

Reconfigurable Intelligent Surfaces for Localization: Position and Orientation Error Bounds

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Abstract—Next-generation cellular networks will witness the creation of smart radio environments (SREs), where walls and objects can be coated with reconfigurable intelligent surfaces (RISs) to strengthen the communication and localization coverage by controlling the reflected multipath. In fact, RISs have been recently introduced not only to overcome communication blockages due to obstacles but also for high-precision localization of mobile users in GPS denied environments, e.g., indoors. Towards this vision, this paper presents the localization performance limits for communication scenarios where a single next generation NodeB base station (gNB), equipped with multiple-antennas, infers the position and the orientation of a user equipment (UE) in a RIS-assisted SRE. We consider a signal model that is valid also for near-field propagation conditions, as the usually adopted far-field assumption does not always hold, especially for large RISs. For the considered scenario, we derive the Cramér-Rao lower bound (CRLB) for assessing the ultimate localization and orientation performance of synchronous and asynchronous signaling schemes. In addition, we propose a closed-form RIS phase profile that well suits joint communication and localization. We perform extensive numerical results to assess the performance of our scheme for various localization scenarios and RIS phase design. Numerical results show that the proposed scheme can achieve remarkable performance, even in asynchronous signaling and that the proposed phase design approaches the numerical optimal phase design that minimizes the CRLB.

Index Terms—Reconfigurable intelligent surfaces; smart radio environment; single-anchor localization; attitude estimation, orientation estimation; near-field localization, Cramér-Rao lower bound.

I. INTRODUCTION

Recently, smart radio environments (SREs) have been conceived as a new paradigm where the traditional radio environment is turned into a smart reconfigurable space that plays an active role in transferring and processing the information [1], [2]. Indeed, key performance indicators (KPIs) for the next sixth generation mobile networks (6G) promote continuous connection availability, strong reliability, huge device density (10^7 devices per km^2) and air interface latency of sub-milliseconds (e.g., $10\mu\text{s}$), etc. [3], [4]. To meet these requirements, reconfigurable intelligent surfaces (RISs) might represent a key solution, allowing to enhance not only wireless communications but also imaging- and localization-based applications thanks to the augmented ambient awareness [4], [5]. In this regard, RISs can aid in establishing a line-of-sight

(LOS) link between the transmitter and the receiver even in the presence of obstructions or when the received power from the direct path does not enable a robust connection [6].

In analogy with software defined radios, RISs are often referred to as software defined surfaces (SDSs), where the electromagnet response to the incident wave can be controlled by a software [7]. The realization of such a technology might be enabled by metamaterials, which are a class of artificial materials whose physical properties, e.g., permittivity and permeability, can be engineered to exhibit some desired characteristics [8]–[10]. When such metamaterials are deployed in metasurfaces, their effective parameters can be tailored to realize a desired transformation on the transmitted, received, or impinging waves [11]–[14]. With the availability of new degrees of freedom useful to improve the network performance, the environment will be no more perceived as a passive entity, but as a meaningful support for wireless communications based applications [15]–[19], e.g., energy transfer [20], vehicular networks [21], unmanned aerial vehicle (UAV) communications [22], physical layer security [20], cognitive radio [23], electromagnetic fields (EMF)-aware beamforming [24], and many others [25]. In this context, wireless localization with RISs [26], [27] has not yet received a large attention, albeit they represent a promising candidate for enhancing positioning and orientation estimation capabilities in next-generation cellular networks for various 6G applications, e.g., augmented reality and self-driving cars [21], [28]–[30]. This is of great help in GPS-denied environments, and it allows to avoid the use of a dedicated infrastructure, usually made of multiple anchors. Indeed, the possibility to localize with antenna arrays is not new, and has been investigated in the last few years [31]–[36]. In particular, fifth generation mobile networks (5G) and beyond foresee the use of millimeter-waves (mm-wave) to enable the integration of arrays with a large number of antennas (massive arrays) into small areas. By enabling such an architecture capable to realize near-pencil beam antennas, it becomes feasible not only to boost communication but also single-anchor localization capabilities at an unprecedented scale [37]–[41].

Current state-of-the-art for intelligent surfaces-based localization considers studies employing RIS either in receive mode [42] or in reflection mode [27], [43]. When exploited in receive mode, a large intelligent surface is used to localize a user in front of it, both in near-field and far-field [42], [44]. When instead operating in reflection mode, in [45] it is proposed an approach exploiting the modification of the RIS reflection coefficient such that the experienced received signal strength

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(RSS) at different points is enlarged and the localization accuracy is improved. Differently, in [27], authors exploit a RIS for supporting the positioning and communication in the mm-wave frequency bands. This paper assumes that the mobile is in far-field with respect to the RIS [27], but this approximation is not always valid, especially when large surfaces and arrays are considered with respect to the distance. Consequently, the entailed models are no more accurate, as the mobile is not in the Fraunhofer region but in the Fresnel region, and the planar wavefront approximation does not hold. Additionally, ignoring the spherical wavefront discard essential information regarding the location and orientation of the mobile [46]–[48].

To the best of authors' knowledge, no paper has considered a general model accounting for 3D RIS-assisted localization and orientation estimation in near-field, as current papers for near-field positioning only refer to the adoption of a large intelligent surface in receive mode and not as a mean for controlling the multipath. To this purpose, in this paper we consider the localization scenario depicted in Fig. 1, where we propose an ad-hoc model for joint communication and localization, accounting for the incident spherical wavefront. Indeed, in SRE, the next generation NodeB base station (gNB) augments its environment awareness, as it allows also to achieve a knowledge of the environment in terms of inferring the location in the 3D space and the orientation (i.e., roll, pitch, and yaw) of the user equipment (UE). In this context, we derive the Cramér-Rao lower bound (CRLB) to investigate the ultimate positioning and orientation estimation performance in the presence of the RIS. Then, we analyze the geometric dilution of precision (GDOP) to evaluate the impact of the geometry on the UE localization. Finally, we derive a suboptimal phase design for the RIS in closed-form to enhance both the localization and communication performance by maximizing the signal-to-noise ratio (SNR).

The main contributions can be summarized as follows.

- We propose an architecture where the RIS is used to assist mobile localization (position and orientation estimation) at the gNB, which can provide surveillance solutions or assist the communication process, while in the literature the UE estimates its own position.
- We consider that the gNB, RIS, and the UE are equipped with multiple-antennas with arbitrary array configurations and geometry including planar arrays that will be adopted for beyond 5G systems, especially the RIS, allowing 3D beamforming in both the azimuth and elevation, while most of the literature considers linear arrays with 2D beamforming that significantly simplifies the analysis to the conventional steering vectors.
- We consider a general model valid for both near- and far-field localization and attitude (i.e., orientation) estimation in 3D space, unlike the literature that either imposes the far-field assumption or it considers simplified 2D geometry.
- Differently from the state-of-the-art that analyzes synchronous systems, we consider two general signaling schemes (i.e., synchronous and asynchronous) and compare their localization error performance.
- We derive the ultimate bound on the localization per-

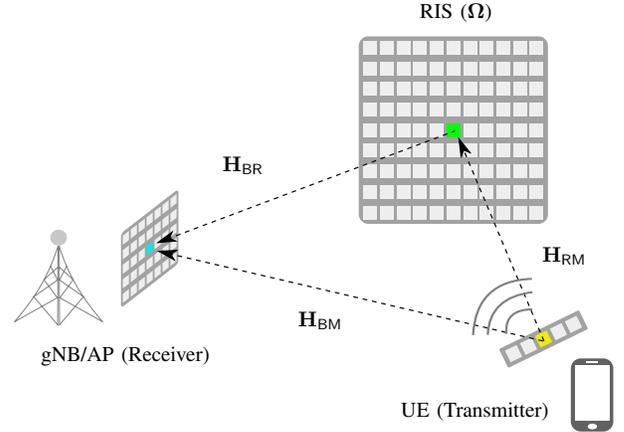


Fig. 1: Pictorial representation of the considered RIS-aided positioning scenario.

formance in terms of the CRLB. Furthermore, in order to get more insights on the effect of the geometry (e.g., the locations and orientations of the gNB, RIS, and UE) on the localization performance, we consider the GDOP metric.

- We propose a closed-form RIS phase design, that accounts for the spherical wavefront, and we compare it to other strategies accounting also for the presence of quantization errors;
- We perform extensive simulations and numerical results that provide insights into the problem and shed light on the benefits offered by the adoption of the RIS in terms of localization performance.

The rest of the manuscript is organized as follows. Sec. II describes the signal model for both synchronous and asynchronous cases. Sec. III investigates the position and orientation performance limits and the impact of the system geometry on localization, whereas Sec. IV discusses a possible design for the RIS phase profile. In Sec. V simulation results are reported and final conclusions are drawn in Sec. VI.

Scalars (e.g., x) are denoted in italic, vectors (e.g., \mathbf{x}) in bold, and matrices (e.g., \mathbf{X}) in bold capital letters. $\nabla_x(a) = \partial a / \partial x$ is the partial derivative of a with respect to the scalar x . $\nabla_{\mathbf{x}}(\cdot)$ is the gradient operator with respect to the vector \mathbf{x} . Transpose and Hermitian operators are represented as \cdot^T and \cdot^H , respectively. The $N \times N$ matrix with all elements being zeros and the $N \times N$ identity matrix are denoted by $\mathbf{0}_{N \times N}$ and $\mathbf{I}_{N \times N}$, respectively. The operator $\text{tr}(\mathbf{X})$ denotes the trace of a matrix \mathbf{X} , while $\text{diag}(\mathbf{x})$ denotes a diagonal matrix with diagonal elements identified by \mathbf{x} . A probability density function is denoted by $p(\cdot)$, and $\mathbb{E}\{\mathbf{x}\}$ is the expectation of a random vector \mathbf{x} with respect to its distribution. $j = \sqrt{-1}$ is the imaginary unit.

II. SYSTEM MODEL

A. Localization Scenario

In this paper, we consider a localization scenario as in Fig. 1 where a gNB, equipped with an antenna array with center located in position $\mathbf{p}_B = [x_B, y_B, z_B]^T$, performs the

spherical wavefront, even if no information can be retrieved from the time-of-arrival (TOA).

1) *Synchronous System*: We here consider that a synchronization procedure has been performed between the gNB and the UE prior the localization step. Once synchronized, the positioning information can be retrieved by jointly processing temporal and angular information of the received signal. By extending (10) to its scalar notation, the general model of the received signal can be rewritten as

$$y_b[n] = \mu_b[n] + w[n], \quad \forall b \in \{1, 2, \dots, N_B\}, \quad (11)$$

with N_B being the number of antennas at the gNB and $w[n]$ being the circularly symmetric zero-mean Gaussian noise with power spectral density σ^2 . The useful part of the signal, without the noise, is

$$\begin{aligned} \mu_b[n] \triangleq & \sqrt{P} \sum_{m=1}^{N_M} x_m[n] e^{-j2\pi f_n \xi_{BM}} \left(\rho_{BM} e^{-j2\pi f_n (\tau_{bm} + \eta_m)} + \right. \\ & \left. + \rho_{BRM} \sum_{r=1}^{N_R} \omega_r e^{-j2\pi f_n (\tau_{br} + \tau_{rm} + \eta_r + \eta_m)} \right), \quad (12) \end{aligned}$$

where $f_n = nB/N$ is the considered sub-carrier, B is the signal bandwidth, and τ_{bm} , τ_{br} , τ_{rm} are the delays for each couple of antenna (e.g., τ_{bm} is the delay between the b th antenna at the gNB and the m th antenna at the UE), ξ_{BM} is a synchronization residual (negligible for accurate synchronization procedures), and η_m and η_r are array non-idealities. The signal attenuation coefficients due to propagation are indicated with ρ_{BM} and ρ_{BRM} for the direct and the relayed paths, respectively.¹ Since the planar wavefront approximation is not valid due to the large size of the RIS, a spherical model is considered where the following relations hold

$$\tau_{bm}(d_{BM}, \theta_{BM}, \phi_{BM}) = d_{bm}/c, \quad (13)$$

$$\tau_{br}(d_{BR}, \theta_{BR}, \phi_{BR}) = d_{br}/c, \quad (14)$$

$$\tau_{rm}(d_{RM}, \theta_{RM}, \phi_{RM}) = d_{rm}/c, \quad (15)$$

$$\rho_{BM} = \frac{\lambda}{4\pi} \frac{1}{d_{BM}}, \quad (16)$$

$$\rho_{BRM} = \frac{\lambda}{4\pi} \frac{1}{d_{RM} + d_{BR}}, \quad (17)$$

where c is the speed of light, $(d_{BM}, \theta_{BM}, \phi_{BM})$, $(d_{BR}, \theta_{BR}, \phi_{BR})$ and $(d_{RM}, \theta_{RM}, \phi_{RM})$ are the distances and angles between the gNB-UE, gNB-RIS, RIS-UE centroids, respectively, and where

$$d_{bm} = \sqrt{d_m^2 + d_b^2 + d_{BM}^2 - 2 \left(G_{bm}^{(1)} + d_{BM} G_{bm}^{(2)} \right)}, \quad (18)$$

with $G_{bm}^{(1)}$ and $G_{bm}^{(2)}$ containing the information of the geometry at the transmitter and at the receiver, that is

$$G_{bm}^{(1)} = x_b x_m + y_b y_m + z_b z_m, \quad (19)$$

$$\begin{aligned} G_{bm}^{(2)} = & (x_m - x_b) \cos \theta_{BM} \cos \phi_{BM} + \\ & + (y_m - y_b) \cos \theta_{BM} \sin \phi_{BM} \\ & + (z_m - z_b) \sin \theta_{BM}. \quad (20) \end{aligned}$$

¹According to the considered system geometry, the signal amplitude is about the same at each antenna as its variations are negligible.

The distances of arrival between the b th gNB antenna and the r th RIS antenna and between the r th RIS antenna and the m th UE antenna, namely d_{br} and d_{rm} , can be found using (18) with appropriate substitutions, as done in (1) and (4).

Differently from traditional schemes that make the assumption of incident planar wavefront, in (18) we infer jointly the ranging and bearing information from the spherical waveform curvature. Notably, it is possible to write (12) only when the clocks of the gNB, RIS and UE have been synchronized. The accurate synchronization might entail several and long procedures. In the following, we consider an asynchronous alternative where it is still possible to retrieve the UE position from the relative phases.

2) *Asynchronous System*: As evidenced in (11), from the received signal it is possible to infer the TOA estimate, which is possible in all those situations where a synchronization procedure has been performed. In this case, the system is no more able to directly estimate the information of the distance from the TOA. Instead, the incident waveform curvature, i.e.,

$$\Delta d_{bm} = d_{bm} - d_{BM} = c \Delta \tau_{bm}, \quad (21)$$

$$\Delta d_{br} = d_{br} - d_{BR} = c \Delta \tau_{br}, \quad (22)$$

can be exploited for UE localization.

In this case, (10) can be written as

$$\begin{aligned} y_b[n] = & \sqrt{P} \sum_{m=1}^{N_M} x_m[n] \left(\rho_{BM} e^{-j\chi_{BM}} e^{-j2\pi f_n (\Delta \tau_{bm} + \eta_m)} \right. \\ & \left. + \rho_{BRM} e^{-j\chi_{BRM}} \sum_{r=1}^{N_R} \omega_r e^{-j2\pi f_n (\Delta \tau_{br} + \Delta \tau_{rm} + \eta_r + \eta_m)} \right) + w[n], \quad (23) \end{aligned}$$

where χ_{BM} and χ_{BRM} are uniformly distributed random variable from 0 to 2π representing the phase offsets between the gNB, the UE and the RIS due to the lack of synchronization.

Given the proposed models for synchronous and asynchronous systems, in the following we derive the attainable fundamental performance limits.

III. RIS-AIDED POSITION AND ORIENTATION ERROR BOUNDS

In this section, we derive the ultimate performance limits for the considered localization scenario. To this end, the CRLB is a useful metric that represents the minimum variance of the estimation error from any unbiased estimator, and it can be represented with the inverse of the Fisher information matrix (FIM) [50], [51]. Then, we investigate the impact of the geometry on the error through the GDOP metric analysis.

A. The CRLB on UE position and orientation

Given the signal models in (11) and (23), we distinguish two possible estimation approaches: (i) the first one exploits a direct localization approach, and it is used for the asynchronous case; (ii) the second one is a two-stage approach that considers that the location and orientation are estimated from a set of features extracted from the signal, e.g., TOAs, angle-of-arrivals

(AOAs), and received signal strength indicators (RSSIs) [52]. In both cases, the parameter vector to be estimated is

$$\mathbf{s} = [\mathbf{p}_M, \phi_M]^T, \quad (24)$$

where \mathbf{p}_M , and ϕ_M contain the UE position and orientation parameters, as indicated in Section II-A. On the other hand, the measurement vector can be written as either

$$\mathbf{\Gamma} = \mathbf{s}, \quad (25)$$

or

$$\mathbf{\Gamma} = [\rho_{BM}, \theta_{BM}, \phi_{BM}, \tau_{BM}, \rho_{BRM}, \theta_{RM}, \phi_{RM}, \tau_{RM}, \phi_M]^T, \quad (26)$$

for the *Direct* and *Two-stage* approaches, respectively, where τ_{BM} and τ_{RM} , ρ_{BM} and ρ_{BRM} , and θ_{BM} , ϕ_{BM} , θ_{RM} ϕ_{RM} are the TOAs, RSSI, and AOAs, respectively. All the main parameters that are required to infer the location and orientation of the UE are included in (26).²

Note that the two approaches are the same from a CRLB perspective. Still, we distinguish two cases. For synchronous and asynchronous signaling, we adopt the direct and two-stage approaches, respectively. This can be attributed to a twofold reason: (i) The vector of measurements in the synchronous case allows to emphasize the parameters of the received signal which depend on the position and orientation and to quantify the error in estimating these parameters; (ii) On the contrary, if a two-stage approach is used in the asynchronous case, where only difference of TOAs are present in (21), then the measurement vector would consist of all the TOA pairs between the gNB-UE and RIS-UE, leading to a dimensional issue for the FIM. Thus, a direct approach is adopted in which the position is directly inferred from the signals received at each antenna of the gNB, allowing the measurement vector to be written in a more compact way.

Starting from (25)-(26), the CRLB on the UE state vector can be written from [50, (178)] as

$$\mathbf{\Lambda}(\mathbf{s}) \triangleq \left[\sum_{n=1}^N \mathbf{I}_n(\mathbf{s}) \right]^{-1}, \quad (27)$$

where $\mathbf{I}_n(\mathbf{s})$ is the FIM of the state vector relative to the n -th subcarrier. Hence, the position error bound (PEB) and orientation error bound (OEB) can be written as

$$\text{PEB} = \sqrt{\text{tr}([\mathbf{\Lambda}(\mathbf{s})]_{1:3,1:3})}, \quad \text{OEB} = \sqrt{\text{tr}([\mathbf{\Lambda}(\mathbf{s})]_{4:6,4:6})}, \quad (28)$$

where $[\cdot]_{a:b,c:d}$ indicates the sub-matrix located between rows (a, b) and columns (c, d) .

The FIM can be obtained by the chain rule as [53]

$$\mathbf{I}_n(\mathbf{s}) = (\nabla_{\mathbf{s}} \mathbf{\Gamma}) \mathbf{I}_n(\mathbf{\Gamma}) (\nabla_{\mathbf{s}} \mathbf{\Gamma})^T, \quad (29)$$

where $\mathbf{I}_n(\mathbf{\Gamma})$ is the FIM of the parameter vector in (25), given by [50] as

$$\mathbf{I}_n(\mathbf{\Gamma}) = \mathbb{E} \left\{ (\nabla_{\mathbf{\Gamma}} \log p(\mathbf{y}[n]; \mathbf{\Gamma}))^H \nabla_{\mathbf{\Gamma}} \log p(\mathbf{y}[n]; \mathbf{\Gamma}) \right\}, \quad (30)$$

²If the localization is only based on TOA and AOAs, then the parameters related to RSSI (i.e., ρ_{BM} and ρ_{BRM}) can be neglected in (26). Further, when the direct path is obstructed, all the sub-elements related to it (i.e., τ_{BM} , ϕ_{BM} , θ_{BM} , and ρ_{BM}) can be discarded. Nevertheless, the resulting bound will be an upper bound on the CRLB derived directly from the signal.

with $\mathbf{J} \triangleq \nabla_{\mathbf{s}} \mathbf{\Gamma}$ being the Jacobian matrix and $\log p(\mathbf{y}[n]; \mathbf{\Gamma})$ is the log-likelihood function of the received signal vector. The log-likelihood function is computed from (11) as

$$\log p(\mathbf{y}[n]; \mathbf{\Gamma}) = -(\mathbf{y}[n] - \boldsymbol{\mu}[n])^H \boldsymbol{\Sigma}^{-1} (\mathbf{y}[n] - \boldsymbol{\mu}[n]) - N_B \log(\pi \sigma^2), \quad (31)$$

where $\boldsymbol{\Sigma} = \sigma^2 \mathbf{I}_{N_B \times N_B}$ is the covariance matrix of the noise. For the Jacobian matrix, it can be written as

$$\mathbf{J} = \mathbf{I}_{6 \times 6}, \quad \text{Direct approach} \quad (32)$$

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{\mathbf{p}_M} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \end{bmatrix}, \quad \text{Two-stage approach} \quad (33)$$

with $\mathbf{J}_{\mathbf{p}_M}$ indicating the term relative to the UE position, and it is given in Appendix A.

The elements of the FIM in (30) can be written as [53]

$$[\mathbf{I}_n(\mathbf{\Gamma})]_{i,j} = \frac{2}{\sigma^2} \text{Re} \left\{ \sum_{b=1}^{N_B} \frac{\partial \mu_b^*[n]}{\partial \Gamma_i} \frac{\partial \mu_b[n]}{\partial \Gamma_j} \right\}, \quad (34)$$

and their derivations are reported in Appendix B.

Since we are considering the curvature of the wavefront in (18), the bound is valid also for the near-field localization that is essential when the size of arrays are sufficiently large, within the Fraunhofer distance [54]. Also, the CRLB in (27) accounts for the errors due to the receiver noise (i.e., in σ^2) and the geometry of the localization scenario.

B. Localization Algorithm

One possible solution for estimating the location and orientation of the UE is through the maximum likelihood estimator (MLE). The UE location and orientation, \mathbf{s} , that maximize the log-likelihood function in (31) can be estimated as

$$\hat{\mathbf{s}} \triangleq \underset{\mathbf{p}_M \in \mathbb{R}^3, \phi_M \in [0, 2\pi]^3}{\text{argmax}} \log p(\mathbf{y}[n]; \mathbf{s}). \quad (35)$$

The previous optimization problem can be solved by a grid search or by iterative methods such as Newton-Raphson and expectation-maximization algorithms, however, the convergence of iterative methods to the global maximum is not guaranteed.

The MLE is known to approach the CRLB derived in Section III-A for asymptotically high SNRs. Also, if there exists an efficient estimator with a variance that coincides with the CRLB, it will be the MLE, which can be found by simultaneously solving the following equations [53]

$$\nabla_{\mathbf{s}} p(\mathbf{y}[n]; \mathbf{s}) = \mathbf{0}_{6 \times 1}. \quad (36)$$

For asymptotically large number of measurements, i.e., snapshots \mathbf{y} in (10), or high SNR, the error in estimating the location and orientation tends in distribution to a zero mean Gaussian distribution (indicated with “d”) with covariance matrix $\mathbf{\Lambda}(\mathbf{s})$ expressed in (27), i.e.,

$$\hat{\mathbf{s}} - \mathbf{s} \xrightarrow{d} \mathcal{N}(\mathbf{0}_{6 \times 1}, \mathbf{\Lambda}(\mathbf{s})), \quad (37)$$

where $\hat{\mathbf{s}}$ is a random vector representing the estimated parameters through the MLE [53].

Alternatively, the two-stage approach can be adopted, where the signal attenuation coefficients can be estimated from the RSSI. On the other hand, the bearing angles (i.e., AOAs) can be estimated through variations of MUSIC algorithms or compressive sensing for both on- and off-grid methods with guaranteed recovery under some mild conditions. The compressive sensing based estimators have the advantage that the angles can be recovered even from a single snapshot of \mathbf{y} , while the performance can be further enhanced by considering multiple measurement vectors (i.e., several snapshots) [55]–[58]. For TOA estimation, two possible schemes can be considered based on correlators (i.e., matched filters) or on energy-based solutions (i.e., energy detectors) [59]–[62]. The second is more practical as it can operate at sub-Nyquist rate.

C. Geometry Impact on Direct RIS-aided Localization

The CRLB derived in Sec. III-A does not explicitly quantify the impact of the system geometry on the performance, because it includes also the effect of the input noise [63]. Hence, we now investigate the solely impact of the geometry on the localization performance using a GDOP analysis.

In particular, as a GDOP metric, we consider the ratio between the root mean square error (RMSE) of position and the RMSE of measurement (ranging) error, i.e., [64]

$$\text{GDOP} = \frac{\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}}{\sigma_M} \triangleq \frac{\text{RMSE}(\mathbf{p})}{\sigma_M}, \quad (38)$$

where σ_M is the RMSE of the measurements, e.g., in GPS positioning it is the standard deviation of ranging measurements. Since the RMSE is lower bounded by the CRLB, the GDOP can be also defined as a function of the PEB as

$$\sigma_M \cdot \text{GDOP} = \text{RMSE}(\mathbf{p}) \geq \sqrt{\text{tr}(\text{CRB}(\mathbf{p}))} = \text{PEB}. \quad (39)$$

Differently from the parameter vector in (25), in direct localization, position and orientation are directly estimated from the received signals at the receiver, with $\boldsymbol{\Gamma} = \mathbf{s}$ as in (26). In this specific case, the measurement noise standard deviation is the same for all the antennas and corresponds to the thermal noise, i.e. $\sigma_M = \sigma$.

Given such signal-related measurements, the GDOP can be computed from the CRLB expression in (27) where the generic element in the FIM is given by

$$[\mathbf{I}_n(\mathbf{s})]_{i,j} = \frac{2}{\sigma^2} \text{Re} \left\{ \sum_{b=1}^{N_B} \frac{\partial \mu_b^*[n]}{\partial \ell_i} \frac{\partial \mu_b[n]}{\partial \ell_j} \right\}, \quad (40)$$

where $\ell_i \in \mathbf{s}$ is either related to the position or to the orientation of the UE. Therefore, we can write the GDOP for position and orientation as [64]

$$\text{GDOP}_{\text{PM}} = \frac{1}{\sigma \kappa_{\text{PM}}} \sqrt{\text{tr} \left(\left[\sum_{n=1}^N \mathbf{I}_n(\mathbf{s}) \right]_{1:3,1:3}^{-1} \right)} = \frac{\text{PEB}}{\sigma \kappa_{\text{PM}}}, \quad (41)$$

$$\text{GDOP}_{\phi_M} = \frac{1}{\sigma \kappa_{\phi_M}} \sqrt{\text{tr} \left(\left[\sum_{n=1}^N \mathbf{I}_n(\mathbf{s}) \right]_{4:6,4:6}^{-1} \right)} = \frac{\text{OEB}}{\sigma \kappa_{\phi_M}}, \quad (42)$$

where $\kappa_{\text{PM}} [\text{m}/\sqrt{\text{Watt}}]$ and $\kappa_{\phi_M} [\text{radians}/\sqrt{\text{Watt}}]$ are the normalization factors for the GDOP to become dimensionless. For example, in our settings the normalization factors κ_{PM} and κ_{ϕ_M} can be designed as d_{BM}/\sqrt{P} and $1/\sqrt{P}$, respectively. With this definition, the position and orientation errors are proportional to the GDOP, i.e., $\text{PEB} \propto \sigma \text{GDOP}_{\text{PM}}$ and $\text{OEB} \propto \sigma \text{GDOP}_{\phi_M}$ [64].

IV. RIS PHASE DESIGN

An important concern, when using RIS, is the proper design of the phase profile in order to exploit as much as possible the RIS potentialities but, unfortunately, it usually entails planar wavefronts incident to the RIS [30]. Thus, in the following we consider possible alternatives for the design of the phase profile accounting for spherical wavefronts.

1) *Optimal RIS Phase Design:* The first possibility considers the optimal phase shifts induced at the RIS that minimize the position or orientation error bounds, i.e.,

$$\text{PEB}^* \triangleq \underset{\boldsymbol{\Theta} \in [0, 2\pi]^{N_R}}{\text{minimize}} \text{PEB}(\boldsymbol{\Theta}), \quad (43)$$

$$\text{OEB}^* \triangleq \underset{\boldsymbol{\Theta} \in [0, 2\pi]^{N_R}}{\text{minimize}} \text{OEB}(\boldsymbol{\Theta}). \quad (44)$$

Notably, if from one side this approach is complex as it involves the minimization of the inverse of the FIM, from the other side it represents the optimal configuration that allows to minimize the position and orientation error bounds.

2) *Proposed RIS Phase Design:* Another possibility is to consider an ad-hoc approach that maximizes the sum of the SNRs at each gNB antenna as

$$\underset{\boldsymbol{\Theta} \in [0, 2\pi]^{N_R}}{\text{Maximize}} \text{SNR}(\boldsymbol{\Theta}), \quad (45)$$

where

$$\begin{aligned} \text{SNR}(\boldsymbol{\Theta}) &= \frac{P}{\sigma^2} \mathbf{b}^T \left| \mathbf{H}_{\text{BM}} \mathbf{w} + \mathbf{H}_{\text{BR}} \boldsymbol{\Omega}(\boldsymbol{\Theta}) \mathbf{H}_{\text{RM}} \mathbf{w} \right|^2 \\ &\leq \text{SNR}_{\text{DL}} + \text{SNR}_{\text{MP}}(\boldsymbol{\Theta}), \end{aligned} \quad (46)$$

with

$$\begin{aligned} \text{SNR}_{\text{DL}} &\triangleq \frac{P}{\sigma^2} \mathbf{b}^T \left| \mathbf{H}_{\text{BM}} \mathbf{w} \right|^2, \\ \text{SNR}_{\text{MP}}(\boldsymbol{\Theta}) &\triangleq \frac{P}{\sigma^2} \mathbf{b}^T \left| \mathbf{H}_{\text{BR}} \boldsymbol{\Omega}(\boldsymbol{\Theta}) \mathbf{H}_{\text{RM}} \mathbf{w} \right|^2, \end{aligned} \quad (47)$$

where $\mathbf{b} = \mathbf{1}_{N_B \times 1}$ is a vector of all ones, SNR_{DL} is the sum of the SNRs from the direct link between the gNB and the UE, while SNR_{MP} represents the sum of the SNRs from the multipath component travelling through the RIS. Regarding (46), it results from the Cauchy–Schwarz inequality with equality iff the phase of the direct path coincides with the phase of the reflected path. In order to design the RIS phase profile, we operate as follows: (i) first, we maximize the upper bound on the SNR in (46); (ii) second, we design an additional constant phase shift for the RIS phase profile such that the Cauchy–Schwarz inequality is satisfied with equality that is, the direct link and the multipath component are coherently summed up at each antenna.

According to the aforementioned considerations, we have

$$\underset{\Theta \in [0, 2\pi]^{N_R}}{\text{Maximize}} \quad \text{SNR}_{\text{DL}} + \text{SNR}_{\text{MP}}(\Theta), \quad (48)$$

that can be further simplified to

$$\underset{\Theta \in [0, 2\pi]^{N_R}}{\text{Maximize}} \quad \mathbf{b}^T |\mathbf{H}_{\text{BR}} \mathbf{\Omega}(\Theta) \mathbf{H}_{\text{RM}} \mathbf{w}|^2, \quad (49)$$

that gives

$$\underset{\Theta \in [0, 2\pi]^{N_R}}{\text{Maximize}} \quad \sum_{b=1}^{N_B} \left| \sum_{r=1}^{N_R} \sum_{m=1}^{N_M} e^{j\theta_r} e^{j\beta_m} e^{-j2\pi f_0(\tau_{br} + \tau_{rm})} \right|^2, \quad (50)$$

where f_0 is the central sub-carrier.

Unfortunately, the number of degrees of freedoms, i.e., the number of controllable phase shifts at the RIS, is not enough to perfectly adjust the phase of the signals at the gNB. To combat such an issue, we relax the problem by minimizing the sum of square distance of the phases from their related centroid $\bar{\phi}(\Theta)$, inspired by the k-means algorithm [65]. Thus we write

$$\underset{\Theta \in [0, 2\pi]^{N_R}}{\text{Minimize}} \quad \gamma(\Theta) \triangleq \sum_{r=1}^{N_R} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} [\theta_r + \beta_m - 2\pi f_0(\tau_{br} + \tau_{rm}) - \bar{\phi}(\Theta)]^2, \quad (51)$$

where $\gamma(\Theta)$ is the objective function of interest, and the centroid $\bar{\phi}(\Theta)$ is given by

$$\begin{aligned} \bar{\phi}(\Theta) &= \frac{1}{N_R N_M N_B} \sum_{r=1}^{N_R} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} [\theta_r + \beta_m - 2\pi f_0(\tau_{br} + \tau_{rm})] \\ &= \frac{1}{N_R} \theta_k + \frac{1}{N_R} \sum_{r=1, r \neq k}^{N_R} \theta_r + \frac{1}{N_M N_B N_R} \sum_{r=1}^{N_R} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} C_{brm}, \end{aligned} \quad (52)$$

where $C_{brm} = \beta_m - 2\pi f_0(\tau_{br} + \tau_{rm})$. It can be verified that $\gamma(\Theta)$ is a convex function. More precisely, $\gamma(\Theta)$ is convex, as the composition of a convex function with an affine mapping is convex, and the positive weighted sum of convex functions preserves the function convexity [66, 3.2.2] and [66, 3.2.1].

The objective function can be expressed for $k \in \{1, 2, \dots, N_R\}$ as

$$\begin{aligned} \gamma(\Theta) &= \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} [\theta_k + C_{bkm} - \bar{\phi}(\Theta)]^2 + \\ &+ \sum_{r=1, r \neq k}^{N_R} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} [\theta_r + C_{brm} - \bar{\phi}(\Theta)]^2. \end{aligned} \quad (53)$$

Since the objective function is convex, the optimal solution can be found by solving the following equations in θ_k [66],

$$\begin{aligned} \frac{\partial \gamma(\Theta)}{\partial \theta_k} &= \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} 2 \left(1 - \frac{1}{N_R}\right) [\theta_k + C_{bkm} - \bar{\phi}(\Theta)] + \\ &+ \sum_{r=1, r \neq k}^{N_R} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} -\frac{2}{N_R} [\theta_r + C_{brm} - \bar{\phi}(\Theta)] = 0, \end{aligned} \quad (54)$$

for each $k \in \{1, 2, \dots, N_R\}$. After some manipulations, we get

$$\begin{aligned} &\left(1 - \frac{1}{N_R}\right) \theta_k - \frac{1}{N_R} \sum_{r=1, r \neq k}^{N_R} \theta_r + \frac{1}{N_M N_B} \\ &\times \left(\sum_{b=1}^{N_B} \sum_{m=1}^{N_M} C_{bkm} - \frac{1}{N_R} \sum_{r=1}^{N_R} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} C_{brm} \right) = 0. \end{aligned} \quad (55)$$

Operating like this, the N_R linear equations in N_R unknowns (i.e., the RIS phases) can be simultaneously solved to obtain the phase shifts optimized for this specific problem. An optimal, albeit not unique, phase profile that minimizes the convexified objective function in (51) can be written in closed-form as

$$\hat{\theta}_k = \frac{2\pi f_0}{N_M N_B} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} \left[\tau_{bk} + \tau_{km} - \frac{1}{N_R} \sum_{r=1}^{N_R} (\tau_{br} + \tau_{rm}) \right], \quad (56)$$

where $\hat{\Theta} \triangleq \{\hat{\theta}_1, \dots, \hat{\theta}_r, \dots, \hat{\theta}_{N_R}\}$. Notably, the solution $\hat{\Theta}$ is not unique, since adding a constant phase shift ϕ_c to the RIS phases yields to the same value for the objective function in (51), as it will be accounted for also in the centroid $\bar{\phi}(\Theta)$. Indeed, even the objective function in (50) will not be changed by adding a constant phase shift because of the absolute operator.

Having maximized the upper bound on the SNR, now we derive the constant phase ϕ_c such that the Cauchy–Schwarz inequality is satisfied with equality and, hence, the SNR is maximized. More precisely, substituting the optimal phases from (56) in (45) and maximizing it with respect to ϕ_c we get

$$\underset{\phi_c \in [0, 2\pi]}{\text{Maximize}} \quad \frac{P}{\sigma^2} \mathbf{b}^T \left| \mathbf{H}_{\text{BM}} \mathbf{w} e^{j\phi_c} \mathbf{H}_{\text{BR}} \mathbf{\Omega}(\hat{\Theta}) \mathbf{H}_{\text{RM}} \mathbf{w} \right|^2. \quad (57)$$

Again, the number of degrees of freedom is not sufficient for adjusting the phase of the direct and reflected links for all the receiving antennas at the gNB. Hence, ϕ_c can be designed to minimize the difference between the phases of the direct and reflected link, i.e.,

$$\begin{aligned} \underset{\phi_c \in [0, 2\pi]}{\text{Minimize}} \quad \gamma_d(\phi_c) &\triangleq \sum_{r=1}^{N_R} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} \left[(\beta_m - 2\pi f_0 \tau_{bm}) + \right. \\ &\left. - (\phi_c + \hat{\theta}_r + \beta_m - 2\pi f_0(\tau_{br} + \tau_{rm})) \right]^2. \end{aligned} \quad (58)$$

The optimal ϕ_c can be found by solving the following equation

$$\begin{aligned} \frac{\partial \gamma_d(\phi_c)}{\partial \phi_c} &= \sum_{r=1}^{N_R} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} -2 [-2\pi f_0 \tau_{bm} + \beta_m + \\ &- (\phi_c + \hat{\theta}_r + \beta_m - 2\pi f_0(\tau_{br} + \tau_{rm}))] = 0, \end{aligned} \quad (59)$$

leading to

$$\hat{\phi}_c = \frac{2\pi f_0}{N_R N_M N_B} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} \sum_{r=1}^{N_R} (-\tau_{bm} + \tau_{br} + \tau_{rm}), \quad (60)$$

obtained by substituting the derived RIS phases in (56) and distribute the summations.

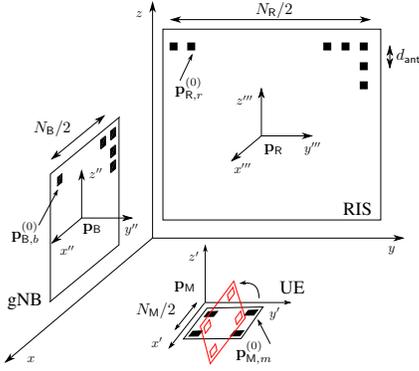


Fig. 3: Considered 3D localization scenario. An example of rotated UE is depicted in red.

Finally, by combining (56) and (60), the designed phases for the RIS that accounts for the direct and reflected paths can be written, for each $k \in \{1, 2, \dots, N_R\}$, as

$$\theta_k^* = \hat{\theta}_k + \hat{\phi}_c = \frac{2\pi f_0}{N_M N_B} \sum_{b=1}^{N_B} \sum_{m=1}^{N_M} (\tau_{bk} + \tau_{km} - \tau_{bm}). \quad (61)$$

V. NUMERICAL RESULTS

A. Simulation Parameters

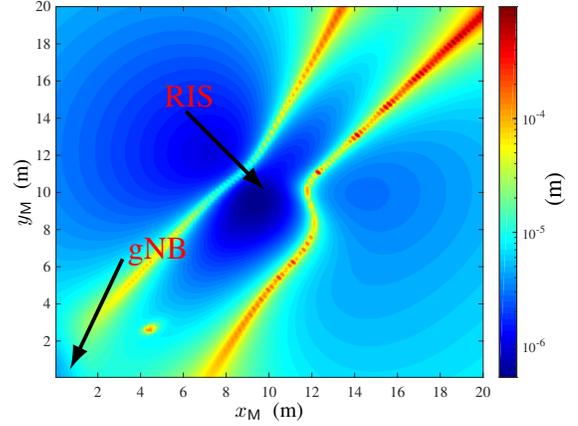
According to the previous analysis, we now evaluate the attainable localization and orientation performance limits for different scenarios. More specifically, we here focus on planar antenna array configuration,³ as they allow compact deployment of massive arrays in gNBs and UE, as well as 3D beam-focusing capabilities [67], [68]. Moreover, in the perspective to place RIS on walls, planar geometry represents a practical solution [1], [30].

The gNB is assumed to be located at the origin, i.e., at $\mathbf{p}_B \triangleq [x_B, y_B, z_B]^T$ (m) = (0, 0, 0), if not otherwise indicated and the initial positions of the antennas (in absence of rotations) can be represented as in Fig. 3 where the gNB, RIS, and UE are lying on the XZ - and YZ -, XY - planes, respectively, and the coordinates of the array elements are given by

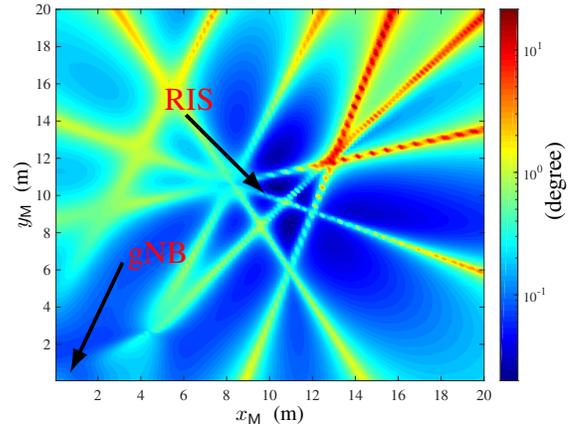
$$\begin{aligned} \mathbf{p}_{B,i}^{(0)} &= d_{\text{ant}} \left[\left[\frac{i}{\sqrt{N_B}} \right], 0, (i \bmod \sqrt{N_B}) \right]^T, i \in \{1, \dots, N_B\}, \\ \mathbf{p}_{R,i}^{(0)} &= d_{\text{ant}} \left[0, \left[\frac{i}{\sqrt{N_R}} \right], (i \bmod \sqrt{N_R}) \right]^T, i \in \{1, \dots, N_R\}, \\ \mathbf{p}_{M,i}^{(0)} &= d_{\text{ant}} \left[\left[\frac{i}{\sqrt{N_M}} \right], (i \bmod \sqrt{N_M}), 0 \right]^T, i \in \{1, \dots, N_M\}, \end{aligned} \quad (62)$$

where mod is the modulo operator, $d_{\text{ant}} = \lambda/2$ is the inter-antenna spacing, and the rotated antenna elements for a given roll, pitch and yaw angles can be defined as in (7). In particular, while the gNB and the RIS are fixed on the XZ - and YZ - planes (i.e., $\alpha_B = \beta_B = \gamma_B = 0$, $\alpha_R = \beta_R = \gamma_R = 0$), respectively, the UE can freely rotate around x -, y -, and z -

³Note that the previous analysis is valid for any geometric configuration, i.e., any antennas spatial deployment and arrays orientation.



(a) PEB



(b) OEB

Fig. 4: (a) PEB in meters and (b) OEB in degrees for different mobile locations in an area of 20×20 m² on the XY -plane. The orientation of the UE is set to $\phi_M = (\pi/6, \pi/6, \pi/6)$.

axis with angles γ_M , β_M , and α_M , respectively, according to Fig. 2.

At the transmitter, we considered OFDM signaling and for each simulation scenario we specify the number N of adopted subcarriers, with unity transmitted power $P = 1$ Watt, a carrier frequency $f_0 = 28$ GHz and a signal bandwidth $B = N \Delta f$, where $\Delta f = 240$ kHz being the subcarrier spacing [69]. At the receiver, we set a noise figure $F = 3$ dB.

Concerning the RIS phase profile in (9), in the next, we use the following *labels* according to the type of design: (i) *Mirror*, when the RIS does not induce any phase shift, that is $\Theta = \mathbf{0}_{1 \times N_R}$; (ii) *Random*, when the RIS phase shifts are uniformly distributed between 0 and 2π ; (iii) *Proposed*, according to the analysis of Sec. IV-2 maximizing the SNR; (iv) *Optimized CRLB*, according to the minimization of the CRLB reported in Sec. IV-1, and; (v) *Quantized*, that accounts for 4 quantization levels in the representation of the optimized CRLB.

B. Numerical Results

a) PEB and OEB for Different Mobile Positions:

Fig. 4 shows the position and orientation errors in RIS-assisted architecture by varying the UE location in different

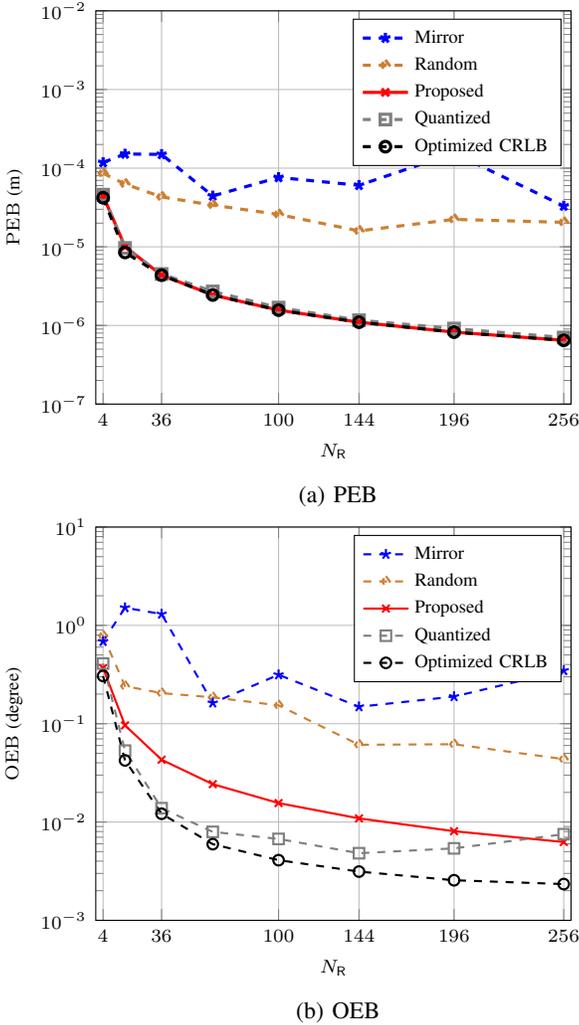


Fig. 5: PEB and OEB vs. number of RIS elements, namely N_R , for different RIS phase design strategies.

points of the area. In particular, the location of the RIS is $\mathbf{p}_R \triangleq [x_R, y_R, z_R]^T$ (m) = (10, 10, -1), whereas the gNB is placed in $\mathbf{p}_B = (0, 0, 0)$, unless stated otherwise. The gNB, RIS, and UE are equipped with planar antenna arrays with $N_B = 36$, $N_R = 100$, $N_M = 4$ antennas, respectively. The UE altitude is set to $z_M = -3$ m, the UE orientation to $\phi_M \triangleq [\alpha_M, \beta_M, \gamma_M]^T$ (rad) = $(\pi/6, \pi/6, \pi/6)$, $N = 1$, and the *proposed* phase design is adopted for the RIS phases. As it can be seen in Fig. 4a and Fig. 4b, the PEB and the OEB are lower in proximity of the gNB and of the RIS, with an error of about 6×10^{-6} m for the position and of 0.2° for the orientation when the UE is placed at $\mathbf{p}_M \triangleq [x_M, y_M, z_M]^T$ (m) = (4, 4, -3). Notably, the achieved errors depend not only on the distance from the gNB and from the RIS, but also on the relative UE location with respect to them, e.g., the UE location has an effect on the actual bearing angles ϕ_{RM} and ϕ_{BM} and, in turns, on the localization.

b) *PEB and OEB for Different RIS Configurations*: Fig. 5 reports the localization and orientation errors for different number of antennas at the RIS and different phase design strategies. We set the number of gNB antennas to $N_B = 16$,

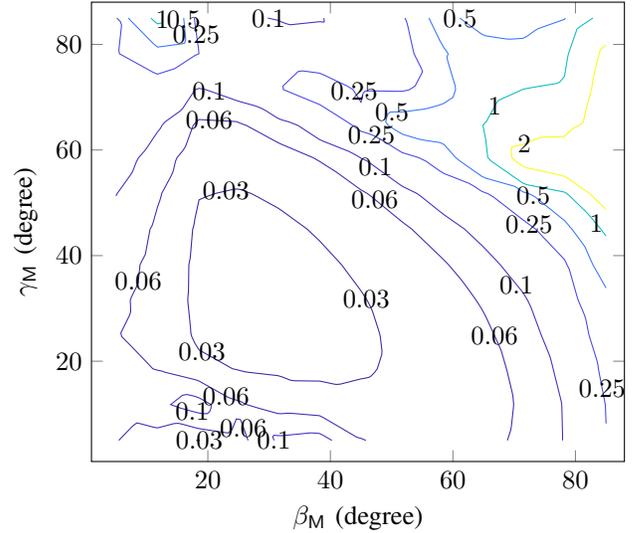


Fig. 6: OEB contours as a function of the orientation angles β_M and γ_M in degrees for a fixed $\alpha_M = 30^\circ$.

the number of UE antennas to $N_M = 4$, the RIS and UE centroids to $\mathbf{p}_R = (4, 3, 1)$ and $\mathbf{p}_M = (5, 2, -1)$, respectively. The UE orientation around the z -axis is $\alpha_M = \pi/6$ and a single sub-carrier is used, i.e. $N = 1$. As previously discussed, the *Optimized CRLB* phase design strategy is obtained by numerically minimizing the PEB and OEB. Because the CRLB optimization problem is non-convex, the algorithm could converge to a local minimum if the initial point is far from the true solution. Therefore, we included the proposed closed-form phase design in (61) as a possible initialization for the optimization algorithm in the *Optimized CRLB* phase design. For the PEB in Fig. 5a, we can see that the *proposed* design almost coincides with the *Optimized CRLB*, and that the quantization does not significantly decrease the performance. Regarding the OEB in Fig. 5b, the *optimized CRLB* and its *quantized* version allow to outperform the *proposed* scheme. Another interesting aspect is that the error tends to slowly decrease for $N_R \geq 100$, thus permitting to relax the number of antennas at the RIS side while obtaining the good localization performance.

c) *Analysis of the UE Orientation*: We now analyze the impact of the mobile orientation angle on the OEB when the location of the mobile and its orientation with respect to the z -axis are fixed, i.e., $\mathbf{p}_M = (15, 5, -3)$ and $\alpha_M = \pi/6$, respectively, while the orientation of the UE around both x - and y - axis (i.e., β_M and γ_M) are varied from 0 to $\pi/2$. The number of antennas at the gNB, UE and RIS are $N_B = 36$, $N_M = 4$, and $N_R = 256$, respectively. The RIS position is $\mathbf{p}_R = (10, 10, -1)$, and the number of sub-carriers is $N = 8$. In this sense, according to the results reported in Fig. 6, we can observe that the OEB increases when the mobile is parallel and/or perpendicular to the RIS and the gNB. On the other hand, the OEB decreases when γ_M and β_M are close to 30° .

d) *Synchronous vs. Asynchronous Signaling*: We now compare the achievable performance of synchronous and asynchronous systems in an environment with and without RIS.

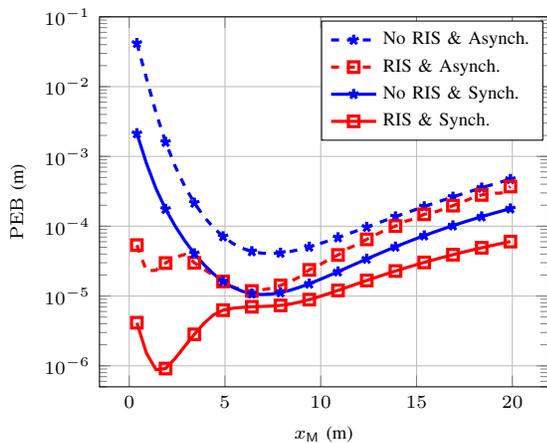
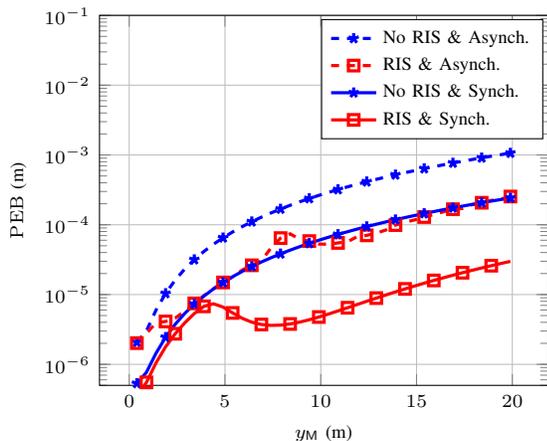
(a) $\mathbf{p}_M = (x_M, 5, 1)$ (b) $\mathbf{p}_M = (5, y_M, 1)$

Fig. 7: PEB for synchronous and asynchronous signal models, for UE orientation fixed to $\phi_M = (\pi/4, \pi/2, 0)$.

More specifically, we evaluated the PEB as a function of the x/y -coordinates of the mobile for a fixed UE orientation, i.e., $\phi_M = (\pi/4, \pi/2, 0)$ (see Fig. 7), and for averaged orientations, i.e., for 36 configurations where both α_M and β_M are varied between 0° and 90° degrees with a step of 15° degree (see Fig. 8). In both configurations, the gNB and the RIS are located at $\mathbf{p}_B = (5, 0, 1.5)$ and $\mathbf{p}_R = (0, 5, 2)$, respectively. The number of antennas and of sub-carriers are $N_B = 36$, $N_R = 64$, $N_M = 4$, and $N = 8$, respectively. In particular, Fig. 7-(a) and Fig. 8-(a) are obtained by fixing $y_M = 5$ m, with x_M spanning from 0 to 20 m, whereas in Fig. 7-(b) and in Fig. 8-(b) we set $x_M = 5$ m and y_M is changed from 0 to 20 m. We can see that the PEB decreases in the area between the gNB and the RIS for the synchronous case, whereas there is not a significant variation for the asynchronous one. Also, it can be noticed that the RIS improves the performance of the localization with up to one- and two- orders of magnitude for the synchronous and asynchronous signaling, respectively. When the orientation is fixed, it is evident that the location of the minimum error coincides with the RIS position for the synchronous case, i.e., $x_R = 5$ m.

The orientation of the mobile impacts the localization per-

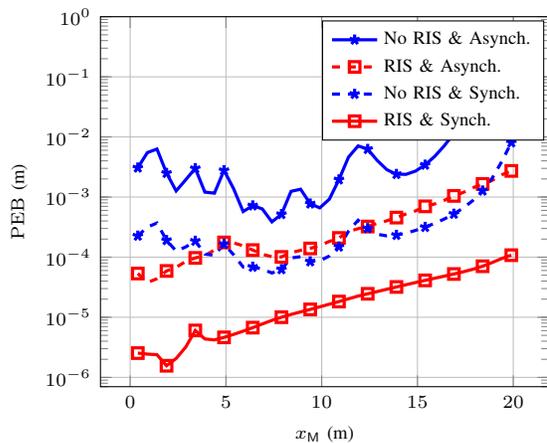
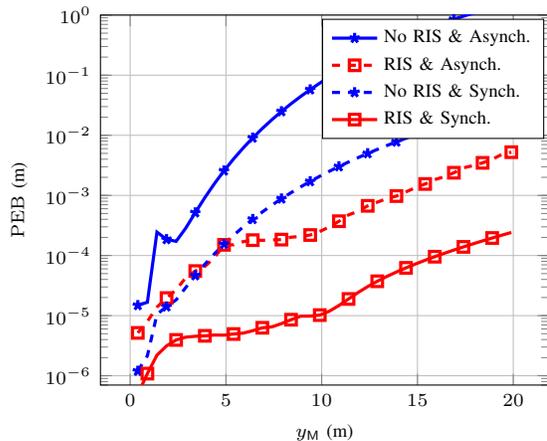
(a) $\mathbf{p}_M = (x_M, 5, 1)$ (b) $\mathbf{p}_M = (5, y_M, 1)$

Fig. 8: PEB for synchronous and asynchronous signal models, averaged for different UE orientation.

formance, as it can be seen in Fig. 8. Hence, we averaged over several mobile orientations to study the effect of the distance on the localization, regardless of the mobile orientation. As expected, when increasing the UE distance from the gNB, e.g., by varying the y -coordinate, the localization error increases faster to what happens by moving along the x -axis (i.e., far from the RIS).

e) Two-stage Localization: In Fig. 9, the localization accuracy is investigated for the case that the system can estimate only a subset of the parameters in (26).

We differentiate between two cases: i) the RSSI and AOA are estimated; ii) the TOAs and the AOA are estimated. Then, the two scenarios are compared with the benchmark, where the system can estimate all the parameters in (26), and the corresponding PEB is calculated as in (28). The PEB for the three cases is depicted in Fig. 9 for various values of mobile locations, along the x -axis. In the considered scenario, we set $\mathbf{p}_M = (x_M, 2, -3)$, $\mathbf{p}_R = (4, 4, -1)$, $N_B = 16$, $N_R = 36$, $N_M = 4$, and $N = 8$. We can see that discarding the RSSIs from the parameter vector (i.e., not relying on measuring the RSSIs for positioning purposes) has negligible impact on the PEB. On the contrary, if the system is able only to estimate the

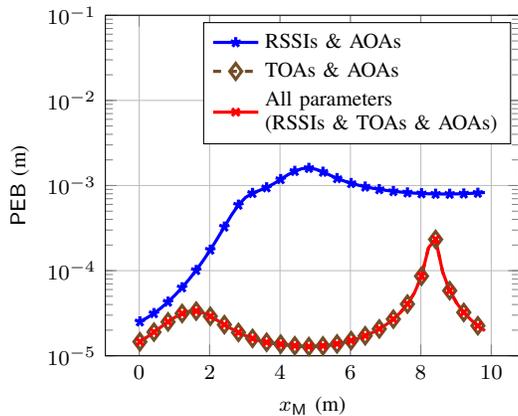
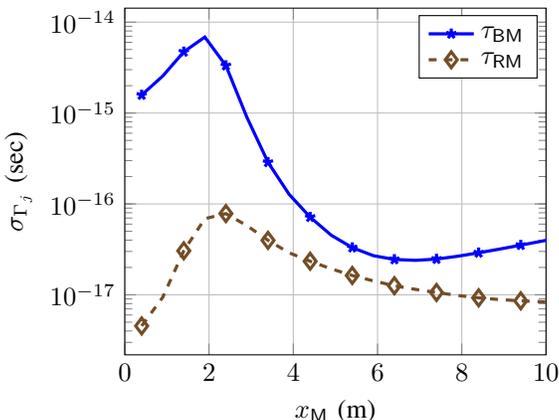
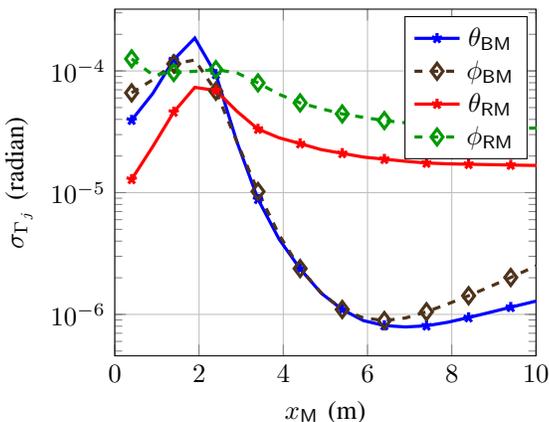


Fig. 9: PEB for different x -coordinates of a UE located in $\mathbf{p}_M = (x_M, 2, -3)$. The RIS is located at $\mathbf{p}_R = (4, 4, -1)$.



(a) Error bounds on the TOA estimation



(b) Error bounds on the angles estimation

Fig. 10: The error in estimating the quantities in the parameters vector vs the x -coordinates of the mobile location, for $\mathbf{p}_B = (5, 0, 1.5)$, $\mathbf{p}_R = (0, 5, 2)$, $\phi_M = (\pi/4, \pi/2, 0)$, $N_B = 36$, $N_R = 64$, $N_M = 4$, and $N = 8$.

RSSI and not the TOAs, the localization error increases up to two order of magnitude. Therefore, in our considered setting, localization systems with accurate TOA estimation can achieve higher performance compared to those relying on RSSIs.

For the two-stage approach in estimating the location, it is beneficial to quantify the minimum possible error for estimating the parameters in (26). To this purpose, the error bound on the parameters can be written as

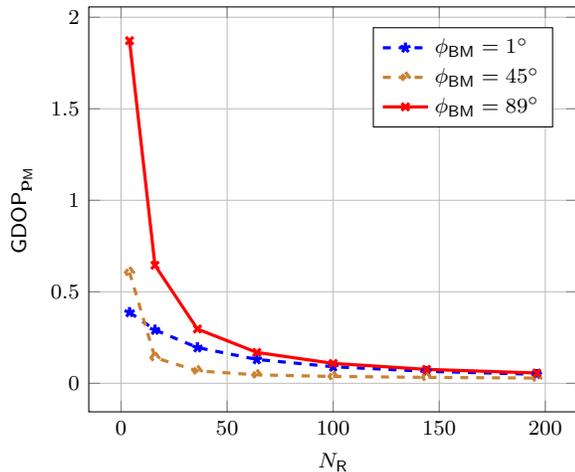
$$\sigma_{\Gamma_j} \triangleq \sqrt{\left[\sum_{n=1}^N \mathbf{I}_n(\Gamma) \right]_{j,j}^{-1}}, \quad \forall j \in \{1, 2, \dots, |\Gamma|\}, \quad (63)$$

where $\mathbf{I}_n(\Gamma)$ is the FIM of the parameters for a given subcarrier n as expressed in (30) and $|\Gamma|$ is the number of parameters in Γ . In this regard, we depict in Fig. 10 the error for estimating the parameters $\theta_{BM}, \phi_{BM}, \tau_{BM}, \theta_{RM}, \phi_{RM}, \tau_{RM}$. In particular, the errors in estimating the time, i.e., $\sigma_{\tau_{BM}}$ and $\sigma_{\tau_{RM}}$, are shown in Fig. 10a, while the standard deviation of the estimation errors of the angles, i.e., $\sigma_{\theta_{BM}}, \sigma_{\phi_{BM}}$ and $\sigma_{\theta_{RM}}, \sigma_{\phi_{RM}}$, are presented in Fig. 10b. We can see that the parameters that depend on the gNB, i.e., $\sigma_{\tau_{BM}}, \sigma_{\theta_{BM}}, \sigma_{\phi_{BM}}$, has minimum estimation errors near the gNB location. On the other hand, the parameters were the RIS is involved increases as UE gets far from the RIS, then decreases again as the it approaches the gNB.

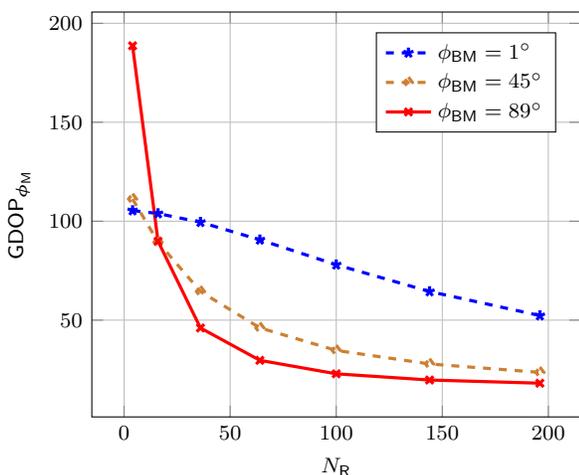
f) Geometric dilution of precision: In Fig. 11, the impact of the geometry on the localization and orientation estimation errors is investigated as a function of the number of elements in the RIS. The GDOP value can be considered as an amplification of the estimation error due to the geometry. Therefore, smaller values of the GDOP indicate a favorable geometry of the mobile with respect to both the gNB and the RIS. The GDOP for the position, $\text{GDOP}_{\mathbf{p}_M}$, is depicted in Fig. 11a as a function of the number of RIS elements for various UE locations with different azimuth angles between the gNB and UE, ϕ_{BM} , while the corresponding distance and the elevation angle are fixed to $d_{BM} = 3$ m and $\theta_{BM} = 30^\circ$, respectively. It can be noticed that increasing the number of RIS elements tends to enhance the geometry of the problem and, thus, it can reduce the positioning error. Also, the GDOP depends strongly on the azimuth angle for small N_R and, consequently, on the UE orientation. The same behavior can be seen in Fig. 11b for the GDOP related to the orientation, i.e., GDOP_{ϕ_M} . The main difference is that it is more harder to estimate the orientation angle with a small number of RIS elements compared to the position estimation. In fact, the GDOP can be interpreted as a mapping from the standard deviation of the PEB and OEB. For example, with $N_R = 4$, we have $\text{PEB} \approx 1.9 d_{BM} \sigma = 5.7 \sigma$, while $\text{OEB} \approx 190 \sigma$, from (41) and (42). The reason is that the orientation estimation relies on the curvature of the wavefront in the near-field. Hence, for a larger number of elements, the effective size of the RIS increases along with the Fraunhofer distance [54].

VI. CONCLUSIONS

In this paper, we propose an architecture for joint communication and UE localization and orientation estimation in a RIS-assisted environment. We derive the ultimate performance in terms of PEB and OEB, accounting for both near- and far-field



(a) The GDOP for the mobile position.



(b) The GDOP for the orientation angles.

Fig. 11: The GDOP for various N_R and ϕ_{BM} , for $N_B = 36$, $N_M = 16$, $\mathbf{p}_R = (4, 3, 1)$, $d_{BM} = 3$ m, $\alpha_M = 30$, and $N = 2400$.

propagation conditions. The RIS phases are designed to maximize the SNR towards the desired UE for both communication and localization enhancement. Indeed, we obtained that the RIS with the proposed phase design can significantly increase the localization performance by focusing the incident spherical wavefront from the UE toward the gNB. The proposed scheme, when compared to a conventional system without RIS, can achieve up to two orders and one order of magnitude reduction in PEB and OEB, respectively. Also, the localization accuracy strongly depends on the considered geometry and the orientation of the UE. The achieved results open the door towards the adoption of RISs as an effective meaning for supporting mobile wireless localization and, thus, to boost the communication performance. A step forward will be the analysis of the localization performance limits in presence of multiple RISs.

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mization problem.

APPENDIX A THE JACOBIAN MATRIX

In this appendix, we report the elements of the Jacobian matrix for the CRLB derivation in (27). The Jacobian matrix of the mobile location is given by (64) in the top of next page where for each $a_M \in \{x_M, y_M, z_M\}$ and $S \in \{B, R\}$, the following relationships holds

$$\nabla_{a_M} \tau_{SM} = \frac{\nabla_{a_M} d_{SM}}{c} = \frac{1}{c} \frac{a_M - a_S}{d_{SM}}, \quad (65)$$

$$\nabla_{a_M} \phi_{SM} = \frac{1}{1 + \left(\frac{y_M - y_S}{x_M - x_S}\right)^2} \nabla_{a_M} \left(\frac{y_M - y_S}{x_M - x_S}\right), \quad (66)$$

$$\nabla_{a_M} \theta_{SM} = \frac{1}{\sqrt{1 - \left(\frac{z_M - z_S}{d_{SM}}\right)^2}} \nabla_{a_M} \left(\frac{z_M - z_S}{d_{SM}}\right), \quad (67)$$

$$\nabla_{a_M} \rho_{BM} = -\frac{\lambda}{4\pi} \frac{1}{(d_{BM})^2} \nabla_{a_M} (d_{BM}), \quad (68)$$

$$\nabla_{a_M} \rho_{BRM} = -\frac{\lambda}{4\pi} \frac{1}{(d_{RM} + d_{BR})^2} \frac{a_M - a_R}{d_{RM}}. \quad (69)$$

After some manipulation, (66)-(67) can be simplified as

$$\begin{cases} \nabla_{x_M} \phi_{SM} = -\frac{1}{d_{SM}} \frac{\sin \phi_{SM}}{\cos \theta_{SM}} \\ \nabla_{y_M} \phi_{SM} = \frac{1}{d_{SM}} \frac{\cos \phi_{SM}}{\cos \theta_{SM}} \\ \nabla_{z_M} \phi_{SM} = 0 \end{cases} \quad \begin{cases} \nabla_{x_M} \theta_{SM} = -\frac{\sin(\theta_{SM}) \cos(\phi_{SM})}{d_{SM}} \\ \nabla_{y_M} \theta_{SM} = -\frac{\sin(\theta_{SM}) \sin(\phi_{SM})}{d_{SM}} \\ \nabla_{z_M} \theta_{SM} = \frac{\cos(\theta_{SM})}{d_{SM}} \end{cases} \quad (70)$$

APPENDIX B FIM ELEMENTS

In order to derive the elements of the FIM in (34), the derivatives of the mean received signal with respect to the parameters, i.e., $\partial \mu_b[n] / \partial \Gamma_j$, should be derived for each $\Gamma_j \in \Gamma$. Let us first rewrite (12) as

$$\mu_b[n] = \mu_{b,BM}[n] + \mu_{b,BRM}[n], \quad (71)$$

with

$$\mu_{b,BM}[n] \triangleq \sqrt{P} \rho_{BM} \sum_{m=1}^{N_M} \mu_{bm}[n], \quad (72)$$

$$\mu_{b,BRM}[n] \triangleq \sqrt{P} \rho_{BRM} \sum_{m=1}^{N_M} \sum_{r=1}^{N_R} \mu_{brm}[n]. \quad (73)$$

The signal inside the summation is

$$\mu_{bm}[n] \triangleq x_m[n] \exp\left(-j2\pi f_n \left(\tilde{\tau}_{bm} + \tilde{\xi}_{BM} + \eta_m\right)\right), \quad (74)$$

$$\begin{aligned} \mu_{brm}[n] &\triangleq x_m[n] \omega_r \\ &\times \exp\left(-j2\pi f_n \left(\tilde{\tau}_{br} + \tilde{\tau}_{rm} + \eta_r + \tilde{\xi}_{BRM} + \eta_m\right)\right), \end{aligned} \quad (75)$$

where we have the following definitions for the synchronous signalling: $\tilde{\xi}_{BM} \triangleq \xi_{BM}$, $\tilde{\xi}_{BRM} \triangleq \xi_{BRM}$, $\tilde{\tau}_{bm} \triangleq \tau_{bm}$, $\tilde{\tau}_{rm} \triangleq \tau_{rm}$, and $\tilde{\tau}_{br} \triangleq \tau_{br}$; whereas for the asynchronous signalling it is: $\tilde{\xi}_{BM} \triangleq \chi_{BM}/2\pi f_n$, $\tilde{\xi}_{BRM} \triangleq \chi_{BRM}/2\pi f_n$, $\tilde{\tau}_{bm} \triangleq \Delta\tau_{bm}$, $\tilde{\tau}_{rm} \triangleq \Delta\tau_{rm}$, and $\tilde{\tau}_{br} \triangleq \Delta\tau_{br}$.

$$\mathbf{J}_{\mathbf{P}_M} = \begin{bmatrix} \nabla_{x_M} \rho_{BM} & \nabla_{x_M} \theta_{BM} & \nabla_{x_M} \phi_{BM} & \nabla_{x_M} \tau_{BM} & \nabla_{x_M} \rho_{BRM} & \nabla_{x_M} \theta_{RM} & \nabla_{x_M} \phi_{RM} & \nabla_{x_M} \tau_{RM} \\ \nabla_{y_M} \rho_{BM} & \nabla_{y_M} \theta_{BM} & \nabla_{y_M} \phi_{BM} & \nabla_{y_M} \tau_{BM} & \nabla_{y_M} \rho_{BRM} & \nabla_{y_M} \theta_{RM} & \nabla_{y_M} \phi_{RM} & \nabla_{y_M} \tau_{RM} \\ \nabla_{z_M} \rho_{BM} & \nabla_{z_M} \theta_{BM} & \nabla_{z_M} \phi_{BM} & \nabla_{z_M} \tau_{BM} & \nabla_{z_M} \rho_{BRM} & \nabla_{z_M} \theta_{RM} & \nabla_{z_M} \phi_{RM} & \nabla_{z_M} \tau_{RM} \end{bmatrix}, \quad (64)$$

Two-stage localization: The derivatives for the indirect approach can be found now as

$$\nabla_{\rho_{BM}} \mu_b[n] = \mu_{b,BM}[n]/\rho_{BM}, \quad \nabla_{\rho_{BRM}} \mu_b[n] = \mu_{b,BRM}[n]/\rho_{BRM},$$

$$\nabla_{\theta_{BM}} \mu_b[n] = -j2\pi f_n \sqrt{P} \rho_{BM} \sum_{m=1}^{N_M} \mu_{bm}[n] \nabla_{\theta_{BM}} \tau_{bm},$$

$$\nabla_{\theta_{RM}} \mu_b[n] = -j2\pi f_n \sqrt{P} \rho_{BRM} \sum_{m=1}^{N_M} \sum_{r=1}^{N_R} \mu_{brm}[n] \nabla_{\theta_{RM}} \tau_{rm},$$

$$\nabla_{\phi_{BM}} \mu_b[n] = -j2\pi f_n \sqrt{P} \rho_{BM} \sum_{m=1}^{N_M} \mu_{bm}[n] \nabla_{\phi_{BM}} \tau_{bm},$$

$$\nabla_{\phi_{RM}} \mu_b[n] = -j2\pi f_n \sqrt{P} \rho_{BRM} \sum_{m=1}^{N_M} \sum_{r=1}^{N_R} \mu_{brm}[n] \nabla_{\phi_{RM}} \tau_{rm},$$

$$\nabla_{\tau_{BM}} \mu_b[n] = -j2\pi f_n \sqrt{P} \rho_{BM} \sum_{m=1}^{N_M} \mu_{bm}[n] \nabla_{\tau_{BM}} \tau_{bm},$$

$$\nabla_{\tau_{RM}} \mu_b[n] = -j2\pi f_n \sqrt{P} \rho_{BRM} \sum_{m=1}^{N_M} \sum_{r=1}^{N_R} \mu_{brm}[n] \nabla_{\tau_{RM}} \tau_{rm},$$

As regards the derivatives with respect to the rotational angles of the mobile, we have $\forall \phi_M \in \{\alpha_M, \beta_M, \gamma_M\}$

$$\begin{aligned} \nabla_{\phi_M} \mu_b[n] = & -j2\pi f_n \sqrt{P} \left(\rho_{BM} \sum_{m=1}^{N_M} \mu_{bm}[n] \nabla_{\phi_M} \tilde{\tau}_{bm} \right. \\ & \left. + \rho_{BRM} \sum_{m=1}^{N_M} \sum_{r=1}^{N_R} \mu_{brm}[n] \nabla_{\phi_M} \tilde{\tau}_{rm} \right). \quad (76) \end{aligned}$$

The derivatives of the TOAs with respect to the parameters can be written for each $S \in \{B, R\}$ and the corresponding antenna index $s \in \{b, r\}$ as

$$\begin{aligned} \nabla_{\theta_{SM}} \tau_{sm} = & \frac{d_{SM}}{c d_{sm}} [(x_m - x_s) \sin \theta_{SM} \cos \phi_{SM} + \\ & + (y_m - y_s) \sin \theta_{SM} \sin \phi_{SM} - (z_m - z_s) \cos \theta_{SM}], \quad (77) \end{aligned}$$

$$\begin{aligned} \nabla_{\phi_{SM}} \tau_{sm} = & \frac{d_{SM}}{c d_{sm}} [(x_m - x_s) \cos \theta_{SM} \sin \phi_{SM} + \\ & - (y_m - y_s) \cos \theta_{SM} \cos \phi_{SM}], \quad (78) \end{aligned}$$

$$\begin{aligned} \nabla_{\phi_M} \tau_{sm} = & \frac{1}{c d_{sm}} [x_m \nabla_{\phi_M} x_m + y_m \nabla_{\phi_M} y_m + z_m \nabla_{\phi_M} z_m \\ & - (x_s \nabla_{\phi_M} x_m + y_s \nabla_{\phi_M} y_m + z_s \nabla_{\phi_M} z_m + \\ & + d_{SM} (\nabla_{\phi_M} x_m \cos \theta_{BM} \cos \phi_{BM} + \\ & + \nabla_{\phi_M} y_m \cos \theta_{BM} \sin \phi_{BM} + \nabla_{\phi_M} z_m \sin \theta_{BM})], \quad (79) \end{aligned}$$

$$\nabla_{\tau_{DM}} \tau_{sm} = \frac{1}{d_{sm}} [d_{SM} - G_{dm}^{(2)}], \quad (80)$$

$$\nabla_{\phi_M} \Delta \tau_{sm} = \nabla_{\phi_M} \tau_{sm}, \quad (81)$$

$$\begin{aligned} \nabla_{\theta_{SM}} \Delta \tau_{sm} &= \nabla_{\theta_{SM}} d_{sm}, \\ \nabla_{\phi_{SM}} \Delta \tau_{sm} &= \nabla_{\phi_{SM}} d_{sm}, \end{aligned} \quad (82)$$

$$\nabla_{\tau_{DM}} \Delta \tau_{sm} = \nabla_{\tau_{DM}} \tau_{sm} - 1, \quad (83)$$

where

$$\begin{aligned} \nabla_{\alpha_M} x_m &= -s_{\alpha_M} c_{\beta_M} x_m^{(0)} + [-s_{\alpha_M} s_{\beta_M} s_{\gamma_M} - c_{\alpha_M} c_{\gamma_M}] y_m^{(0)} \\ &+ [c_{\alpha_M} s_{\gamma_M} - s_{\alpha_M} s_{\beta_M} c_{\gamma_M}] z_m^{(0)}, \\ \nabla_{\beta_M} x_m &= -c_{\alpha_M} s_{\beta_M} x_m^{(0)} + c_{\alpha_M} c_{\beta_M} s_{\gamma_M} y_m^{(0)} + c_{\alpha_M} c_{\beta_M} c_{\gamma_M} z_m^{(0)}, \\ \nabla_{\gamma_M} x_m &= [c_{\alpha_M} s_{\beta_M} c_{\gamma_M} + s_{\alpha_M} s_{\gamma_M}] y_m^{(0)} + \\ &+ [s_{\alpha_M} c_{\gamma_M} - c_{\alpha_M} s_{\beta_M} s_{\gamma_M}] z_m^{(0)}, \\ \nabla_{\alpha_M} y_m &= c_{\alpha_M} c_{\beta_M} x_m^{(0)} + [-c_{\gamma_M} s_{\alpha_M} + c_{\alpha_M} s_{\beta_M} s_{\gamma_M}] y_m^{(0)} + \\ &+ [c_{\gamma_M} c_{\alpha_M} s_{\beta_M} + s_{\alpha_M} s_{\gamma_M}] z_m^{(0)}, \\ \nabla_{\beta_M} y_m &= -s_{\alpha_M} s_{\beta_M} x_m^{(0)} + s_{\alpha_M} c_{\beta_M} s_{\gamma_M} y_m^{(0)} + c_{\gamma_M} s_{\alpha_M} c_{\beta_M} z_m^{(0)}, \\ \nabla_{\gamma_M} y_m &= [-s_{\gamma_M} c_{\alpha_M} + s_{\alpha_M} s_{\beta_M} c_{\gamma_M}] y_m^{(0)} + \\ &- [s_{\gamma_M} s_{\alpha_M} s_{\beta_M} + c_{\alpha_M} c_{\gamma_M}] z_m^{(0)}, \\ \nabla_{\alpha_M} z_m &= 0, \\ \nabla_{\beta_M} z_m &= -c_{\beta_M} x_m^{(0)} - s_{\beta_M} s_{\gamma_M} y_m^{(0)} - s_{\beta_M} c_{\gamma_M} z_m^{(0)}, \\ \nabla_{\gamma_M} z_m &= c_{\beta_M} c_{\gamma_M} y_m^{(0)} - c_{\beta_M} s_{\gamma_M} z_m^{(0)}, \end{aligned}$$

where $c_{x_M} \triangleq \cos(x_M)$ and $s_{x_M} \triangleq \sin(x_M)$.

Direct localization: When a direct localization approach is used, the signal can be rewritten as

$$\mu_b = \sum_{m=1}^{N_M} \left(f_{bm}(\rho_{BM}, d_{bm}) + \sum_{r=1}^{N_R} g_{brm}(\rho_{BRM}, d_{rm}) \right), \quad (84)$$

where f_{bm} and g_{brm} are two non-linear functions depending on the parameters to be estimated,⁴ defined as⁵

$$f_{bm}(\rho_{BM}, d_{bm}) \triangleq \sqrt{P} x_m[n] \rho_{BM}(\mathbf{P}_M) \exp\left(-j2\pi \frac{f_n}{c} \tilde{d}_{bm}\right), \quad (85)$$

where $\tilde{d}_{bm} = \tilde{d}_{bm}(\mathbf{P}_M, \phi_M) \triangleq d_{bm}$ and $\tilde{d}_{bm} \triangleq \Delta d_{bm}$ in synchronous and asynchronous cases, respectively, and

$$\begin{aligned} g_{brm}(\rho_{BRM}, d_{rm}) &\triangleq \sqrt{P} x_m[n] \rho_{BRM}(\mathbf{P}_M) \omega_r \cdot \\ &\times \exp\left(-j2\pi \frac{f_n}{c} (\tilde{d}_{br} + \tilde{d}_{rm})\right), \quad (86) \end{aligned}$$

where $\tilde{d}_{rm} = \tilde{d}_{rm}(\mathbf{P}_M, \phi_M) \triangleq d_{rm}$ and $\tilde{d}_{rm} \triangleq \Delta d_{rm}$ in synchronous scheme; whereas $\tilde{d}_{br} = d_{br}$ and $\tilde{d}_{br} = \Delta d_{br}$ in asynchronous scheme.

⁴Generally, the optimal design of RIS phases can depend on the UE location and orientation. For convenience, such a dependence is neglected in (84).

⁵Here we have dropped the synchronization mismatches and array errors.

The gradient vector with respect to the position, \mathbf{p}_M , and orientation, ϕ_M , can be written from (84) as

$$\begin{aligned}\nabla_{\mathbf{p}_M}(\mu_b) &= [\nabla_{x_M} \mu_b, \nabla_{y_M} \mu_b, \nabla_{z_M} \mu_b] = \\ &= \sum_{m=1}^{N_M} \nabla_{\mathbf{p}_M} f_{bm} + \sum_{r=1}^{N_R} \nabla_{\mathbf{p}_M} g_{brm},\end{aligned}\quad (87)$$

$$\begin{aligned}\nabla_{\phi_M}(\mu_b) &= [\nabla_{\alpha_M} \mu_b, \nabla_{\beta_M} \mu_b, \nabla_{\gamma_M} \mu_b] = \\ &= \sum_{m=1}^{N_M} \nabla_{\phi_M} f_{bm} + \sum_{r=1}^{N_R} \nabla_{\phi_M} g_{brm},\end{aligned}\quad (88)$$

where for the direct path we have

$$\begin{aligned}\nabla_{\mathbf{p}_M} f_{bm} &= \sqrt{P} x_m [\nabla_{d_{BM}} \rho_{BM} \nabla_{\mathbf{p}_M} d_{BM} + \\ &\quad - j 2 \pi \frac{f_n}{c} \rho_{BM} \nabla_{\mathbf{p}_M} \tilde{d}_{bm}] e^{-j 2 \pi \frac{f_n}{c} \tilde{d}_{bm}}, \\ \nabla_{\phi_M} f_{bm} &= -j 2 \pi \frac{f_n}{c} \sqrt{P} \rho_{BM} x_m e^{-j 2 \pi \frac{f_n}{c} \tilde{d}_{bm}},\end{aligned}\quad (89)$$

while for the RIS-relayed path it is

$$\begin{aligned}\nabla_{\mathbf{p}_M} g_{brm} &= \sqrt{P} x_m \omega_r [\nabla_{d_{rM}} \rho_{BRM} \nabla_{\mathbf{p}_M} d_{rM} + \\ &\quad - j 2 \pi \frac{f_n}{c} \rho_{BRM} \nabla_{\mathbf{p}_M} \tilde{d}_{rM}] e^{-j 2 \pi \frac{f_n}{c} (\tilde{d}_{rM} + \tilde{d}_{br})}, \\ \nabla_{\phi_M} g_{brm} &= -j 2 \pi \frac{f_n}{c} \sqrt{P} \rho_{BRM} x_m \omega_r e^{-j 2 \pi \frac{f_n}{c} (\tilde{d}_{rM} + \tilde{d}_{br})} \\ &\quad \times \nabla_{\phi_M} \tilde{d}_{rM},\end{aligned}\quad (90)$$

The derivatives of the path-loss amplitudes with respect to the distances between array centers are

$$\nabla_{d_{BM}} \rho_{BM} = -\frac{\lambda}{4\pi d_{BM}^2}, \quad \nabla_{d_{RM}} \rho_{BRM} = -\frac{\lambda}{4\pi(d_{BR} + d_{RM})^2}.\quad (91)$$

By denoting with $a_M \in \{x_M, y_M, z_M\}$, $S \in \{B, R\}$ and $s \in \{b, r\}$, we can obtain

$$\begin{aligned}\nabla_{a_M} \Delta d_{sm} &= \nabla_{a_M} d_{sm} - \nabla_{a_M} d_{SM}, \\ \nabla_{a_M} d_{sm} &= \\ &= \frac{1}{2 d_{sm}} \nabla_{a_M} (d_m^2 + d_s^2 + d_{SM}^2 - 2 (G_{sm}^{(1)} + d_{SM} G_{sm}^{(2)})), \\ &= \frac{1}{d_{sm}} (d_{SM} \nabla_{a_M} d_{SM} - \nabla_{a_M} d_{SM} G_{sm}^{(2)} - d_{SM} \nabla_{a_M} G_{sm}^{(2)}),\end{aligned}\quad (93)$$

where $\nabla_{a_M} d_{SM}(a_M) = \frac{a_M - a_S}{d_{SM}}$ and where

$$\begin{aligned}\nabla_{a_M} G_{sm}^{(2)} &= -(x_m - x_s) \sin(\theta_{SM}) \cos(\phi_{SM}) \nabla_{a_M} \theta_{SM} \\ &\quad - (x_m - x_s) \cos(\theta_{SM}) \sin(\phi_{SM}) \nabla_{a_M} \phi_{SM} \\ &\quad - (y_m - y_s) \sin(\theta_{SM}) \sin(\phi_{SM}) \nabla_{a_M} \theta_{SM} \\ &\quad + (y_m - y_s) \cos(\theta_{SM}) \cos(\phi_{SM}) \nabla_{a_M} \phi_{SM} \\ &\quad + (z_m - z_s) \cos(\theta_{SM}) \nabla_{a_M} \theta_{SM},\end{aligned}\quad (94)$$

with $\nabla_{a_M} \phi_{SM}$ and $\nabla_{a_M} \theta_{SM}$ as in Appendix A. Thus, the derivatives in (94) becomes

$$\begin{aligned}\nabla_{x_M} G_{sm}^{(2)} &= \frac{x_m - x_s}{d_{SM}} [\sin^2 \theta_{SM} \cos^2 \phi_{SM} + \sin^2 \phi_{SM}] \\ &\quad + \frac{y_m - y_s}{d_{SM}} [\sin^2 \theta_{SM} \cos \phi_{SM} \sin \phi_{SM} - \sin \phi_{SM} \cos \phi_{SM}] \\ &\quad - \frac{z_m - z_s}{d_{SM}} \sin \theta_{SM} \cos \phi_{SM} \cos \theta_{SM},\end{aligned}\quad (95)$$

$$\begin{aligned}\nabla_{y_M} G_{sm}^{(2)} &= \frac{x_m - x_s}{d_{SM}} [\sin^2 \theta_{SM} \sin \phi_{SM} \cos \phi_{SM} \\ &\quad - \sin \phi_{SM} \cos \phi_{SM}] + \frac{y_m - y_s}{d_{SM}} [\sin^2 \theta_{SM} \sin^2 \phi_{SM} \\ &\quad + \cos^2 \phi_{SM}] - \frac{z_m - z_s}{d_{SM}} [\sin \theta_{SM} \cos \theta_{SM} \sin \phi_{SM}],\end{aligned}\quad (96)$$

$$\begin{aligned}\nabla_{z_M} G_{sm}^{(2)} &= -\frac{x_m - x_s}{d_{BM}} \sin \theta_{SM} \cos \theta_{SM} \cos \phi_{SM} \\ &\quad + \frac{y_m - y_s}{d_{SM}} [\sin \theta_{SM} \cos \theta_{SM} \sin \phi_{SM}] \\ &\quad + \frac{z_m - z_s}{d_{SM}} \cos^2 \theta_{SM}.\end{aligned}\quad (97)$$

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