BER Reduction Using Partial-Elements Selection in IRS-UAV Communications with Imperfect Phase Compensation

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Abstract—This work considers minimizing the communications bit error rate (BER) of unmanned aerial vehicle (UAV) when assisted by intelligent reflecting surfaces (IRSs). By noting that increasing the number of IRS elements in the presence of phase errors does not necessarily improve the system’s BER, it is crucial to use only the elements that contribute to reducing such a parameter. To this end, we propose an efficient algorithm to select the elements that can improve BER. The proposed algorithm has lower complexity and comparable BER to the optimum selection process which is an NP-hard problem. The accuracy of the estimated phase is evaluated by deriving the probability distribution function (PDF) of the least-square (LS) channel estimator, and showing that the PDF can be closely approximated by the von Mises distribution at high signal-to-noise ratios (SNRs). The obtained analytical and simulation results show that using all the available reflectors can significantly deteriorate the BER, and thus, partial element selection is necessary. It is shown that, in some scenarios, using about 26% of the reflectors provides more than 10 fold BER reduction. The number of selected reflectors may drop to only 10% of the total elements. As such, the unassigned 90% of the elements can be allocated to serve other users, and the overhead associated with phase information is significantly reduced.

Index Terms—Intelligent reflecting surfaces (IRS), phase estimation, imperfect phase, phase compensation, unmanned aerial vehicle (UAV), bit error rate (BER), phase error.

I. INTRODUCTION

Unmanned aerial vehicles (UAVs) based communications have been on a rise with the aim of providing higher transmission rates and better link reliability for wireless communications systems [1]–[5]. However, the promised reliability and transmission rates may degrade if a line-of-sight (LoS) link is not guaranteed. To ensure auspicious reliability, intelligent reflecting surface (IRS) assisted UAVs can be a promising solution as IRS can be designed to provide high quality communications links [1], [6]–[10]. To achieve this goal, the reflected signals from each IRS element should be added coherently by adjusting the phase of each IRS element. Optimization of resources for UAV based communication has been discussed in [4], [5]. Most of the work related to IRS reported in the literature demonstrates that increasing the number of reflecting elements improves the signal quality with perfect phase estimation and compensation [7], or when all reflectors have equal phase errors statistics [8]–[10]. However, when the statistical properties of the phase error for reflectors are not identical, then increasing the number of reflecting elements does not necessarily improve the link quality [7].

Generally speaking, the phase estimation and compensation of IRS cascaded channels are challenging processes and cannot be performed perfectly. Therefore, the impact of phase alignment on IRS performance has been considered widely in the literature [8], [9], [11]–[15]. For example, bit error rate (BER) and outage probability expressions have been derived in the presence of phase errors for a single-user IRS under different channel models including LoS [8] and Nakagami- m fading [15]. The achievable capacity for IRS-assisted UAV communications under LoS assumption is analyzed in [9]. The impact of the phase quantization errors on the achievable capacity is discussed in [11], where it is shown that phase quantization and phase noise can severely degrade the achievable rate.

A. Motivation and Contributions

As can be noted from the cited literature and references listed therein, the IRS system error performance is typically analyzed while considering that all reflecting elements have the same phase error statistics. For example, Al-Jarrah et al. [7]–[9] modeled the phase error of each reflecting element as a von Mises random variable with a particular concentration factor $\kappa$, but the performance is evaluated while assuming that the $\kappa$ value is equal for all reflecting elements. Therefore, increasing the number of elements always improves the BER. However, practically this may not be the case because phase estimation accuracy can vary for different reflecting elements based on the link quality between the base station (BS), reflecting element, and destination receiver [16]. Consequently, the contribution of each reflecting element to the overall signal quality depends on its phase estimation accuracy. Therefore, in this article we capitalize on our work [8] by considering a more general and realistic model where different reflectors...
may have different phase-error characteristics. We demonstrate that reflecting elements with low phase estimates accuracy can degrade the overall signal quality [9] and increase the BER. As such, only a subset of elements with a particular phase accuracy should be activated, which can be performed through an optimization process. To clarify the concept even further, consider a simple case where an IRS panel has only two elements. If the phase error between the two elements is uniformly distributed in $[-\pi, \pi)$, then the overall channel becomes hyper-Rayleigh, and the performance will be much worse than the case if only one element is used [17].

To the best of the authors’ knowledge, there is no work in the literature that considers the impact of the individual reflecting elements or applies elements’ selection based on their contribution to the BER improvement. Therefore, this work considers optimizing the IRS elements’ selection process to reduce the average BER. In addition to BER reduction, the number of elements reduces the overhead associated with phase information feedback to the IRS panel, and allows serving other users in the network. Moreover, the phase error distribution is derived for the case of LoS communications in closed-form, and it is shown that it can be closely approximated to the von Mises distribution, which has been widely used to model the phase error, but without proper justification. The optimum solution of the considered problem is initially obtained using brute-force search, then a lower complexity optimum solution is developed by noting the relation between the BER and link quality for different IRS elements. Finally, an efficient selection rule called sort-and-drop selection (SDS) is proposed to provide near-optimum results at low complexity. The SDS results are compared to the optimum solution and to the results obtained and using Genetic algorithm (GA) [18].

**B. Paper Organization**

The rest of the paper is structured as follows. The system and channel models are presented in Sec. II. The phase error distribution is derived in Sec. III. The problem formulation, proposed solution, and complexity evaluation are presented in Sec. IV. Sec. V presents the numerical results, and finally, the conclusion is provided in Sec. VI.

**II. SYSTEM AND CHANNEL MODELS**

This work considers IRS-enabled high-frequency UAV communications as shown in Fig. 1. The BS is transmitting to a low altitude UAV (LAUAV) with the assistance of an IRS panel attached to a relatively higher altitude platform (HAP). The IRS panel consists of $L$ reflecting elements $\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_L$, and their indices form the set $\mathbb{L} = \{1, 2, \ldots, L\}$. A microcontroller is employed to control the phase shifts of the IRS elements. The direct link is assumed to be blocked between the BS and LAUAV due to large obstacles such as high-rise buildings [1]. Therefore, the signal is transmitted from the BS to the LAUAV with the assistance of an IRS panel attached to HAP. The reflecting elements of the IRS panel introduce phase shifts to the signals of the BS and reflect them to the LAUAV. The passband BS signal arriving at the $i$th IRS reflecting element can be expressed as

$$\tilde{r}_i(t) = ah_i \cos(2\pi f_c t + \varphi - \psi_i), \quad i \in \mathbb{L}$$

where $a$ is the amplitude and $\varphi$ is the phase of the transmitted information symbol, $f_c$ is the carrier frequency, $h_i$ is the channel gain from the BS to the $i$th reflecting element, and $\psi_i$ is the phase shift introduced by the channel. The signal reflected by each IRS element is attenuated by a factor $g_i \in [0, 1]$ and its phase is shifted by $\theta_i \in [-\pi, \pi)$. The signal received at the destination LAUAV from the $L$ reflecting elements of the HAP is given as

$$y(t) = \sum_{i=1}^{L} h_i h_i g_i h_i a \cos(2\pi f_c t + \varphi - \psi_i - \phi_i + \theta_i) + n(t)$$

where $n(t)$ is $\mathcal{CN} \sim (0, \sigma_n^2)$, $h_i$ and $\phi_i$ are respectively the gain and phase of IRS to LAUAV channel. The values of $\theta_i \forall i$ should be selected such that the phases of all reflected signals are aligned at the receiver, i.e., $\theta_i = \psi_i + \phi_i$. However, the estimated cascaded channel phase $\theta_i$ can not be estimated and compensated perfectly, which leads to phase alignment errors that can be defined as $\epsilon_i \triangleq \theta_i - \theta_i$, $\epsilon_i \in [-\pi, \pi)$. The received signal with imperfect phase compensation can be expressed as

$$y(t) = a \sum_{i=1}^{L} A_i \cos(2\pi f_c t + \varphi + \epsilon_i) + n(t)$$

where $A_i = h_i h_i g_i \in (-\infty, \infty)$. For phase shift keying (PSK) modulation, the information symbol amplitude $a = 1$. After down conversion to baseband, and considering the Sinusoidal Addition Theorem [19], [20], the received signal inphase and quadrature components can be written as

$$y_I = B_L \cos(\varphi + \epsilon_i) + n_I$$

and

$$y_Q = B_L \sin(\varphi + \epsilon_i) + n_Q$$

where $B_L$ is the signal envelope, $n_I$ and $n_Q$ are the inphase and quadrature components of the additive noise, respectively.
In UAV communications, ground-to-air (G2A) and air-to-air (A2A) channels, $h_i$ and $h_s$, typically have a dominant LoS component, and thus can be modeled using the Rician distribution with a large $K$ factor [21]. It was experimentally found that the Rician factor for air-to-air and ground-to-air can be about 12 dB for C-band and L-band signaling [22], [23]. Consequently, free space pathloss dominates the channel gain $A_i$ [8], [9]. Moreover, the transmitted signals may experience different atmospheric distortions caused by the spatial temperature variations given that the distance between the IRS elements is larger than the coherence distance, which may experience different atmospheric distortions caused by the IRS elements is larger than the coherence distance, which thus can be modeled using the Rician distribution with a large $K$ factor, and thus can be modeled using the Rician distribution with a large $K$ factor, and thus can be modeled using the Rician distribution with a large $K$ factor.

Channel estimation in IRS can be performed by dividing the time interval $T$ into two time slots $T_1$ and $T_2$. The time interval $T_1 \ll T_2$ is allocated for phase estimation, where $\hat{\theta}_i$ is estimated and fed to the corresponding IRS element controller. After phase estimation and compensation, the data transmission is performed in $T_2$. For channel estimation, we consider the protocol proposed in [25]–[27]. Therefore, $T_1$ is divided into $L$ sub-slots each of which has a duration of $T_1/L$, where only one reflector is switched on in each sub-slot while all other IRS elements are switched off. Therefore, the received complex baseband signal during the channel estimation of the $l$th IRS element is $y_l = A_l s_l \exp(j\theta_l) + n_l$, where $s_l$ is pilot symbol. For least square estimation (LSE), the channel can be estimated as $\hat{H}_l = y_l/s_l$, and thus, $\hat{\theta}_l = \arctan (\hat{H}_Q/\hat{H}_I)$ where $\hat{H}_I$ and $\hat{H}_Q$ are the in-phase and quadrature components of $\hat{H}_l$.

III. PHASE ERROR DISTRIBUTION

As discussed in Sec. II, the channel gain is dominated by a LoS component. Since the derivation of the phase error distribution is similar for all reflectors, we omit the subscript $i$ from the equations in this section to simplify the notation. For a deterministic complex channel, the channel can be represented as $H = H_I + jH_Q \triangleq \beta e^{j\theta}$. To estimate the channel phase, a pilot symbol $s$ is transmitted, and the received signal is given by

$$y = Hs + w$$

where $w \sim CN(0, \sigma^2_w)$ is the additive white Gaussian noise (AWGN). Given that $s = 1$, then $y = (H_I + w_I) + j(H_Q + w_Q)$. Consequently, the phase estimate is obtained as

$$\hat{\theta} = \arctan \left( \frac{H_Q + w_Q}{H_I + w_I} \right).$$

Due to the presence of the AWGN, there will be a phase error $\epsilon = \hat{\theta} - \theta$. By noting that $y_I$ and $y_Q$ are independent and identically distributed (i.i.d.) Gaussian random variables, $y_I \sim N(H_I, \sigma^2_I)$ and $y_Q \sim N(H_Q, \sigma^2_Q)$, then their joint PDF is given by

$$f(y_I, y_Q) = \frac{1}{2\pi \sigma^2_w} e^{-\frac{y_I^2 + y_Q^2}{2\sigma^2_w}}.$$ (8)

The PDF of $\epsilon$ can be obtained by converting the PDF to polar coordinates $y_I = r \cos \theta$ and $y_Q = r \sin \theta$ to obtain $f(r, \theta)$, and then averaging over $r$. The joint PDF in polar coordinates can be found as

$$f_{R, \Theta}(r, \theta) = \frac{r}{2\pi \sigma^2_w} e^{-\frac{r^2 + \beta^2 - 2(rH_I \cos \theta + H_Q \sin \theta)}{2\sigma^2_w}}.$$ (9)

Then, $f_{R, \Theta}(r, \theta)$ needs to be averaged over $r$ to obtain the marginal PDF of $\theta$, $f_{\Theta}(\theta)$

$$f_{\Theta}(\theta) = \frac{\gamma e^{-\gamma^2\theta^2}}{\pi^{\frac{1}{4}}} \int_0^\infty r e^{-\gamma (r^2 - 2r(H_I \cos \theta + H_Q \sin \theta))} dr$$

where $\gamma = 1/(2\sigma^2_w)$. Using [28], (2.13.57), pp 344 and after some manipulations we obtain,

$$f_{\Theta}(\theta) = \frac{e^{-\gamma^2\theta^2}}{2\pi} \left( 1 + \sqrt{\gamma \pi} z(\theta) e^{\gamma^2 z^2(\theta)} \text{erfc} \left( -\sqrt{\gamma} z(\theta) \right) \right)$$

(11)

where $z(\theta) = H_I \cos \theta + H_Q \sin \theta$ and $\text{erfc}(x)$ is the complementary error function which is defined as $\text{erfc}(x) \triangleq \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.

Finally, since $\epsilon = \hat{\theta} - \theta$, the phase error distribution $f_{\epsilon}(\epsilon) = f_{\Theta}(\epsilon + \theta)$ can be expressed as (11), except that $z(\theta)$ is replaced by $\hat{z}(\theta)$, where $\hat{z}(\theta) = H_I \cos(\epsilon + \theta) + H_Q \sin(\epsilon + \theta)$. Thus,

$$f_{\epsilon}(\epsilon) = \frac{e^{-\gamma^2\hat{z}^2}}{2\pi} \left( 1 + \sqrt{\gamma \pi} \hat{z}(\theta) e^{\gamma^2 \hat{z}^2(\theta)} \text{erfc} \left( -\sqrt{\gamma} \hat{z}(\theta) \right) \right).$$ (12)
Interestingly, by using the trigonometric identities $\cos(a+b) = \cos a \cos b - \sin a \sin b$ and $\sin(a+b) = \sin a \cos b + \cos a \sin b$, it can be proven that $\bar{z}(\theta) = \beta \cos(\epsilon)$. For ease of notation, we use $f(\epsilon)$ as $f(\epsilon)$. Consequently, $f(\epsilon)$ can be expressed as

$$f(\epsilon) = \frac{e^{-\gamma \beta^2}}{2\pi} \left[1 + \sqrt{\gamma \beta} \cos(\epsilon) e^{\gamma \beta^2 \cos^2(\epsilon)} \text{erfc}(\sqrt{\gamma / \beta} \cos(\epsilon))\right]. \tag{13}$$

### A. High SNR Scenario

It can be noted that at high SNRs, the second term inside the brackets in (13) is $\gg 1$, $\beta$ can be normalized to 1 and $\text{erfc}(.)$ can be approximated as

$$\text{erfc}(\sqrt{\gamma / \beta} \cos(\epsilon)) = 2\Phi(\epsilon) \tag{14}$$

where $\Phi(\epsilon) = u(\epsilon + \frac{\pi}{2}) - u(\epsilon - \frac{\pi}{2})$ and $u(.)$ is the Heaviside unit step function. Consequently,

$$f(\epsilon) \approx \sqrt{\frac{\gamma}{\pi}} \Phi(\epsilon) e^{-\gamma(1-\cos^2(\epsilon)) + \ln \cos(\epsilon)}. \tag{15}$$

In addition, it can be noted that $-\gamma(1-\cos^2(\epsilon))$ and $\ln(\cos(\epsilon))$ are both negative valued functions for $\epsilon \in (\frac{\pi}{2}, \frac{\pi}{2})$ and $-\gamma(1-\cos^2(\epsilon)) \ll \ln(\cos(\epsilon))$ at high SNRs. Thus, $f(\epsilon)$ can be approximated as

$$f(\epsilon) \approx \sqrt{\frac{\gamma}{\pi}} \Phi(\epsilon) e^{-\gamma(1-\cos^2(\epsilon))}. \tag{16}$$

By applying the trigonometric identity $\cos^2(\epsilon) = \frac{1}{2}(\cos(2\epsilon) + 1)$, the approximated $f(\epsilon)$ can be further simplified to

$$f(\epsilon) \approx \frac{\Phi(\epsilon)}{2\pi c} e^{\frac{\gamma}{2} \cos(2\epsilon)} \tag{17}$$

where $c = \frac{e^2}{2\sqrt{\pi}}$. By defining $\nu \equiv \frac{\epsilon}{2}$ and using the modified Bessel function approximation $I_0(\nu)$ for large argument [29, eq. (9.7.1)], we obtain

$$I_0(\nu) \approx \frac{1}{\sqrt{\sqrt{\pi}}} e^{\nu} \equiv 2c. \tag{18}$$

Consequently, $f(\epsilon)$ can be written as

$$f(\epsilon) \approx \frac{2\Phi(\epsilon)}{2\pi I_0(\nu)} e^{\nu \cos(2\epsilon)}. \tag{19}$$

It is worth noting that the approximation in (18) provides a normalized error less than $5 \times 10^{-2}$ for $\nu > 3$. Moreover, the normalized error approaches 0 when $\nu$ approaches $\infty$. The accuracy of the approximation can be also concluded from Fig. 2 where exact and approximated PDFs are presented for low and moderate SNRs.

### B. The von Mises Distribution

To find the relationship between $f(\epsilon)$ as the standard form of von Mises distribution, we equate $f(\epsilon)$ and the von Mises given in (23). Thereafter, the approximation used in (18) for $I_0(\cdot)$ can be applied to obtain

$$2\sqrt{\nu} e^{\nu \cos(2\epsilon) - 1} = \sqrt{\kappa} e^{\kappa \cos(\epsilon) - 1}. \tag{20}$$

Applying the trigonometric identity $\cos(2\epsilon) = 2\cos^2(\epsilon) - 1$, and then the difference between two squares rule yields,

$$2\sqrt{\nu} e^{(\cos(2\epsilon) - 1)(2\cos(\epsilon) + 1) - \kappa} = \sqrt{\kappa}. \tag{21}$$

Since we aim at equating the two functions in (21), it can be observed that for any value of $\epsilon \in (\frac{\pi}{2}, \frac{\pi}{2})$, the equality is satisfied if and only if $4\nu = \kappa$. Finally, by setting $\kappa = 4\nu = 2\gamma$, $f(\epsilon)$ can be written in von Mises form as

$$f(\epsilon) \approx \frac{\Phi(\epsilon)}{2\pi I_0(\kappa)} e^{\kappa \cos(\epsilon)}. \tag{22}$$

As can be noted from (13), the PDF of phase error has several nonlinear elements, and it does not have an analytical solution due to $\text{erfc}(.)$, therefore using it for further analysis would mostly yield intractable solutions and can be closely approximated by the von Mises PDF [9, 15] for a wide range of SNRs. Therefore, we consider that

$$f(\epsilon_i) = \frac{1}{2\pi I_0(\kappa_i)} e^{\kappa_i \cos(\epsilon_i - \mu_i)} \tag{23}$$

where $\mu_i$ is the mean of the phase estimation error associated with the $i$th reflector, $\mu_i = 0$ for unbiased estimators, and $\kappa_i$ is the concentration parameter of the von Mises PDF, which captures the phase estimation accuracy. Higher values of $\kappa$ correspond to more accurate phase estimates. Ultimately, $\epsilon \rightarrow 0$ when $\kappa \rightarrow \infty$. Moreover, although the channel estimation and data transmission are typically performed within the channel coherence time, the channel estimation process time may exceed the coherence time. Consequently, the SNR can be different during time slots $T_1$ and $T_2$. It is worthy to notice that $\kappa_i$ is a function of SNR, i.e., $\kappa_i = 2\gamma_i$, and thus the value of $\kappa_i$ can be determined at the microcontroller of IRS based on previous knowledge of the value of SNR.

Fig. 2 shows the derived distribution in (13), von Mises approximation, and simulated phase error at SNR = 15 dB. It can be observed that the von Mises distribution perfectly matches the exact PDF. For the case of SNR = 8 dB, the accuracy of the approximation is still high, but slightly worse than the case of 15 dB.

### IV. BER Aware IRS Element Selection

#### A. BER Analysis

Given that the microcontroller can switch off the $l$th IRS element by setting $g_l = 0$, then the BER at the LAUAV can be approximated as [8]

$$P_E(\alpha) \approx \frac{\log_2(M)}{\omega_L \sqrt{2\pi \sigma_L^2} \left[Q(\frac{\sqrt{2} \omega_L \sqrt{2} \sigma_L^2}{\sqrt{2} \sigma_L^2}) - \frac{(\sqrt{2} \omega_L \sqrt{2} \sigma_L^2)^2}{2 \pi \sigma_L^2} \right] dy_L \tag{24}$$

where $M$ is the modulation order, $\omega_L$ is the truncated PDF normalization factor, which can be approximated by $1$ for large number of reflectors, $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_L], \alpha_l \in \{0, 1\}$ \forall l, $Q(\epsilon) = 1/2 \text{erfc}(\epsilon/\sqrt{2})$, $X = \sqrt{C_2 Y_L^2 / \sigma_L^2}$, $C_1$ and $C_2$ are constants that depend on the modulation scheme and order [30, Table 6.1, pp. 167], $Y_L$ is the squared signal envelope, i.e., $Y_L \equiv B_L^2$, which can be approximated as a Gaussian distribution using the central limit theorem (CLT) for large $L$ values,
\(\mu_{Y_L}(\alpha)\) and \(\sigma^2_{Y_L}(\alpha)\) are the mean and variance of \(Y_L\), respectively, which are functions of \(\alpha\). Henceforth, we will omit \(\alpha\) from the notation and denote \(P_E(\alpha), \mu_{Y_L}(\alpha)\) and \(\sigma^2_{Y_L}(\alpha)\) as \(P_E, \mu_{Y_L}\) and \(\sigma^2_{Y_L}\), respectively, to simplify the notation. The squared signal envelope is \(Y_L = B_L^2 \in [B_{L,m}^2, B_{L,M}^2]\), where \(B_{L,m}^2 = 0\) and \(B_{L,M}^2 = \sum_{k=1}^{L} \alpha_k A_k\). The mean \(\mu_{Y_L}\) and variance \(\sigma^2_{Y_L}\) can be represented as [8]

\[
\mu_{Y_L} = \sum_{k=1}^{L} \alpha_k A_k^2 + 2 \sum_{j=2}^{L-j+1} \sum_{k=1}^{\min(j,j-1)} \alpha_j \alpha_k A_j A_k I_1(\kappa_j) I_1(\kappa_k)
\]

(25)

and

\[
\sigma^2_{Y_L} = E[Y_L^2] - \mu^2_{Y_L}
\]

(26)

where \(E[\cdot]\) is the expectation operation, and \(E[Y_L^2]\) is given by

\[
E[Y_L^2] = \left(\sum_{k=1}^{L} \alpha_k A_k \cos \theta_k \right)^2 + 4E \left[ \sum_{L_j \geq j, k \geq 1} \alpha_j \alpha_k A_j A_k \cos \theta_k \right] + 4\| \alpha \| \sum_{L_j \geq j, k \geq 1} \alpha_j \alpha_k A_j A_k \cos \theta_k
\]

(27)

where \(\theta_j = \frac{\theta_j}{\sigma_0(\alpha)}\). On the other hand, the term \(T_1\) can be derived as [8]

\[
T_1 = 2 \sum_{L_j \geq j, k \geq 1} \alpha_j A_j A_k (1 + \theta_j \theta_k)
\]

\[
+ \sum_{1 \leq k \leq L} \sum_{l \leq \min(j,k)} \sum_{L_l \leq \min(l,j)} 4\alpha_k A_k A_{j,k,l} E_{j,k,l}
\]

\[
+ \sum_{1 \leq k \leq L} \sum_{l \leq \min(j,k)} \sum_{L_l \leq \min(l,j)} A_{j,k,l} A_{j,k,l} E_{j,k,l}
\]

(28)

where \(\alpha_{j,k,l} = \alpha_j \alpha_i \alpha_l \cos \theta_j = \frac{\theta_j}{\sigma_0(\alpha)} A_{j,k,l} = A_k A_j A_l\) and \(E_{j,k,l} = E[\cos \phi_j \cos \phi_k \cos \varphi_l]\). By observing that \(\theta_j \neq \phi_i\) and \(\phi_i \neq \phi_l\), taking into account all possible scenarios for the equality between \(\phi_i\) and \(\{\phi_i \mid \phi\}\), and between \(\phi_k\) and \(\{\phi_k \mid \phi\}\), the value \(E_{j,k,l}\) can be derived as

\[
E_{j,k,l} = \begin{cases}
\frac{1}{2} \theta_j \theta_l \theta_3 \theta_4, & i \neq k, i \neq j, l \neq k, l \neq j \\
1 + \theta_j \theta_k, & i \neq k, i = j, l = k, l \neq j \\
\frac{1}{2} \theta_j \theta_l (1 + \theta_k), & i \neq k, i \neq j, l = k, l \neq j \\
\frac{1}{2} \theta_j \theta_l (1 + \theta_k), & i \neq k, i \neq j, l = k, l \neq j \\
\frac{1}{2} \theta_j \theta_l (1 + \theta_k), & i = k, i \neq j, l \neq k, l \neq j \\
\frac{1}{2} \theta_j \theta_l (1 + \theta_k), & i = k, i \neq j, l \neq k, l \neq j
\end{cases}
\]

(30)

Based on the BER in (24), it can be numerically demonstrated that \(P_E\) is not strictly decreasing versus \(L\) when the values of \(\kappa_i\) are not equal for all values of \(i\). Consequently, only a subset of elements should be selected while all others should be switched off.

It is worth noting that the BER in (24) can be evaluated in closed-form in terms of the parabolic cylinder function (PCF) as described in [8]. However, evaluating the exact value of the PCF is infeasible because it is generally represented using an infinite series. To obtain an accurate evaluation, two different series representations are used based on the argument value. The closed-form expression of \(P_E\) is given by [8]

\[
P_E \approx \frac{Z}{\log_2(M)} \sum_{i=1}^{n_A} (-1)^{i+1} \frac{\Gamma \left( i + \frac{1}{2} \right)}{\Gamma \left( i + \frac{1}{2} \right)} \left( C_2 \sigma_{Y_L} \right)^{\frac{1}{2}}
\]

\[
\times D_{-\frac{i-1}{2}} \left( \sigma_{Y_L} \left( C_2 \frac{\sigma_{Y_L}^2}{2 \sigma_n^2} - \frac{\mu_{Y_L}}{2 \sigma_n} \right) \right)
\]

(31)

where

\[
Z = \frac{C_1 \sigma_n}{2 \pi \sqrt{C_2 \sigma_{Y_L}}} \exp \left( \frac{\sigma_{Y_L}^2}{4} \left( \frac{\mu_{Y_L}}{2 \sigma_n} - \frac{C_2 \sigma_{Y_L}^2}{2 \sigma_n^2} - \frac{\mu_{Y_L}}{2 \sigma_n} \right) \right)
\]

(32)

and \(D_{-\frac{i-1}{2}}(\cdot)\) is the PCF expressed in terms of Whittaker’s function \(D_{-\frac{i-1}{2}}(\cdot)\). To numerically evaluate (31) we used \(n_A = 30\). For computing the PCF, the implementation given in [31] is used. More specifically, when \(|a, x| < 5\) routine \(pu\) [31, Table I] is used, whereas for \(|x| \gg \) a routine \(pulx\) is used.

### B. IRS Element Selection

To minimize \(P_E\) by selecting a certain subset of IRS elements, the values of \(\alpha\) should be selected such that

\[
\arg \min_{\alpha} P_E(\alpha).
\]

(33)

Because the objective function is non-convex and the solution requires integer programming, then the optimization problem is NP-hard [32]. To solve the problem, the following approaches are considered.

1) **Exhaustive search:** By noting that the optimization problem in (33) is an integer selection problem, then the optimal solution can be obtained using exhaustive search. Therefore, \(P_E(\alpha)\) is computed for all possible elements-selection combinations, and the set of elements that provide the minimum...
$P_E(\alpha)$ is selected. The complexity of this approach is generally very high because there is a total of $2^L - 1$ combinations that have to be evaluated. Consequently, this approach can be prohibitively complex, particularly for a large number of elements.

2) Low-complexity optimum selection: To explain the low-complexity optimum selection, consider that the reflecting elements are sorted in a descending order based on their $\kappa$ values. Therefore, reflecting element $E_i$ in the sorted set will have concentration coefficient $\kappa_i$ where $\kappa_i > \kappa_j \forall i < j$. Therefore, the sorted set of IRS elements can be written as $E = [E_1, E_2, \ldots, E_L]$. In the brute-force approach, all possible combinations, without repetition, that be formed from $E$ should be evaluated, and the one that minimizes (33) should be selected, which implies that $2^L - 1$ operations are needed. However, it can be observed from (24) that

$$P_E(1,1,\ldots,0) < P_E(1,0,1,\ldots,0)$$
$$P_E(1,0,1,\ldots,0) < P_E(1,0,0,1,\ldots,0)$$
$$\vdots$$
$$P_E(1,0,\ldots,0,1,1,0) < P_E(1,0,\ldots,0,1,0,1). \quad (34)$$

Although in (34) we considered the case where only two elements are selected out of the available $L$ elements, by induction, the same process can be generalized for more than two. Consequently, if we evaluate $\alpha = [1,1,0,\ldots,0]$, then there is no need to evaluate other cases of $\alpha$ which have only two activated elements. The same argument applies to any other number of activated elements. Consequently, we need only to test the cases sequentially starting from the minimum number of elements, and adding the element with the next lowest $\kappa$ in the subsequent step. Therefore, the total number of trials required to find the optimum set of reflectors is $L$, which is much less than $2^L - 1$ in the case of brute force search.

Fig. 3 shows the BER using various SNRs for two different sets of $\kappa$, which are $\kappa$-E = [20, 20, …] and $\kappa$-I = [20, 20, 20, $U(0, 4), U(0, 4), \ldots$], where $U(a, b)$ represents the uniform distribution between $a$ and $b$. BPSK is considered as the modulation scheme, therefore $C_1 = 1$ and $C_2 = 2$. As can be noted from the figure, increasing the number of reflectors when all $\kappa$ values are equal consistently decreases the BER. However, for unequal $\kappa$ values, there is an optimum number of reflectors ($L_O$) that minimizes the BER. The value of $L_O$ depends on several factors such as $L$, SNR and $\kappa$. In Fig. 3, it can be noted that $L_O = 42$ and $40$ for SNR of $-20$ and $-15$ dB, respectively. For higher values of SNR, the BER is not shown because it is very low, $L_O = 33$ and $31$ for SNR of $-10$ and $-5$ dB, respectively.

Fig. 4 is similar to Fig. 3, except that the $\kappa$ values are defined as $\kappa$-II = [20, 20, 20, $U(0, 1), U(0, 1), \ldots$]. Because $\kappa$-II is generally worse than $\kappa$-I, the BER does not decrease steeply as in the case of Fig. 3. Moreover, the figure shows that the objective function might have multiple local minima at high SNRs. For example, for SNR of $= 0$ dB, there are two local minima values when the number of reflectors is 4 and 29.

Fig. 5 shows the BER using a squared quadrature amplitude modulation (QAM) with modulation order 16, and hence, $C_1 = 3$ and $C_2 = 1/5$. As can be noted from the figure, the BER with the 16-QAM generally follows those of the BPSK, but with some differences. For example, at low SNRs of $-20$ and $-15$, the AWGN becomes more impactful on the BER as compared to the phase error. Consequently, the entire set of reflecting elements was used, and the minimum BER is obtained at $L = 50$. Moreover, for the case of SNR $= 0$ dB, there is only one minimum, which actually corresponds to the optimum value.

3) Sub-optimum sort-and-drop selection (SDS) algorithm:

The suboptimum selection algorithm can be directly derived from the low-complexity optimum algorithm, where the search
process stops when the first minimum, local or global, point is found. Therefore, the SDS algorithm initially sorts all IRS elements based on their phase error statistics, which is indicated by $\kappa$, and then evaluates the impact of including each IRS element on the BER. The algorithm stops when adding an element increases the BER, which indicates the first minimum point. Algorithm 1 summarizes the proposed SDS algorithm.

As described in Algorithm 1, all reflectors are sorted in a descending order based on their $\kappa$ values. Then the first element is selected from the sorted set and $P_E([1, 0, 0, \ldots, 0])$ is computed. In the second iteration, the second element in the sorted set is included and $P_E([1, 1, 0, \ldots, 0])$ is computed. Then, $P_E$ in the second iteration $P_E^{(2)}$ is compared with $P_E$ that was obtained in the first iteration $P_E^{(1)}$. If $P_E^{(2)} < P_E^{(1)}$, the algorithm proceeds to the third iteration where $\alpha = [1, 1, 1, 0, 0, \ldots, 0]$. If $P_E^{(3)} < P_E^{(2)}$ then the algorithm starts another iteration, otherwise it stops, and so forth. Although the SDS algorithm may not give the optimum solution, it offers a reasonable BER performance with low complexity and less number of reflectors.

It should be noted that (24) is generally accurate for large values of $L$, roughly $L > 3$, because it is based on the Central Limit Theorem (CLT). Therefore, Algorithm 1 should start at $l = 4$. In the case that a small number of reflectors should be considered, then Algorithm 1 remains unchanged, except that the BER for $l < 4$ should be computed as reported in [8].

### C. Complexity

To find the set of reflecting elements that should be activated, the computation of (24) is required for each trial set of reflectors. Therefore, the exhaustive search would require a total of $2^L - 1$ computations. For the SDS, the worst case scenario would require $L$ computations, which occurs when all $\kappa$ values are roughly equal for all reflectors. However, although performing the linear search is significantly less complex than the brute force search, significant additional complexity reduction can be achieved using the Bisection method, where the number of computations can be expressed as [33, eq. (35)],

$$I = \left\lceil \log_2 \left( \frac{L - 4}{\delta} \right) \right\rceil + 2$$

where $\delta$ is the line search accuracy. Because the number of reflectors is an integer, we set $\delta = 1$. Therefore $I < L$, $\forall L$. For the case of $L = 50$ the number of computations is 8.

### V. Numerical Results

This section presents the performance of the proposed SDS algorithm and compares its performance to the optimum solution as well as to heuristic GA. The optimum solution uses exhaustive search over $2^L - 1$ combinations. For these numerical results, we considered $A_l = 1$, $C_1 = 1$ and $C_2 = 2$ unless specified. In the heuristic GA, the BER is used as the fitness function, which is computed for each selected individual set of reflectors, and the next population is based on the current generation with the best fitness. For the GA implementation, a total of 20 generations and 8 initial solutions are considered.

Table 1 shows the SDS BER and selected number of reflectors where all reflectors have the same value of $\kappa$. The table shows that the total number of selected elements $L_{SDS}$ is generally inversely proportional to SNR and directly proportional to $\kappa$. Such performance is obtained because the BER at low SNRs is dominated by the AWGN, and hence, increasing $L_{SDS}$ reduces the BER. When the SNR is increased to $-6$ dB, $L_{SDS}$ increases with the increase of $\kappa$. In this case,
the BER depends on both the phase noise and SNR. Therefore, $L_{SDS}$ increases when the phase error decreases.

Fig. 6 compares the BER using SDS, the optimum solution, and GA for $L = 15$. Moreover, the BER is presented when all reflectors are used, and when the best 4 reflectors are selected. The used sets of $\kappa$ are, $\kappa-\text{III} = [20, 20, 20, 20, U(0, 1), U(0, 1), \ldots]$ and $\kappa-\text{IV} = [20, 20, 20, 20, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0]$. Both Fig. 6a and Fig. 6b show that the SDS algorithm perfectly matches the optimum. Fig. 6a shows that a noticeable improvement is obtained by the SDS algorithm when compared to the case of $L = 15$ for SNR $> -10$ dB. Fig. 6b shows that the BER using the SDS algorithm provides comparable BER to the all reflectors selection scenario over the considered SNR range. This behavior confirms the fact that the selection of a few reflectors with accurate phase compensation can provide equal or lower BER. It can be seen also that the performance of GA approaches the other algorithms at certain SNR values, while it deviates at other SNR values. Interestingly, the selection of the best 4 reflectors provides worse BER compared to SDS, GA, and the optimum solution. The performance of GA depends on the number of considered initial solutions, which is generally a small fraction of the complete solution set. Therefore, some of the initial solutions may be significantly far from the optimal solution, and the GA might stop before reaching the minimum point. The SDS is based on sorting, and in most scenarios, it converges to the minimum. Consequently, the SDS offers better performance than GA.

Fig. 7 shows the distribution of the number of reflectors selected using the SDS algorithm for low, moderate, and high SNRs. The total number of reflecting elements $L \in \{15, 50\}$, and $\kappa = [20, 20, U(0, 3), U(0, 3), \ldots]$. The figure also compares the impact of varying $g_i$ on the number of selected reflectors, where $g_i \sim U(0.6, 1) \forall i$ [34]. As can be noted from the figure, the number of selected reflectors is inversely proportional to SNR. Additionally, it can be noted that more reflectors are selected when $g_i < 1$. Such behavior is obtained because using $g_i < 1$ is equivalent to reducing the SNR, which is inversely proportional to the number of selected reflectors. For $L = 50$, the average number of selected elements at SNR $= -10$ dB is 27, whereas the average is 16.5 and 4 reflectors for SNR $= 0$ and 10 dB respectively. A similar trend can be noted for $L = 15$ where the average number of reflectors is 11, 7, and 4, respectively. It is also worth noting that the average number of selected elements for the two cases converges to the same value at high SNRs.

Fig. 8 shows the impact of imperfect $\kappa$ estimation on the performance of the SDS algorithm for $L = 15$ and $L = 50$, where $\kappa = [20, 20, 20, U(0, 2), U(0, 2), \ldots]$. The estimation error vector $\mathbf{e} = [e_1, e_2, \ldots, e_L]$, where $e_i \forall i$ are i.i.d. and $e_i \sim U(0, e_{\text{max}})$. The figure presents the BER for three cases $e_{\text{max}} = 0.1$ and 10. The case of $e_{\text{max}} = 0$ corresponds to the perfect estimation scenario. As can be noted from the figure, the SDS demonstrates high tolerance when
The amplitude of the transmitted symbol

\[ \epsilon_{\text{max}} = 1 \] because the \( \kappa \) is generally low for a large number of reflectors. Consequently, the selected number of reflectors marginally changes at certain SNRs. When \( \epsilon_{\text{max}} = 10 \), certain reflectors will erroneously have a high \( \kappa \), and thus will be allocated reflection elements, which may cause severe BER degradation at high SNRs. At low SNRs, the performance is mostly dominated by the AWGN, which makes the impact of the estimation error less significant. Nevertheless, the SDS always provides performance improvement because selecting all elements actually corresponds to the worst-case scenario.

VI. CONCLUSION

This work presented a reflector-selection algorithm for UAV-IRS network with imperfect phase estimation and compensation, where the aim is to minimize the BER. Additionally, the distribution of the phase error is derived in a closed-form, and an accurate approximation was provided using the von Mises PDF with parameter \( \kappa \). The proposed selection algorithm depends on the SNR and \( \kappa \) value of each reflector. The obtained BER results show that the proposed SDS algorithm may provide significant BER enhancement as compared to the all-reflectors case, and a comparable performance with the optimum algorithm. In addition to the BER reduction, the partial element selection allows for increasing the number of UAVs served by the IRS panel. In particular scenarios, the obtained results show that assigning only 10\% of the total reflectors can minimize the BER and allow allocating the remaining 90\% of the elements to other users.

APPENDIX I: LIST OF SYMBOLS

\[ \beta \] Magnitude of \( H \)
\[ \alpha \] Element allocation matrix
\[ \kappa \] Phase error concentration matrix
\[ \mathcal{E} \] Sorted set of IRS elements
\[ \delta \] Line search accuracy
\[ \epsilon \] Phase alignment error
\[ \gamma \] Signal-to-noise ratio
\[ \theta \] Estimated channel phase
\[ h \] Channel gain from the reflecting element to LAUAV
\[ \kappa_i \] Phase error concentration parameter for \( i \)th element
\[ \mathcal{A} \] Set of reflecting elements
\[ \mathcal{E} \] Cascaded channels vector
\[ \alpha \] \( \kappa \) estimation error
\[ \mathcal{U}(a, b) \] Uniform distribution with parameters \( a \) and \( b \)
\[ \omega_L \] Truncated PDF normalization factor
\[ \phi_j \] Phase shift of BS-LAUAV link associated with the \( j \)th IRS element
\[ \psi \] Channel phase shift for the reflecting element
\[ \theta \] Phase shift of the reflecting element
\[ \tilde{\phi}_q \] Ratio of first order and zeroth order modified Bessel function, \( \tilde{\phi}_q = \frac{I_1(q \sqrt{\alpha})}{I_0(q \sqrt{\alpha})} \)
\[ \varphi \] Phase of the transmitted information symbol
\[ \tilde{\varphi}_q \] Ratio of second order and zeroth order modified Bessel function, \( \tilde{\varphi}_q = \frac{I_2(q \sqrt{\alpha})}{I_0(q \sqrt{\alpha})} \)
\[ A \] Cascaded channel gain, \( A = hh_g \)
\[ \alpha \] The amplitude of the transmitted symbol

\[ BL \] Received signal envelope
\[ BL,M \] Maximum value of \( BL \)
\[ BL,m \] Minimum value of \( BL \)
\[ C_1, C_2 \] Modulation dependent constants
\[ D_{\alpha}(\cdot) \] Parabolic cylindrical function (PCF)
\[ D_{-\alpha-2}(x) \] PCF in terms of Whittaker’s function
\[ \epsilon_{\text{max}} \] Maximum estimation error
\[ f(\cdot) \] Phase error PDF
\[ f_c \] Carrier frequency
\[ g \] Reflection coefficient of the reflecting element
\[ H \] Deterministic complex channel
\[ h \] Channel gain from BS to the reflecting element
\[ I_0(\cdot) \] Modified Bessel function of first kind and order zero
\[ L \] Total number of reflecting elements
\[ L_{\text{SDS}} \] Optimal number of selected reflecting elements
\[ M \] Number of reflecting elements selected using SDS
\[ n_A \] Number of terms in the series
\[ P_E \] BER
\[ r \] Magnitude of the received signal \( y \)
\[ y_i \] Received signal inphase component
\[ y_L \] Squared signal envelope, \( Y_L \equiv B_L^2 \)
\[ y_Q \] Received signal quadrature component

REFERENCES


