Joint PP and PS Plane-Wave Wave-Equation Migration Velocity Analysis

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Running head: Joint PP and PS PWMVA

ABSTRACT

The conventional joint PP and PS velocity analysis is based on ray tomography. We now present a joint PP and PS wave-equation migration-velocity-analysis method using plane-wave common-image gathers to produce accurate P- and S-wave velocity models. The objective function of our new method consists of three terms: the first and second terms penalize the moveout residuals computed from PP and PS plane-wave common-image gathers, respectively, and the third term constrains the non-zero relative depth shifts between PP and PS migration images. The moveout of plane-wave common-image gathers is automatically picked using a semblance analysis method, and the relative depth shifts between PP and PS images are automatically computed using dynamic warping or manually picking the depths of certain primary reflectors. Both the moveout residuals and relative depth shifts are transformed into weighted image perturbations, and then are projected into the velocity models to update the P- and S-wave velocity models using the scalar-wave equations and their linearized forms. Numerical tests with synthetic and multi-component field data demonstrate that our method can simultaneously invert for accurate P- and S-wave velocity models for elastic
migration.
Accurate migration of reflection data requires accurate velocity models. One of the most popular velocity inversion methods is migration velocity analysis (MVA) (Stork, 1992; Biondi and Sava, 1999; Mulder and ten Kroode, 2002; Sava and Biondi, 2004; Shen and Symes, 2008). MVA inverts for a migration velocity that maximizes the similarities of a group of migration images at the same horizontal location, which is referred to as common-image gathers (CIGs). Using an accurate migration velocity model for migration, subsurface reflectors can be imaged at the same locations, such that their CIGs are flat. An incorrect velocity model can result in depth shifts of reflector images, such that the reflection events in the CIGs are not flat and so generate non-zero CIG moveout residuals. MVA inverts these CIG moveout residuals for the velocity updates using either a ray-based operator (Stork, 1992), or a wave-equation based operator (Biondi and Sava, 1999; Mulder and ten Kroode, 2002; Sava and Biondi, 2004; Shen and Symes, 2008).

Current MVA methods focus mostly on inverting for the P-wave velocity model using migrated PP CIGs. However, MVA for converted PS waves is more complicated than PP MVA, and often uses CIGs computed from prestack time migration images (Li and Yuan, 2003; Dai and Li, 2007). In practice, migration and the velocity model building are more robust and easier in the depth domain for structurally complex environments (Liu et al., 2007). Recently, Yan and Sava (2010) conducted MVA in the depth domain for inverting the S-wave velocity model using PS data only. Rosales et al. (2008) suggested to make use of PS CIGs in the depth domain to update both the P- and S-wave velocity models. However, flattening the events in the PP and PS CIGs separately is not enough to obtain accurate P- and S-wave velocity models since the PP and PS images of a common reflector often do not match in depth (Foss et al., 2005). To invert both PP and PS data together using MVA, a depth consistency between the PP and PS images is usually enforced as a constraint, particularly
for primary reflectors (Broto et al., 2003; Foss et al., 2005; Du et al., 2012; Cai and Tsvankin, 2013; Mathewson et al., 2013; He and Hilburn, 2015). However, these joint PP and PS MVA methods are implemented using ray tracing, an asymptotic high-frequency approximation of wave propagation.

Wave-equation MVA (WEMVA) is intrinsically more robust than ray-based MVA because the former avoids the asymptotic high-frequency assumption of wave propagation (Biondi and Sava, 1999; Sava and Biondi, 2004). However, to computer CIGs, most WEMVA methods numerically solve the wave equation for each shot and use an imaging condition in the extended dimension or extra processing of the extrapolated wavefields during migration (Sava and Fomel, 2003; Biondi and Symes, 2004; Sava and Fomel, 2006; Xu et al., 2011; Zhang and Biondi, 2013). This typically leads to a high computational cost and a huge memory-storage requirement for large 3D imaging problems.

To mitigate the above problems, we present a novel joint PP and PS plane-wave wave-equation migration velocity analysis (PWEMVA) method for simultaneously inverting for both the P- and S-velocity models. This joint PP and PS PWEMVA method is based on the PWEMVA method for PP data (Guo and Schuster, 2017) and uses plane-wave migration (Whitmore, 1995; Duquet et al., 2001; Zhang et al., 2005; Liu et al., 2006) to form the PP and PS plane-wave CIGs. This technique combines multiple PP and PS shot gathers into PP and PS plane-wave gathers, and migrates the PP and PS plane-wave gathers with different ray parameters $p$. It greatly reduces the computational cost and the memory storage requirements because plane-wave CIGs are obtained directly from prestack plane-wave migration images, and the number of plane waves used for migration is often much smaller than the number of shots (Guo and Schuster, 2017).

The migration of PP reflections in PWMVA uses the scalar-wave equation along with a P-wave velocity model. For the migration of PS reflections, the source wavefield is extrapolated using the
scalar-wave equation with the P-wave velocity model, and the receiver wavefield is backpropagated using the scalar-wave equation with a S-wave velocity model (Sun and McMechan, 2001; Sun et al., 2006). The objective function of the our joint PP and PS PWEMVA consists of three terms: the residual moveouts of the PP and PS plane-wave CIGs and the relative depth shifts between the same reflectors in the PP and PS images. The last term assumes that the PP and PS images have these common reflectors (Foss et al., 2005).

The joint PP and PS PWEMVA method computes the residual moveouts of PP and PS CIGs by choosing parabolas that best fit the local curvature of the plane-wave CIGs (Zhang and Biondi, 2013; Zhang et al., 2015; Guo and Schuster, 2017), and computes the relative depth shifts between the PP and PS images computed using either dynamic image warping (Hale, 2013; He and Hilburn, 2015), or manually picking of the depths of some primary reflectors. Previously, Guo and Schuster (2017) linked the residual moveout of PP CIGs to the P-wave migration velocity update using a connective function and the implicit function theorem (Luo and Schuster, 1991; Guo and Schuster, 2017). In our joint PP and PS PWMVA method, we further link the residual moveout of PS CIGs and relative depth shifts between the PP and PS images to the P- and S-wave migration velocity updates. During implementation, we weight the PP and PS migration images to construct the image perturbations using the residual moveouts and the relative depth shifts, and project these weighted image perturbations into the P- and S-wave velocity models to update the velocities. The projection uses the scalar-wave equations for the downward wavefield and their linearized equations for the upward wavefield.

After the introduction, the theory section introduces the objective function used in our joint PP and PS PWEMVA method, derives its associated velocity gradient, gives the formulas for updating the P- and S-wave velocity models, and presents a work flow for implementation. The section on numerical examples provides the PP and PS PWEMVA results of two synthetic seismic datasets and
a field land dataset recorded at Kevin Dome, Montana, US. The last two sections include discussions and conclusions.

**THEORY**

The objective function $J$ for our joint PP and PS PWMVA method consists of three terms: the vertical shift $\Delta w_p^j(x)$ of a plane-wave migration image $m_p^j$ and the reference image $m_0^p$ for PP data, the vertical shift $\Delta w_s^j(x)$ for traces in the PS CIGs, and the vertical shift $w^d(x)$ between the primary reflectors in the PP and PS images:

$$J = \frac{1}{2} \sum_x \left\{ \frac{\alpha_p}{n_p} \sum_{j=1}^{n_p} (\Delta w_p^j(x))^2 + \frac{\alpha_s}{n_s} \sum_{j=1}^{n_s} (\Delta w_s^j(x))^2 + \alpha_d (\Delta w^d(x))^2 \right\},$$  \hspace{1cm} (1)$$

where $j$ denotes the plane-wave index, $n_p$ and $n_s$ represent the number of PP and PS plane waves respectively, and $\alpha_p$, $\alpha_s$ and $\alpha_d$ are the weights. Term $\Delta w_p^j(x)$ is the local vertical shift that aligns $m_0^p(x, z + \Delta w_p^j(x))$ with $m_p^j(x, z)$ for $x = (x, z)$, where the reference image $m_0^p$ is chosen to be the plane-wave migration image with the ray direction perpendicular to the subsurface interface in practice (Guo and Schuster, 2017). The term $\Delta w_s^j(x)$ is the local vertical shift that aligns $m_0^s(x, z + \Delta w_s^j(x))$ with $m_s^j(x, z)$ for PS data, and $\Delta w^d(x)$ is the vertical shift term that aligns the PS image $m_s^s(x, z + \Delta w^d(x))$ with the PP image $m_p^p(x, z)$, where the PS image $m_s^s(x)$ is chosen to be the reference image. The term $\Delta w^d(x)$ also aligns the PP image $m_p^p(x, z - \Delta w^d(x))$ with the PS image $m_s^s(x, z)$, when the PP image $m_p^p(x)$ is chosen to be the reference image. Here, $m_p^p(x)$ and $m_s^s(x)$ are the stacked plane-wave migration images,

$$m_p^p(x) = \sum_{j=1}^{n_p} m_p^j(x) \quad \text{and} \quad m_s^s(x) = \sum_{j=1}^{n_s} m_s^j(x).$$ \hspace{1cm} (2)
The gradients of the objective function with respect to the P- and S-wave slownesses $s_p(x')$ and $s_s(x')$ (reciprocal of the P- and S-wave migration velocities), respectively, are

$$\frac{\partial J}{\partial s_p(x')} = \sum_x \left\{ \frac{\alpha_p}{n_p} \sum_{j=1}^{n_p} \frac{\partial \Delta w^p_j(x)}{\partial s_p(x')} \Delta w^p_j(x) + \frac{\alpha_s}{n_s} \sum_{j=1}^{n_s} \frac{\partial \Delta w^s_j(x)}{\partial s_p(x')} \Delta w^s_j(x) + \alpha_d \frac{\partial \Delta w^d(x)}{\partial s_p(x')} \Delta w^d(x) \right\},$$

$$\frac{\partial J}{\partial s_s(x')} = \sum_x \left\{ \frac{\alpha_s}{n_s} \sum_{j=1}^{n_s} \frac{\partial \Delta w^s_j(x)}{\partial s_s(x')} \Delta w^s_j(x) + \alpha_d \frac{\partial \Delta w^d(x)}{\partial s_s(x')} \Delta w^d(x) \right\}. \tag{3}$$

The terms on the right-hand side of Equation (3) are given by (see Appendix C for derivation):

$$\sum_{j=1}^{n_p} \sum_x \frac{\partial \Delta w^p_j(x)}{\partial s_p(x')} \Delta w^p_j(x) = \sum_{j=1}^{n_p} \sum_x f^{pp}_j \left( \frac{G_p(x'|x) \mathcal{M}_j^{pp}(x) R_j^{pp}(\omega, x)}{m^{pp}_j(x)} \right), \tag{4}$$

where $f^{pp}_j = \Re \left\{ \sum_{\omega} 2\omega^2 s_p(x') S_j^p(\omega, x') \left[ G_p(x'|x) \mathcal{M}_j^{pp}(x) R_j^{pp}(\omega, x) \right] * \right\}$, \hspace{1cm} \text{upward PP receiver wavefield}

and $g^{pp}_j = \Re \left\{ \sum_{\omega} 2\omega^2 s_p(x') G_p(x'|x) \mathcal{M}_j^{pp}(x) S_j^p(\omega, x) R_j^{pp}(\omega, x') * \right\}$, \hspace{1cm} \text{upward PP source wavefield}

where $\mathcal{M}_j^{pp}(x) = -\Delta w^p_j(x) m^{pp}_j(x)$. \hspace{1cm} \tag{5}

$$\sum_{j=1}^{n_s} \sum_x \frac{\partial \Delta w^s_j(x)}{\partial s_p(x')} \Delta w^s_j(x) = \sum_{j=1}^{n_s} \sum_x f^{ps}_j \left( \frac{G_p(x'|x) \mathcal{M}_j^{ps}(x) R_j^{qs}(\omega, x)}{m^{ps}_j(x)} \right), \tag{6}$$

where $f^{ps}_j = \Re \left\{ \sum_{\omega} 2\omega^2 s_p(x') S_j^p(\omega, x') \left[ G_p(x'|x) \mathcal{M}_j^{ps}(x) R_j^{qs}(\omega, x) \right] * \right\}$, \hspace{1cm} \text{downward PS source wavefield}

and $g^{ps}_j = \Re \left\{ \sum_{\omega} 2\omega^2 s_p(x') G_p(x'|x) \mathcal{M}_j^{ps}(x) S_j^p(\omega, x) R_j^{qs}(\omega, x') * \right\}$, \hspace{1cm} \text{downward PS receiver wavefield}

where $\mathcal{M}_j^{ps}(x) = -\Delta w^s_j(x) m^{ps}_j(x)$. \hspace{1cm} \tag{7}
\[
\sum_{j=1}^{n_s} \sum_{x} \frac{\partial \Delta w^s_j(x)}{\partial s_s(x')} \Delta w^s_j(x) = \sum_{j=1}^{n_s} \sum_{x} \frac{g^{ps}_j}{m^s_0(x, z + \Delta w^s_j(x))m^s_j(x)}.
\] (9)

where \( g^{ps}_j = \Re \left\{ \sum_{\omega} 2\omega^2 s_s(x') G_s(x'|x) M^{ps}(x) S^{p}(\omega, x) R^s(\omega, x')^* \right\} \).

\[
\sum_{x} \frac{\partial \Delta w^d(x)}{\partial s_p(x')} \Delta w^d(x) = \sum_{j=1}^{n_p} \sum_{x} f^{dp}_j + g^{dp}_j
\]
\[
= \sum_{j=1}^{n_s} \sum_{x} \frac{f^{ds}_j}{m^p(x, z - \Delta w^d(x))m^s(x)}.
\] (11)

where \( f^{dp}_j = \Re \left\{ \sum_{\omega} 2\omega^2 s_p(x') S^{p}(\omega, x') \left[ G_p(x'|x) M^{dp}(x) R^s(\omega, x) \right]^* \right\} \),

\[
\text{upward PP source wavefield}
\]

and \( g^{dp}_j = \Re \left\{ \sum_{\omega} 2\omega^2 s_p(x') G_p(x'|x) M^{dp}(x) S^{p}(\omega, x) R^s_j(\omega, x')^* \right\} \),

\[
\text{downward PP source wavefield}
\]

and \( f^{ds}_j = \Re \left\{ \sum_{\omega} 2\omega^2 s_p(x') S^{p}(\omega, x') \left[ G_p(x'|x) M^{ds}(x) R^s(\omega, x) \right]^* \right\} \),

\[
\text{upward PS source wavefield}
\]

and \( g^{ds}_j = \Re \left\{ \sum_{\omega} 2\omega^2 s_s(x') G_s(x'|x) M^{ds}(x) S^{p}(\omega, x) R^s_j(\omega, x')^* \right\} \),

\[
\text{downward PS source wavefield}
\]

where \( M^{dp}(x) = -\Delta w^d(x) \dot{m}^s(x, z + \Delta w^d(x)) \) and

\( M^{ds}(x) = \Delta w^d(x) \dot{m}^p(x, z - \Delta w^d(x)). \)

Here, \( G_p(x'|x) \) and \( G_s(x'|x) \) represent the P- and S-wave Green’s functions, respectively,

where the receiver is at \( x' \), and the source at \( x \) oscillates with a specific angular frequency \( \omega \).

The gradient terms in Equations (4) to (15) can be divided into two categories. In the first category,
\( f_j^{pp}, f_j^{ps}, f_j^{dp} \) and \( f_j^{ds} \) can be interpreted as representing the source wavepath, which is computed by the dot product of the downward-propagated source wavefield \( S(\omega, x') \) at \( x' \) with the upward-propagated receiver wavefield \( G(x'|x)^* \mathcal{M}(x)R(\omega, x) \) (omitting the superscripts and subscripts).

The upward-propagated receiver wavefield is generated by a virtual source \( \mathcal{M}(x)R(\omega, x) \) at the image point \( x \), which is redatumed from the receivers on the surface. In the other category, \( g_j^{pp}, g_j^{ps}, g_j^{dp} \) and \( g_j^{ds} \) can be interpreted as representing the receiver wavepath, which is the dot product of the downward-propagated receiver wavefield \( R(\omega, x') \) at \( x' \) with the upward-propagated source wavefield \( G(x'|x)\mathcal{M}(x)S(\omega, x) \). The upward-propagated source wavefield is excited by a virtual source \( \mathcal{M}(x)S(\omega, x) \) at the image point \( x \), which is redatumed from the sources on the surface. Here the virtual sources \( \mathcal{M}(x)R(\omega, x) \) and \( \mathcal{M}(x)S(\omega, x) \) emit waves using image perturbations \( \mathcal{M}(x) \), which are mainly obtained by weighting the aligned migration images using the residual moveouts and relative depth shifts.

The source wavefields \( S_j^p \) and \( S_j'^{pp} \) are generated by plane-wave sources with specified ray parameters, and the receiver wavefields \( R_j^p \) and \( R_j^s \) are generated by the corresponding PP and PS plane-wave gathers, respectively. The migration slowness is updated by smearing the energy of the local-image shifts at \( x \) along its plane-wave paths associated with the sources and receivers (Guo and Schuster, 2017). The source wavefields \( S_j^p \) and \( S_j'^{pp} \) are computed using the P-wave velocity model, and the receiver wavefields \( R_j^p \) and \( R_j^s \) are computed using the P-wave velocity model and the S-wave velocity model, respectively. After computing the gradients, an iterative gradient method such as steepest-descent gradients is used to iteratively update the P and S migration slownesses until the shifts in the plane-wave CIGs and those between the PP and PS images are sufficiently small.
WORK FLOW

The work flow of our joint PP and PS PWEMVA method consists of three computational steps: compute the objective function, the gradients, and the step lengths. The implementations of the first two steps are described in the following.

1. Compute the objective function

We first transform the shot profile data for the PP and PS reflections into PP and PS plane-wave gathers, respectively, and then apply plane-wave prestack reverse time migration (RTM) to each PP and PS plane-wave gather to obtain plane-wave CIGs using the scalar-wave equation (Sun and McMechan, 2001). The RTM of PS data uses the background P-wave velocity model for the source wavefield and background S-wave velocity model for the receiver wavefield.

We follow Guo and Schuster (2017) to compute the local shifts in CIGs at every image point for the first two terms in the objective function (Equation 1) by choosing parabolas that best fits the moveouts of the PP and PS plane-wave CIGs. We first compute the semblance spectra of PP and PS plane-wave CIGs using a semblance analysis method (Taner and Koehler, 1969; Neidell and Taner, 1971), and then pick their curvatures automatically using the method developed by Fomel (2009). The third term of the objective function requires computation of the relative depth shifts between the stacked PP and PS plane-wave migration images. We employ a dynamic warping technique to align the PP image to the PS image in depth (Hale, 2013). However, the dynamic warping method is defined by the waveform differences between two traces. This means that if the two traces in the PP and PS images have large differences in amplitude, bandwidth or phases, the dynamic warping method might fail. In these cases, we choose to compute the image shifts by manually picking the depths of certain
primary reflectors in the PP and PS images to compute their depth differences.

2. Compute the gradients

When computing \( f_{pp}^j \), \( g_{pp}^j \), \( f_{ps}^j \) and \( g_{ps}^j \), the weighted image perturbations \( M(x) \) in Equations (4) to (10) are simplified as

\[
\mathcal{M}_{pp}^j(x) = -\Delta w^p_j(x) \dot{m}_j^p(x), \\
\mathcal{M}_{ps}^j(x) = -\Delta w^s_j(x) \dot{m}_j^s(x). \tag{17}
\]

To compute \( f_{dp}^j \), \( g_{dp}^j \), \( f_{ds}^j \) and \( g_{ds}^j \), the weighted image perturbations \( M(x) \) in Equations (11) to (16) are simplified as

\[
\mathcal{M}_{dp}^j(x)/\ddot{m}_s(x, z + \Delta u^d(x)) = \Delta w^d(x) \dot{m}_j^p(x)/\ddot{m}_j^p(x), \\
\mathcal{M}_{ds}^j(x)/\ddot{m}_p(x, z - \Delta u^d(x)) = -\Delta w^d(x) \dot{m}_j^s(x)/\ddot{m}_j^s(x). \tag{18}
\]

before summing over the plane wave index in Equations (11) to (16). The denominators are then replaced with the summation of their root mean squares in this implementation because dividing an image can be numerically unstable.

The Green’s functions \( G_p(x' | x) \) and \( G_s(x' | x) \) in Equations (4) to (15) are computed by solving the two-way scalar-wave equations in the time domain with the P- and S-wave velocity models, respectively (Sun and McMechan, 2001; Sun et al., 2006):

\[
\frac{\partial^2 P(x', t)}{\partial t^2} - \frac{1}{s(x')^2} \nabla^2 P(x', t) = f(x, t), \tag{19}
\]

where \( P(x', t) \) is the scalar-wave function, \( s(x') \) is the slowness with \( s(x') = s_p(x') \) for \( P(x', t) = G_p(x' | x) \) and \( s(x') = s_s(x') \) for \( P(x', t) = G_s(x' | x) \), and \( f(x, t) \) represents a band-limited
point source function at \( x \) with \( f(x, t) = \delta(x)\delta(t) \) for Green’s functions.

During implementation, the downward-propagated source and receiver wavefields are computed by solving the scalar-wave equations forward and backward in time, respectively. The upward-propagated source and receiver wavefields are computed by solving the linearized scalar-wave equations forward and backward in time, respectively. The detail equations are given in Appendix D. The slowness model needs to be smoothed to avoid reflection events in Green’s functions. After computing the gradients, a numerical line search method is used to compute the step lengths and update the P and S slowness models using the steepest-descent or the conjugate gradient methods (Nocedal and Wright, 1999).

**NUMERICAL RESULTS**

We verify the effectiveness of our joint PP and PS PWEMVA method using two synthetic datasets and a field land dataset recorded at Kevin Dome, Montana, US.

**Synthetic test 1**

We first use a simple 2D model to test our new method. We compute synthetic PP and PS shot gathers using finite-difference solutions to the 2D scalar Born modeling equations (Dai and Schuster, 2013) for the velocity and reflectivity models shown in Figures 1a to 1d. The Born modeling for PS data solves the scalar-wave equations with the P-wave velocity model for the source wavefield and the S-wave velocity model for the receiver wavefield. The source wavelet is a Ricker wavelet with a 40-Hz peak frequency. 201 sources and 401 stationary receivers are evenly distributed on the top surface of the model with intervals of 10 m and 5 m, respectively. The PP and PS shot profiles are both computed for 41 plane-wave gathers with \(-0.471 \text{ s/km} \leq p \leq 0.471 \text{ s/km}\) and the shooting
angles range from $-45^\circ$ to $45^\circ$. The PP and PS plane-wave gathers with $p = 0$ s/km is shown in Figures 1e and 1f, respectively. The initial P- and S-wave velocity models are homogeneous with $v_p = 1.5$ km/s and $v_s = 0.93$ km/s, respectively.

The PP and PS plane-wave CIGs for the initial velocity models are shown in Figures 2a and 2b, respectively. Their corresponding semblance spectra and the moveout residuals at 1.0 km in the $x$-direction are displayed in Figures 2c and 2d, respectively. The P- and S-wave velocity models obtained using the joint PP and PS PWMVA method are depicted in Figure 3. The final inverted velocity models mostly resembles the true velocity models in Figures 1a and 1b. The plane-wave CIGs in Figures 4a and 4b obtained from the inverted velocity models are mostly flattened. We also compare the stacked plane-wave images in Figure 5 produced using the homogeneous and inverted velocity models. It is evident that the PP and PS images computed from the inverted velocity models are more focused, continuous and aligned. Meanwhile, the relative depth shifts between the PP and PS images yielded using the inverted velocity models are shown in Figure 6a. They are reduced extensively compared to those in Figure 6b computed from the initial velocity models, particularly in the deep region.

**Synthetic test 2**

The second test inverts synthetic data generated using a time-space staggered-grid solution to the 2D elastic-wave equation (Levander, 1988). The P and S waves are extracted from the synthetic two-component elastic wavefields using the divergence and curl operators of the vector wavefield (Etgen, 1988). The P-wave velocity and density models displayed in Figures 7a and 7c, respectively, are modified from a portion of the BP2004 model. The S-wave velocity model is generated using an S- to P-wave velocity ratio of $1/\sqrt{3}$. The source wavelet is a Ricker wavelet with a 15-Hz peak
frequency. The 701 shots and 1401 stationary receivers are evenly distributed on the surface with spatial intervals of 20 m and 10 m, respectively. The synthetic PP and PS common-shot gathers with the source at $x = 7.0$ km are shown in Figures 8a and 8b, respectively. These data are transformed into 81 plane-wave gathers with the ray parameters ranging from $-0.33$ to $0.33$ s/km and the shooting angles from $-30^\circ$ to $30^\circ$. The PP and PS plane-wave gathers with $p = 0.082$ s/km are depicted in Figures 8c and 8d, respectively.

The laterally homogeneous velocity models shown in Figure 9 are used as the initial P- and S-wave velocity models for joint PP and PS PWMVA, resulting in the PP and PS plane-wave CIGs with strong residual moveouts as depicted in Figure 10. The stacked PP and PS plane-wave images produced using the initial velocity models are shown in Figures 11a and 11b, respectively. The relative depth shifts between them are computed using dynamic warping and the results are displayed in Figure 11c.

The inverted P- and S-wave velocity models after 12 iterations of joint PP and PS PWMVA are shown in Figures 12a and 12b, respectively. The inverted velocity models recover most of the low-wavenumber components of the true-velocity models in Figures 7a and 7b. Figure 12c presents the values of the objective function $J$ in Equation (1) and its three terms at each iteration. The PP and PS plane-wave CIGs associated with the inverted velocity models are mostly flattened in Figure 13. The stacked PP and PS plane-wave images computed from the inverted velocity models are given in Figures 14a and 14b, respectively.

We compare the stacked plane-wave images in Figures 11 and 14 computed from the initial and inverted velocity models, respectively, with the density model in Figure 7c because density perturbations generate most of the reflection data. It is clear that the migration images computed with the inverted velocity models closely resemble the high-wavenumber features of the density.
model with more focused structures in the deep region. Figure 14c displays the relative depth shifts between the PP and PS images computed from the inverted velocity models. The depth shifts are reduced extensively compared to those in images generated using the initial velocity models in Figure 11, particularly in the deep region.

**Field data test**

We test the capability of our joint PP and PS plane-wave MVA method on a 2D slice of a 3D field land seismic dataset acquired at Kevin Dome, Montana, USA. The extracted 2D land dataset consists of 48 shots with a shot interval of approximately 268 m. Each shot has a maximum of 110 geophones with a receiver interval of roughly 33.5 m. The maximum source-receiver offset is approximately 1850 m, and the recording time is 2 s. Figures 15a and 15b show the recorded PP and PS common-shot gathers with the source at $x = 3.5$ km, respectively. The frequency range of the data is from 5 Hz to 85 Hz. We transform the 48 shot gathers into 55 plane-wave gathers with $-0.15 \text{ s/km} \leq p \leq 0.15 \text{ s/km}$. The shooting angles range from $-14^\circ$ to $14^\circ$. Since the source is sparsely distributed, we apply f-k filtering to the plane-wave gathers to mitigate the aliasing problem. The PP and PS plane-wave gathers with $p = 0.073 \text{ s/km}$ are shown in Figures 16a and 16b, respectively.

The laterally homogeneous velocity models shown in Figure 17 are used as the initial P- and S-wave velocity models for our joint PP and PS PWMVA, resulting in the PP and PS plane-wave CIGs with strong residual moveouts as displayed in Figure 18. The stacked PP and PS plane-wave images obtained using the initial velocity models are depicted in Figures 19a and 19b, respectively. Figures 20a and 20b depict the inverted P- and S-wave velocity models, respectively. The PP and PS plane-wave CIGs associated with the inverted velocity models are mostly flattened as shown
in Figure 21. The stacked PP and PS plane-wave images computed from the inverted velocity models are shown in Figures 22a and 22b, respectively. The inverted velocity models produce more focused and continuous PP and PS images. In addition, the average absolute values of the relative depth shifts between the three picked reflectors in Figure 22 (indicated by the dotted lines) are approximately $1/3$ of the average P-wave wavelength at the central frequency.

**DISCUSSION**

Discussions related to the shooting angles, PWEMVA performance, semblance analysis and dynamic warping can be found in Guo and Schuster (2017). In this section, we discuss other issues for implementing our joint PP and PS PWEMVA method.

For separated PP and PS common-shot gathers, the PS common-shot gathers should be processed to correct the polarity reversal problem before migration. For the second synthetic test, the polarity reversal mainly occurs near zero offset in Figure 8b, thus we simply multiply the PS data recorded at negative offsets by $-1$ (Harrison and Stewart, 1993). For the field data test, the data are already processed to solve this issue.

In our field data example, we do not consider anisotropy and only invert for isotropic P- and S-wave velocity models. Ideally, it is necessary to account for anisotropy for joint PP and PS tomography (Grechka and Tsvankin, 2002; Broto et al., 2003; Foss et al., 2005; Szydlik et al., 2007). In our field data test, the PP and PS plane-wave CIGs obtained using the inverted velocity models (Figure 21) are not completely flattened. This is likely caused by anisotropy when aligning the PP and PS images during inversion (Audebert et al., 2001; Foss et al., 2005). For anisotropic media, we need to use decoupled anisotropic wave equations (Xiang et al., 2007; Zhan et al., 2012) to perform joint PP and PS PWEMVA.
One challenging issue for practical applications of our joint PP and PS PWMVA is that PS data usually have low signal-to-noise ratios, especially at near offset. This is because PS reflections are relatively weak around normal incidence (Aki and Richards, 2002) and may be masked by surface waves in land seismic data. In our field data test, near-offset data contain mostly noise, as shown in Figure 15b. We can still use semblance analysis to automatically pick the moveout of PS CIGs because it is computed by stacking all the plane-wave images. However, traces in the PP and PS stacked images have large differences in amplitude, bandwidth and level of noise. To compute the relative depth shifts between the PP and PS stacked images, we manually pick the depths of three primary reflectors during inversion rather than using dynamic warping, as shown by the dotted lines in Figure 22.

For all tests, we use the same number of PP and PS plane-wave gathers \( n_p = n_s \) and set \( \alpha_p = \alpha_s = 1 \) when computing the objective function \( J \) in Equation (1). In the synthetic tests, we also simply set \( \alpha_d = 1 \) and the residuals for the corresponding three terms reduce simultaneously during inversion, as shown in Figure 12c. In the field data test, we first set \( \alpha_d = 0 \) for the first few iterations, because it is impossible to manually pick reflectors from the stacked images when velocity models have significant inaccuracies, as depicted in Figure 19. After the PP and PS CIGs are flattened to a certain level, we pick a few primary reflectors from their stacked images and set \( \alpha_d \sim n_z \) to compute the objective function and continue the velocity update, where \( n_z \) is the number of image points in the \( z \)-direction.

In conventional MVA, the inverted velocity model is considered to be acceptable when the CIGs are flattened. However, reflection tomography is characterized by an inherent nonuniqueness inverse problem (Foss et al., 2005) because of the velocity/depth ambiguity (Bickel, 1990; Ross, 1994). In this paper, accurate P- and S-wave velocity models further requires that the corresponding reflectors in the migration PP and PS images are aligned in depth. We expect that this depth consistency
condition can improve the accuracy of the inverted velocity models, particularly for the S-wave velocity. This is because the wavepaths for updating the P-wave velocity are three times as many as those for updating the S-wave velocity, as shown by Equations (3) to (16). For validation, we further test the joint PP and PS PWEMVA method without the depth consistency ($\alpha_d = 0$) on the synthetic test 1 and the field data.

Figures 23 and 24 show the MVA result without the depth consistency condition for the synthetic test 1. Compared with the results from Figures 3 to 5, aligning the PP and PS images further improves the accuracy of the inverted S-wave velocity model (Figure 3b vs Figure 23b), flattens the PS CIGs (Figure 4b vs Figure 24b) and reduces the relative depth shifts between PP and PS images (Figure 4c vs Figure 24c). We do not see too much improvement in the inverted P-wave velocity model. This may be because there is no noise in the synthetic data and the model is not complex.

Figures 25 to 27 display the joint PP and PS PWMVA results without the depth consistency condition for the field data. Compared with the results from Figures 20 to 22, though the CIGs obtained without the depth consistency condition are slightly more flattened than those generated with the depth consistency (Figure 21 vs Figure 26). In addition, the average absolute value of the relative depth shifts between the three picked reflectors in Figure 27 (indicated by the dotted lines) is approximately twice the average P-wave wavelength at the central frequency, which is 5 times larger than that with the depth consistency (Figure 22 vs Figure 27). In addition, the high velocity zone in P- and S-wave velocity models obtained with the depth consistency are more consistent in depth than those produced without the depth consistency condition (Figure 20 vs Figure 25).
CONCLUSIONS

We have developed a joint PP and PS plane-wave wave-equation migration velocity analysis method for producing accurate P- and S-wave velocity models. This method flattens both the PP and PS plane-wave common-image gathers simultaneously while enforcing depth consistency between the corresponding PP and PS reflectors in migration images. The residual moveouts in the PP and PS plane-wave common-image gathers and the relative depth shifts between PP and PS images are used to construct the weighted image perturbations. The image perturbations are then used to compute the velocity gradients using the scalar-wave-equations with P- and S-wave velocity models. Numerical results with synthetic data verify that the method can invert for accurate P- and S-wave velocity models for migration. The field data test validates that the migration images obtained using the inverted P- and S-wave velocity models can flatten plane-wave common-image gathers and align PP and PS reflector images, suggesting that the inverted velocity models represent a good approximation to the low-wavenumber velocity models. One future research direction is to extend the joint PP and PS plane-wave migration velocity analysis method to anisotropic media.

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APPENDIX A

RELATIONSHIP BETWEEN THE GRADIENT OF THE OBJECTIVE FUNCTION AND THAT OF THE MIGRATION IMAGE

The objective function $J_{ab}$ is defined as the squared summation of vertical shift $\Delta w_{ab}(x)$ that aligns migration image $m_a(x, z + \Delta w_{ab}(x))$ with $m_b(x, z)$ for $x = (x, z)$:

$$J_{ab} = \frac{1}{2} \sum_x \Delta w_{ab}(x)^2. \tag{A-1}$$

The gradient of $J_{ab}$ with respect to the slowness $s(x')$ is

$$\frac{\partial J_{ab}}{\partial s(x')} = \sum_x \frac{\partial \Delta w_{ab}(x)}{\partial s(x')} \Delta w_{ab}(x). \tag{A-2}$$

To compute the Fréchet derivative of the vertical shift centered at $x$ with respect to the slowness perturbation at $x'$, a connective function is defined as the local crosscorrelation between $m_a(x)$ and $m_b(x)$:

$$f(s(x'), w(x)) = \sum_x m_a(x, z + w(x)) m_b(x, z), \tag{A-3}$$

where $w(x)$ is an arbitrary local shift. The correct image shift $\Delta w_{ab}(x)$ aligns $m_a(x, z + \Delta w_{ab}(x))$ with $m_b(x, z)$, so that the connective function in Equation (A-3) is maximized. This means that the derivative of $f$ with respect to $w(x)$ should be zero at $\Delta w_{ab}(x)$:

$$\frac{\partial f}{\partial w(x)} \bigg|_{w(x) = \Delta w_{ab}(x)} = \sum_x \dot{m}_a(x, z + \Delta w_{ab}(x)) m_b(x, z) \tag{A-4}$$

$$= 0,$$
where the dot represents the derivative with respect to $z$. The implicit function theorem gives

$$\frac{\partial \Delta w_{ab}(x)}{\partial s(x')} = -\frac{\partial f/\partial s(x')}{\partial f/\partial \Delta w_{ab}(x)},$$

(A-5)

where the denominator is given by

$$\frac{\partial f}{\partial \Delta w_{ab}(x)} = \sum_x \tilde{m}_a(x, z + \Delta w_{ab}(x))m_b(x, z).$$

(A-6)

Here the double dots represent the second-order derivative with respect to $z$.

If $m_a$ is chosen to be the reference image and only $m_b$ is a function of the migration slowness, the numerator of Equation (A-5) becomes

$$\frac{\partial f}{\partial s(x')} = \sum_x \tilde{m}_a(x, z + \Delta w_{ab}(x)) \frac{\partial m_b(x)}{\partial s(x')}.$$

(A-7)

Inserting Equations (A-5), (A-6) and (A-7) into Equation (3) yields

$$\frac{\partial J_{ab}}{\partial s(x')} = \sum_x -\Delta w_{ab}(x)\tilde{m}_a(x, z + \Delta w_{ab}(x)) \frac{\partial m_b(x)}{\partial s(x')}.$$

(A-8)

This equation links the gradients of the objective function with respective to the slowness to the gradients of the migration image using the weighted image perturbations.
APPENDIX B

GRADIENT OF MIGRATION IMAGE WITH RESPECT TO SLOWNESS

We briefly derive Fréchet derivative of the migration image at \( x \) with respect to the slowness according to Guo and Schuster (2017). For a 2D medium, let \( S(\omega, x) \) be the source wavefield. The receiver wavefield \( R(\omega, x) \) is computed using backward extrapolation of recorded data.

The migration image is obtained by multiplying the source wavefield with the complex conjugate of the receiver wavefield in the frequency domain and summing over all frequencies:

\[
m(x) = \Re \left\{ \sum_{\omega} S(\omega, x) R(\omega, x)^* \right\},
\]

where \( \Re \{ \} \) represents the real part.

Based on the migration imaging condition in Equation (B-1), the Fréchet derivative of the image at \( x \) with respect to the slowness perturbation at \( x' \) consists of two terms:

\[
\frac{\partial m(x)}{\partial s(x')} = \Re \left\{ \sum_{\omega} \frac{\partial S(\omega, x)}{\partial s(x')} R(\omega, x)^* \right\} + \Re \left\{ \sum_{\omega} \frac{\partial R(\omega, x)^*}{\partial s(x')} S(\omega, x) \right\}.
\]

To derive the formula for term \( \gamma_1 \) in Equation (B-2), the source wavefield \( S(\omega, x) \) emitted from a point source at \( x_s \) is defined as

\[
S(\omega, x) = G_{sr}(x|x_s)W(\omega),
\]

where \( G_{sr}(x|x_s) \) represents the source Green’s function at \( x \) from a harmonic point source at \( x_s \), and \( W(\omega) \) is the source spectrum. We assume here the source is zero phase and a single shot gather.
\(S(\omega, x)\) includes an additional summation over different shot gathers for the plane-wave source wavefield.

According to the Born approximation, the derivative of the source Green’s function with respect to the slowness is (Guo and Schuster, 2017)

\[
\frac{\partial G_{sr}(x|x_s)}{\partial s(x')} = 2\omega^2 s_{sr}(x') \frac{\partial s_{sr}(x')}{\partial s(x')} G_{sr}(x|x') G_{sr}(x'|x_s), \tag{B-4}
\]

where \(s_{sr}(x)\) is the source-side slowness associated with the source Green’s function \(G_{sr}(x|x_s)\).

Substituting the combination of Equations (B-3) and (B-4) into the expression of term \(\gamma_1\) in Equation (B-2) yields

\[
\gamma_1 = \Re \left\{ \sum_\omega \frac{\partial G_{sr}(x|x_s)}{\partial s(x')} W(\omega) R(\omega, x)^* \right\},
\]

\[
= \Re \left\{ \sum_\omega 2\omega^2 s_{sr}(x') \frac{\partial s_{sr}(x')}{\partial s(x')} W(\omega) G_{sr}(x'|x_s) G_{sr}(x|x') R(\omega, x)^* \right\},
\]

\[
= \Re \left\{ \sum_\omega 2\omega^2 s_{sr}(x') \frac{\partial s_{sr}(x')}{\partial s(x')} S(\omega, x') [G_{sr}(x'|x)^* R(\omega, x)]^* \right\}. \tag{B-5}
\]

The formula for term \(\gamma_2\) in Equation (B-2) is derived analogously. The wavefield \(R(\omega, x)\) is the backward extrapolated wavefield computed using the time-reversed propagation of the data \(d(\omega, x_g)\) at \(x_g\):

\[
R(\omega, x) = \sum_{x_g} G_{rv}(x|x_g)^* d(\omega, x_g), \tag{B-6}
\]

where \(G_{rv}(x|x_g)\) represents the receiver Green’s function at \(x\) from a harmonic point source at \(x_g\).
Analogous to Equation (B-4), under the Born approximation, the derivative of the receiver Green’s function with respective to the slowness is given by

\[
\frac{\partial G_{rv}(x|x_g)}{\partial s(x')} = 2\omega^2 s_{rv}(x') \frac{\partial s_{rv}(x')}{\partial s(x')} G_{rv}(x|x') G_{rv}(x'|x_g), \tag{B-7}
\]

where \(s_{rv}(x)\) is the receiver-side migration slowness associated with the receiver Green’s function \(G_{rv}(x|x_g)\). Substituting the combination of Equations (B-6) and (B-7) into the expression of term \(\gamma_2\) in Equation (B-2) yields

\[
\gamma_2 = \Re \left\{ \sum_{\omega} \left[ \sum_{x_g} \frac{\partial G_{rv}(x|x_g)}{\partial s(x')} (\omega, x_g) \right]^* S(\omega, x) \right\} = \Re \left\{ \sum_{\omega} \left[ 2\omega^2 s_{rv}(x') \frac{\partial s_{rv}(x')}{\partial s(x')} G_{rv}(x|x')^* \sum_{x_g} G_{rv}(x'|x_g)^* d(\omega, x_g) \right]^* S(\omega, x) \right\} = \Re \left\{ \sum_{\omega} \left[ 2\omega^2 \frac{\partial s_{rv}(x')}{\partial s(x')} \left[ S(\omega, x) G_{rv}(x'|x) \right] R(\omega, x')^* \right\}. \tag{B-8}
\]

Summarizing the aforementioned derivations, we obtain

\[
\frac{\partial m(x)}{\partial s(x')} = \gamma_1 + \gamma_2, \tag{B-9}
\]

where \(\gamma_1 = \Re \left\{ \sum_{\omega} 2\omega^2 s_{sr}(x') \frac{\partial s_{sr}(x')}{\partial s(x')} S(\omega, x')^* \left[ G_{sr}(x'|x)^* R(\omega, x) \right]^* \right\}, \tag{B-10}\)

and \(\gamma_2 = \Re \left\{ \sum_{\omega} 2\omega^2 s_{rv}(x') \frac{\partial s_{rv}(x')}{\partial s(x')} \left[ S(\omega, x) G_{rv}(x'|x) \right] R(\omega, x')^* \right\}. \tag{B-11}\)
APPENDIX C

GRADIENTS OF THE OBJECTIVE FUNCTION WITH RESPECT TO SLOWNESSES

Gradient of $\Delta w_p^j(x)$

The first term $\sum_x \Delta w_p^j(x)^2$ in the objective function is associated with the PP plane-wave CIGs. To derive its gradient with respect to the P-wave slowness $s_p(x')$, let $\Delta w_{ab}(x) = \Delta w_p^j(x)$, $m_a(x) = m_0^p(x)$, $m_b(x) = m_j^p(x)$ and $s(x') = s_p(x')$ in Appendix A. Correspondingly, let $m(x) = m_j^p(x)$, $S(\omega, x) = S_j^p(\omega, x)$, $R(\omega, x) = R_j^p(\omega, x)$, $G_{sr}(x'|x) = G_p(x'|x)$, $G_{rv}(x'|x) = G_p(x'|x)$, $s_{sr}(x') = s_p(x')$, $s_{rv}(x') = s_p(x')$ and $s(x') = s_p(x')$ in Appendix B. Inserting Equation (B-11) into Equation (A-8) yields

$$\sum_x \frac{\partial \Delta w_p^j(x)}{\partial s_p(x')} \Delta w_p^j(x) = \sum_x \frac{f_{pp}^j + g_{pp}^j}{m_0^p(x, z + \Delta w_p^j(x)) m_j^p(x)},$$

where

$$f_{pp}^j = \Re \left\{ \sum_\omega 2 \omega^2 s_p(x') S_j^p(\omega, x') \left[ G_p(x'|x)^* M_j^{pp}(x) R_j^p(\omega, x) \right]^* \right\},$$

and

$$g_{pp}^j = \Re \left\{ \sum_\omega 2 \omega^2 s_p(x') \left[ G_p(x'|x) M_j^{pp}(x) S_j^p(\omega, x) \right] R_j^p(\omega, x')^* \right\},$$

where

$$M_j^{pp}(x) = -\Delta w_j^p(x) m_0^p(x, z + \Delta w_j^p(x)) .$$

Since $\Delta w_j^p(x)$ is a function of PP plane-wave CIGs and is independent of the S-wave slowness $s_s(x')$, the gradient of the first term with respect to the S-wave slowness is zero.
Gradients of $\Delta w^s_j(x)$

The second term in the objective function is associated with the PS plane-wave CIGs. To derive its gradients with respect to the P-wave slowness $s_p(x')$ (or S-wave slowness $s_s(x')$), let $\Delta w_{ab}(x) = \Delta w^s_j(x)$, $m_a(x) = m^s_a(x)$, $m_b(x) = m^s_b(x)$ and $s(x') = s_p(x')$ (or $s(x') = s_s(x')$) in Appendix A. 

Correspondingly, let $m(x) = m^s_j(x)$, $S(\omega, x) = S^p_j(\omega, x)$, $R(\omega, x) = R^s_j(\omega, x)$, $G_{sr}(x'|x) = G_p(x'|x)$, $G_{rv}(x'|x) = G_s(x'|x)$, $s_{sr}(x') = s_p(x')$, $s_{rv}(x') = s_s(x')$ and $s(x') = s_p(x')$ (or $s(x') = s_s(x')$) in Appendix B. Inserting Equation (B-11) into Equation (A-8) yields

$$
\sum_x \frac{\partial \Delta w^s_j(x)}{\partial s_p(x')} \Delta w^s_j(x) = \sum_x \frac{f^ps_j}{m_0(x, z + \Delta w^s_j(x)) m^s_j(x)}, \quad (C-2)
$$

where $f^ps_j = \Re \left\{ \sum_\omega 2\omega^2 s_p(x') S^p_j(\omega, x') \left[ G_p(x'|x) M^ps_j(x) R^s_j(\omega, x) \right]^* \right\}$,

where $M^ps_j(x) = -\Delta w^s_j(x) m_0^s(x, z + \Delta w^s_j(x))$,

or

$$
\sum_x \frac{\partial \Delta w^s_j(x)}{\partial s_s(x')} \Delta w^s_j(x) = \sum_x \frac{g^ps_j}{m_0(x, z + \Delta w^s_j(x)) m^s_j(x)}, \quad (C-3)
$$

where $g^ps_j = \Re \left\{ \sum_\omega 2\omega^2 s_s(x') \left[ G_s(x'|x) M^ps_j(x) S^p_j(\omega, x) \right] R^s_j(\omega, x')^* \right\}$.

Gradient of $\Delta w^d(x)$ with respected to $s_p(x')$

The third term in the objective function is associated with the stacked PP and PS plane-wave images.

Its gradient with respect to the P-wave slowness $s_p(x')$ can be expressed as:

$$
\sum_x \frac{\partial \Delta w^d(x)}{\partial s_p(x')} \Delta w^d(x) = \left. \sum_x \frac{\partial \Delta w^d(x)}{\partial s_p(x')} \Delta w^d(x) \right|^{m^s(x)} + \left. \sum_x \frac{\partial \Delta w^d(x)}{\partial s_p(x')} \Delta w^d(x) \right|^{m^p(x)}, \quad (C-4)
$$
where the first term is obtained by choosing the stacked PS image \( m^s(x) \) as the reference image, while the second term by choosing the stacked PP image \( m^p(x) \) as the reference image.

**Using \( m^s(x) \) as the reference image**

When \( m^s(x) \) is the reference image, \( \Delta w^d(x) \) aligns the PS image \( m^s(x, z + \Delta w^d(x)) \) with the PP image \( m^p(x, z) \), thus we let \( \Delta w_{ab}(x) = \Delta w^d(x) \), \( m_a(x) = m^s(x) \), \( m_b(x) = m^p(x) \) and \( s(x') = s_p(x') \) in Appendix A. The first term on the right-hand side of Equation (C-4) becomes:

\[
\sum_x \frac{\partial \Delta w^d(x)}{\partial s_p(x')} \Delta w^d(x) \bigg|_{m^s(x)} = \sum_x \frac{-\Delta w^d(x) \dot{m}^s(x, z + \Delta w^d(x)) \frac{\partial m^p(x)}{\partial s_p(x')}}{\dot{m}^s(x, z + \Delta w^d(x)) m^p(x)}. \tag{C-5}
\]

According to Equation (2),

\[
\frac{\partial m^p(x)}{\partial s_p(x')} = \sum_{j=1}^{n_p} \frac{\partial m^p_j(x)}{\partial s_p(x')}. \tag{C-6}
\]

The expression of \( \frac{\partial m^p(x)}{\partial s_p(x')} \) is obtained when \( m(x) = m^p_j(x) \), \( S(\omega, x) = S^p_j(\omega, x) \), \( R(\omega, x) = R^p_j(\omega, x) \), \( G_{sr}(x'|x) = G_p(x'|x) \), \( G_{rv}(x'|x) = G_p(x'|x) \), \( s_{sr}(x') = s_p(x') \), \( s_{rv}(x') = s_p(x') \) and \( s(x') = s_p(x') \) in Appendix B. Substituting Equations (B-11) and (C-6) into Equation (C-5) yields

\[
\sum_x \frac{\partial \Delta w^d(x)}{\partial s_p(x')} \Delta w^d(x) \bigg|_{m^s(x)} = \sum_{j=1}^{n_p} \sum_x \dot{m}^s(x, z + \Delta w^d(x)) \frac{f_{jp}^d + g_{jp}^d}{m^p(x)}, \tag{C-7}
\]

where

\[
f_{jp}^d = \Re \left\{ \sum_{\omega} 2 \omega^2 s_p(x') S^p_j(\omega, x') \left[ G_p(x'|x) M^{dp}(x) R^p_j(\omega, x) \right]^* \right\},
\]

and

\[
g_{jp}^d = \Re \left\{ \sum_{\omega} 2 \omega^2 s_p(x') \left[ G_p(x'|x) M^{dp}(x) S^p_j(\omega, x) \right] R^p_j(\omega, x)^* \right\},
\]

where \( M^{dp}(x) = -\Delta w^d(x) \dot{m}^s(x, z + \Delta w^d(x)) \).

27
Using $m^p(x)$ as the reference image

When $m^p(x)$ is the reference image, $\Delta w^d(x)$ aligns the PP image $m^p(x, z - \Delta w^d(x))$ with the PS image $m^s(x, z)$, thus we let $\Delta w_{ab}(x) = -\Delta w^d(x)$, $m_a(x) = m^p(x)$, $m_b(x) = m^s(x)$, $s(x') = s_p(x')$ and $s(x') = s_p(x')$ in Appendix A. The second term on the right-hand side of Equation (C-4) becomes:

\[
\sum_x \frac{\partial \Delta w^d(x)}{\partial s_p(x')} \Delta w^d(x) \bigg|_{m^p(x)} = \sum_x \frac{\Delta w^d(x) \dot{m}^p(x, z - \Delta w^d(x))}{\dot{m}^p(x, z - \Delta w^d(x)) m^s(x)} \frac{\partial m^s(x)}{\partial s_p(x')}. \tag{C-8}
\]

According to Equation (2),

\[
\frac{\partial m^s(x)}{\partial s_p(x')} = \sum^n_{j=1} \frac{\partial m^s_j(x)}{\partial s_p(x')}. \tag{C-9}
\]

The expression of $\frac{\partial m^s_j(x)}{\partial s_p(x')}$ is obtained when $m(x) = m^s_j(x)$, $S(\omega, x) = S^p_j(\omega, x)$, $R(\omega, x) = R^s_j(\omega, x)$, $G_{sr}(x'|x) = G_p(x'|x)$, $G_{rv}(x'|x) = G_s(x'|x)$, $s_{sr}(x') = s_p(x')$, $s_{rv}(x') = s_s(x')$ and $s(x') = s_p(x')$ in Appendix B. Inserting Equations (B-11) and (C-9) into Equation (C-8) yields:

\[
\sum_x \frac{\partial \Delta w^d(x)}{\partial s_p(x')} \Delta w^d(x) \bigg|_{m^p(x)} = \sum^n_{j=1} \sum_x \frac{f^{ds}_j}{\dot{m}^p(x, z - \Delta w^d(x)) m^s(x)} \frac{\partial m^s_j(x)}{\partial s_p(x')}, \tag{C-10}
\]

where $f^{ds}_j = \Re \left\{ \sum_\omega 2\omega^2 s_p(x') S^p_j(\omega, x') \left[ G_p(x'|x)^* M^{ds}(x) R^s_j(\omega, x) \right]^2 \right\}$, \tag{C-11}

where $M^{ds}(x) = \Delta w^d(x) \dot{m}^p(x, z - \Delta w^d(x))$. 

28
Gradient of $\Delta w^d(x)$ with respected to $s_s(x')$

To derive the gradient of the third term with respect to the S-wave slowness $s_s(x')$, we only let $m^p(x)$ as the reference image because $m^p(x)$ is not a function of $s_s(x')$, thus $\Delta w^d(x)$ aligns the PP image $m^p(x, z - \Delta w^d(x))$ with the PS image $m^s(x, z)$. Let $\Delta w_{ab}(x) = -\Delta w^d(x)$, $m_a(x) = m^p(x)$, $m_b(x) = m^s(x)$ and $s(x') = s_s(x')$ in Appendix A, we obtain

$$\sum_x \frac{\partial \Delta w^d(x)}{\partial s_s(x')} \Delta w^d(x) \biggl|_{m^p(x)} = \sum_x \frac{\Delta w^d(x) \hat{m}^p(x, z - \Delta w^d(x)) \frac{\partial m^s(x)}{\partial s_s(x')}}{\hat{m}^p(x, z - \Delta w^d(x)) m^s(x)}. \quad (C-12)$$

According to Equation (2),

$$\frac{\partial m^s(x)}{\partial s_s(x')} = \sum_{j=1}^{n_s} \frac{\partial m^s_j(x)}{\partial s_s(x')}.$$ \quad (C-13)

The expression of $\frac{\partial m^s_j(x)}{\partial s_s(x')}$ can be obtained when $m(x) = m^s_j(x)$, $S(\omega, x) = S^p_j(\omega, x)$, $R(\omega, x) = R^s_j(\omega, x)$, $G_{sr}(x'|x) = G_p(x'|x)$, $G_{rv}(x'|x) = G_s(x'|x)$, $s_{sr}(x') = s_p(x')$, $s_{rv}(x') = s_s(x')$ and $s(x') = s_s(x')$ in Appendix B. Substituting Equations (B-11) and (C-13) into Equation (C-12) yields:

$$\sum_x \frac{\partial \Delta w^d(x)}{\partial s_s(x')} \Delta w^d(x) = \sum_x \sum_{j=1}^{n_s} \frac{\partial m^s_j(x)}{\partial s_s(x')} = \sum_{j=1}^{n_s} \sum_x \frac{g^{ds}_j}{\hat{m}^p(x, z - \Delta w^d(x)) m^s(x)}, \quad (C-14)$$

where $g^{ds}_j = \Re \left\{ \sum_\omega 2\omega^2 s_s(x') \left[ G_s(x'|x) \mathcal{M}^{ds}(x) S^p_j(\omega, x) \right] R^s_j(\omega, x')^* \right\}.$ \quad (C-15)

where $\mathcal{M}^{ds}(x)$ is the same as in Equation (C-11).
APPENDIX D

EQUATIONS FOR COMPUTING THE UPWARD- AND DOWNWARD-PROPAGATED SOURCE AND RECEIVER WAVEFIELDS

The downward-propagated source and receiver wavefields $S_{p}^{j}(\omega, x')$ and $R_{p}^{j}(\omega, x')$ in Equations (4) to (16) are computed using

$$\frac{\partial^2 S_{p}^{j}(\omega, x', t)}{\partial t^2} - \frac{1}{s_{p}(x'')^2} \nabla^2 S_{p}^{j}(\omega, x', t) = f_{p}^{j}(\omega, x_s, t),$$  \hspace{1cm}  \text{(D-1)}$$

and

$$\frac{\partial^2 R_{p}^{j}(\omega, x', t)}{\partial t^2} - \frac{1}{s_{p}(x'')^2} \nabla^2 R_{p}^{j}(\omega, x', t) = d_{p}^{j}(\omega, x_g, t),$$  \hspace{1cm}  \text{(D-2)}$$

respectively. Here $S_{p}^{j}(\omega, x', t)$ and $R_{p}^{j}(\omega, x', t)$ are the time-domain wavefield of $S_{p}^{j}(\omega, x')$ and $R_{p}^{j}(\omega, x')$, respectively, $f_{p}^{j}(\omega, x_s, t)$ is the plane-wave source function at $x_s$ and $d_{p}^{j}(\omega, x_g, t)$ is the recorded plane-wave PP(PS) gather at $x_g$ for the $j$th plane-wave index. Equation (D-2) is solved backward in time.

The upward-propagated source wavefield $S_{u,j}^{p}(\omega, x') = G(x'|x) \mathcal{M}(x) S_{p}^{j}(\omega, x)$ is computed using

$$\frac{\partial^2 S_{u,j}^{p}(\omega, x', t)}{\partial t^2} - \frac{1}{s_{p}(x'')^2} \nabla^2 S_{u,j}^{p}(\omega, x', t) = \mathcal{M}(x) S_{p}^{j}(\omega, x, t),$$  \hspace{1cm}  \text{(D-3)}$$

where $S_{u,j}^{p}(\omega, x', t)$ is the time-domain wavefield of $S_{u,j}^{p}(\omega, x')$ and $S_{p}^{j}(\omega, x, t)$ is computed using Equation (D-1). Here the superscripts and subscripts of $\mathcal{M}(x)$ are omitted.

The upward-propagated receiver wavefield $R_{u,j}^{p}(\omega, x') = G(x'|x)^* \mathcal{M}(x) R_{p}^{j}(\omega, x)$ is com-
puted using

\[
\frac{\partial^2 R_{u,j}^{(s)}(x', t)}{\partial t^2} - \frac{1}{s_{p(s)}(x')^2} \nabla^2 R_{u,j}^{p(s)}(x', t) = M(x) R_{j}^{p(s)}(x, t),
\]

(D-4)

where \( R_{u,j}^{p(s)}(x', t) \) is the time-domain wavefield of \( R_{u,j}^{p(s)}(\omega, x') \). Equation (D-4) is solved backward in time and \( R_{j}^{p(s)}(x, t) \) is computed using Equation (D-2).
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LIST OF FIGURES

1 Synthetic test 1: True (a) P- and (b) S-wave velocity models, true (c) PP and (d) PS reflectivity models, together with Born modeling results of (e) PP and (f) PS plane-wave gathers with $p = 0$ s/km computed using velocity and reflectivity models in (a-d).

2 Synthetic test 1: (a) PP and (b) PS plane-wave CIGs computed using homogeneous velocity models. The computed semblance spectra and the picked curvatures at 1.0 km for (c) PP and (d) PS CIGs. (e) The relative depth shifts between the stacked PP and PS images shown in Figures 5a and 5b. The relative depth shifts are computed using dynamic warping. A positive depth shift means the PS image needs to be shifted downward to match the PP image.

3 Synthetic test 1: Inverted (a) P- and (b) S-wave velocity models obtained using our joint PP and PS PWMVA. They mostly resemble the true velocity models in Figures 1a and 1b.

4 Synthetic test 1: (a) PP and (b) PS plane-wave CIGs computed using the inverted velocity models shown in Figure 3.

5 Synthetic test 1: Stacked plane-wave RTM images produced using the (a and b) initial homogeneous and (c and d) inverted velocity models shown in Figure 3.

6 Synthetic test 1: The relative depth shifts computed using (a) the stacked PP and PS images in Figures 5a and 5b, and (b) the stacked PP and PS images in Figures 5c and 5d. The relative depth shifts in (b) are much smaller than those in (a).

7 Synthetic test 2: True (a) P- and (b) S-wave velocity models, and (c) density model modified from a portion of the BP2004 model for generating the synthetic elastic wavefields to test our joint PP and PS PWMVA.

8 Synthetic test 2: (a) PP and (b) PS common-shot gathers extracted from the synthetic elastic wavefields with a source at $x = 7.0$ km, (c) PP and (d) PS plane-wave gathers with $p = -0.082$ s/km (shooting angle = $-7.1^\circ$) computed from the shot gathers.

9 Synthetic test 2: Initial (a) P- and (b) S-wave velocity models for joint PP and PS PWMVA.

10 Synthetic test 2: (a) PP and (b) PS plane-wave CIGs computed using the initial velocity models in Figure 9. There exist strong residual moveouts.

11 Synthetic test 2: Stacked (a) PP and (b) PS plane-wave RTM images obtained using the initial velocity models in Figure 9, together with (c) the relative depth shifts between (a) and (b).

12 Synthetic test 2: Inverted (a) P- and (b) S-wave velocity models after 12 iterations of joint PP and PS PWMVA. Panel (c) plots the normalized value of the objective function $J$ and its three terms at each iteration. The inverted velocity models recover most of the low-wavenumber components of the true-velocity models in Figures 7a and 7b.

13 Synthetic test 2: (a) PP and (b) PS plane-wave CIGs computed using the inverted velocity models in Figure 12. They are mostly flattened compared to the plane-wave CIGs in Figure 10 computed using the initial velocity models.

14 Synthetic test 2: Stacked (a) PP and (b) PS plane-wave RTM images produced using the inverted velocity models in Figure 12, together with (c) the relative depth shifts between (a) and (b). The RTM images closely resemble the high-wavenumber features of the density model in Figure 7c.

15 Field data test: (a) PP and (b) PS common-shot gathers for a source at $x = 3.5$ km. This is a land seismic dataset acquired at Kevin Dome, Montana, USA.

16 Field data test: (a) PP and (b) PS plane-wave gathers with $p = -0.073$ s/km (shooting angle = $-6.7^\circ$) transformed from the shot profiles in Figure 15.

17 Field data test: Initial (a) P- and (b) S-wave velocity models for joint PP and PS PWMVA.

18 Field data test: (a) PP and (b) PS plane-wave CIGs with strong residual moveouts computed using the initial velocity models in Figure 17.

19 Field data test: Stacked (a) PP and (b) PS plane-wave RTM images generated using the
initial velocity models in Figure 17.

20 Field data test: Inverted (a) P- and (b) S-wave velocity models obtained using joint PP and PS PWMVA.

21 Field data test: (a) PP and (b) PS plane-wave CIGs computed using the inverted velocity models in Figure 20. They are mostly flattened compared to the plane-wave CIGs in Figure 18 generated using the initial velocity models.

22 Field data test: Stacked (a) PP and (b) PS plane-wave RTM images produced using the inverted velocity models models in Figure 20. The dotted lines indicate the three reflectors manually picked for computing the relative depth shifts during inversion. The RTM images are more focused and continuous than those in Figure 17 generated using the initial velocity models. The average absolute value of relative depth shifts of the three reflectors is approximately 35 m (about 1/3 of the average P-wave wavelength at the central frequency).

23 Synthetic test 1: Inverted (a) P- and (b) S-wave velocity models obtained using joint PP and PS PWMVA without enforcing the depth consistency, together with corresponding stacked (c) PP and (d) PS plane-wave RTM images obtained using the inverted velocity models in (a) and (b).

24 Synthetic test 1: (a) PP and (b) PS plane-wave CIGs computed using the inverted velocity models in Figures 23a and 23b. (c) The relative depth shifts between the stacked PP and PS images in Figures 23c and 23d.

25 Field data test: (a) P- and (b) S-wave velocity models obtained using joint PP and PS PWMVA without enforcing the depth consistency.

26 Field data test: (a) PP and (b) PS plane-wave CIGs computed using the inverted velocity models obtained without the depth consistency in Figure 20.

27 Field data test: Stacked (a) PP and (b) PS plane-wave RTM images produced using the inverted velocity models models in Figure 25. The dotted lines indicate the same three reflectors as those in Figure 22. The dotted lines in these figures are just for visualization and comparison. The average absolute value of the relative depth shifts of these three reflectors is approximately 170 m, which is roughly two average P-wave wavelengths at the central frequency.
Figure 1: Synthetic test 1: True (a) P- and (b) S-wave velocity models, true (c) PP and (d) PS reflectivity models, together with Born modeling results of (e) PP and (f) PS plane-wave gathers with $p = 0$ s/km computed using velocity and reflectivity models in (a-d).
Figure 2: Synthetic test 1: (a) PP and (b) PS plane-wave CIGs computed using homogeneous velocity models. The computed semblance spectra and the picked curvatures at 1.0 km for (c) PP and (d) PS CIGs. (e) The relative depth shifts between the stacked PP and PS images shown in Figures 5a and 5b. The relative depth shifts are computed using dynamic warping. A positive depth shift means the PS image needs to be shifted downward to match the PP image.
Figure 3: Synthetic test 1: Inverted (a) P- and (b) S-wave velocity models obtained using our joint PP and PS PWMVA. They mostly resemble the true velocity models in Figures 1a and 1b.
Figure 4: Synthetic test 1: (a) PP and (b) PS plane-wave CIGs computed using the inverted velocity models shown in Figure 3.
Figure 5: Synthetic test 1: Stacked plane-wave RTM images produced using the (a and b) initial homogeneous and (c and d) inverted velocity models shown in Figure 3.
Figure 6: Synthetic test 1: The relative depth shifts computed using (a) the stacked PP and PS images in Figures 5a and 5b, and (b) the stacked PP and PS images in Figures 5c and 5d. The relative depth shifts in (b) are much smaller than those in (a).
Figure 7: Synthetic test 2: True (a) P- and (b) S-wave velocity models, and (c) density model modified from a portion of the BP2004 model for generating the synthetic elastic wavefields to test our joint PP and PS PWMVA.
Figure 8: Synthetic test 2: (a) PP and (b) PS common-shot gathers extracted from the synthetic elastic wavefields with a source at $x = 7.0$ km, (c) PP and (d) PS plane-wave gathers with $p = -0.082$ s/km (shooting angle $= -7.1^\circ$) computed from the shot gathers.
Figure 9: Synthetic test 2: Initial (a) P- and (b) S-wave velocity models for joint PP and PS PWMVA.
Figure 10: Synthetic test 2: (a) PP and (b) PS plane-wave CIGs computed using the initial velocity models in Figure 9. There exist strong residual moveouts.
Figure 11: Synthetic test 2: Stacked (a) PP and (b) PS plane-wave RTM images obtained using the initial velocity models in Figure 9, together with (c) the relative depth shifts between (a) and (b).
Figure 12: Synthetic test 2: Inverted (a) P- and (b) S-wave velocity models after 12 iterations of joint PP and PS PWMVA. Panel (c) plots the normalized value of the objective function $J$ and its three terms at each iteration. The inverted velocity models recover most of the low-wavenumber components of the true-velocity models in Figures 7a and 7b.
Figure 13: Synthetic test 2: (a) PP and (b) PS plane-wave CIGs computed using the inverted velocity models in Figure 12. They are mostly flattened compared to the plane-wave CIGs in Figure 10 computed using the initial velocity models.
Figure 14: Synthetic test 2: Stacked (a) PP and (b) PS plane-wave RTM images produced using the inverted velocity models in Figure 12, together with (c) the relative depth shifts between (a) and (b). The RTM images closely resemble the high-wavenumber features of the density model in Figure 7c.
Figure 15: Field data test: (a) PP and (b) PS common-shot gathers for a source at $x = 3.5$ km. This is a land seismic dataset acquired at Kevin Dome, Montana, USA.
Figure 16: Field data test: (a) PP and (b) PS plane-wave gathers with $p = -0.073$ s/km (shooting angle = $-6.7^\circ$) transformed from the shot profiles in Figure 15.
Figure 17: Field data test: Initial (a) P- and (b) S-wave velocity models for joint PP and PS PWMVA.
Figure 18: Field data test: (a) PP and (b) PS plane-wave CIGs with strong residual moveouts computed using the initial velocity models in Figure 17.
Figure 19: Field data test: Stacked (a) PP and (b) PS plane-wave RTM images generated using the initial velocity models in Figure 17.
Figure 20: Field data test: Inverted (a) P- and (b) S-wave velocity models obtained using joint PP and PS PWMVA.
Figure 21: Field data test: (a) PP and (b) PS plane-wave CIGs computed using the inverted velocity models in Figure 20. They are mostly flattened compared to the plane-wave CIGs in Figure 18 generated using the initial velocity models.
Figure 22: Field data test: Stacked (a) PP and (b) PS plane-wave RTM images produced using the inverted velocity models in Figure 20. The dotted lines indicate the three reflectors manually picked for computing the relative depth shifts during inversion. The RTM images are more focused and continuous than those in Figure 17 generated using the initial velocity models. The average absolute value of relative depth shifts of the three reflectors is approximately 35 m (about 1/3 of the average P-wave wavelength at the central frequency).
Figure 23: Synthetic test 1: Inverted (a) P- and (b) S-wave velocity models obtained using joint PP and PS PWMVA without enforcing the depth consistency, together with corresponding stacked (c) PP and (d) PS plane-wave RTM images obtained using the inverted velocity models in (a) and (b).
Figure 24: Synthetic test 1: (a) PP and (b) PS plane-wave CIGs computed using the inverted velocity models in Figures 23a and 23b. (c) The relative depth shifts between the stacked PP and PS images in Figures 23c and 23d.
Figure 25: Field data test: (a) P- and (b) S-wave velocity models obtained using joint PP and PS PWMVA without enforcing the depth consistency.
Figure 26: Field data test: (a) PP and (b) PS plane-wave CIGs computed using the inverted velocity models obtained without the depth consistency in Figure 20.
Figure 27: Field data test: Stacked (a) PP and (b) PS plane-wave RTM images produced using the inverted velocity models in Figure 25. The dotted lines indicate the same three reflectors as those in Figure 22. The dotted lines in these figures are just for visualization and comparison. The average absolute value of the relative depth shifts of these three reflectors is approximately 170 m, which is roughly two average P-wave wavelengths at the central frequency.