Searching the parameter space for resolution and uniqueness in elastic anisotropic waveform inversion: What can redatuming and machine learning offer?

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Summary

Full waveform inversion (FWI) can retrieve high-resolution subsurface medium parameters from the observed data. However, the inverse problem is typically ill-posed and non-unique, especially for the multi-parameter elastic FWI (EFWI) in complex media. Besides, high-resolution EFWI is computationally expensive because it requires fine discretization for the whole computational domain. The redatuming approach allows us retrieve the virtual data at the target level using mainly a kinematically accurate overburden, thus, focusing the high-resolution inversion on the target zone to reduce the computational cost. In multi-parameter inversion, even at the target zone, we will need to utilize a prior information and we do that through deep learning to find the connection between well information and the a prior needed by FWI. In such a framework, we take into consideration the proper parameter makeup for reducing the ill posedness of the problem. Numerical tests on the synthetic SEAM model are used to demonstrate the performance of the proposed inversion scheme, and its robustness in the multi parameter inversion case.
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Introduction

The quantitative estimation of the medium parameters, including anisotropy, from Elastic full waveform inversion (EFWI) allows for better characterization of reservoirs and monitoring their time-lapse changes. Considering that high-resolution inversion for the whole survey area is computationally expensive, the inversion can be oriented to target the zone of interest (e.g., the reservoir or the monitor zone). Redatuming allows us to retrieve the virtual data at the target level by projecting the recorded seismic data from the Earth’s surface to a datum level. A waveform-fitting based redatuming approach has been developed to provide the virtual data at the datum level for target-oriented high-resolution FWI and time-lapse seismic monitoring (Li et al., 2021; Li and Alkhalifah, 2021). It is necessary to account for anisotropy in the elastic redatuming process, especially when the target zone underlie complex media with anisotropic features. Thus, we use an anisotropic elastic wave equation as our modeling operator for the redatuming to account for anisotropy. Once the virtual elastic data for the full band is retrieved by the elastic redatuming, we apply EFWI to recover the elastic properties in the target zone.

To make use of the detailed measurements of media parameters from wells, we will employ a deep learning (DL) algorithm to map the well information to the target inversion area. The derived prior model can be used to regularize the inversion process. The DL-assisted regularization approach will be applied using target-oriented time-lapse EFWI to show its performance.

Theory

We account for anisotropy of seismic wave propagation in the elastic anisotropic overburden in the redatuming process to allow the retrieved virtual data to appropriately represent the reflection properties in the underlying. The redatuming is an inverse problem that retrieves the virtual elastic data at the datum level from surface seismic data \( \mathbf{d} \), its corresponding objective function is given by:

\[
\min_R J = \frac{1}{2} \sum_s \left\| \Gamma(m_o) + u^D(m_o, R) - d_s \right\|_2^2, \tag{1}
\]

where we use the projection operator \( \Gamma \) to map the simulated elastic wavefield, \( \mathbf{u} \), in the overburden model \( m_o \), and the wavefield, \( u^D \), generated by the datum-based modeling operator:

\[
F u^D_i(x, x_s, t) = \int_{S_D} R^\phi(x_{y}, x_{s}, t; x_{y}, x_{s}, t) u_p(x_{y}, x_{s}, t) dx_{y}, \tag{2}
\]

where, \( F \) represents the anisotropic elastic wave equation operator, \( x_{y} \) and \( x_{s} \) refer to the virtual receiver and source, \( S_D \) refers to the surface at the datum level, \( u_p \) is the downgoing pressure wavefield from surface to datum, and \( R^\phi \) is the Green’s function to be solved for a P-wave source. A gradient-based optimization method is applied to retrieve \( R^\phi \), and the gradient is expressed as:

\[
\frac{\partial J}{\partial R^\phi_{i,p}}(x_{s} + h, x_{s}, t) = -\sum_j \left( u_p(x_{y}, x_{s}, t), u^\phi_j(x_{y}, x_{s}, t) \right) _i. \tag{3}
\]

where the wavefield \( u^\phi_j \) is simulated by back-propagating the data residuals at the receivers. The retrieved virtual data at the datum level provide input data for target-oriented inversion.

Assume that well data provide direct measurements of subsurface property in the target zone, this prior knowledge can be introduced to regularize the inverse problem. The regularized objective function is:

\[
J(m) = J_d(m) + \beta J_m(m; m_{pri}), \tag{4}
\]

where \( \beta \) is used to balance the contributions from the data \( J_d \) and the model \( J_m \) terms. To build the prior model \( m_{pri} \), we first learn the mapping relationship between seismic estimation and the well information from the model samples at well locations by training a deep neural network (DNN). Then, the well information can be mapped to the inversion zone to derive the prior model by using the trained network.
Examples

We apply the target-oriented inversion with elastic anisotropic redatuming on a 2D VTI elastic SEAM model. Using the smoothed overburden, we retrieve the virtual elastic data from the observed data by employing the redatuming. We then implement a target-oriented EFWI to recover the elastic properties in the target zone by using the redatumed data. Starting from the target model in Figures 1a and 1f, \( v_p \) and \( v_s \) are estimated with improved resolution as shown in Figures 1b and 1g. Considering that it is almost impossible to estimate \( \delta \) without a well, we design a scenario where \( v_{nmo}, \eta \) and \( v_s \) are known while \( \delta = 0 \). In this case, the inverted model, in Figures 1c and 1i, reflects a shift to deeper depths, a common occurrence in seismic-well misties. If we had well information down to the datum level, we can use the ratio of the average NMO to vertical velocity to correct for the misties and obtain the inverted model in Figures 1d and 1j. A closer look through the vertical profiles in Figures 1f and 1l for the \( v_p \) and \( v_s \) demonstrate the improved matching after this correction. Thus, by ignoring \( \delta \) and then using a well to correct for the misties, we can obtain a reasonably accurate result.

![Figure 1](image1)

**Figure 1** (a)(g) Initial \( v_p \) and \( v_s \) for target-oriented EFWI; (b)(h) the recovered \( v_p \) and \( v_s \) with anisotropy considered in redatuming; (c)(i) the recovered \( v_p \) and \( v_s \) with overburden \( \delta = 0 \) for redatuming; (d)(j) True \( v_p \) and \( v_s \); (e)(k) the corrected \( v_p \) and \( v_s \) with a available well; (f)(l) \( v_p \) and \( v_s \) profiles at the distance 7 km.

We also test how deep learning help target-oriented time-lapse (TL) EFWI by using a 2D SEAM time-lapse model. Figures 2a and 2d show the inverted time-lapse changes of \( v_p \) and \( v_s \) in the target zone by using the redatumed data. Three wells pointed by white lines in Figure 2c are assumed available. Figures 2b and 2e show the time-lapse inversion result by introducing the prior model predicted by DL. Figures 2c and 2f are the true time-lapse changes. We can see that the inversion performance is improved by using the DL-assisted regularization.

![Figure 2](image2)

**Figure 2** Target-oriented time-lapse EFWI result (a) \( v_p \) and (d) \( v_s \), the inverted (b) \( v_p \) and (e) \( v_s \) changes by using DL-assisted regularization, and true time-lapse changes of (c) \( v_p \) and (f) \( v_s \) in the target zone.

References
