Blind Estimation of Central Blood Pressure Using Least-Squares with Mean Matching and Box Constraints

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***Abstract*—Central aortic blood pressure (CABP) is a very-well recognized source of information to asses the cardiovascular system conditions. However, the clinical measurement protocol of this pulse wave is very intrusive and burdensome as it requires expert staff and complicated invasive settings. On the other hand, the measurement of peripheral blood pressure is much more straightforward and easy-to-get non-invasively. Several mathematical tools have been employed in the past few decades to reconstruct CABP waveforms from distorted peripheral pressure signals. More specifically, the cross-relation approach together with the widely used least-squares method, are shown to be effective as a way to estimate CABP waves. In this paper, we propose an improved cross-relation method that leverages the values of the diastolic and systolic pressures as box constraints. In addition, a mean-matching criterion is introduced to relax the need for the input and output mean values to be strictly equal. Using the proposed method, the root mean squared error is reduced by approximately 20% while the computational complexity is not significantly increased.**

***Keywords*—Central aortic blood pressure, peripheral blood pressure, optimization, multi-channel blind system identification, box constraints, eigenvalues, eigenvectors.**

I. INTRODUCTION

CENTRAL aortic blood pressure (CABP) wave carries valuable clinical information which can help in the diagnosis of cardiovascular diseases [1]–[3]. The clinical measurement procedure of CABP is deemed to be very complicated and intrusive for patients as it is usually measured invasively with a catheter. For that reason, alternative methods for the estimation of CABP signals has gained an increasing attention from researchers and clinicians over the past few decades. The detection of peripheral blood pressure waveforms from some distal sites, such as radial, femoral and finger, is accessible non-invasively [4], [5]. However, the pulse at a distal level is not directly useful as the wave gets distorted significantly compared to the CABP due to factors such as the length and the stiffness of the arteries [3], [6].

Several research works have investigated the estimation of CABP signal from peripheral pressure pulse waves. The most common technique is based on the transfer function (TF) derivation that relates the CABP wave to one or more peripheral pulse waves. These works can be classified into two classes; extracting generalized TF [2], [6], [7], and individual TF that depends on some physical parameters [8]–[13]. A CABP signal cannot be accurately estimated in the case of the generalized TF because of the significant variability of the physiological parameters from patient to patient. On the other hand, multiple measurements have to be collected to obtain an individual TF, some of which are not easy to obtain.

To avoid these complicated measurements, multi-channel blind system identification (MBSI) techniques have been proposed. MBSI is based on the estimation of a common input from one or more output signals that propagate through different channels [14], [15], [18], [20]. These techniques are used in many fields such as communication, image restoration and bio-signal estimation [16], [17], [19]. In the field of blood pressure estimation, linear and nonlinear techniques are used. A simple cross-relation approach with least-squares is applied in [15]. Although this method is attractive due to its low computational complexity, it does not incorporate other information apart from the two peripheral pressure signals, even those simple cues. A more advanced nonlinear method is used in [14] where the system is modeled to have linear and nonlinear parts which increases the complexity significantly. The advantage of the method presented in this paper over the previously mentioned algorithms is that no measurement-based calibrations are needed. In addition, the system is approximated as a linear time-invariant (LTI) system, which reduces the complexity of the calculations while offering improved performance compared to the basic linear methods.

The rest of the paper is organized as follows. In section II, the proposed approach is described along with the basic cross-relation approach. Section III presents results and discussions. Finally, Section IV concludes the paper with future work recommendations.

II. MATERIALS & METHODS

Similar to [14], [15], the MBSI technique is applied to model the arterial system, where the input is the CABP signal denoted by $x[n]$ and the outputs are two peripheral blood pressure signals denoted by $y_1[n]$ and $y_2[n]$, as shown

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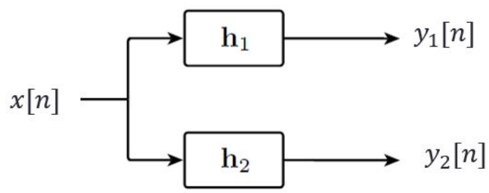

Fig. 1: Single-input-two-outputs arterial system.

in Fig. 1. The objective is to use the two peripheral pressure waveforms to reconstruct the central blood pressure. For simplicity, both channels are assumed to have an equal and known length L. As a result, the two outputs should have the same length N.

A number of assumptions with a physiological insight have been adopted as follows:

- Assumption 1: Each channel is linear time-invariant (LTI) for a small time window.
- Assumption 2: The channels are co-prime, i.e., their transfer functions do not share common zeros. Otherwise, the common zeros will be eliminated in the cross-relation. This is strictly impossible in the blood pressure context since the proximal aortic path is common for all peripheral sites. However, selecting two sites with a very small common path is sufficient to approximately satisfy this assumption.
- Assumption 3: $N \geq 2L$. This is required for the channel estimation as will be shown later.

Under these assumptions, the input-output relation is represented by the following expression:

$$y_i[n] = x[n] * h_i[n] = \sum_{k=0}^{L-1} x[n]h_i[n-k], \quad i = 1, 2, \tag{1}$$

where $*$ denotes the linear convolution operator and $h_i[n]$ is the i-th channel response. In the absence of noise, the following relationship holds [16]:

$$\begin{aligned} y_1[n] * h_2[n] &= (h_1[n] * x[n]) * h_2[n] \\ &= h_1[n] * (x[n] * h_2[n]) \\ &= h_1[n] * y_2[n] = y_2[n] * h_1[n]. \end{aligned} \tag{2}$$

The above relationship can be rewritten in a matrix-vector format as:

$$\mathbf{Y}_1\mathbf{h}_2 = \mathbf{Y}_2\mathbf{h}_1, \tag{3}$$

where

$$\mathbf{h}_i = \begin{bmatrix} h_i[0] & h_i[1] & \dots & h_i[L-1] \end{bmatrix}^T. \tag{4}$$

Here T denotes matrix transpose, $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are N-by-L rank-L convolution or filtering matrices and are expressed as:

$$\mathbf{Y}_i = \begin{bmatrix} y_i[0] & y_i[-1] & \dots & y_i[-L+1] \\ y_i[1] & y_i[0] & \dots & y_i[-L+2] \\ \vdots & \vdots & \vdots & \vdots \\ y_i[N-1] & y_i[N-2] & \dots & y_i[N-L] \end{bmatrix}. \tag{5}$$

In order to find the coefficients of the two channels, (3) is used to formulate the following optimization problem [15]:

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} ||\mathbf{Y}\mathbf{h}||_2^2, \tag{6}$$

where

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y_2} & -\mathbf{Y_1} \end{bmatrix}, \tag{7}$$

and

$$\mathbf{h} = \begin{bmatrix} \mathbf{h_1} \\ \mathbf{h_2} \end{bmatrix}. \tag{8}$$

Based on Assumption 3, the number of columns of Y is guaranteed to, at least, be equal to the number of rows, hence avoiding an underdetermined model. To avoid an all-zero solution, a nontrivial constraint is applied, usually a quadratic constraint on the Euclidean norm of the vector $\mathbf{h}$, as follows:

$$\begin{aligned} \hat{\mathbf{h}} &= \arg\min_{\mathbf{h}} ||\mathbf{Y}\mathbf{h}||_2^2 \\ \text{s.t.} &\quad ||\mathbf{h}||_2^2 = \alpha^2. \end{aligned} \tag{9}$$

A. The basic cross-relation method with least-squares (LS)

For consistency and reader's convenience, the cross-relation with least-square (LS) approach [15] shall be reviewed hereafter. In this approach, the vector $\mathbf{h}$ can be obtained up to a scaling factor by setting the value of α^2 in (9) to one. The optimization (9) is then converted into an eigenvalue problem that has a closed-form solution which is the unit-norm eigenvector associated with the minimum eigenvalue of the matrix $\mathbf{Y}^T\mathbf{Y}$.

After estimating the scaled channel's coefficients, the input-output relation is expressed as follows:

$$\begin{bmatrix} \mathbf{y_1} \\ \mathbf{y_2} \end{bmatrix} = \begin{bmatrix} \mathbf{H_1} \\ \mathbf{H_2} \end{bmatrix} \mathbf{x} + \mathbf{n}. \tag{10}$$

In (10), $\mathbf{y_1}$ and $\mathbf{y_2}$ are vectors containing the samples of one period of the two peripheral pressure signals and $\mathbf{n}$ is the noise vector. The vector $\mathbf{x}$ and the matrices $\mathbf{H_1}$ and $\mathbf{H_2}$ are defined as:

$$\mathbf{x} = \begin{bmatrix} x[-L+1] & \dots & x[0] & \dots & x[N-1] \end{bmatrix}^T, \tag{11}$$

$$\mathbf{H}_i = \begin{bmatrix} h_i[L-1] & \dots & h_i[0] & 0 & \dots & 0 \\ 0 & \dots & h_i[1] & h_i[0] & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & h_i[L-1] & \dots & h_i[0] \end{bmatrix}, \tag{12}$$

each has a size of $N \times (N+L-1)$.

For readability, the following variables are defined,

$$\mathbf{y} = \begin{bmatrix} \mathbf{y_1} \\ \mathbf{y_2} \end{bmatrix}, \tag{13}$$

and

$$\mathbf{H} = \begin{bmatrix} \mathbf{H_1} \\ \mathbf{H_2} \end{bmatrix}, \tag{14}$$

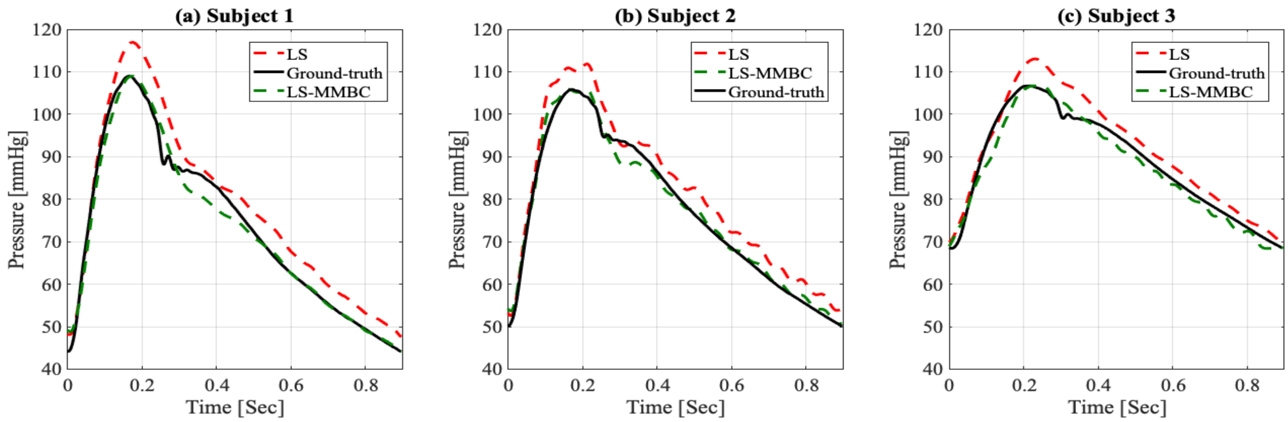

Fig. 2: True and reconstructed CABP signals using cross-relation with LS and LS-MMBC.

the common input can be estimated by minimizing the energy of the noise, that leads to the following least-squares problem:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} ||\mathbf{y} - \mathbf{H}\mathbf{x}||_2^2. \tag{15}$$

The relevant components of the vector $\mathbf{x}$ are those located between the indices L and $N+L-1$. Using a unit Euclidean norm as a constraint in (9) results in a scaled version of the input. To obtain the scaling factor, the fact that only a small change occurs in the mean value of the blood pressure as the signal propagates [15] is used. Thus, the scaling factor can be obtained by adjusting the mean of the estimated input signal to match the mean of one of the output pressure signals as follows:

$$\hat{\mathbf{x}}_s = \hat{\mathbf{x}} \, \frac{\sum_{n=0}^{N-1} y_i(n)}{\sum_{n=L}^{N+L-1} \hat{x}(n)}. \tag{16}$$

B. The proposed cross-relation method using least-squares with mean matching and box constraints (LS-MMBC)

As opposed to the least-squares based cross-relation method, in the proposed cross-relation method using least-squares with mean matching and box constraints (LS-MMBC), the variable α^2 is not assigned a value in (9), i.e. kept as an optimization variable in the input estimation stage. In this case, (9) can be altered to the following form:

$$\begin{aligned} \hat{\mathbf{h}} &= \arg\min_{\mathbf{h}} ||\mathbf{Y}\mathbf{h}||_2^2 = \alpha\mathbf{q} \\ &\text{s.t.} \quad ||\mathbf{h}||_2^2 = \alpha^2, \end{aligned} \tag{17}$$

where $\mathbf{q}$ is the the eigenvector associated with the minimum eigenvalue of the matrix $\mathbf{Y}^T\mathbf{Y}$. The first L indices of $\mathbf{q}$ contain the scaled coefficients of the first channel (denoted by $\mathbf{q}_1$), whereas the rest of the vector is the scaled coefficients of the second channel (denoted by $\mathbf{q}_2$).

While $\mathbf{x}$ has the same definition as in (11), $\mathbf{H}_i$ is redefined as:

$$\tilde{\mathbf{H}}_i = \begin{bmatrix} q_i[L-1] & \dots & q_i[0] & 0 & \dots & 0 \\ 0 & \dots & q_i[1] & q_i[0] & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & q_i[L-1] & \dots & q_i[0] \end{bmatrix}. \tag{18}$$

This converts the least-squares problem in (15) into a two-variable minimization problem as follows:

$$(\hat{\mathbf{x}}, \hat{\alpha}) = \arg\min_{\mathbf{x},\alpha} ||\mathbf{y} - \alpha\tilde{\mathbf{H}}\mathbf{x}||_2^2, \tag{19}$$

where

$$\tilde{\mathbf{H}} = \begin{bmatrix} \tilde{\mathbf{H}}_1 \\ \tilde{\mathbf{H}}_2 \end{bmatrix}. \tag{20}$$

Another way to estimate the CABP signal is to minimize the distance between the central pressure mean value and the peripheral one as the change in mean pressure values as the signal propagates is very small. This leads to the following optimization:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{N} ||\mathbf{u}^T\mathbf{x} - \mathbf{v}^T\mathbf{y}_1||_2^2, \tag{21}$$

where $\mathbf{u}$ is a vector containing zeros in the first $L-1$ places and ones in the other N places, $\mathbf{v}$ is an $N \times 1$ all-ones vector.

The systolic and the diastolic pressure, which are the maximum and the minimum values of the CABP signal respectively, can be accurately approximated using some additional, but simple, measurements [21], and can be used as constraints. Considering this with (19) and (21) leads to the following joint optimization problem:

$$\begin{aligned} (\hat{\mathbf{x}}, \hat{\alpha}) &= \arg\min_{\mathbf{x},\alpha} ||\mathbf{y} - \alpha\tilde{\mathbf{H}}\mathbf{x}||_2^2 + \frac{1}{N}||\mathbf{u}^T\mathbf{x} - \mathbf{v}^T\mathbf{y}_1||_2^2 \\ &\text{s.t.} \qquad \mathbf{x}^{min} \leq \mathbf{x} \leq \mathbf{x}^{max}, \end{aligned} \tag{22}$$

where $\mathbf{x}^{min}$ and $\mathbf{x}^{max}$ are two $N+L-1$ vectors whose elements are the diastolic and the systolic blood pressure values, respectively. Using a simple substitution:

$$\mathbf{z} = \alpha\mathbf{x}, \tag{23}$$

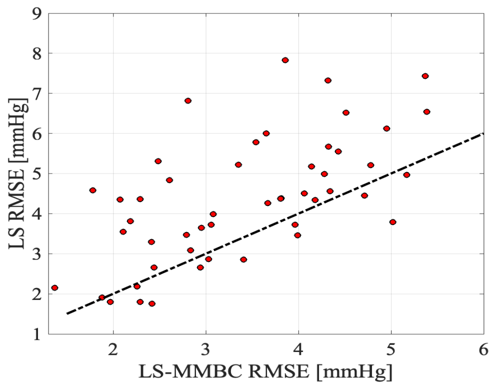

Fig. 3: The RMSE of LS versus the RMSE of LS-MMBC.

converts the problem into the following form:

$$(\hat{\mathbf{z}}, \hat{\alpha}) = \arg\min_{\mathbf{z},\alpha} ||\mathbf{y} - \tilde{\mathbf{H}}\mathbf{z}||_2^2 + \frac{1}{N}||\mathbf{u}^T\mathbf{z} - \alpha\mathbf{v}^T\mathbf{y}_1||_2^2 \quad \text{s.t.} \quad \alpha\mathbf{x}^{min} \leq \mathbf{z} \leq \alpha\mathbf{x}^{max}, \tag{24}$$

which is convex and hence can be solved using commonly used iterative optimization algorithms such as interior point or active set methods.

C. In-silico Virtual Population

To evaluate the performance of the proposed method, we utilize a virtual, pre-validated database of simulated pulse waves (PW) [22], which is publicly available. [1]. This dataset is considered as a useful resource to evaluate the pre-clinical assessment of PW analysis algorithms. The database encompasses mainly these arterial PWs, blood flow and luminal area at different sites of the arterial network. The database represents samples of $3,225$ virtual healthy adults. The pulse waves were created by varying specific cardiac and arterial parameters like the arterial stiffness and heart rate within normal ranges. In this study, blood pressure signals at the level of the radial, femoral and aortic arteries have been investigated to test the proposed methodology. The cardiac outputs vary between 3.5 and 7.2 l/min, depending on the values of the heart rate (53, 63, and 72 beats/min) and stroke volume (66, 83, and 100 ml) prescribed.

III. Results and Discussions

Two criteria are used in our performance evaluation; the root mean squared error (RMSE) and the correlation coefficient. Both metrics are calculated using the estimated and the true central blood pressure signals. The RMSE is a sample-by-sample error metric that can be evaluated using the following formula:

$$\text{RMSE} = \sqrt{\frac{\sum_{n=0}^{N-1}(\hat{x}[n+L] - x[n])^2}{N}}. \tag{25}$$

[1] https://peterhcharlton.github.io/pwdb/index.html

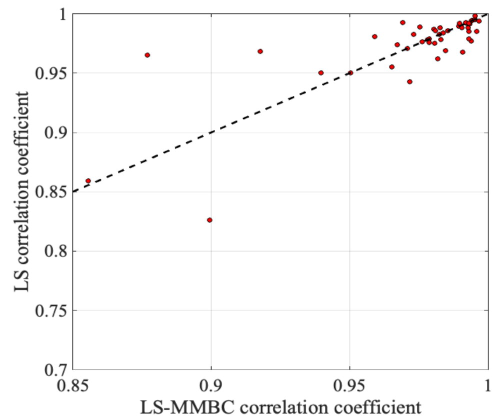

Fig. 4: Correlation coefficients of LS versus correlation coefficients of LS-MMBC.

On the other hand, the correlation coefficient measures the closeness of the shape of two signals and is evaluated as:

$$r = \frac{\sum_{n=0}^{N-1}(x[n] - \bar{x})(\hat{x}[n+L] - \bar{\hat{x}})}{\sqrt{\sum_{n=0}^{N-1}(x[n] - \bar{x})^2}\sqrt{\sum_{n=0}^{N-1}(\hat{x}[n+L] - \bar{\hat{x}})^2}}, \tag{26}$$

where $\bar{.}$ represents the statistical average operator.

Using the dataset described Section II and CVX MATLAB Toolbox [23], [24], fifty subjects are randomly chosen and the RMSEs and the correlation coefficients are examined. Since obtaining the optimal channel length is outside the scope of this work, and to draw a fair comparison between the basic cross-relation approach with LS and that with LS-MMBC, for each subject, the RMSE is calculated as the channel order varies from one to one-hundred and the estimated signal that has the lowest RMSE is chosen to be the reconstructed signal, i.e., the channel length is optimized manually. Fig. 2 shows examples of the reconstructed CABP signals using both methods along with the true signals for three subjects. The RMSE and correlation coefficients of all fifty subjects are plotted for cross-relation with LS versus cross-relation with LS-MMBC in Fig. 3 and 4, respectively. From Fig. 3, It is obvious that most of the points are concentrated in the upper part of the graph, which indicates that the LS-MMBC method achieves lower RMSEs.

In contrast, achieving a higher correlation coefficients is desirable, this requires most of the points to be below the line for LS-MMBC to be better which is the case in Fig. 4.

TABLE I: The mean and standard deviation of the RMSE and correlation coefficients for LS and LS-MMBC.

Method	LS	LS − MMBC
RMSE mean [mmHg]	4.40	3.50
RMSE standard deviation [mmHg]	1.55	1.25
Correlation coefficients mean	0.96	0.97
Correlation coefficients standard deviation	0.09	0.02

A summary of the mean and standard deviation of the two metrics is available in table I. This table shows that while the correlation coefficients are comparable, the RMSE is reduced by 20% when LS-MMBC is used.

Finally, it should be noted that solving (24) is generally more computationally expensive than solving the least-squares (15). However, LS-MMBC is computationally simpler than the wiener based method in [16], and the parameters estimation in several transfer function models. The other advantage of the proposed method is that no measurement-based calibration is required to estimate the CABP signals.

IV. Conclusions

The results presented in this paper demonstrate that the proposed cross-relation method based on LS-MMBC method provides a better performance in estimating the central blood pressure pulse wave than the basic cross-relation method using pure LS. This is attributed to two main factors; incorporating a mean matching criterion in the cost function, and applying diastolic and systolic pressure measurements as constraints to limit the solution space. Using LS-MMBC, the RMSE has been reduced by 20% compared to the pure LS method. The two methods, however, maintain comparable correlation coefficients. Both the pure LS method and the proposed LS-MMBC method assume a priori knowledge of channel order, which is not the case in practice. Finding a way to estimate the optimal channel length is an open problem for future studies. Moreover, leveraging measurements from more than two peripheral sites constitutes another promising future direction.

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