

Rejoinder on the discussion of ‘A high-resolution bilevel skew-t stochastic generator for assessing Saudi Arabia’s wind energy resources’

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We would like to thank all four discussants for their discussions on different aspects of our paper. We were particularly pleased to see comments across the entire spectrum of our work, from theoretical properties of the model to geoscience, engineering and wind energy aspects that could provide a pathway to implement a wind energy portfolio in Saudi Arabia, truly reflecting the mission of *Environmetrics*. Throughout this rejoinder, we will denote the discussion of Adelichi Azzalini as A, Sándor Baran as B, Emilio Porcu, Jonas Rysgaard and Valerie Eveloy as PRE and Andrew Zammit-Magion as Z.

This rejoinder covers the different points raised by the discussants in different sections. Section 1 discusses theoretical and methodological aspects of our model. Section 2 focuses on inference and also shows new results with a Bayesian implementation with Stan (Carpenter et al. 2017). Section 3 replies to discussion points on validation. Finally, Section 4 covers aspects related to geoscience, engineering and wind energy stemming from this work.

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19 1 Modeling aspects

20 The choice of the model has been motivated by the need to strike a balance between the flexibility
21 of a theoretical construct for a new class of parametric families and practical necessities in the
22 big picture of a large interdisciplinary project. As remarked in the introductory comment by
23 A, it is challenging to propose models in environmental statistics with theoretically justified
24 models, which can also be realistically employed in applications whose focus is beyond marginal
25 distributions. As also noted by A, our model indeed can be seen as conceptually similar to Zhang
26 & El-Shaarawi (2010), with a random denominator allowing for a skew- t marginal distribution.

27 PRE has proposed a classification of both this model and the alternative formulation of
28 Tagle et al. (2020) in terms of a more general class of additive models, with a common random
29 effect across a partition of the domain. As a result, they highlighted how one of the main
30 shortcomings of the model, namely the presence of discontinuity at the borders of the domain,
31 in this model is shared across this more general class. This limitation, also discussed by Z
32 is practically unavoidable if one is to avoid the latent Gaussian model approach, and has the
33 appealing advantage of allowing to express moments in closed form.

34 PRE also provided a detailed discussion about the choice of the clusters, advocating for
35 the use of topic-relevant information (when available). To support this point, we would like to
36 offer an example of a multi-resolution model belonging to the proposed class in the context of
37 functional Magnetic Resonance Imaging (fMRI), using a notation similar (not identical due to
38 the presence of a convex combination) to that PRE for the ease of comprehension, and removing
39 any issue of time dependence and trends to simplify the notation. In Castruccio et al. (2018) a
40 mean-zero bi-resolution Gaussian process was proposed

$$X(\mathbf{s}) = (1 - \omega_{r(\mathbf{s})})V_{r(\mathbf{s})} + \omega_{r(\mathbf{s})}\xi_{r(\mathbf{s})}(\mathbf{s}),$$

41 where the partition $r(\mathbf{s})$ was provided by the Regions of Interest (ROIs) in the brain, and the

42 fine scale resolution represented the voxel intensity. The spatial process was expressed as a
 43 convex combination with parameters $\omega_{r(\mathbf{s})} \in [0, 1]$ (depending on the ROI) between a ROI-specific
 44 Gaussian random effect $V_{r(\mathbf{s})} \sim \mathcal{N}_R(\mathbf{0}, \mathbf{\Sigma}_V)$ and a fine-scale Gaussian field $\xi_{r(\mathbf{s})}(\mathbf{s}) \sim \mathcal{N}_{|\mathcal{D}_\ell|}(\mathbf{0}, \mathbf{\Sigma}_\xi)$,
 45 where $|\mathcal{D}_\ell|$ is the cardinality of the region \mathcal{D}_ℓ . Inference was performed in a two-step fashion with
 46 graphical LASSO to estimate the connectivity among ROIs. While this model is more limited in
 47 its marginal properties than the one we propose here, it is a good example of a bi-resolution model
 48 in the general class proposed by PRE where the choice of region is not dictated by a clustering
 49 algorithm, but it already provided by the ROIs, regions in the brain localizing different motor
 50 and cognitive tasks. Even in this case, however, one may argue that the choice of the partition
 51 (*parcellation*) is not unique, and it should be chosen depending on the particular investigation a
 52 scientist is interested in.

53 Z also remarked that the model does not allow for temporal varying dynamics, and while
 54 this is appropriate in the presented work (as shown in the Figure S3), this may be an issue for
 55 higher resolution (hourly or sub-hourly) wind. While we ourselves were relatively surprised by
 56 the regularity of the temporal dynamics, this seems consistent with the results from other data
 57 sets for multi-decadal wind projections, which do not show significant trends across years. It is
 58 likely that this could become an issue for hourly data, and several possible generalizations of our
 59 model could accommodate for that. Perhaps the simplest approach would be to detect change
 60 points in the dynamics and assume a piecewise constant dynamics. While this solution would
 61 induce discontinuity, it would preserve a computationally convenient expression of the likelihood.

62 Z also pointed out that covariances could be used to improve the model’s explanatory power.
 63 The main modeling challenge compared to a Gaussian (or latent Gaussian) model is that the
 64 addition of covariates would impact not just the mean of the model, but also the covariance and
 65 other higher-order properties. Therefore, even the simplest approach of modifying equation (2)

66 so that

$$Y_r(\mathbf{s}) = \sum_{i=1}^p \beta_i X_i(\mathbf{s}) + \frac{\lambda_r |U_r| + \eta(\mathbf{s})}{\sqrt{Z_r}},$$

67 would imply a nontrivial recomputation of the moments. Nevertheless, since height is expected
68 to play a key role in the determination of both mean and variability of wind fields, we agree
69 that the use of static and possibly dynamic covariates could improve the fidelity of the stochastic
70 generator.

71 **2 Inferential aspects**

72 The use of the EM algorithm for inference has made necessary the development of semi-closed
73 form expression for both the expectation and the maximization step, with numerical approx-
74 imation schemes of some terms, hence resulting in non-negligible computationally demanding
75 resources. Several discussants have suggested alternative methods to ameliorate the computa-
76 tional burden. A proposed the determination of the initial estimates by using sample quantiles
77 instead of sample moments, a more stable choice for long-tailed distribution (Azzalini & Salehi
78 2020).

79 Z discussed Laplace approximation and stochastic variational Bayes as alternatives to our EM
80 algorithm. Both choices have relative merits in their ability to achieve approximate Bayesian
81 inference, either with or without a Gaussian approximation. Additionally to Z's proposals we
82 would like to offer an alternative, more automatic albeit less computationally efficient approach
83 with Hamiltonian Monte Carlo inference with the Stan software in R. As part of this new imple-
84 mentation, we also offer a comparison with a Gaussian-log-Gaussian (GLG) model (Palacios &
85 Steel 2006), which models non Gaussianity with a lognormal scale mixing of a Gaussian process.

2.1 A Bayesian bi-resolution skew- t model with Hamiltonian Monte Carlo Inference

Similarly to Tagle et al. (2020), we consider a partition \mathcal{R} of cardinality R , where for each region $r \in \mathcal{R}$, and each $\mathbf{s} \in r$ we have the following model, which we denote as SKT

$$Y(\mathbf{s}) = \sigma_r \frac{\rho_r U_{0,r} + \lambda_r |U_{1,r}| + \eta_{2,r}(\mathbf{s})}{\sqrt{Z_r}}, \quad (1)$$

where $\sigma_r, \rho_r > 0$, $\lambda_r \in \mathbb{R}$. The model assumes that there exist scalar random variables $U_{1,r}$ and Z_r across each element r of the partition, and $\eta_{2,r}$, a stationary Gaussian process, with mean zero and correlation function $C_{\theta_r}(\cdot)$ with the associated covariance matrix $\Sigma(\theta_r)$. We assume independence of $U_{1,r}$ and Z_r across \mathcal{R} , as well as for the Gaussian process $\eta_{2,r}$. The large-scale effect, whose regional component is denoted by $U_{0,r}$ induces the desired spatial dependence, encoded in the correlation matrix Σ_0 .

If we denote by $\mathbf{Y}_r = \{Y(\mathbf{s}), \mathbf{s} \in r\}$, by $\mathbf{U}_0 = (U_{0,1}, \dots, U_{0,R})^\top$, and by n_r the number of sampled sites in the region r , an equivalent hierarchical representation of (1) is given by,

$$\begin{aligned} \mathbf{Y}_r | U_{0,r} = u_{0,r}, U_{1,r} = u_{1,r}, Z_r = z_r &\sim \mathcal{N}_{n_r} \left(\sigma_r \frac{\rho_r u_{0,r} + \lambda_r |u_{1,r}|}{\sqrt{z_r}}, \frac{\sigma_r^2}{z_r} \Sigma(\theta_r) \right), \\ \mathbf{U}_0 &\sim \mathcal{N}_R(\mathbf{0}, \Sigma_0), \\ U_{1,r} &\stackrel{iid}{\sim} \mathcal{N}(0, 1), \\ Z_r &\stackrel{iid}{\sim} \text{Gamma}(\nu_r/2, \nu_r/2). \end{aligned} \quad (2)$$

The vector of parameters for each region is $\gamma_r = (\sigma_r, \rho_r, \lambda_r, \nu_r, \theta_r^\top)^\top$, with θ_r collecting the parameters of the region-specific correlation function, and the total set of parameter consists of $\theta = (\{\gamma_r, r \in \mathcal{R}\}, \Sigma_0)$. The non-negativity constraints on σ_r , ρ_r and ν_r motivate the use of prior distributions with support on \mathbb{R}^+ . We use non-informative priors for all parameters, following weak prior suggestions in the Stan documentation. In particular, we choose normal random variables centered at the parameter true values with large variances (Stan automatically adds a truncation when a lower bound is specified). We run 3 chains for 2,000 iterations each, discarding the first 1000 as warm-up. Inference is performed on simulated data considering $N = 15, 50$, and 200 spatial replicates.

107 Computations were performed on a cluster with 2.3Ghz AMD EPYC 7451 24-Core Processors
 108 (48 threads), of which only 3 were used for any of the scripts, one for each of the noted 3 chains
 109 of the Stan inference procedure. Running the 2,000 iterations took between 90 to 120 minutes.

110 2.2 Simulation study

111 We firstly explore various parameter and regional configurations, with varying degrees of marginal
 112 skewness and kurtosis, as well as spatial dependence, to offer some guidance regarding the number
 113 of spatial replicates and the spatial design that leads to posterior distributions that are reasonably
 114 well-concentrated. Secondly, we investigate the degradation in predictive performance of a model
 115 which only allows for flexibility in capturing heavy-tailed behavior. In particular, we consider
 116 the GLG model of Palacios & Steel (2006)

$$Y(\mathbf{s}) = \mathbf{x}(\mathbf{s})^\top \boldsymbol{\beta} + \frac{\eta(\mathbf{s})}{\sqrt{\xi(\mathbf{s})}} + \varepsilon(\mathbf{s}),$$

117 where the first term represents a spatial trend, $\eta(\mathbf{s})$ is a stationary Gaussian process with mean 0,
 118 variance σ^2 and isotropic correlation function $C_\theta(h)$, $h = \|\mathbf{s} - \mathbf{s}'\| > 0$, $\mathbf{s}, \mathbf{s}' \in \mathcal{D}$, and the last term
 119 is an independent and uncorrelated Gaussian process with mean 0 and variance τ^2 representing
 120 the nugget effect. Here, $\xi(\mathbf{s})$ is such that $\log\{\xi(\mathbf{s})\}$ follows an independent Gaussian process with
 121 constant mean surface at $-\nu/2$ and covariance function $\nu C_\theta(h)$, with $\nu > 0$. Marginally, $\xi(\mathbf{s})$
 122 follows a lognormal distribution with mean 1 and variance $\exp(\nu) - 1$. The marginal kurtosis
 123 of $Y(\mathbf{s})$ is given by $3 \exp(\nu)$; thus large values of ν correspond to heavy-tailed behavior, while
 124 Gaussian tails are recovered in the limit as $\nu \rightarrow 0$. The covariance function $C(h)$, without a
 125 nugget effect, for the GLG model is given by

$$C(h) = \sigma^2 C_\theta(h) e^{\nu(1 + \frac{1}{4}[C_\theta(h) - 1])}.$$

126 2.2.1 Parameter estimation

127 We compare the proposed model’s posterior performance against the competing GLG model
128 based on simulations drawn from the former model with varying degrees of skewness. Of course,
129 we expect to see a degradation in performance for the GLG model but it is instructive to assess
130 along which aspects such failings occur. We generate synthetic spatial replicates from the SKT
131 model assuming a 12 region setup, each region a unit square, arranged in a 4 by 3 lattice
132 configuration, and choose 15 points randomly from each unit square, for a total of 170 points. We
133 assume a fixed value for the large-scale effect parameter, $\rho_r = 0.3$, as well as its range parameter,
134 ϕ_0 , set at 2. We consider a control simulation with no skewness, i.e., $\lambda_r = 0$, and test simulations
135 with values $\lambda_r = 1.7, 1, 0.28$ for the first 3 regions, implying marginal skewness values of 0.15,
136 0.5 and 0.75, respectively, and 0 for the remaining regions. Because of the parameterization of
137 the skew- t distribution, ν_r needs to be modified accordingly to keep the excess kurtosis fixed at 2
138 across all regions. In particular $\nu_r = 9.2, 7.8, 7$ for the first 3 regions and $\nu_r = 7$ for the remaining
139 9 regions. Lastly, we consider different values of the regional range parameter $\phi_r = 0.1, 0.5, 1$, to
140 partially offset the increase in intra-regional spatial dependence that accompanies the increases
141 in λ_r . From the control simulation we derive two fitted models, GLG-control and SKT-control,
142 and similarly, we denote those from test simulations as GLG-skew and SKT-skew.

143 Figure 1 displays the 95% and 80% credible intervals for the fitted GLG models for simulations
144 with $\phi_r = 0.1$. The posterior marginal distributions of ϕ_r for the first 3 regions of the GLG-skew
145 model do not show clear departures in their median with the other regions nor with those of
146 GLG-control, and a similar assessment can be made of their variability. Slight differences emerge
147 in ν , as the presence of skewness results in reduced posterior medians for the affected regions.
148 The first region, in particular, achieves the lowest regional value, falling below 0.1, implying a
149 reduction in the implied kurtosis and spatial dependence. In contrast, the posterior medians of
150 ρ_r for the skewness-affected regions show marked differences, with values exceeding 2 and 1 in

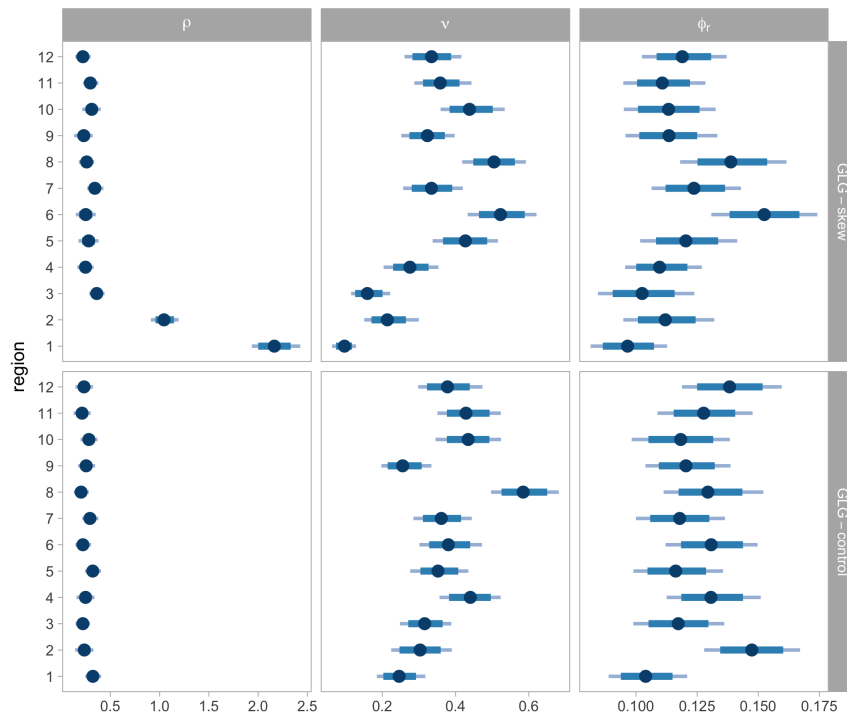


Figure 1: The posterior median, and the 95% (light blue) and 80% (dark blue) credible intervals for ρ , ν and ϕ for the each of the 12 regions, for the GLG-control and GLG-skew models.

151 the first and second region, respectively, while the rest are closely concentrated in the vicinity of
 152 0.4. This represents a significant increase in the strength of the regional spatial dependence.

153 Figure 2 distills the impact of these parameter value differences on the implied spatial depen-
 154 dence, by displaying the difference between the correlation functions evaluated at the posterior
 155 samples and the true correlation functions. Due to space considerations, we only show the differ-
 156 ences pertaining to the SKT simulation with $\phi_r = 1$ for the first 3 regions, and choose the 4th as
 157 representative of the remaining 9 regions. Significant biases can be seen for the correlation func-
 158 tions of the GLG-skew model in the first two regions, while the differences are indistinguishable
 159 from those of the fitted SKT-skew model for the third and beyond. These biases at distances
 160 larger than 0.2 approximately vary between 0.2 and 0.3 in the first region, and 0.1 and 0.25 for
 161 the second, across the 3,000 posterior samples.

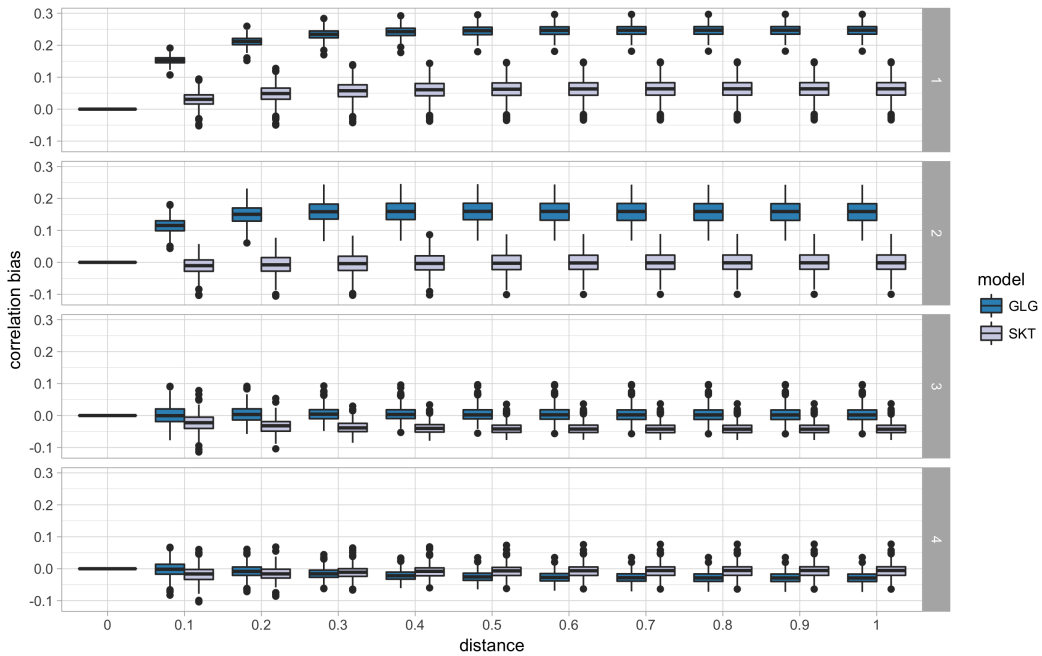


Figure 2: Difference between the correlation functions for the GLG-skew and SKT-skew models evaluated at the posterior samples of the SKT simulation with $\phi_r = 0.1$, and the true correlation functions underlying said simulation (displayed in Fig. S.1) for the first 4 regions.

162 2.2.2 Predictive performance

163 We compare posterior predictive performance using standard metrics, the continuous ranked
 164 probability score (CRPS; Gneiting et al. (2007)) and the mean absolute error (MAE). Table 1
 165 shows the out-of-sample posterior predictive performance based on the CRPS and MAE for the
 166 GLG and SKT models for the various values of ϕ_r , for the first 3 regions and the range of values
 167 of the remaining 9. The effect of diminishing ϕ_r is clearly reflected in the predictive performance
 168 across statistics and fitted models: the greater the strength of spatial dependence, the better the
 169 predictive ability of the model. Comparing the performance of the models, the statistics clearly
 170 favor the SKT model, across all values of ϕ_r . Interestingly, a comparison using in-sample values
 171 did not show any degradation in predictive performance in neither of the two statistics, despite
 172 the GLG's inherent inability to match the marginal structure. An inspection of the posterior
 173 samples of the large scale effect U_0 revealed that in contrast with the model assumption of a
 174 centered normal distribution, the means were consistently positive across the replicates used for

175 the fitting, with an average of 0.72, underscoring the model misspecification.

Table 1: Out-of-sample continuous ranked probability score (CRPS) and mean-absolute error (MAE) values for the first three regions and the minimum and maximum of the other 9 regions.

Model	CRPS ₁	CRPS ₂	CRPS ₃	CRPS _{min}	CRPS _{max}	MAE ₁	MAE ₂	MAE ₃	MAE _{min}	MAE _{max}
GLG-skew ($\phi_r = 0.1$)	1.36	0.76	0.66	0.57	0.71	3.39	2.89	1.39	1.59	2.50
GLG-skew ($\phi_r = 0.5$)	0.45	0.36	0.40	0.33	0.47	4.67	3.52	1.91	1.84	4.40
GLG-skew ($\phi_r = 1.0$)	0.31	0.26	0.29	0.24	0.34	4.91	3.70	1.95	1.91	5.00
SKT-skew ($\phi_r = 0.1$)	0.95	0.66	0.64	0.57	0.72	2.77	2.40	1.37	1.53	2.42
SKT-skew ($\phi_r = 0.5$)	0.40	0.34	0.39	0.32	0.44	4.57	3.41	1.86	1.79	4.17
SKT-skew ($\phi_r = 1.0$)	0.28	0.24	0.29	0.23	0.33	4.87	3.64	1.92	1.89	4.88

176 Lastly, we compare the marginal posterior predictive distributions with the out-of sample
 177 marginal distributions of the simulated data. Figure 3 compares the marginal posterior predictive
 178 distribution for a point chosen at random from the first four regions of our spatial design, of the
 179 GLG-skew model, and the out-of-sample marginal distribution of the associated SKT simulation.
 180 Here the anticipated discrepancies emerge, as the GLG-skew model with its symmetry constraint
 181 is incapable of reproducing the skewness of the SKT simulation, particularly for the first region,
 and gradually vanishing through the subsequent regions.

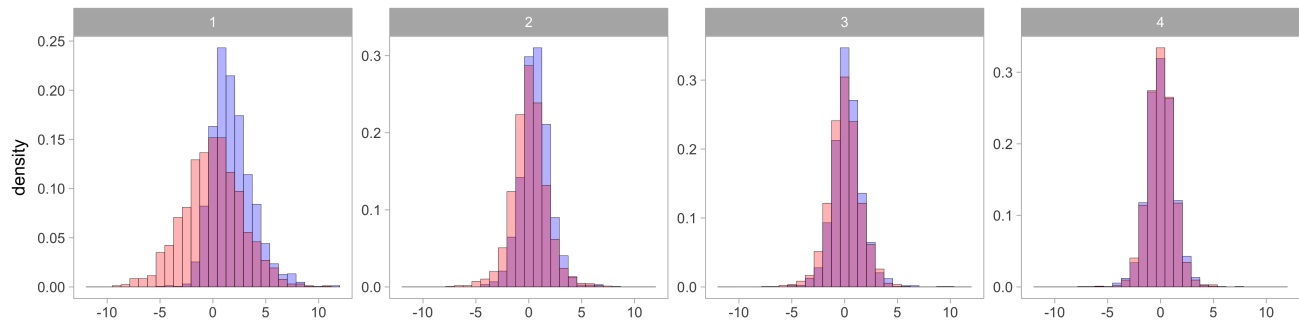


Figure 3: The marginal posterior predictive distribution for the (red) GLG-skew model for a point chosen at random for regions 1 through 4, and the associated out-of-sample marginal distribution (blue) of the SKT simulation.

182

183 3 Validation aspects

184 The use of statistical models as stochastic approximations for expensive, deterministic models
 185 (or parts thereof) has a long history in statistics and environmental science. However, despite
 186 the sheer number of applications ranging from emulation, data assimilation, stochastic weather

187 generation and more recently data compression, a fundamental point raised by A is: how do
188 we decide to which extent the approximation is good for our purpose? In other words, would
189 statistical surrogates from our model pass a ‘Turing test’ and be indistinguishable from the
190 climate/weather model output? While this question is at the center of the development of
191 validation methods for stochastic weather generators, it has been mostly addressed in the context
192 of time series. The push for new quantitative methods to compress data, led by the National
193 Center for Atmospheric Research (NCAR), as well as the wide availability of spatio-temporal
194 models, is prompting a renewed discussion on which validation methods are appropriate as Turing
195 tests in space and space/time.

196 On this topic, we would like to offer an image/video processing perspective. As argued
197 by Castruccio et al. (2019), determination of similarity between maps of physical quantities
198 from the weather model and its approximation can be regarded as a visual problem. While
199 image processing has developed quantitative metrics of similarity, empirical evidence suggests
200 that the perception of similarity is strongly determined by the level of expertise of the subject
201 performing the test. In other words, a subject expert is expected to be considerably more skilled
202 in determining differences between the original output and the statistical model than a casual
203 subject.

204 **4 Geoscience, engineering and wind energy aspects**

205 While this work indeed does represent a cross-disciplinary effort intended to go beyond the
206 statistical community, as noticed by A, several important points were raised regarding extension
207 of the present methodology to provide a practical pathway towards the implementation of this
208 plan, from both a geoscience and engineering perspective. An important point raised by PRE
209 and also by Z relates to the role of a stochastic approximation for a single run as a representative
210 reproduction of the uncertainty. We are now in a position to have much more robust evidence
211 from an ensemble and a more articulated use of wind extrapolation at hub height and different

212 turbines (see the points below), but this is an important question to address in the case of
213 a large cross-disciplinary study such as this where multiple simulations controlling for all the
214 desired sources of uncertainty can only be provided in the span of years, if ever.

215 Is then one run sufficient to capture some of the most important sources of uncertainty? In
216 this particular case, we believe that it is case, as we did not see evidence of inter-annual trends
217 in the wind patterns from the (short) WRF simulations, but also for multi-decadal reanalysis
218 (MERRA-2, Gelaro et al. (2017)) and present and future simulations (MENA CORDEX, Jones
219 et al. (2011)). Therefore, given the regional nature of the problem, one may regard the year-to-
220 year variability as a surrogate for the internal variability of the model, hence lending support for
221 the use of *stochastic generators* originally used for multi-decadal global climate model ensembles
222 (Jeong et al. 2018, 2019, Tagle et al. 2019, Fuglstad & Castruccio 2020).

223 Further points strictly related to wind energy production have been raised by PRE, and given
224 the importance of these remarks, we address them on a point-by-point basis:

225 (i) In collaboration with the Atmospheric Chemistry and Aerosol Modeling Group at the
226 University of Notre Dame, we have now generated a new WRF ensemble comprising of four
227 runs between 2013-2016, with two different planetary boundary layer (PBL) schemes and
228 two different surface layer schemes. The simulation is nested and comprises of resolutions
229 from 9km to 3km in Saudi Arabia, and the output is hourly data for most of the domain
230 and half-hourly for the western part of the country. The simulations have 40 vertical levels
231 with approximately 20 meters spacing in the lower levels up to 200 meters to have high
232 detail in the region of interest for wind turbines, see Giani et al. (2020) for a full description.
233 This also addresses the point raised by Z and B on the use of a single WRF run and the
234 need of uncertainty quantification: the ensemble represents the first attempt to control for
235 variability of PBL and surface layer schemes, which were identified by our collaborators
236 as two of the most important sources of uncertainty to quantify. The sheer amount of

237 computational power required for just four runs (240,000 core hours), along with the cost
238 of storage of the considerable amount of data generated (111 Tb) has prevented us further
239 sensitivity analysis against other sources of interest, most noticeably boundary conditions.
240 An alternative, useful approach would be to consider off-the-shelves forecast data such as
241 ECMWF-HRES, as pointed out by B, which could be used for additional validation of the
242 results.

243 (ii) We agree that the functional form of extrapolation, i.e. the power law, has been subject
244 to increased scrutiny and alternative modeling strategies have been currently sought to
245 decrease the extrapolation error. One of the most recent work on the topic (Vassallo et al.
246 2020) successfully applied a feedforward neural network, providing up to 52% improvement
247 in extrapolation over the power law. The use of such approach is however here problematic
248 as 1) no observational data at high altitude were available at the time of this work and 2)
249 even assuming such data were available, more than 80,000 neural networks should be trained
250 for each location, unless another a full domain convolutional neural network approach would
251 be deployed.

252 (iii) In our siting work (Giani et al. 2020) we constrained the search of sites to locations with-
253 out a complex terrain precisely to avoid these effects. One of our group member is also
254 currently developing simulations with WRF runs acting as boundary conditions for Large
255 Eddy Simulations (LES) models in order to bring the simulation resolutions below 1km,
256 resolve at least some of the turbulence and hence be able to capture the effects of complex
257 terrain and high altitude.

258 (iv) One of our group members is currently finalizing a work specifically targeted at finding the
259 best generalization of the power law for hourly data for the ensemble mentioned in part (i).
260 Our new results indicate that a model with a heteroscedastic (due to atmospheric stability),
261 location-specific, hourly-varying power law is indeed considerably more appropriate. Such

262 a study was not possible in the work object of this discussion, as the WRF run did not
263 have sufficiently many pressure levels near the surface to test the power law.

264 (v) We again refer to our siting work (Giani et al. 2020), where we have used a database
265 comprising of a wide array of wind turbines with different power curves. The same work
266 also addresses the issue of the use of different optimal turbines for different sites.

267 (vi) In the siting work (Giani et al. 2020) we have considered error propagation generated by
268 wind, as well as uncertainty on the building and maintenance costs, as well as connection
269 to the electric grid when providing the final levelized cost of energy.

270 (vii) In yet another work in progress we have focused on some areas of interest in Saudi Arabia
271 and performed simulations at 1 km with different wind farm configurations. Preliminary
272 results indicate configurations between 5 to 7 rotor diameters imply a loss in the range of
273 10% to 20% due to the wake effect.

274 An additional point raised by Z was the change of wind patterns over relatively short time
275 spans observed over many regions of the world. A WRF simulation over decades would provide
276 strong evidence, but it is presently impossible due to computational and most importantly storage
277 constraints. We have however preliminary evidence from an analysis of the Middle East North
278 Africa - Coordinated Regional Climate Downscaling Experiment (MENA-CORDEX, Jones et al.
279 (2011)) that there is no significant trend in the vast majority of the country across a past (1980-
280 2005) and a future (2025-2050) 26-years period. This analysis was performed at a considerably
281 coarser resolution ($0.44^\circ \times 0.44^\circ$), but still lends some support on the claim that the siting provided
282 in this work or in Giani et al. (2020) could be still more or less unchanged in the coming decades.

283 Finally, while the issue of extreme wind was not raised by the discussants, we nevertheless
284 like to point out that there is a non-negligible risk of turbine disruption operation due to strong
285 wind. Our group has addressed this topic in a recent work (Chen et al. 2020).

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