Probabilistic Shaping Based Spatial Modulation for Spectral-Efficient VLC

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Abstract—Visible light communication (VLC) is a promising technology for 6th-generation (6G) networks because of its attractive feature such as a wide unlicensed spectrum. In this paper, a novel adaptive coded spatial modulation scheme with probabilistic shaping (PS) is proposed to approach the capacity of the spatial modulation (SM) in VLC channels with intensity modulation and direct detection (IM/DD). In the proposed scheme, spatial and constellation symbols are probabilistically shaped depending on the user’s location inside the room and the optical signal-to-noise ratio (OSNR). Moreover, we optimize the channel coding rate to maximize further the achievable rate of the proposed scheme for a given OSNR. Finally, we propose an algorithm to compute the capacity-achieving distribution of the proposed scheme with unipolar M-ary pulse amplitude modulation (PAM) signaling. The proposed scheme outperforms uniform and an orthogonal frequency-division multiplexing (OFDM) based scheme in terms of spectral efficiency (SE) and/or frame error rate (FER). For example, for 8-PAM signaling with N = 8 transmit antennas, the proposed scheme operates within 0.2 dB from the unipolar M-PAM SM VLC channel signaling capacity and outperforms the uniform and OFDM based schemes in terms of FER by at least 1.1 dB and 1.3 dB at a normalized data rate of 1.33 bits per channel use per sub-carrier (b/cu/sc), respectively.

Index Terms—VLC, spatial modulation, probabilistic shaping

I. INTRODUCTION

With the recent development of light emitting diodes (LEDs) technology, visible light communication (VLC) has attracted significant research interest as a promising candidate to solve the problem of spectrum scarcity in today’s exponentially growing wireless communication demand. The key advantages of VLC over radio frequency (RF) communication counterpart are better area spectral efficiency, more secure indoor wireless access across unregulated bandwidth, higher wireless transmission speed and lower electromagnetic field exposure [1]–[6]. Besides, VLC potentially will have a significant role in the development of 6th-generation (6G) because of its ability to provide massive and dense connectivity [7].

VLC systems have several unique properties relative to other systems, operating in different radio frequency bands. First, VLC performs illumination and communication simultaneously; in other words, it controls the light intensity to convey information [7]. Second, the average optical power of VLC should remain almost the same with time [8]. Third, the input signals should be positive only because the illumination intensity cannot be negative [7], [8]. Hence, capacity analysis, modulation schemes, and algorithms derived in traditional RF communication systems are not usually applicable in VLC systems. The low-cost digital modulation schemes in VLC typically relies on intensity modulation and direct detection (IM/DD) techniques because of the above physical properties of the conventionally employed front-end devices. LEDs modulate the intensity of the wireless optical signals, and photo-detectors (PDs) directly detect them [9]. Despite a huge unlicensed bandwidth in VLC, the bandwidth of lens of LEDs, which are used in optical wireless data links, is limited to 245 MHz. Therefore, it is important to consider spectral efficiency (SE) in designing modulation schemes for VLC. So, in order to explore the potential of VLC systems, specifically tailored modulation schemes need to be designed.

One of the best technologies to improve SE is to use multiple sources on both transmitter and receiver sides or only on the transmitter side, which are known as multiple-input multiple-output (MIMO) and multiple-input single-output (MISO), respectively. Hence, extensive work has been done to implement MIMO and MISO technologies for VLC systems in recent years [10]. For instance, authors in [11] achieved transmit diversity and data multiplexing in indoor VLC by using multiple LEDs and PDs. According to [12], increasing the number of LEDs at the transmitter side increases the SE linearly in the spatial multiplexing.

However, MIMO techniques experience synchronization problems, and inter-channel interference (ICI). Therefore, the spatial modulation (SM) technique has been proposed as a low complex solution to the above problems [13]. The optical SM has multiple LEDs, which are light sources, at the transmitter side. At each time slot, binary information is used to activate only one LED out of all available LEDs to transmit a signal. Hence, a unique binary sequence is assigned to each LED. The corresponding LED is activated when the incoming binary sequence at each time slot matches the pre-located sequence, and the index of active LED is called a spatial symbol. Hence, the LED index is used as an additional dimension to send information, which helps to improve the SE. In [14], [15], authors show that the SM outperforms performance benchmarks in some indoor frameworks.

Many works on optical SM have been proposed in the literature [14], [16]–[18] to improve the SE of VLC systems. Most of these studies consider uniformly distributed transmitted symbols, i.e., the transmitted symbols belonging to the signal-
Probabilistic shaping (PS) can induce non-uniform distribution of the input signal [22]–[24]. One of the practical PS methods is the probabilistic amplitude shaping (PAS) architecture, where the distribution dematching is performed after the forward error correction (FEC) decoding, i.e., reverse concatenation [23], [24]. Unlike normal (non-reverse) concatenation architecture, where the distribution dematching is performed before the FEC decoding, reverse concatenation is simple to implement for long codes and has very low frame error rate (FER) [25]. The three main properties of PAS architecture that differentiate it from other PS schemes are the following: first, it integrates existing FEC with shaping; second, it adapts the rate by changing the input distribution, while the FEC part remains unchanged; third, it achieves the channel capacity [26]. However, in PAS architecture, the optimal probability mass function (PMF) should be symmetric, and the signal should be non-unipolar [23]. Therefore, PAS schemes designed in [23], [26], [27] cannot be directly considered for VLC with IM/DD.

Some works have implemented PS in optical communication. For example, in [28], probability density function (PDF) of the input signal has been optimized to maximize the secrecy rate of SM based indoor VLC; however, a specific coded modulation scheme is not considered. Authors in [29] consider uniform, exponential, Maxwell–Boltzmann, and Pareto source distributions for shaping color shift keying (CSK) symbols to maximize the received signal-to-noise ratio (SNR) gain. However, in [29], the channel coding and the reverse concatenation architecture, which are essential for PS based system due to the noise sensitivity of the distribution dematching, are not considered [23], [24]. Implementing the reverse concatenation architecture in optical communication is a problem because the optical signal is unipolar and has a non-symmetric optimal PMF. An adaptive coded modulation scheme with PS for free space optics (FSO) communication is proposed in [30], [31]. The results from the above works show that PS gives the performance gain. Hence, to further increase SE in VLC systems, we propose a novel adaptive coded SM scheme which deploys PS.

A. Main Contribution

In this work, a novel adaptive coded SM scheme with PS is proposed to improve the SE of VLC. The scheme allows increasing the throughput while using cheaper LEDs with limited bandwidth. The proposed scheme considers the optical power and non-negative \( M \)-pulse amplitude modulation (PAM) signaling constraints and can approach the IM/DD VLC channel capacity with fine granularity. At the transmitter side of the proposed scheme, a constant composition distribution matcher (CCDM) for PS is concatenated with an FEC encoder. On the receiver side, the reverse concatenation technique takes place where distribution dematching is performed after FEC decoding. In this case, the parity bits at the output of the FEC encoder are uniformly distributed, regardless of the distribution of the bits at its input. The parity bits together with some part of uniformly distributed information bits have to be converted either to spatial or constellation symbols. Thus, the spatial and constellation symbols cannot be simultaneously probabilistically shaped. In this regard, we propose that the CCDM probabilistically shapes either constellation or spatial symbols depending on the user’s location and optical signal-to-noise ratio (OSNR). Hence, part of information bits is non-uniformly distributed while the remained part of information bits and parity check bits generated by FEC encoder are uniformly distributed. In this paper, we calculate the maximum achievable rate (AR) (i.e., capacity) of the proposed scheme at various OSNRs using both practical low-complex bit metric decoders (BMDs) and symbol metric decoders (SMDs).

The main contributions of this work are summarized as follows:

- We design a novel adaptive coded SM scheme with PS, where depending on the user’s location and OSNR either the input PMF of unipolar \( M \)-PAM constellation symbols and spacing between them are optimized or the input PMF of spatial symbols is optimized, to approach the SM VLC channel capacity with IM/DD.
- We convexify the optimization problem and provide an algorithm to find the optimal distribution of the input symbols that leads to the maximum AR of the proposed scheme using optimized FEC coding rate, SMDs and BMDs.
- The transmission rate, channel coding rate, and the proposed scheme (constellation or spatial shaping of the symbols) are adapted.
- We provide extensive numerical and simulation results to demonstrate the proposed scheme performance compared to uniform signaling and the SM VLC channel capacity bound with IM/DD.

B. Paper Organization

The rest of the paper is organized as follows. Section II describes the signal model of SM based indoor VLC system. In section III, we introduce the proposed adaptive coded SM scheme for VLC systems, and the capacities for SMD and BMD are obtained. In section IV, an algorithm is designed to find optimal input PMFs for SMD and BMD cases. Numerical results are provided in section V, and finally, the work is concluded in section VI.

C. Notations

The following notations are used throughout this paper. Small bold letters \( \mathbf{x} \) and capital bold letters \( \mathbf{X} \) represent row vectors and matrices, respectively. The transpose of vectors or matrices is represented by \((\cdot)^T\). The probability of the event is denoted as \( P(\cdot) \). The expectation of a random variable (r.v.) is denoted as \( E(\cdot) \). The logarithm function with base two is denoted as \( \log(\cdot) \), and it acts as entry-wise on vectors. The natural logarithm is denoted as \( \ln(\cdot) \), and the optimal solution is denoted by \((\cdot)^*\).
For indoor VLC, an average optical power constraint of the input signal is
\[
\mathbb{E}[S] = \sum_{i=1}^{M} s_i p_i = \sum_{j=1}^{\Delta} \Delta i p_i = s p^T \leq P,
\]
(3)
where \(s = [\Delta, 2\Delta, \ldots, M\Delta]\), and \(P\) denotes the average optical signal power limit. In this paper, we use the instantaneous OSNR defined as \(hP/\sigma\) rather than electrical SNR because OSNR is more relevant for VLC systems [1], [21], [37].

A. Optical Channel Gains

In this work, the MISO VLC channels are simulated inside a 4.0 m × 4.0 m × 3.0 m room shown in Fig. 1. The PD is located at the height of \(z = 0.75\) m (e.g., the height of a table). Since transmitting LEDs perform illumination and communication simultaneously, an equal distribution of the light inside the room and a good communication performance should be guaranteed. In [15], authors investigate different static setups with varying spacing between the LEDs. According to [15], if the spacing between the LEDs is small, the channel gains are quite similar, whereas if the spacing gets larger, the differences between the channel links increase. Since the performance of SM depends on the differences between the channel links, SM is more robust and has a lower bit error rate (BER) performance if the spacing gets larger [15]. In this paper, we consider only one scenario when transmitting LEDs are equidistantly distributed on the ceiling because, in that case, the spacing between the single LEDs is the largest, and the light is distributed equally inside the room.

The optical channel gain entries \(h_{ij} \in \mathbb{H}\) are calculated as [15]
\[
h_{ij} = \begin{cases}
\frac{(k+1)A}{2\pi d_j^2} \cos^k (\phi_j) \cos (\psi_j), & 0 \leq \psi_j \leq \Psi_c, \\
0, & \psi_j > \Psi_c,
\end{cases}
\]
(4)
where \(k = \frac{-\ln(2)}{\ln(\cos(\Phi_{1/2}))}\) is the Lambert’s mode number, \(\Phi_{1/2}\) is the half-power angle of LED, \(\phi_j\) and \(\psi_j\) are the angle of emergence and angle of incidence from \(j\)th LED to the PD with respect to their normal axes, respectively, \(d_j\) is the distance from \(j\)th LED to the PD, and \(\Psi_c\) and \(A\) are the field-of-view and the area of PD, respectively.

B. Mutual Information Analysis

If the transmission rate of the modulation scheme is less than its AR, then reliable VLC with the arbitrarily low probability of error can be achieved. The AR of the most modulation schemes depends on the channel input signal distribution. Therefore, in order to get the maximum AR for SM with \(M\)-PAM (here referred to as the capacity), the input distribution should be optimized.

1Please note that without loss of generality, we consider only a Line-of-Sight (LOS) channel model as in [13]–[15], [38]. Since our proposed system is agnostic to the channel model, a channel with some reflections as in [34] can also be used.
Entropy can be written as symbols are independent. Hence, the entropy and conditional AR

\[ C_{\text{SMD}}(\Delta^*, \mathbf{p}^*, \mathbf{q}^*) = \maximize_{\Delta > 0, \mathbf{p}, \mathbf{q}} \mathbb{I}(X; Y) \]  

subject to \[ s^T \mathbf{p} \leq P \]  

\[ \sum_{i=1}^{M} p_i = 1 \]  

\[ \sum_{j=1}^{N} q_j = 1 \]  

\[ p_i \geq 0, \forall i \in \{1, 2, \ldots, M\} \]  

\[ q_j \geq 0, \forall j \in \{1, 2, \ldots, N\} \].

where \( \mathbb{I}(X; Y) = [\mathcal{H}(X) - \mathcal{H}(X|Y)] \) is the mutual information (MI) between received signal \( Y \) and transmitted signal \( X \). In this work, we consider that the spatial and constellation symbols are independent. Hence, the entropy and conditional entropy can be written as

\[ \mathcal{H}(X) = -\int_{-\infty}^{\infty} f_k \log(f_k) \, dk = -\sum_{i=1}^{M} \sum_{j=1}^{N} p_i q_j \log(p_i q_j) \]  

\[ \mathcal{H}(X|Y) = -\int_{-\infty}^{\infty} g_y(y) \sum_{k=1}^{K} g_{X|Y}(x_k|y) \log(g_{X|Y}(x_k|y)) \, dy \]  

where PDF of \( Y \) and conditional PDF of \( X|Y \) can be derived as

\[ g_y(y) = \sum_{k=1}^{K} f_k \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-x_k)^2}{2\sigma^2}\right) \]  

\[ = \sum_{i=1}^{M} \sum_{j=1}^{N} p_i q_j \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-h_j s_i)^2}{2\sigma^2}\right) \]  

\[ g_{X|Y}(x_k|y) = \frac{g_y(y|x_k) g_x(x_k)}{g_y(y)}. \]

Hence, the conditional entropy is shown below as

\[ \mathcal{H}(X|Y) = -\int_{-\infty}^{\infty} f_k \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-x_k)^2}{2\sigma^2}\right) \]  

\[ \log\left( \sum_{k=1}^{K} f_k \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-x_k)^2}{2\sigma^2}\right) \right) \, dy \]  

\[ = \int_{-\infty}^{\infty} \sum_{i=1}^{M} \sum_{j=1}^{N} p_i q_j \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-h_j s_i)^2}{2\sigma^2}\right) \]  

\[ \log\left( \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{i'=1}^{M} \sum_{j'=1}^{N} p_i' q_j' \exp\left(-\frac{(y-h_i' s_j')^2}{2\sigma^2}\right) \right) \, dy. \]

As it can be seen, the rate depends on the optimization parameters \( \Delta > 0, \mathbf{p}, \mathbf{q} \).

It is clear that the highest MI in (5) is achieved by probabilistically shaping constellation and spatial symbols simultaneously, i.e., \( C_{\text{SMD}}(\Delta^*, \mathbf{p}^*, \mathbf{q}^*) \). However, as we stated earlier in section I, our proposed novel adaptive coded SM scheme with PS cannot perform this simultaneous PS because of the uniform parity bits generated by FEC, non-symmetric optimal PMF, and unipolar M-PAM signaling. Therefore, in order to determine whether it is better to probabilistically shape constellation or spatial symbols, we have to analyze (5) for different scenarios.

Fig. 2 illustrates the behavior of maximum AR as a function of OSNR for \( M = 8 \) and \( N = 8 \). In Fig. 2a and Fig. 2b, the user’s locations are close to the center of the room with coordinates \((x=0.01, y=0.05, z=1)\) and far from the center of the room with coordinates \((x=1.5, y=0.5, z=1)\), respectively. According to Fig. 2, \( C_{\text{SMD}}(\Delta^*, \mathbf{p}^*, \mathbf{q}^*_u) \), where
q_u \triangleq [1/N, 1/N, \ldots, 1/N]$, considers that only constellation symbols are probabilistically shaped. In $C_{\text{SMD}}(\Delta_u, p_u, q_u^*)$, where $\Delta_u = \frac{2M}{M+1}$ and $p_u \triangleq [1/M, 1/M, \ldots, 1/M]$, only spatial symbols are probabilistically shaped. $C_{\text{SMD}}(\Delta_u, p_u, q_u)$ considers that both spatial and constellation symbols are uniformly distributed. As can be seen from Fig. 2a, for a certain range of OSNR, it is better to use the scheme with PS constellation symbols when the user is almost at the center of the room since it gives higher maximum AR while, according to Fig. 2b, the scheme with PS spatial symbols is more efficient when the user is far from the center of the room. It happens because VLC channels gains depend on the location of the user. For example, when the user is at the center, some VLC channels gains are almost the same. Therefore, in order to achieve maximum capacity, we need to develop a coded modulation scheme, where either spatial or constellation symbols are probabilistically shaped depending on the location of the user inside the room and OSNR.

### III. Proposed Adaptive SM Scheme with Probabilistic Shaping for Indoor VLC

In order to approach the capacity, a novel adaptive coded SM scheme that improves the SE of indoor VLC by probabilistically shaping the distribution of the input signal is proposed. In the scheme shown in Fig. 3a, the constellation symbols are non-uniformly distributed, and the spatial symbols are uniformly distributed. Therefore, we will call it a scheme with probabilistically shaped constellation symbols (ProCon). In the scheme shown in Fig. 3b, on the contrary, constellation symbols are uniformly distributed, and spatial symbols are non-uniformly distributed. Hence, we will refer to it as a scheme with probabilistically shaped spatial symbols (ProSpa). The channel gains are available at both transmitter and receiver. The proposed ProCon and ProSpa schemes are composed of source, serial-to-parallel converter, distribution matching (DM), symbols-to-bits converter, redundancy bits generating parity matrix $P$ (i.e., FEC encoder), multiplexer, and bits-to-symbols converter. The channel coding at the encoder is performed to reliably extract information bits from the noisy received signal at the decoder. In the rest of the section, we describe the encoders and decoders of ProCon and ProSpa. Moreover, the ARs and optimal feasible rates of the schemes are discussed.

#### A. Encoder

1) ProCon Scheme: The source outputs $k + \alpha n \log(N)$ information bits, where $n$ is the frame size (the number of channel uses over a discrete-time VLC channel), $k = n \mathcal{H}(S)$ and $\alpha$ (will be quantified later in Section III-C) represents the fraction of spatial bits used for information. For the encoder in Fig. 3a, one part of the information bits is converted into probabilistically shaped unipolar $M$-PAM symbols, while the other part together with parity bits are kept as uniformly distributed spatial bits. The optimized distribution, $p^*$ and symbol spacing, $\Delta^*$, are computed as in Algorithm 1, presented in Section IV-A1, to achieve the maximum reliable transmission rate for a given OSNR. The DM, described in Section III-B, transforms the uniformly distributed bits, $d_i \in [0, 1]^k$, into independent, identically distributed (i.i.d.) unipolar $M$-PAM symbols, $s \in \mathbb{S}^{n \alpha \frac{\log(M)}{n}}$, with $p^*$ and $\Delta^*$. The fraction of spatial bits used for information, $d_i \in [0, 1]^{n\alpha \frac{\log(M)}{n}}$, and the bits from the probabilistically shaped $M$-PAM symbols, $b_i \in [0, 1]^{n \log_2(M)}$, are encoded by the parity matrix $P$ with a channel coding rate $R_{\text{FEC}}$ (derived in Section III-C). The output of the parity matrix is uniformly distributed parity bits, $b_{\text{par}} \in [0, 1]^{(1-\alpha) n \log_2(N)}$. After that, multiplexer outputs uniformly distributed spatial bits, $b_h \triangleq [d_i, b_{\text{par}}] \in [0, 1]^{n \log_2(N)}$.

In LED mapper block in Fig. 3a, a group of $\log(N)$ bits from $b_h$ is used to activate only one LED out of $N$ available LEDs to transmit constellation symbol from $s$.

2) ProSpa Scheme: The source outputs $k + \bar{\alpha} n \log(M)$ uniformly distributed information bits, where $k = n \mathcal{H}(S)$ and $\bar{\alpha}$ is the fraction of constellation bits used for information. The encoder shown in Fig. 3b converts one part of information bits into probabilistically shaped spatial symbols and keeps the other part of information bits together with the parity bits as uniformly distributed constellation bits. Capacity-achieving distribution, $q^*$, is computed to achieve the highest reliable transmission rate for a given OSNR. The DM described in section III-B transforms the uniformly distributed bits, $d_i \in [0, 1]^k$, into spatial symbols, $\hat{h} \in \mathbb{H}^{n \alpha \frac{\log(M)}{n}}$, that are i.i.d. according to $q^*$. The bits from the probabilistically shaped spatial symbols, $\hat{b}_n \in [0, 1]^{n \log_2(N)}$, and the fraction of constellation bits used for information bits, $d_i \in [0, 1]^{n \log_2(M)}$, are encoded by the parity matrix $P$ of a channel coding rate $R_{\text{FEC}}$ (derived in Section III-C). The output of parity matrix is uniformly distributed parity bits, $b_{\text{par}} \in [0, 1]^{(1-\bar{\alpha}) n \log_2(N)}$. After that, multiplexer outputs uniformly distributed constellation bits, $b_h \triangleq [d_i, b_{\text{par}}] \in [0, 1]^{n \log_2(M)}$. These bits are then, mapped to uniformly distributed unipolar $M$-PAM symbols, $s \in \mathbb{S}^n$. In the LED mapper block in Fig. 3b, a group of $\log(N)$ bits from $b_h$ is used to activate only one LED out of $N$ available LEDs to transmit one constellation symbol from $s$. To transmit another constellation symbol, another LED may get activated as per optimized PMF of spatial symbols, $q^*$.

#### B. Distribution Matching

In the literature, there are various designs of variable length distribution matchers [23], [40], [41]. However, these matchers can cause synchronization loss, overflow and error propagation [41]. Hence, in this work, we use a fixed-length matcher CCDM proposed in [24]. The CCDM outputs a fixed-length sequence of probabilistically shaped symbols from a fixed-length sequence of uniformly distributed bits. For CCDM, all output sequences have an identical empirical distribution. Hence, the target distribution is achieved, to some extent, by every output sequence. In this paper, we use $n$-type CCDM, which means that the target distribution is quantized such that the probability of any symbol from the constellation is an integer multiple of $1/n$, where $n$ is the frame size. To be more specific, the PMF of $S$, $p^*$, is distributed in the following form $\hat{p} \triangleq [u_1/n, u_2/n, \ldots, u_n/n]$, where $n_i$ represents the number of times the $i$th symbol appears in output sequence,
and \( \sum_{i=1}^{M} n_i = n \). For very large output sequence length, the quantization error is negligible, i.e., \( \lim_{n \to \infty} \hat{p} = \hat{p}^* \).

The number of input bits, \( k \), that need to be mapped to the output sequence of \( n \) symbols can be calculated as

\[
k = \left\lfloor \log \left( \frac{n}{n_1!n_2! \cdots n_M!} \right) \right\rfloor .
\]

The floor operator \( \lfloor \cdot \rfloor \) is required because the cardinality of the set of all permutations may not be a power of two. For asymptotically large output sequence length, we observe the convergence of the CCDM rate, which is the number of input bits per output symbol, to the source entropy such that \( \lim_{n \to \infty} \frac{k}{n} = H(S) \). Again, the rate loss is very negligible for a very large output sequence length.

### C. Transmission Rate

At the beginning of section III, we stated that we use binary FEC encoder with channel coding rate \( R_{\text{FEC}} \) in order to achieve reliable communication. According to the proposed ProCon scheme in Fig. 3a, the channel coding rate can be derived in terms of \( M, N \) and \( \alpha \) as

\[
R_{\text{FEC}} = \frac{n \log(M) + \alpha n \log(N)}{n \log(M) + \alpha n \log(N) + (1 - \alpha)n \log(N)}
= \frac{\log(M) + \alpha \log(N)}{\log(M) + \alpha \log(N)},
\]

(12)
to ensure that the number of parity bits plus the number of spatial bits used for information equals the overall number of spatial bits, whereas the number of spatial symbols should be equal to the number of constellation symbols. The parameter \( \alpha \) is used to provide more control over the transmission rates of the proposed schemes. For a given \( R_{\text{FEC}} \), the fraction \( \alpha \) is given by

\[
\alpha = \frac{R_{\text{FEC}}(\log(M) + \log(N)) - \log(M)}{\log(N)}
\]

(13)

where \( 0 \leq \alpha \leq 1 \). Hence, the minimum coding rate that we can use in ProCon scheme is

\[
R_{\text{FEC}} = \frac{\log(M)}{\log(M) + \log(N)}.
\]

(14)

Now, the transmission rate of ProCon scheme can be written as

\[
T(R_{\text{FEC}}, p) = \frac{k + \alpha n \log(N)}{n} = \frac{nH(S) + \alpha n \log(N)}{n}
= H(S) + \alpha \log(N),
\]

(15)
where $k + \alpha n \log(N)$ is the number of information bits. After substituting $\alpha$ in (15) with (13),

$$T(\bar{R}_{\text{FEC}}, p) = H(S) + R_{\text{FEC}} \log(M) \log(N) - \log(M).$$

The transmission rate of proposed ProSpa scheme in Fig. 3b can be derived by following the same procedure, and it is equal to

$$\bar{T}(\bar{R}_{\text{FEC}}) = H(H) + \bar{R}_{\text{FEC}} \log(N) \log(N) - \log(N),$$

where

$$\bar{R}_{\text{FEC}} = \frac{n \log(N) + \alpha n \log(M)}{n \log(N) + \alpha n \log(M) + (1 - \alpha)n \log(M)} \frac{\log(N) + \alpha \log(M)}{\log(M) + \log(N)},$$

and $0 \leq \alpha \leq 1$.

### D. Bit Metric Decoder

On the receiver side, we apply the reverse concatenation method, where the distribution dematching is performed after the FEC decoding, in order to prevent the common errors burst problem after the distribution dematching, because of the erroneous symbols received from the channel [42]. For the FEC decoding, the optimal SMD can be used to achieve $C_{\text{SMD}}(\Delta^*, p^*, q_0)$ and $C_{\text{SMD}}(\Delta_u, p_u, q^*)$. Nevertheless, SMD has very high computational complexity. Therefore, we propose to use BMD, which has low complexity and can provide a rate close to the capacity of the scheme (i.e., $C_{\text{SMD}}$). For BMD with soft decisions, the maximum a posteriori probability (MAP) of each bit level is computed given the received symbol.

The binary representation of the transmitted symbol (constellation and spatial) at time instant $t$ is $b(X_t) = [B_{t,1}^L, B_{t,2}^L, \ldots, B_{t,L\log(K)}]$, where $B_{t, l}^L \in \{0, 1\}$ is a r.v representing the $l$th bit level of the $t$th transmitted symbol, and $b(\cdot)$ is a function that maps symbols to bits. The binary vector mapping for X indicating bits used to select the intensity level and the LED is

$$B_x = b(X) \triangleq [B_x, B_h] = [B_1^L, B_2^L, \ldots, B_{L\log(K)}^L],$$

where $B_x$ and $B_h$ are the binary vectors representing constellation and spatial symbols, respectively. The $B_l^L \in \{0, 1\}$, $B_h^L \in \{0, 1\}$, and $B_{t, l}^L \in \{0, 1\}$ are the r.v.s representing the bit level of the constellation symbol, spatial symbol and the transmitted symbol, respectively. Please note that $B_{l, t}^L = B_{t, l}^L$ for $l \in L_s^L \triangleq \{1, 2, \ldots, \log(M)\}$ while $B_{l, t}^L = B_{t, l}^L$ for $l \in L_s^L \triangleq \{\log(M) + 1, \log(M) + 2, \ldots, \log(M) + \log(N)\}$. Hence, the MAP of the $l$th bit level given the received signal at time instant $t \in \{1, 2, \ldots, n\}$, i.e., $y_t$, is derived as

$$L_{l, t} = \log \left( \frac{\sum_{y_t \in X_l^h} f_{Y_t|X_t}(y_t|x) \mathbb{P}(X_t = x)}{\sum_{y_t \in X_l^h} f_{Y_t|X_t}(y_t|x) \mathbb{P}(X_t = x)} \right) = \log \left( \frac{\sum_{x \in X_l^h} f_{Y_t|X_t}(y_t|x) \mathbb{P}(X_t = x)}{\sum_{x \in X_l^h} f_{Y_t|X_t}(y_t|x) \mathbb{P}(X_t = x)} \right),$$

where $X_l^h$ and $X_l^h$ are the sets that include all the values of $x$ whose $l$th bit level representation equals 0 and 1, respectively.

The MAP of the ProCon is

$$L_{l, t} = \log \left( \frac{\sum_{y_t \in Y_l^h} f_{Y_t|X_t}(y_t|x) \mathbb{P}(X_t = x)}{\sum_{y_t \in Y_l^h} f_{Y_t|X_t}(y_t|x) \mathbb{P}(X_t = x)} \right),$$

for $l \in L_s^h$

$$L_{l, t} = \log \left( \frac{\sum_{y_t \in Y_l^h} f_{Y_t|X_t}(y_t|x) \mathbb{P}(X_t = x)}{\sum_{y_t \in Y_l^h} f_{Y_t|X_t}(y_t|x) \mathbb{P}(X_t = x)} \right),$$

for $l \in L_s^h$

where $\mathbb{P}\{H_t = h_j\} = 1/N, S_l^h$ and $S_l^h$ are the sets that include constellation symbols whose $l$th bit level, $B_l^h$, equals to 0 and 1, respectively. Similarly, the sets $H_l^h$ and $H_l^h$ include spatial symbols, whose $l$th bit level, $B_l^h$, equals to 0 and 1, respectively.

Since $L_{l, t}$ is a sufficient statistic to estimate the bit level $l$ from the channel output $y_t$ [23], MAPs are provided to the FEC soft decoder for error correction. Hence, the output of the FEC soft decoder shown in Fig. 3a is $\hat{b}_t, \hat{d}_t \in [0, 1)^{n \log(M) + \alpha n \log(N)}$. After that, the corresponding $M$-PAM symbols, $s$, are obtained from the first $n \log(M)$ estimated bits of the FEC soft decoder output, $\hat{b}_t$. The distribution dematching maps these symbols into their $k$ associated bits, $\hat{d}_t$, belonging to the first part. The remained $\alpha n \log(N)$ estimated bits of the FEC soft decoder output, $\hat{d}_t$, belong to the second part, i.e., the information bits that are not probabilistically shaped. The first and second parts are combined to recover information bits.

The ProSpa scheme has a similar decoder as the ProCon scheme, but the MAP is slightly different. It can be written as

$$\hat{L}_{l, t} = \log \left( \frac{\sum_{H_t \in H_l^h} f_{Y_t|H_t}(y_t|x, h_j) \mathbb{P}(H_t = h_j)}{\sum_{H_t \in H_l^h} f_{Y_t|H_t}(y_t|x, h_j) \mathbb{P}(H_t = h_j)} \right),$$

for $l \in L_s^h$

$$\hat{L}_{l, t} = \log \left( \frac{\sum_{H_t \in H_l^h} f_{Y_t|H_t}(y_t|x, h_j) \mathbb{P}(H_t = h_j)}{\sum_{H_t \in H_l^h} f_{Y_t|H_t}(y_t|x, h_j) \mathbb{P}(H_t = h_j)} \right),$$

for $l \in L_s^h$

where $\mathbb{P}\{S_t = s_i\} = 1/M, \hat{L}_h^h \triangleq \{1, 2, \ldots, \log(N)\}, \hat{L}_s^h \triangleq \{\log(N) + 1, \log(N) + 2, \ldots, \log(N) + \log(M)\}$.

Generalized mutual information (GMI) is a metric to quantify an achievable information rate for receivers using mismatched decoding while MI is the highest achievable information rate for coded modulation [42], [43]. When PS is added to coded modulation, an achievable information rate is BMD rate [23], [39]. Therefore, in our proposed scheme, we are interested in successfully decoding at a transmission rate close to $R_{\text{SMD}}$ by using a BMD. In other words, at the receiver side, we use a bitwise decoder [23], [39]. For ProCon scheme, the AR with BMD coincides with the GMI, and it
can be written as
\[
R_{BMD}(\Delta, \mathbf{p}, \mathbf{q}_u) = \left[ H(B_x) - \sum_{l=1}^N H(B_i^l | Y) \right] +
\]
\[
= \left[ H(B_x) - \sum_{l=1}^N H(B_i^l | Y) \right] + \left[ H(B_h) - \sum_{l=1}^N H(B_i^h | Y) \right] +
\]
\[
= \left[ H(S) - \sum_{l=1}^N H(B_i^l | Y) \right] + \left[ H(H) - \sum_{l=1}^N H(B_i^h | Y) \right],
\]
where \([\cdot]^+\) denotes max(0, \cdot). The conditional entropy functions due to one-to-one mapping are derived as
\[
H(B_i^l | Y) = \sum_{\infty < s_l \in \{0, 1\} \subseteq \mathbb{F}} \sum_{j=1}^M \frac{P(s_i)}{N\sqrt{2\pi \sigma^2}} \exp \left( -\frac{(y-h_js_l)^2}{2\sigma^2} \right)
\]
\[
\log \left( \sum_{j=1}^M \frac{P(s_i)}{N} \exp \left( -\frac{(y-h_js_l)^2}{2\sigma^2} \right) \right) dy
\]
and
\[
H(B_i^h | Y) = \sum_{\infty < s_h \in \{0, 1\} \subseteq \mathbb{F}} \sum_{j=1}^M \frac{P(s_i)}{N\sqrt{2\pi \sigma^2}} \exp \left( -\frac{(y-h_js_h)^2}{2\sigma^2} \right)
\]
\[
\log \left( \sum_{j=1}^M \frac{P(s_i)}{N} \exp \left( -\frac{(y-h_js_h)^2}{2\sigma^2} \right) \right) dy.
\]

For BMD, the maximum AR of ProCon scheme can be obtained by maximizing \(R_{BMD}(\Delta, \mathbf{p}, \mathbf{q}_u)\) provided that the average optical power constraint is satisfied. Hence, the maximum AR of ProCon scheme with BMD is denoted as \(C_{BMD}(\Delta^*, \mathbf{p}^*, \mathbf{q}_u) \triangleq R_{BMD}(\Delta^*, \mathbf{p}^*, \mathbf{q}_u)\). Even though the AR with BMD is less than that obtained with SMD, the loss in the rate is negligible, as shown in section V. Therefore, in section IV, we maximize \(R_{SMD}(\Delta, \mathbf{p}, \mathbf{q}_u)\). For ProSpa scheme, the AR with BMD is derived using similar equations from (23) to (25) where we need to maximize \(R_{BMD}(\Delta_u, \mathbf{p}_u, \mathbf{q})\) instead of \(R_{BMD}(\Delta, \mathbf{p}, \mathbf{q}_u)\).

### IV. Practical Transmission Rate Adaptation

This section proposes an algorithm to get the highest achievable transmission rate of ProCon scheme for any given OSNR by optimizing the input distribution \(\mathbf{p}\) and symbol spacing \(\Delta\). The transmission rate is adapted by adjusting the channel coding rate of the FEC encoder, \(R_{FEC}\), and the source entropy.

#### A. Optimization Problem

The maximum transmission rate of ProCon scheme for any OSNR can be found by solving the following optimization problem

\[
\text{maximize}_{\Delta > 0, \mathbf{p}} \quad R_{FEC}(\mathbf{p})
\]

subject to

\[
\begin{align*}
\sum_{i=1}^M p_i & = 1 \\
p_i & \geq 0, \quad \forall i \in \{1, 2, \ldots, M\} \\
R_{FEC}(\mathbf{p}) & \leq R_{SMD}(\Delta, \mathbf{p}, \mathbf{q}_u) - R_{\text{backoff}}
\end{align*}
\]

where \(R_{\text{backoff}}\) is the back-off rate accounting for the AR reduction due to BMD, which is set in Algorithm 1. The last constraint in (26) is required to ensure that the transmission rate is feasible for given OSNR.

Unfortunately, solving (26) by optimizing \(\mathbf{p}\) and \(\Delta\) jointly is involved. Hence, we propose an iterative algorithm that alternately optimizes one variable while fixing the other. For a fixed \(\Delta\), the problem (26) is non-convex due to the last constraint. However, it can be convexified with the help of a method inspired by the classical Blahut-Arimoto algorithm (BBA), which uses a surrogate function for convexification [44]. In this regard, we can rewrite \(R_{SMD}\) as

\[
R_{SMD}(\Delta, \mathbf{p}, \mathbf{q}_u) = g_1(\mathbf{p}) - g_2(\mathbf{p}),
\]

where

\[
\begin{align*}
\Phi(\mathbf{p}, \mathbf{q}_u) & \triangleq \sum_{i=1}^M p_i \log \left( \frac{N}{p_i} \right) = H(S) + \log(N) \\
g_1(\mathbf{p}) & \triangleq \sum_{i=1}^M p_i \log \left( \frac{N}{p_i} \right), \\
g_2(\mathbf{p}) & \triangleq \sum_{i=1}^M p_i W(\mathbf{p}, i),
\end{align*}
\]

and

\[
W(\mathbf{p}, i) \triangleq \sum_{j=1}^N \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left( -\frac{(y-h-js_l)^2}{2\sigma^2} \right)
\]

\[
\log \left( \sum_{j=1}^M \sum_{l=1}^N p_i \exp \left( -\frac{(y-h-js_l)^2}{2\sigma^2} \right) \right) dy.
\]

Let us introduce a surrogate function, \(\Phi(\hat{\mathbf{p}}, \mathbf{p})\), that approximates \(R_{SMD}(\Delta, \mathbf{p}, \mathbf{q}_u)\) around \(\mathbf{p} = \hat{\mathbf{p}}\). More precisely, we set

\[
\Phi(\hat{\mathbf{p}}, \mathbf{p}) \triangleq g_1(\mathbf{p}) - g_2'(\hat{\mathbf{p}}, \mathbf{p}),
\]

where

\[
g_2'(\hat{\mathbf{p}}, \mathbf{p}) \triangleq \sum_{i=1}^M p_i W(\hat{\mathbf{p}}, i).
\]

From the relative entropy properties, we have \(g_2'(\hat{\mathbf{p}}, \mathbf{p}) \geq g_2(\mathbf{p})\) with equality if \(\hat{\mathbf{p}} = \mathbf{p}\) [44].

Now, for fixed \(\Delta\) and initial probability vector \(\mathbf{p}\), the
Lemma IV.1 (Optimal solution for (33)). For the given transmit power, $P$, back off rate, $R_{\text{backoff}}$, constellation spacing, $\Delta$, and initial distribution, $\hat{p}$.

\[
p^*_\Delta = \begin{cases} 
2^{-\eta/\Delta} & \text{if } \sum_{i=1}^{M} A_i 2^{-\eta/\Delta} = 0 \text{ and } \sum_{i=1}^{M} B_i 2^{-\eta/\Delta} < 0 \\
\frac{1}{M} & \text{if } \sum_{i=1}^{M} (\Delta - M) P < 0 \text{ and } \sum_{i=1}^{M} W(\hat{p},i) - MC < 0 \\
\frac{1}{M} \sum_{i=1}^{M} 2^{-W(\hat{p},i)\tau - \eta/\Delta} & \text{if } \sum_{i=1}^{M} A_i 2^{-W(\hat{p},i)\tau - \eta/\Delta} = 0 \text{ and } \sum_{i=1}^{M} B_i 2^{-W(\hat{p},i)\tau - \eta/\Delta} = 0 \\
\frac{1}{M} \sum_{i=1}^{M} 2^{-W(\hat{p},i)\tau} > 0 & \text{if } \sum_{i=1}^{M} A_i 2^{-W(\hat{p},i)\tau} < 0 \text{ and } \sum_{i=1}^{M} B_i 2^{-W(\hat{p},i)\tau} = 0, 
\end{cases}
\]

where $A_i \triangleq \Delta - P$, $B_i \triangleq W(\hat{p},i) - C$, $C \triangleq (1 - R_{\text{FEC}}) \log(M) + \log(N) - R_{\text{backoff}}$, $\eta$ and $\tau$ can be any positive number.

Proof. See the Appendix.

Algorithm 1 Proposed algorithm optimizing $p$ to solve (26) for a fixed $\Delta$

1: Input $\Delta$, $R_{\text{FEC}}$, $\sigma$ and $\gamma_{st}$, $\% \gamma_{st}$ is the stopping criteria tolerance
2: Initialize $\hat{p}$ as arbitrary vector that satisfies probability constraints
3: repeat
4: Step 1: Calculate $W(\hat{p},i)$ for each $i \in \{1, 2, \ldots, M\}$
5: Step 2: Calculate new input probability $p^*_{\Delta}$ according to Lemma IV.1
6: $\gamma_{c} = ||\hat{p} - p^*_{\Delta}||^2$
7: Update: $\hat{p} := p^*_{\Delta}$
8: until $\gamma_{c} < \gamma_{st}$
9: Output $p^*_{\Delta}$, $R_{\text{SMD}}(\Delta, p^*_{\Delta}, q_{\text{u}})$ and $R_{\text{BMD}}(\Delta, p^*_{\Delta}, q_{\text{u}})$

The optimization problem (26) can be convexified as

\[
p^*_{\Delta} = \max_{p} T(R_{\text{FEC}}, p) \quad (33a)
\]

subject to

\[
s^T p \leq P \quad (33b)
\]

\[
\sum_{i=1}^{M} p_i = 1 \quad (33c)
\]

\[
p_i \geq 0, \quad \forall j \in \{1, 2, \ldots, M\} \quad (33d)
\]

\[
T(R_{\text{FEC}}, p) \leq \Phi(\hat{p}, p) - R_{\text{backoff}} \quad (33e)
\]

Note that the constraint (33e) is not violated by constraint (33e) because $\Phi(\hat{p}, p) \leq \Phi(p, p)$ and $\Phi(p, p) = R_{\text{SMD}}(\Delta, p, q_{\text{u}})$. The problem (33) is convex, and it can be solved with Karush-Kuhn-Tucker (KKT) optimality conditions.

1) Optimization Algorithm: Algorithm 1 describes a procedure for finding capacity-achieving distribution for a fixed $\Delta$ with optimal FEC coding rate. From the power constraint in (3), the range of $\Delta$ can be derived as $0 \leq \Delta \leq P^4$. The golden section method is used to find optimal constellation spacing, $\Delta^*$, that maximizes $R_{\text{SMD}}$ for a fixed $p$. The corresponding optimal probabilities $p^* \triangleq p^*_{\Delta^*}$, which lead to the maximum feasible transmission rate of ProCon scheme, are found as algorithm 1.

B. FEC Coding Rate Adaptation

This work considers DVB-S2 receiver with low-density parity-check (LDPC) channel encoder, which has predefined coding rates. Algorithm 1 is run repeatedly for each supported FEC coding rate from the DVB-S2 receiver. Please note that, in proposed ProCon scheme, you can use only $R_{\text{FEC}} \geq R_{\text{FEC min}}$, defined in (14). The optimal $R_{\text{FEC}}$ for a given OSNR is the one that gives the highest achievable transmission rate for the proposed scheme.

The computational complexity of the ProCon scheme is mainly because of the computation of the capacity-achieving distribution. Let us assume that $k_1$ is the number of iterations until we get $\gamma_{c} < \gamma_{st}$ in Algorithm 1, $k_2$ is the number of iterations until we find the optimal $\Delta$ by the golden section algorithm, and $k_3$ is the number of channel coding rates supported by the FEC coder for given $M$-ary modulation and $N$ LEDs. Therefore, we need to solve $k_1 k_2 k_3$ convex problems to find the optimal distribution for given $M$ and $N$. For example, using the interior point method to solve the convex problems, the complexity of the proposed algorithm is $k_1 k_2 k_3 O(M^3)$ for ProCon scheme while it is $k_1 k_2 k_3 O(N^3)$ for ProSpa scheme. All the $M$-ary modulation orders supported by the encoder can be tested with the above procedure, and $M$-ary modulation that gives the maximum feasible transmission rate for given OSNR is selected.

V. NUMERICAL RESULTS

This section presents numerical results to assess the SE and FER performance of our proposed scheme. We choose FER rather than BER as a performance metric due to the error sensitivity of the distribution matching and dematching [23], [24]. FER is a more robust metric [23]. The main performance benchmarks are uniform signaling and the capacity of SM with $M$-PAM signaling. The optimal distribution obtained from Algorithm 1 for our proposed scheme is used in all numerical results. The LDPC DVB-S2 shown in section IV-B is used for channel coding. All the considered codes for DVB-S2 have the same block length of 64800 bits.

Fig. 4 depicts the transmission rates of proposed scheme and uniform-based scheme versus OSNR for two different locations of the users with $M = 8$, $N = 8$ and adaptive coding rates. In Fig. 4a, we compare the transmission rates of ProCon scheme and uniform-based scheme when the user...
is close to the center of the room. The FEC coding rate is adapted with the OSNR for both schemes. As can be seen from Fig. 4a, the spatial modulation capacity of 8-PAM signaling with $N = 8$ antennas is taken as a benchmark because it is regarded as an upper bound for the transmission rates of the proposed scheme. The optimal FEC coding rate, which maximizes the transmission rate of ProCon scheme and keeps it feasible, tends to be $R_{\text{FEC},\min}$ for low OSNR and increases with the OSNR until it reaches the highest supported practical FEC coding rate. Such behavior is resulted from the fact that $R_{\text{BMD}}(\Delta^*,p^*,q_u)$ is low at low OSNR region and increases with OSNR, and transmission rate of ProCon scheme, which increases with $R_{\text{FEC}}$, is feasible when it is lower or equal to $R_{\text{BMD}}$. For example, the optimal FEC coding rate at 4 dB is $1/2$ while the optimal FEC coding rate at 11 dB is $4/5$. The optimal FEC rate for the uniform case is selected to maximize the transmission rate, $R_{\text{FEC}}(\log(M) + \log(N))$, while keeping it lower than the capacity of the uniform

---

![Fig. 4: Transmission rate of the proposed scheme, spatial modulation capacity and uniform signaling vs OSNR with adaptive coding rates for a) ProCon and b) ProSpa.](image-url)
signaling, i.e., $C_{SMD}(\Delta_u, p_u, q_u)$. At transmission rate 1.33 bpcu, ProCon scheme operates within 0.05 dB and 0.2 dB from BMD and SMD capacities, respectively. In addition, it outperforms uniform signaling by 1 dB and 0.3 dB at rates 1.33 bpcu and 3 bpcu, respectively. The function $g_2(p)$ in (28) tends to zero for high OSNR. Thus, at high OSNRs, $R_{SMD}$ is just an entropy of the overall transmitted symbol, $X$. The transmission rate of the proposed scheme, $T(R_{FEC}, p)$, is also just the entropy of $X$ at high OSNRs because the channel coding rate, $R_{FEC}$, tends to 1 for high OSNR. For fixed optimal $\Delta^*$ and high OSNRs, the solution to (33) is uniform distribution because the entropy of the constellation symbols is maximum in that case. It explains why the gap between transmission rates of ProCon scheme and uniform based scheme decreases at high OSNRs for fixed $M$ and $N$. So we should switch to higher $M$ and $N$ for high OSNR.

Fig. 4b displays the OSNR gap of the transmission rate of the proposed ProSpa scheme to SMD, BMD capacities and the transmission rate of the uniform based scheme for $M=8$ and $N=8$ when the user is located far from the center of the room. The optimal FEC coding rates for ProSpa scheme and uniform based scheme are selected according to the same concept as in Fig. 4a. As can be seen, the optimal FEC rate for ProSpa scheme follows the same pattern as in proposed ProCon scheme: FEC rate increases with OSNR. At transmission rate 1.5 bpcu, ProSpa scheme operates within 0.1 dB from $C_{SMD}(\Delta_u, p_u, q_u)$. It outperforms the uniform based scheme by almost 2 dB and 1 dB at rates 1.5 bpcu and 4 bpcu, respectively. Similar theoretical reasoning as in Fig. 4a can be applied to explain why the gap between transmission rates of ProSpa scheme and uniform based scheme also decreases at high OSNRs for fixed $M$ and $N$.

Table V.1 depicts the performance gains in terms of OSNR at different transmission rates for proposed ProCon and ProSpa schemes relative to the uniform based scheme when the user is close to the center of the room and far from the center of the room, respectively. When the user is far from the center of the room, the channel gains (spatial symbols) are quite different, whereas, when the user is near the center of the room, the channel gains (spatial symbols) are pretty similar. So, the minimum distance between spatial symbols for the ProSpa scheme is larger than for the ProCon scheme. The rate is better for the larger minimum distance. It explains why the ProSpa scheme performs better with an increased transmission rate in Table V.1.

The optimized PMFs of probabilistically shaped symbols in proposed scheme for three different OSNRs are shown in Fig. 5. According to Fig. 5a, where we plot optimized PMF of constellation symbols in ProCon scheme when the user is located near to the center of the room, higher probabilities are assigned to symbols with lower amplitudes at low OSNRs. Hence, large inter symbol spacing $\Delta^*$ is possible at low OSNRs satisfying the average optical power constraint. However, for high OSNRs, the inter symbol spacing $\Delta^*$ is small since the distribution of constellation symbols is almost uniform. It happens because the MI approaches source entropy, which is maximized with uniform distribution, at asymptotically high OSNRs (see Fig. 2a).

The optimized probability distribution of spatial symbols in ProSpa scheme for three different OSNRs is illustrated in Fig. 5b when the user is located near to the edge of the room. At 0 dB, one spatial symbol has probability of almost 1 while other spatial symbols have very negligible probabilities. This means that no information is transmitted on spatial symbols at low OSNR since the source entropy, $H(H)$, is almost zero. Hence, for low OSNR region, it is better to use just IM/DD without conveying information data in spatial domain. At 10 dB, the probability distribution of spatial symbols is more or less equal; however, there is still some spatial symbols, whose probability is around zero. At 50 dB, the distribution of spatial symbols is uniform because, as we can see from Fig. 2b, the gap between $C_{SMD}(\Delta_u, p_u, q_u)$ and $C_{SMD}(\Delta_u, p_u, q_u)$ is very negligible at high OSNRs.

Fig. 6 displays the performance comparison between the proposed schemes with $M=8$, $N=8$, uniform based scheme, and asymmetrically clipped optical OFDM (ACO-OFDM) with index modulation [45] in terms of FER using Monte Carlo simulation. All systems under consideration have the same normalized data rate, i.e. $R_{norm}$ bits per channel use per sub-carrier (b/cu/sc). In Fig. 6, the optimal FEC rate, back-off rate, probability distribution of the symbols of the proposed scheme are obtained from the procedures indicated in previous sections. The soft-demappers consider (21) and (22) for detection in ProCon and ProSpa schemes, respectively. The following parameters are assumed for ACO-OFDM with index modulation: number of orthogonal frequency-division multiplexing (OFDM) subblocks is 4, number of active subcarriers is 3, Fast Fourier Transform (FFT) size is 128, cyclic prefix length is 8. In order to achieve $R_{norm} = 0.67$ b/cu/sc and $R_{norm} = 1.33$ b/cu/sc, ACO-OFDM with index modulation use constellation of size 64 and 128 and FEC coding rate 1/2 and 9/10, respectively.

In Fig. 6a, at FER $= 10^{-3}$ and for $R_{norm} = 1.33$ b/cu/sc, ProCon scheme outperforms ProSpa and uniform based schemes by around 0.9 dB and 1.1 dB, respectively. This happens because, as can be noticed from Fig. 2a, optimizing constellation rather than spatial symbols offers better result when the user is located close to the center of the room. For example, for ProCon scheme, the optimal values are $R_{FEC} = 1/2$, $R_{backoff} = 0.1$ bpcu, $p^* = [0.0761, 0, 0.0249, 0, 0, 0.0897, 0.1065, 0.1028]$ and $\Delta^* = 0.35$, respectively. For $R_{norm} = 0.67$ b/cu/sc, the OSNR gain of ProCon scheme over ACO-OFDM with index modulation is around 0.8 dB, and it increases with $R_{norm}$. This is due to the fact that the modulation size in ACO-OFDM with index modulation has to be significantly increased compared to the BMD capacity.
According to Fig. 6b, at FER $\leq 10^{-3}$ and for $R_{\text{norm}} = 1.33$ bpcu/sc, ProSpa scheme outperforms ProCon scheme, uniform based scheme and ACO-OFDM with index modulation by around 0.8 dB, 1.8 dB and 2.5 dB, respectively. For $R_{\text{norm}} = 0.67$ bpcu/sc, the OSNR gain of ProSpa scheme over ACO-OFDM with index modulation is around 2 dB.

Fig. 7a illustrates the difference between the ARs of ProCon scheme and uniform based scheme, i.e. $R_{\text{SMD}}(\Delta^s, p^s, q_u) - R_{\text{SMD}}(\Delta_u, p_u, q_u)$, for different locations of the user inside the room. Thus, as can be noticed from Fig. 7a, ProCon scheme outperforms the uniform based scheme in terms of the AR when the user’s location is near to the center of the room because the difference in that region is around 0.5 bpcu. However, when the user is located near the edges of the room, the performance of the ProCon scheme and uniform based scheme in terms of the AR is almost the same since the difference is close to zero.

Fig. 7b depicts the difference between the ARs of ProSpa scheme and uniform based scheme, i.e. $R_{\text{SMD}}(\Delta_u, p_u, q_u) - R_{\text{SMD}}(\Delta^s, p^s, q_u)$, for different locations of the user inside the room. If the user is located near to the center of the room, the channel gains are quite similar, whereas if the user is located near to the edge of the room, the differences between the channel links increase. The performance of SM depends on the differences between the channel links [15]. Thus, ProSpa scheme significantly outperforms the uniform based scheme in terms of the AR when the user is located near to the edges of the room, and the difference in that region varies from around 0.5 bpcu until 1 bpcu. Nevertheless, when the position of the user is close to the center of the room, the channel gains are quite similar, and the ProSpa scheme assigns an almost equal probability for these channel gains. Hence, the performance of the ProSpa scheme and uniform based scheme in terms of the AR is almost identical around the center of the room, and the difference is close to zero in that region. As can be noticed, Fig. 7b depicts the opposite scenario compared to Fig. 7a.

Fig. 8 shows the advantages of adapting ProCon and ProSpa schemes to the location of the user; in other words, we choose the one which gives higher AR in that specific location of the user. Fig. 8a represents the difference between the ARs of proposed adaptive scheme and uniform based scheme, i.e. $\max\{R_{\text{SMD}}(\Delta^s, p^s, q_u), R_{\text{SMD}}(\Delta_u, p_u, q_u)\} - R_{\text{SMD}}(\Delta_u, p_u, q_u)$, for different locations of the user inside the room. For example, when the user is located near to the center of the room, we use ProCon while ProSpa is used when the user is located near to the edges of the room. As can be seen from Fig. 8a, the difference in the rate is at least 0.4 bpcu for almost all the locations of the user.

In Fig. 8b, we show the difference between the capacity of SM with $M$-PAM signaling and AR of proposed adaptive scheme, $C_{\text{SMD}}(\Delta^s, p^s, q_u)$, for different locations of the user inside the room. According to Fig. 8b, the difference is negligible for almost all the locations of the user. It means that the proposed adaptive scheme operates within around 0.2 dB from the capacity, $C_{\text{SMD}}(\Delta^s, p^s, q_u)$, regardless of the position of the user inside the room.

VI. CONCLUSION

To conclude, we propose novel adaptive coded SM scheme with PS for VLC communications. The encoder of the proposed scheme probabilistically shapes either constellation

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\[^5\text{Although we select ACO-OFDM with index modulation as a benchmark, there are other OFDM based schemes such as direct current biased optical OFDM (DC-OFDM) and several variants of OFDM with index modulation [45], [47]–[49].}\]
symbols or spatial symbols, which depend on the location of the user inside the room, using DM with low complexity in order to approach the SM VLC channel capacity with IM/DD. In addition, the channel coding rate is adapted to OSNR, and the algorithm is proposed to obtain the optimal probability distribution, which approaches the proposed scheme capacity. Simulation results show that the proposed scheme approaches the capacity of SM VLC channel signaling with fine granularity. Moreover, the error performance results show that at FER = 10^{-3} and transmission rate 1.33 bpcu, the proposed
adaptive coded SM scheme outperforms the uniform-based scheme by around 1.1 dB and 1.8 dB when the user is located close to and far from the center of the room, respectively.

APPENDIX

The lagrangian of the convex optimization problem (33) is

\[
L = - \sum_{i=1}^{M} p_i \log \left( \frac{1}{p_i} \right) - R_{\text{FEC}} [\log(M) + \log(N)] \\
+ \log(M) + \mu \left( \sum_{i=1}^{M} p_i - 1 \right) + \eta \left( \sum_{i=1}^{M} \Delta_i p_i - P \right) \\
+ \tau \left( \sum_{i=1}^{M} p_i \left[ \log \left( \frac{1}{N} \right) + W(\hat{p}, i) \right] \right) \\
+ R_{\text{FEC}} [\log(M) + \log(N)] - \log(M) + R_{\text{backoff}},
\]

(34)
where $\mu$, $\eta$ and $\tau$ are lagrangian multipliers. Now, let us take the derivative of the lagrangian $L$ with respect to $p_i$ and $\mu$ as
\[
\frac{dL}{dp_i} = -\log \left( \frac{1}{p_i} \right) + 1 + \mu + \eta \Delta i + \tau \left[ \log \left( \frac{1}{N} \right) + W \left( \hat{p}, i \right) \right] = 0 \tag{35}
\]
and
\[
\frac{dL}{d\mu} = \sum_{i=1}^{M} p_i - 1 = 0, \tag{36}
\]
respectively. We have four different conditions due to lagrangian multipliers $\eta$ and $\tau$, which account for non-equality constraints.

**Condition 1**: $\eta > 0$, $\tau = 0$

From (35), we get
\[
p_i = 2^{-1-\mu-\eta \Delta i}. \tag{37}
\]
Now substituting $p_i$ in (36) with (37), we can express $\mu$ as
\[
\mu = -1 + \log \left( \sum_{i'=1}^{M} 2^{-\eta \Delta i'} \right). \tag{38}
\]
Hence, substituting $\mu$ in (37) with (38), $p_i$ can be written as
\[
p_i = 2^{-\eta \Delta i - \log \left( \sum_{i'=1}^{M} 2^{-\eta \Delta i'} \right)} = \frac{2^{-\eta \Delta i}}{\sum_{i'=1}^{M} 2^{-\eta \Delta i'}}. \tag{39}
\]
Since $\eta > 0$, we should have power constraint satisfied with equality such that
\[
\sum_{i=1}^{M} \Delta i p_i - P = 0. \tag{40}
\]
Substituting $p_i$ in (40) with (39), we get
\[
\sum_{i=1}^{M} (\Delta i - P) 2^{-\eta \Delta i} = 0. \tag{41}
\]
Since $\tau = 0$, we should have rate constraint satisfied with strict inequality such that
\[
\sum_{i=1}^{M} p_i \left[ -\log \left( N \right) + W \left( \hat{p}, i \right) \right] + R_{\text{FEC}} \log(M) + \log(N)] - \log(M) + R_{\text{backoff}} < 0. \tag{42}
\]
Substituting $p_i$ in (42) with (39), we have
\[
\sum_{i=1}^{M} \left[ W \left( \hat{p}, i \right) - C \right] 2^{-\eta \Delta i} < 0, \tag{43}
\]
where $C = (1 - R_{\text{FEC}}) \log(M) + \log(N) - R_{\text{backoff}}$.

**Condition 2**: $\eta = 0$, $\tau = 0$

The derivation of optimal probabilities is the same as in condition 1. The only difference is that $\eta = 0$. Hence, $p_i$ can be written as
\[
p_i = 2^{-\log(M)} = \frac{1}{M} > 0. \tag{44}
\]
Since $\eta = 0$, we should have power constraint satisfied with strict inequality such that
\[
\sum_{i=1}^{M} \Delta i p_i - P < 0. \tag{45}
\]
Substituting $p_i$ in (45) with (44), we get
\[
\sum_{i=1}^{M} \Delta i - MP < 0. \tag{46}
\]
Since $\tau = 0$, we should have rate constraint with strict inequality such that
\[
\sum_{i=1}^{M} p_i \left[ -\log \left( N \right) + W \left( \hat{p}, i \right) \right] + R_{\text{FEC}} \log(M) + \log(N)] - \log(M) + R_{\text{backoff}} < 0. \tag{47}
\]
Substituting $p_i$ in (47) with (44), we get
\[
\sum_{i=1}^{M} W \left( \hat{p}, i \right) - MC < 0. \tag{48}
\]

**Condition 3**: $\eta > 0$, $\tau > 0$

From (35), we get
\[
p_i = 2^{-1-\mu-\eta \Delta i}. \tag{49}
\]
Now substituting $p_i$ in (36) with (49), we can express $\mu$ as
\[
\mu = \log(N) \tau - 1 + \log \left( \sum_{i'=1}^{M} 2^{-W(\hat{p},i') \tau - \eta \Delta i'} \right). \tag{50}
\]
Hence, substituting $\mu$ in (49) with (50), $p_i$ can be written as
\[
p_i = 2^{-W(\hat{p},i) \tau - \Delta i \eta - \log \left( \sum_{i'=1}^{M} 2^{-W(\hat{p},i') \tau - \eta \Delta i'} \right)} = \frac{2^{-W(\hat{p},i) \tau - \eta \Delta i}}{\sum_{i'=1}^{M} 2^{-W(\hat{p},i') \tau - \eta \Delta i'}} > 0. \tag{51}
\]
Since $\eta > 0$, we should have power constraint satisfied with equality such that
\[
\sum_{i=1}^{M} \Delta i p_i - P = 0. \tag{52}
\]
Substituting $p_i$ in (52) with (51), we get
\[
\sum_{i=1}^{M} (\Delta i - P) 2^{-W(\hat{p},i) \tau - \eta \Delta i} = 0. \tag{53}
\]
Since $\tau > 0$, we should have rate constraint satisfied with equality such that
\[
\sum_{i=1}^{M} p_i \left[ -\log \left( N \right) + W \left( \hat{p}, i \right) \right] + R_{\text{FEC}} \log(M) + \log(N)] - \log(M) + R_{\text{backoff}} = 0. \tag{54}
\]
Substituting $p_i$ in (54) with (51), we get
\[
\sum_{i=1}^{M} \left[ W \left( \hat{p}, i \right) - C \right] 2^{-W(\hat{p},i) \tau - \eta \Delta i} = 0. \tag{55}
\]

**Condition 4**: $\eta = 0$, $\tau > 0$

The derivation of optimal probabilities is the same as in condition 3. The only difference is that $\eta = 0$. Hence, $p_i$ can be written as
\[
p_i = 2^{-W(\hat{p},i) \tau - \log \left( \sum_{i'=1}^{M} 2^{-W(\hat{p},i') \tau} \right)} = \frac{2^{-W(\hat{p},i) \tau}}{\sum_{i'=1}^{M} 2^{-W(\hat{p},i') \tau}} > 0. \tag{56}
\]
Since $\eta = 0$, we should have power constraint satisfied with
strict inequality such that
\[ \sum_{i=1}^{M} (\Delta_i - P_i)^2 W(\hat{p}_i T) < 0. \]  
(57)
Since \( \tau > 0 \), we should have rate constraint satisfied with equality such that
\[ \sum_{i=1}^{M} [W(\hat{p}_i T) - C_i] 2^{-W(\hat{p}_i T)} \tau = 0. \]  
(58)

REFERENCES


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