( 1 + \frac{G_0 p_b x^{-\zeta}}{m} \right)^{-m} \right) x \, dx \right) , \tag{A.13}
\]

where step (a) follows from definition of a PGFL, and step (b) follows from the MGF of a Gamma distributed random variable.

\[ \square \]

**Remark 1.** The proof of Lemma 4 is skipped because it is a similar to the proof of Lemma 3 with \( G = 1 \).
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