Multi-dimensional wave steering with higher-order topological phononic crystal

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ABSTRACT

The recent discovery and realizations of higher-order topological insulators enrich the fundamental studies on topological phases. Here, we report three-dimensional (3D) wave-steering capabilities enabled by topological boundary states at three different orders in a 3D phononic crystal with nontrivial bulk topology originated from the synergy of mirror symmetry of the unit cell and a non-symmorphic glide symmetry of the lattice. The multitude of topological states brings diverse possibilities of wave manipulations. Through judicious engineering of the boundary modes, we experimentally demonstrate two functionalities at different dimensions: 2D negative refraction of sound wave enabled by a first-order topological surface state with negative dispersion, and a 3D acoustic interferometer leveraging on second-order topological hinge states. Our work showcases that topological modes at different orders promise diverse wave steering applications across different dimensions.

1. Introduction

Since its incipience in the 1980s, the study of topological notion in physics has attracted tremendous attention across diverse disciplines [1–3]. We have witnessed the emergence of a kaleidoscope of topological phases, such as quantum Hall system [1–3], Chern insulators [4–6], and the relatively recent discovery of higher-order topological phases [7–14]. As a hallmark, the nontrivial topology of the system can lead to the existence of certain types of excitations that are localized at the boundaries of the system. Some of these topologically protected states have fascinating properties, such as being immune to backscattering [15–25].

While many topological phases originate from the electronic systems [1–3], they quickly gained the attention from other realms, spanning from optics, photonics [26,27], electromagnetism [4–6,15–17], to acoustics and phononics [18–25]. Due to the versatility offered by these classical systems, they rapidly became platforms to realize novel topological phases and to investigate the physics therein. However, relatively few efforts were devoted to the exploration of novel topological states for wave manipulation applications. This work aims at applying topological states to achieve novel wave steering at different dimensions. It is based on a simple realization of a 3D higher-order topological phononic crystal (PC) possessing a large bandgap that can be characterized by the nontrivial quantized bulk polarization. Different from the acoustic analog of the Su-Schrieffer-Heeger (SSH) model [10] or the Kagome model [8], our PC maintains nontrivial topology after a reversion of the center and corner of the unit cell, which means our PC does not have a topologically trivial counterpart. As a result of the topological protection, 2D topological surface states (TSSs), 1D topological hinge states (THSs), and 0D topological corner states (TCSs) are observed. We then present the dispersion of the TSSs and show that it can lead to negative refraction at PC-air interfaces. In addition, we exploit the THSs as tailorable transport channels to realize a 3D acoustic interferometer. Our work showcases that topological states can be tailored for versatile wave steering applications across multiple dimensions.

2. Results and discussions

2.1. Phononic crystal with higher-order topology

Our PC comprises a cubic array of orthogonally aligned aluminum rods (treated as sound-hard objects) along the x-, y-, z-directions, respectively, in an air background. All rods have a square cross-section with a side length $L = 1.8$ cm, and their axes are separated by $a/2$, where $a = 4$ cm is the lattice constant. The PC belongs to a non-symmorphic space group no. 223 [28] and has a glide symmetry $G_{pm} = \{M_{yz}|_{\frac{1}{2}}\}$: $(\mu, \nu, \xi) \rightarrow (-\nu, -\mu, \xi + \frac{1}{2})$. 

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where the subscripts represent spatial coordinates. It transforms a rod at \( \mu = (x, y, z) \) to \( v = (y, z, x) \) through a reflection over \((-\mu, v)\) plane followed by a translation of \(a/2\) along \(\xi \equiv (z, x, y)\) [29–31]. A selection of unit cell labeled as “uc1” is made such that mirror symmetry \(m_h\) with respect to all three bisecting planes. The PC’s glide symmetry implies that shifting the unit cell along one of the diagonal directions by a distance \(\Delta x, \Delta y, \Delta z = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\) produces a different choice of unit cell, which also possesses the same mirror symmetry \(m_h\). This unit cell’s center coincides with the corner of the original unit cell. For presentation purpose, we shift the uc1 by \((\Delta x, \Delta y, \Delta z) = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\), and label the new unit cell as “uc2” in Fig. 1.a.

The band structure along high symmetry lines of the first Brillouin zone is calculated by the acoustic module in COMSOL MULTIPHYSICS and plotted in Fig. 1b. The combination of lattice symmetry and geometry parameters results in a large complete bandgap from 5.0 to 6.8 kHz. Protected by the glide symmetry, the two bands below the bandgap are doubly degenerate at the Brillouin zone boundaries [30,32]. Meanwhile, the unit cell’s mirror symmetry indicates the possibility of quantized band topology [10]. Here, we characterize the band topology using the Wannier bands [33], which consists of Wannier centers \(v\) as functions of wave vector \(k\). The Wannier centers are obtained by computing the Wilson loop operator defined as:

\[
\mathcal{W}_{\mu}(k) = -\frac{1}{2\pi} \int_0^{2\pi/a} u^{m_i}(k) \partial_k u^n(k) dk, \tag{1}
\]

where \(m\) and \(n\) are the band indices, \(k\) defines the starting point of the loop, \(\mu = (x, y, z)\) indicates the loop direction, and \(u^i(k)\) is the periodic part of the Bloch wavefunction. The integration is carried out by discretizing the Brillouin Zone into 20 segments in each direction of \(k\). We then diagonalize the Wilson loop operator \(\mathcal{W}_{\mu}(k)\) as \(\mathcal{W}_{\mu}(k) | \mathcal{R}_i \rangle = e^{\text{ii}2\pi/n} | \mathcal{R}_i \rangle\). The phase \(\mathcal{R}_i\), which depends on the band index and the Wilson loop, is the Wannier center of the Bloch wavefunctions. Fig. 1c shows the Wannier band \(v_i (k_x, k_y)\) plane, where \(\xi, \mu, v \in (x, y, z)\) in cyclic order. The \(v\) component of the bulk polarization can be obtained as [10]:

\[
P_v = \frac{1}{(2\pi)^2} \int \mathcal{W}_{\mu}(k_x, k_y) dk_x dk_y, \tag{2}
\]

where \(\mathcal{W}_{\mu}(k_x, k_y)\) represents the Wannier band and \(S\) is the projection area of the Brillouin zone. Our computation yields two gapped and quantized Wannier centers \(v_1 = (0, 0, 0)\) and \(v_2 = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\) for these two bands, indicating non-zero quantized bulk dipole moment \(P = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\), which characterizes higher-order topological insulators and gives rise to multiple orders of topological states [10,11,34–36]. The nonzero quantized bulk polarizations indicate a mismatch between the Wannier centers and the center of the unit cell, which means our PC maintains a nontrivial topology [37,38]. Because the eigenvalues of crystalline symmetry operators of bulk states at high-symmetry points are rigorously related to topological invariants [37–39], we can analyze the bulk topology by comparing the parities of Bloch wavefunctions at the high-symmetry points of the Brillouin zone [10,40]. In a system with three mirror symmetries \(m_{\mu}(\mu = x, y, z)\) and a \(C_2\) rotation symmetry along the \([111]\) direction, when the wavefunctions of a Bloch band at \(k_0, a = 0\) and \(k_0, a = \pi\) have same (opposite) parities, the corresponding polarization component should be \(P_{\mu} = 0 (\pm)\) [10]. The Bloch wavefunctions at the high-symmetry points labeled in Fig. 1b are plotted in Fig. 1d–k. It is seen that, at the Brillouin zone center, the wavefunctions have identical parity (Fig. 1d, e). In contrast, since the glide symmetry mandates the two bands to be degenerate at the Brillouin zone boundaries, the corresponding wavefunctions must have opposite parities. Interestingly, as previously mentioned, the glide symmetry also implies that the second choice of the unit cell (uc2), which also preserves mirror symmetry \(m_h\) gives identical bulk dipole moment \(P = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\). In other words, the nontrivial bulk polarization of the first bandgap exists for both choices of the unit cell. Consequently, under the premise of \(m_{\mu}\), the glide symmetry of our PC leads to the robustness of bulk polarization to different selections of the unit cell. This characteristic distinguishes our PC from certain higher-order topological insulators, such as the ‘breathing’ Kagome lattice [8] and the 3D SSH model [10], in which the bulk topology depends on the selection of unit cell.

2.2. Negative refraction at PC-air interfaces by TSS

We consider a lattice truncation, which gives rise to topological boundary modes [8,41,42]. Shown in Fig. 2a is the supercell, consisting of 7 unit cells in the z-direction, and periodic in x- and y-directions. The top unit cell is truncated by a plane at a distance \(dh \in [0.4, 0.4]\) cm away from the top, as illustrated in Fig. 2a. Two sound-hard boundaries cover the top and bottom ends of the supercell. Fig. 2b shows the dispersion of the TSS along high symmetry lines of the surface Brillouin zone at \(dh = 2.9\) cm. The parameter \(dh\) affects the frequency of the TSS. Here we choose \(dh = 2.9\) cm to keep the frequency of the first TSS around the middle of bandgap, which is desirable for its application. Note that the group velocity of this TSS is negative. The isofrequency contours (IFC) are plotted in Fig. 2c. The black arrow represents the wavevector of the incident wave, and the white arrow is along the direction of the group velocity of the refracted wave in the PC. The IFC implies negative refraction, where a single incident beam couples to a refracted beam on the same side of the surface normal.
TSS-enabled negative refraction is demonstrated with the setup shown in Fig. 2d. A PC sample consisting 20 × 6 identical supercells with fixed truncation position (dh = 2.9 cm) is bounded in a cuboid aluminum box. Two planar waveguides of 5 mm thickness are connected to the top surface via two thin slits. The top plate of the waveguides and the PC is removed to show the microphone and the PC. A waveform generator (Keysight 33500B) is used to generate a monochromatic signal to drive a loudspeaker through an audio power amplifier, emits a near-Gaussian beam at a frequency of 6.06 kHz and at an incident angle of 15°. The beam propagates inside the waveguide and incident to the top surface of the PC to excite the TSS. With a house-built motorized translation stage and a microphone, we raster-scan the acoustic field inside both the waveguides on the incident and transmitted sides. The scanning regions are marked by the dashed black boxes. In Fig. 2e, the experimental and simulation results of the acoustic pressure field are plotted. The incident field and transmitted field in the experimental result are normalized by their respective maximum, respectively. The signals are detected by a 1/4-inch microphone (PCB Piezotronics Model-378C10) and are then recorded by a digital oscilloscope (Keysight DSO2024A). As expected, both the simulated and the measured acoustic pressure fields show that the output beam is right-shifted, verifying the negative refraction.

2.3. Observation of higher-order topological states

We consider a PC cube consisting 5 × 5 × 5 unit cells (shown in Fig. 3a) enclosed by sound-hard boundaries at all sides. The truncations are chosen as dh = 1.0 cm on the three adjacent boundary surfaces (visible in Fig. 3a) and dh = 3.0 cm on the other three boundary surfaces (hidden in Fig. 3a). Such truncation scheme results in two different types of hinges, as respectively indicated by black and cyan dashed lines in Fig. 3a. We calculate the eigen-spectrum of this PC cube and plot eigenfrequencies versus the solution number in Fig. 3b, in which the first-order TSSs, second-order THSs, and third-order TCSs are identified in the bulk bandgap. The in-gap THSs only exist at the black hinges. Likewise, the in-gap TCSs exist at the crossing point of three cyan hinges. The THSs cluster in a frequency range of 5.0–5.8 kHz, whereas TCSs are found at 6.69 kHz. The acoustic pressure field distributions at 5.14 and 6.69 kHz are plotted in Fig. 3c and d, manifesting the typical THSs’ and TCSs’ features, respectively.

To verify the simulated results, we fabricated this 5 × 5 × 5 PC cube enclosed in aluminum plates, whose picture is shown in Fig. 3e without the top aluminum plate for viewing purposes. Arrays of holes are drilled on the aluminum plates. The holes are blocked when not in use. Sound sources can be placed at different positions and excite the PC cube through the holes. First, we insert a microphone well inside the PC through the holes to measure the bulk response. The normalized pressure field is plotted in the black curve in Fig. 3f, exhibiting a large bandgap in the frequency range 4.9–7.0 kHz as predicted by the simulation. Then the hinge response is also observed, which exhibits a dominant peak near 5.14 kHz followed by a plateau extending to about 6.8 kHz, as shown in the green curve in Fig. 3f, implying the existence of localized states on the surfaces or at the hinge. The response at the corner is plotted in Fig. 3f in red curve. Only one dominant peak at 6.72 kHz is observed, agreeing with the numerical simulation. To further confirm the existence of THSs and TCSs, we mapped out the acoustic field distribution on the surfaces, hinges, and corners by inserting the microphone to all small holes on the claddings. The pressure field maps obtained at 5.20 and 6.72 kHz are shown in Fig. 3g and h, and clearly demonstrate the localization of the sound wave on the hinges and at the corner, respectively, affirming the coexistence of second-order and third-order topological states in our PC. The stars in Fig. 3g and h represent the position of sound sources.

2.4. Wave transport leveraging THSs

Higher-order topological states give rise to new possibilities for wave manipulation. For example, THSs naturally offer tailorable...
wave transport channels that can be tailored to versatile shapes. As a proof-of-principle demonstration of potential practical applications of the higher-order topological states, an interferometer based on the PC cube are fabricated and characterized. As illustrated in Fig. 4a, three rectangular waveguides, labeled port 1, 2, and 3, are connected to three corners of the aforementioned $5 \times 5 \times 5$ PC cube. The location of these waveguides ensures three-fold rotational symmetry about the diagonal axis of the PC cube. The cross-section of each waveguide is set as $1.6 \times 0.1$ cm. Fig. 4b shows the simulated acoustic field distribution when a wave with frequency $5.16$ kHz incidents from port 1. The THSs are excited and indeed functions as waveguiding channels, which can be exploited to construct an interferometer. The inset in Fig. 4b shows the dispersion of a THS when a pair of parallel truncations on the PC cube in Fig. 3a are removed (i.e., the PC becomes infinite in one of the three directions). We emphasize that the THS is a propagative state, which is the foundation of wave manipulation applications. Fig. 4c plots the result of two in-phase waves incident to the cube from ports 1 and 2. Owing to the constructive interference, the amplitude of the outgoing sound at port 3 is doubled. In contrast, when two out-of-phase waves incident onto the same ports, destructive interference occurs, and consequently the suppression of the outgoing sound at port 3 is observed, as shown in Fig. 4d. To better present our finding, we study the hinge-state interferometer as a three-port network with a scattering matrix.
\[
\begin{pmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{pmatrix}
\begin{pmatrix}
\psi_{\text{in}}^0 \\
\psi_{\text{in}}^7 \\
\psi_{\text{in}}^2
\end{pmatrix} = 
\begin{pmatrix}
\psi_{\text{out}}^0 \\
\psi_{\text{out}}^7 \\
\psi_{\text{out}}^2
\end{pmatrix}.
\]

Using COMSOL Multiphysics, we numerically obtain 
\( S_{11} = -0.2 - 0.28i \) (\( \zeta = 1, 2, 3 \)), 
\( S_{22} = 0.54 + 0.38i \) (\( \mu = 1, 2, 3 \)), 
\( S_{33} \) are elements of the scattering matrix. The interference between the incoming waves \( \psi_{\text{in}}^0 \) and \( \psi_{\text{in}}^2 \) can be therefore calculated by the scattering matrix. For the in-phase case, the input wave can be described as \( \psi_{\text{in}} = (1, 1, 0)\). The output of the three-port network is \( \psi_{\text{out}} = (0.34 + 0.10i, 0.34 + 0.10i, 0.18 + 0.76i) \), indicating a constructive interference at port 3. For the out-of-phase case, the input wave is \( \psi_{\text{in}} = (1, -1, 0)\); the output of the three-port network is \( (-0.74 - 0.67i, 0.74 + 0.67i, 0) \), indicating a destructive interference with zero amplitude at port 3. Fig. 4(e) gives a picture of the experimental setup of such a THS interferometer. Two speakers are used as sound sources, placed at the end of two aluminum waveguides with a rectangular cross-section. The other ends of the waveguides are connected to the two corners of the PC cube. We measure the sound amplitude in the output waveguide at the upper right corner. The transmittance for \( w = 1 \) and \( w = 2 \) is in-phase, while it is minimized at the same frequency when the two speakers are out-of-phase. These results are strong evidence of the constructive and destructive interferences contributed to two separate but equivalent hinge paths.

3. Conclusion

We present a simple design of an acoustic 3D high-order TI that can support topological states at three different dimensional hierarchies. The negative dispersion of the TSS makes negative refraction easily attainable. Note that our negative refraction occurs at the PC-air interfaces instead of between different PC boundaries, making it feasible for wave-steering applications. The presence of higher-order topological states, in particular, the THSS, brings even more intriguing effects. First, THSSs along different directions can relay the transport of sound waves, guiding the propagation to bend around corners towards different directions. As a result, the inputs and output are not on the same plane. Similar effects can only be attained by using a 3D double-zero-index medium previously [31], which relies on the stringent tuning of system parameters. In comparison, the topological protection of THSSs endows additional robustness. Second, the design of the THS interferometer possesses versatility. For example, by choosing the positions of input and output ports, the interferometer can easily guide waves towards different directions in the 3D space (as shown in Fig. 4b). Such functionality is unobtainable for any 2D TIs.

In summary, our work presents convincing cases that higher-order topological wave crystals can benefit wave-steering applications. As topological notions can be universally applied to other realms of physics, such as photonics and electromagnetism, we believe our work is an important step for a broad area of next-generation technology and devices.

Declaration of Competing Interest

The authors declare that they have no conflict of interest.

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Author contributions

Changqing Xu designed the phononic crystal. Changqing Xu and Ze-Guo Chen performed the numerical simulations and designed the experiment. Ze-Guo Chen and Guangqing Zhang set up the experiment and carried out the measurements. Changqing Xu, Ze-Guo Chen, Guancong Ma and Ying Wu wrote the manuscript. The project was supervised by Guancong Ma and Ying Wu.

References


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