Reply to Comment on “Scattering Cancellation-Based Cloaking for the Maxwell-Cattaneo Heat Waves”

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Comment [1] points out possible inconsistencies in the notations of our paper [2] and, based on these remarks, it questions the validity of our conclusions. In this Reply, we demonstrate the general validity of all conclusions in [2], and we take the opportunity to clarify our notation and our results, and to discuss their domain of validity.

Equation derivation—The remarks in Comment [1] are rooted into a normalization that we implicitly applied in the definition of Eq. (3) of [2]. More explicitly, the author of [1] points out that Eq. (3) should read $\rho c \frac{\partial T}{\partial t} = -\nabla \cdot \Phi$, whereas in our notation the coefficient ρc is missing. In our paper [2], for simplicity of notation we have normalized the term κ_0, using the normalized quantity $\tilde{\kappa}_0 = \kappa_0/(\rho c)$, which represents the thermal diffusivity, with units of $m^2 s^{-1}$. This normalization explains the reason for the missing term in Eq. (3). Also the following Eq. (5) and the paragraph below it, as well as Eq. (13) are consistent with this normalization, where κ in those equations needs to be read again as the thermal diffusivity $\tilde{\kappa}$. In Sections III.B, III.C, and IV, similarly, κ denotes the diffusivity $\tilde{\kappa}$, including in Eq. (16), which makes its units consistent. Along with this normalization, also σ_0 has units of $m^2 s^{-1}$. Given the form of Eq. (3), we assumed that this normalization would be pretty straightforward for the average reader, and the reviewers of our paper had no problem with it. However, it is clear that the author of [1] was confused by it, and we are glad to have the opportunity to clarify it for all readers. It is important to stress, however, that this normalization does not change any of the results in our paper, since all our simulations and derivations properly treated κ as a thermal diffusivity term.

The source term that the author of [1] would like us to include in our equation is not needed in our theory, since our problem considers the scattering of a passive object from an external excitation, without the presence of sources in the scattering region of interest. Similarly, in Eq. (4) the source term δ is not necessary for the problem at hand.

Finally, we may have introduced a potential ambiguity in our paper [2] by using the same symbol T for the temperature in Eq. (4) and Eq. (5), where these two values are proportional by a phase $e^{-i\omega t}$; this simplified notation is very common in the broad scientific literature to lighten the formulas in the many instances in which the analysis is carried out in the frequency domain, so we are confident that the text is sufficiently clear for the average reader.

Diffusive term—Classically, the Maxwell-Cattaneo (MC) equations are derived without a diffusive term (σ_0 in Eq. (3) of [2]), and we agree with the author of [1] that the implications of our theory in the special case σ_0=0 are interesting to be explored. Our work considers more generally also a diffusive term in these equations. Inspired by the Comment [1], in Fig. 1 we have numerically compared our cloaking condition derived in [2] against the scenarios in which σ_0 is zero and in which σ_0 is ten times smaller in comparison to the value used in [2], keeping all other parameters the same. We note that these results agree very well with the results in the paper. This numerical comparison confirms that our findings apply also in the limit $\sigma_0 = 0$, which is consistent with the classical MC equations, as seen in Figs. 1(a)-(b). In order to avoid any source of confusion, we reiterate here that in our paper [2] we work with pseudo-heat waves that possess complex wavenumbers or diffuse photon density waves (DPDW), corresponding to the generalized form of MC equations used in the paper.

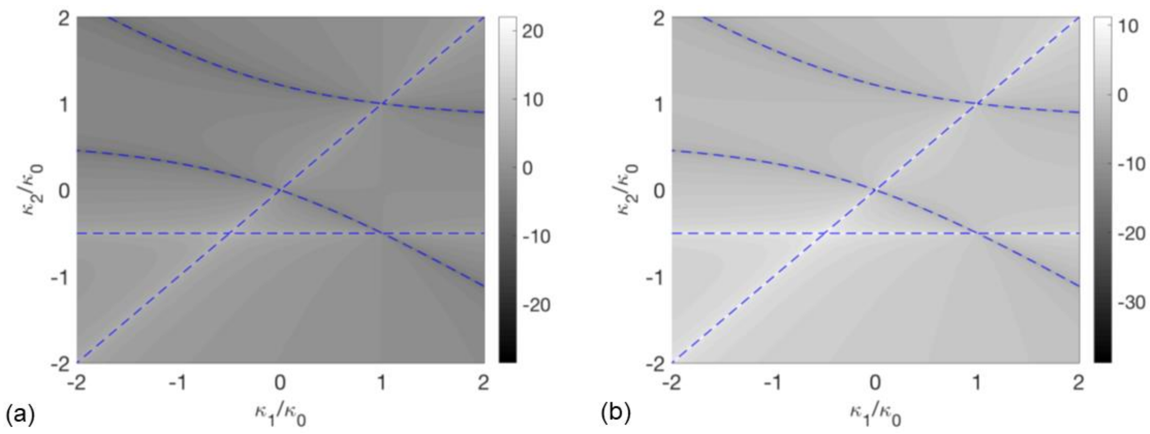

Fig. 1: Landscape of the scattering cross section in logarithmic scale, i.e., $10\log_{10}|\Sigma_2^{sc}/\Sigma_1^{sc}|$ for (a) $\sigma = 0.1\sigma_0$ and (b) $\sigma = 0$, with σ_0 being the value used in [2]. The dashed blue line corresponds to the cloaking condition derived from our model in [2].

Flux definition—We have used the expression $\kappa\nabla T$ to express the flux for the continuity at the boundaries between different layers. As outlined in [1], a more general expression for the flux is $(1 + i\omega\tau_0)/(1 + \omega^2\tau_0^2)\kappa\nabla T$. We are well aware of this, but we made this approximation rightfully, because in [2] we deal with the quasistatic regime. Indeed, the term $\omega\tau_0$ in the regime considered throughout our paper is lower than 10^{-3}, so all calculations are not affected by using either expression. The suggested modification starts having an impact for much larger frequencies, but this is not at all the subject of our paper [2], since in such regimes many other scattering harmonics would contribute to the overall response, and our theory would not be applicable.

To validate our claims, we compare our simulations using both expressions for the flux in Fig. 2(a) (solid lines correspond to the results in [2], whereas dashed lines correspond to the dispersive definition of the flux proposed in [1]). In the frequency range of interest for [2] the results are identical for $\omega\tau_0 = 0.1$, and nearly identical for $\omega\tau_0 = 0.4$ (there is a tenuous mismatch between the curves at specific conductivities), as seen in Fig. 2. Also, in this case, all results and conclusions in [2] are not at all affected, since we operate in the quasi-static limit ($\omega\tau_0 \ll 1$), as clearly spelled out in the paper [2]. In Fig. 2(b), we consider a larger frequency, for which we start observing some slight deviations from our results. Again, our results are very accurate even in this regime, which in any case starts deviating from the quasi-static assumption at the basis of our work.

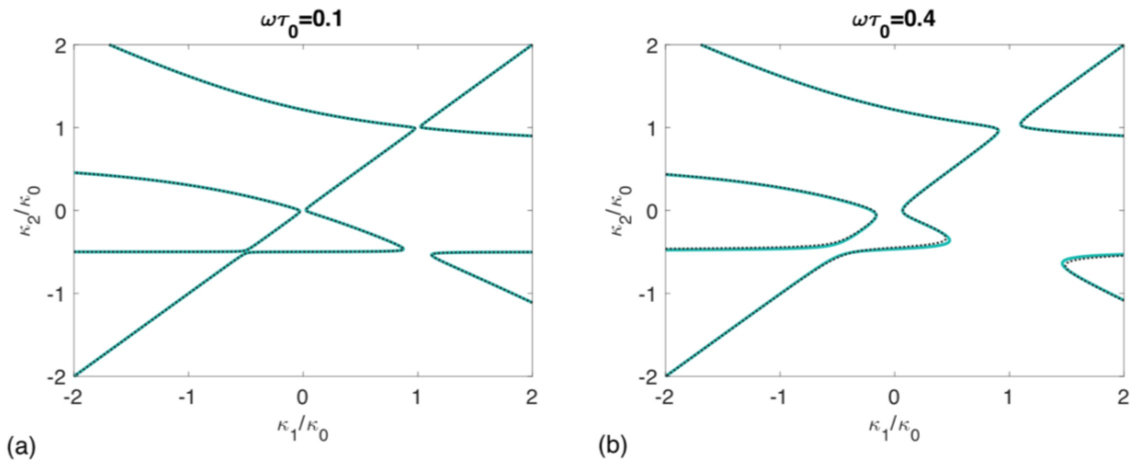

Fig. 2: Contourplot of conductivities that make cloaking possible for (a) $\omega\tau_0 = 0.1$ and (b) $\omega\tau_0 = 0.4$. The dashed curves correspond to the flux expression in [1], while the solid lines correspond to the flux expression used in [2].

We hope that these clarifications may help clarify the results of our paper for the author of [1] and all other interested readers. We stand by all conclusions and remarks in our work, which are indeed correct and accurate, as shown in this Reply.

References

[1] Comment by I. C. Christov.

[2] M. Farhat, S. Guenneau, P.-Y. Chen, A. Alù, and K. N. Salama, "Scattering Cancellation based Cloaking for the Maxwell-Cattaneo Heat Waves," Physical Review Applied 11, 044089 (2019).