Analysis of Electromagnetic Scattering from Composite Objects using a Multi-trace Surface Integral Equation Method

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Abstract

The multi-trace domain-decomposition surface integral equation (MT-DD-SIE) originally proposed to analyze electromagnetic scattering from dielectric composite objects is extended to efficiently account for perfect electrically conducting (PEC) bodies. This extension uses a special form of Robin transmission conditions (RTCs) on the surfaces of the PEC bodies to “couple” them to the dielectric bodies. Rao-Wilton-Glisson (RWG)-based discretization of these RTCs, which are the only governing equations on PEC surfaces, yields a well-conditioned matrix block. Therefore, PEC bodies are accounted for within the MT-DD-SIE solver without negatively affecting the convergence of the iterative solution. The accuracy and the efficiency of this approach are demonstrated by numerical examples.

I. INTRODUCTION

Various types of surface integral equations (SIEs) can be solved to analyze the electromagnetic (EM) scattering from composite objects consisting of dielectric and/or perfect electrically conducting (PEC) bodies. For example, one can couple the electric field and the Poggio-Miller-Chang-Harrington-Wu-Tsai integral equations (EFIE-PMCHWT) \[^{[1]}\]. However, discretizing this
coupled system of equations using the Rao-Wilton-Glisson (RWG) [2] functions yields an ill-conditioned matrix equation whose solution calls for a large number of iterations, especially for electrically large scatterers [3]. One can use combined field integral equations (CFIEs) to increase the convergence rate of the iterations [4], [5], but this comes at the cost of accuracy [6].

In [7], a multi-trace domain decomposition method (MT-DD-SIE) has been developed for analyzing scattering from composite objects consisting of multiple dielectric bodies (subdomains). This method uses EFIE and the magnetic field integral equation (MFIE) as the governing equations (in unknown equivalent currents) in each subdomain. These currents introduced on the subdomain surfaces are locally coupled via the Robin transmission conditions (RTCs) [8]. The RWG-based discretization of this locally coupled system yields a matrix equation that does not suffer from any ill-conditioning. In addition, well-conditioned matrix blocks pertinent to the subdomains are used to generate a preconditioner that further accelerates the iterative solution of the locally-coupled final matrix equation [9]. The accuracy of this method is same as that of EFIE-PMCHWT, but its memory requirement is significantly less.

In this work, the MT-DD-SIE proposed in [7] is extended to efficiently account for PEC bodies. This extension uses a special form of RTCs on the surfaces of the PEC subdomains to couple them to dielectric subdomains. RWG-based discretization of these RTCs, which are the only governing equations on PEC subdomains, yields a well-conditioned matrix block. This approach maintains the well-conditioning of MT-DD-SIE matrix system, which can still be solved efficiently and accurately using an iterative scheme.

II. Formulation

Let $\Omega$ represent the support of a PEC/dielectric composite scatterer: $\Omega = \Omega_1 \cup \Omega_2$, where $\Omega_1$ and $\Omega_2$ represent the subdomains for the PEC and the dielectric bodies, respectively (Fig. 1). An EM field $\{E^{inc}, H^{inc}\}$ which originates in subdomain $\Omega_0$ (background medium), is incident on $\Omega$. The permittivity, permeability, and intrinsic impedance in $\Omega_0$ and $\Omega_2$ are $\{\varepsilon_0, \mu_0, \eta_0\}$ and $\{\varepsilon_2, \mu_2, \eta_2\}$, respectively. The boundaries of these subdomain are $\partial \Omega_0$, $\partial \Omega_1$, and $\partial \Omega_2$, and $\hat{n}_0$, $\hat{n}_1$ and $\hat{n}_2$ are the inward pointing unit normal vectors on these boundaries. Equivalent unknown (electric and normalized magnetic) currents $\{J_0, M_0\}$, $J_1$, and $\{J_2, M_2\}$ are introduced on $\partial \Omega_0$, $\partial \Omega_1$, and $\partial \Omega_2$, respectively.

Let $\Gamma_{ij}$, $i, j \in \{0, 1, 2\}$ denote an interface between any two subdomains $\Omega_i$ and $\Omega_j$. On $\Gamma_{20}$ and $\Gamma_{02}$, i.e., the interfaces between the subdomains of free space and the dielectric body,
RTCs described in [7] are used. On $\Gamma_{01}, \Gamma_{10}, \Gamma_{21},$ and $\Gamma_{12},$ i.e., the interfaces between the PEC subdomain and the other two subdomains, the following RTCs are enforced:

\[
\eta_0 \mathbf{J}_0 - \eta_0 \hat{n}_0 \times \mathbf{M}_0 + \eta_0 \mathbf{J}_1 = 0, \text{ on } \Gamma_{01} \tag{1}
\]

\[
\eta_0 \hat{n}_0 \times \mathbf{J}_0 + \eta_0 \mathbf{M}_0 - \eta_0 \hat{n}_1 \times \mathbf{J}_1 = 0, \text{ on } \Gamma_{01} \tag{2}
\]

\[
\eta_0 \mathbf{J}_1 + \eta_0 \mathbf{J}_0 + \eta_0 \hat{n}_0 \times \mathbf{M}_0 = 0, \text{ on } \Gamma_{10} \tag{3}
\]

\[
\eta_2 \mathbf{J}_2 - \eta_0 \hat{n}_2 \times \mathbf{M}_2 + \eta_2 \mathbf{J}_1 = 0, \text{ on } \Gamma_{21} \tag{4}
\]

\[
\eta_2 \hat{n}_2 \times \mathbf{J}_2 + \eta_0 \mathbf{M}_2 - \eta_2 \hat{n}_1 \times \mathbf{J}_1 = 0, \text{ on } \Gamma_{21} \tag{5}
\]

\[
\eta_0 \mathbf{J}_1 + \eta_0 \mathbf{J}_2 + \eta_0 \hat{n}_2 \times \mathbf{M}_2 = 0, \text{ on } \Gamma_{12}. \tag{6}
\]

In $\Omega_1,$ (3) and (6) are used as the governing equations. For $\Omega_0$ and $\Omega_2,$ RTCs on $\Gamma_{01}, \Gamma_{02},$ and $\Gamma_{20}, \Gamma_{21}$ are respectively combined with EFIE and MFIE to yield the governing equations (see [7] for details). RWG-based discretization of the RTCs and governing equations yield

\[
\begin{bmatrix}
A_0 & M_{01}^{ii} & M_{02}^{ii} \\
M_{10}^{ii} & G_1 & M_{12}^{ii} \\
M_{20}^{ii} & M_{21}^{ii} & A_2
\end{bmatrix}
\begin{bmatrix}
\mathbf{I}_0 \\
\mathbf{I}_1 \\
\mathbf{I}_2
\end{bmatrix}
= 
\begin{bmatrix}
V_0 \\
0 \\
0
\end{bmatrix}. \tag{7}
\]
Here, $\mathbf{I}_0$, $\mathbf{I}_1$, and $\mathbf{I}_2$ are the vectors that store the unknown coefficients of the RWG expansions of $\{\mathbf{J}_0, \mathbf{M}_0\}$, $\mathbf{J}_1$, and $\{\mathbf{J}_2, \mathbf{M}_2\}$, respectively. $\mathbf{V}_0 = [\mathbf{V}_0^j, \mathbf{V}_0^m]$ is the vector of tested incident fields. $\mathbf{A}_0$ and $\mathbf{A}_2$, which are dense and diagonally-dominant just like the matrices that arise from the discretization of CFIE, correspond to the self-interactions in $\Omega_0$ and $\Omega_2$, respectively. Similarly, sparse and well-conditioned Gram matrix $\mathbf{G}_1$ corresponds to the self-interactions on $\Omega_1$. $\mathbf{M}_{mn}$, which are sparse and well-conditioned, correspond to the coupling between subdomains. FGMRES, an inner-outer iteration scheme described in [7], [9] is used to solve (7).

### III. Numerical Results

In this section, the EM scattering from a three-layer plate (Fig. 2) is investigated using the extended MT-DD-SIE solver and the contact region modeling (CRM) scheme that solves EFIE-PMCHWT [1]. The relative permittivity of the dielectric layers (located on top and bottom of the PEC layer) are 3.0 and 2.0, respectively. The average edge length in the discretization is 1 cm. MT-DD-SIE decomposes the scatterer into four subdomains with unknown numbers of $155 010$, $141 746$, $71 136$, and $142 272$. The number of unknowns for EFIE-PMCHWT is $355 154$. The FGMRES iterations for MT-DD-SIE and the GMRES iterations for CRM are terminated when the relative residual reaches 0.001. The excitation is a plane-wave propagating in the $-z$ direction with $\hat{p}$-polarized and unit-amplitude electric field. The frequency is $f = 3.0$ GHz. Two sets of simulations are carried out with $\hat{p} \in \{\hat{x}, \hat{y}\}$. The RCS computed by MT-DD-SIE and CRM on the $xz$-plane for two different polarizations are shown in Fig. 2. The results agree very well. The computation time and the memory required by the two methods are shown in Table I. The table clearly shows the benefits of MT-DD-SIE over CRM.

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Fig. 2. The bistatic RCS of the three-layer composite plate.

REFERENCES


