1. Introduction

Non-premixed turbulent jet flames are encountered in a variety of practical scenarios including furnaces, industrial flares, and pressurized fuel tanks in case of a leak. For design purposes, predicting the flame height/length is desirable. Therefore, predictive models have been developed (e.g., [1-8]). Their validity depends whether the flames are buoyancy-controlled, momentum-controlled, or in the buoyancy-momentum transition and is a function of the nozzle geometry [3, 6, 9, 10].

Although introduced three decades ago, a current benchmark model for circular vertical turbulent jet flames that are momentum-controlled or in the buoyancy-momentum transition is that of Delichatsios [2]. In this semi-empirical model, the normalized flame height $H^*$ is predicted by casting a flame Froude Number $Fr_f$ into Eq. (1):

$$H^* = \frac{13.5 Fr_f^{2/5}}{(1+0.07 Fr_f^2)^{1/5}} \quad \text{with} \quad Fr_f^2 = \frac{U^2}{T_{ad} - T_0 (1+S)^3 g D (\rho_f/\rho_o)^{1/2}}$$

where $U$ is the bulk jet velocity, $S$ is the air to fuel stoichiometric mass fraction, $g$ is the gravitational acceleration, $D$ is the nozzle diameter, $T_0$ and $T_{ad}$ are the ambient and adiabatic flame temperatures, and $\rho_f$ and $\rho_o$ are the fuel and ambient air densities. Constants 13.5 and 0.07 were derived empirically
using data of [1]. The temperature rise \( T_{ad} - T_0 \) is here simplified [11] and can be inferred from equilibrium. Equation (1) does not account for non-unity combustion efficiency and radiative heat-losses.

The flame height \( h \) is obtained with Eq. (2):

\[
H^* = \frac{h}{(1+S)D(\frac{\rho_f}{\rho_o})^{1/2}}
\]  

In principle, Delichatsios’ model is valid only for turbulent flames that are momentum-controlled or in the buoyancy-momentum transition if \( T_{ad}/T_0 \sim 5 \) and \( S > 8 \). It was shown to accurately predict flame heights [1, 2, 12-14] at atmospheric pressure for different fuels, a range of flame Froude numbers \( (10^{-2} \leq Fr_f \leq 10^1) \), and bulk jet exit Reynolds numbers up to \( \sim 50,000 \). It was also validated at a reduced pressure of 64 kPa [13], but a corrected flame Froude number was used to account for the impact of Burke-Schumann diffusion on transport at this reduced pressure. However, Delichatsios’ model performed poorly for DME [5] and highly under-expanded hydrogen jet flames [4]. Regardless, the simplicity and large range of applicability of Delichatsios’ model show that it is a useful engineering tool, and it is important to refine the boundaries of its applicability.

Delichatsios’ model was never challenged for pressures above atmospheric. Also, due to lift-off, height measurements are difficult to achieve at atmospheric pressure for attached jet flames with bulk Reynolds numbers above \( \sim 50,000 \). Increasing pressure can relax this limitation [15]. Increasing pressure also allows reducing the flame Froude number at the turbulent-to-laminar transition so that lower flame Froude numbers may be examined.
The objective here is to challenge Delichatsios’ model for subsonic jet flames at elevated pressure (1 to 13 bar) for large ranges of bulk Reynolds numbers (3,100 < Re < 123,000) and flame Froude numbers (10^-2 < Fr_f < 40).

2. Experimental setup and methods

Experiments were conducted at KAUST in the High-Pressure Combustion Duct (HPCD) [16]. A schematic of the experimental setup is provided in supplementary materials. The HPCD accommodates long (> 2 m) vertically-oriented flames without flame-wall interactions. It was operated between 1 and 13 bar. Fuel was supplied by either one of two straight 0.57-m long stainless-steel tubes with inner diameters of 3.4 or 4.6 mm. Pure fuels were investigated, namely hydrogen, methane, and ethane, as well as blends of hydrogen-nitrogen and carbon monoxide-hydrogen-nitrogen. Operating conditions are detailed in Table 1. All flames were subsonic and attached to the nozzle. Mass-flow rates were regulated with an accuracy better than 1 % (Brooks SLA). Because the HPCD is air-cooled, an air co-flow was needed. The air velocity $U_c$ always satisfied $U_c/U < 0.05$ and can be neglected. The burner was mounted on a vertical translation stage so that the flame tip was visible through a HPCD windows.

Flame heights were measured with an intensified CCD camera (Princeton Instruments PI-MAX4) following the procedure of [10, 13]. For each condition, 600 images were recorded, binarized, and averaged to yield the probability $p$ to find a flame at each location. The flame height was defined as the height above the nozzle, along the axis, where $p = 0.5$. This probability and the threshold used for image binarization are arbitrary and 50 possible combinations were tested. The associated scatter will be highlighted as error bars. For non-sooting syngas flames, a bandpass filter centered at 309 nm was used to target OH*. For sooting flames and hydrogen flames, no filter was used. This is not expected to influence the results.
Table 1: Operating conditions.

<table>
<thead>
<tr>
<th>Fuel (% vol)</th>
<th>P [bar]</th>
<th>U [m/s]</th>
<th>D [mm]</th>
<th>Re</th>
<th>S</th>
<th>(T_{ad}/T_0)</th>
<th>Fr_f</th>
<th>H*</th>
<th>H [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH₄</td>
<td>1–13</td>
<td>1.0–13.0</td>
<td>4.6</td>
<td>3,500</td>
<td>17.1</td>
<td>~7.7</td>
<td>0.02–0.4</td>
<td>3.4–8.4</td>
<td>209–519</td>
</tr>
<tr>
<td>C₂H₆</td>
<td>1–12</td>
<td>0.43–5.2</td>
<td>4.6</td>
<td>3,100</td>
<td>16.0</td>
<td>~7.9</td>
<td>0.02–0.1</td>
<td>1.7–6.0</td>
<td>134–476</td>
</tr>
<tr>
<td>H₂</td>
<td>6–12</td>
<td>9.5–19.0</td>
<td>3.4</td>
<td>3,500</td>
<td>34.1</td>
<td>~8.4</td>
<td>0.2–0.4</td>
<td>6.8–9.3</td>
<td>210–287</td>
</tr>
<tr>
<td>75H₂:25N₂</td>
<td>1–12</td>
<td>3.5–41.6</td>
<td>3.4</td>
<td>3,500</td>
<td>6.1</td>
<td>~7.8</td>
<td>0.6–6.4</td>
<td>10–21</td>
<td>131–265</td>
</tr>
<tr>
<td>50H₂:50N₂</td>
<td>1–6</td>
<td>8.8–52.7</td>
<td>3.4</td>
<td>7,000-12,000</td>
<td>2.3</td>
<td>~6.9</td>
<td>4.3–40</td>
<td>16–18</td>
<td>126–144</td>
</tr>
<tr>
<td>49CO:36H₂:15N₂</td>
<td>2.4–12</td>
<td>4.1–76.8</td>
<td>4.6</td>
<td>10,000-94,000</td>
<td>3.2</td>
<td>~8.1</td>
<td>1–18</td>
<td>16–21</td>
<td>218–325</td>
</tr>
<tr>
<td>40CO:30H₂:30N₂</td>
<td>5.1–12</td>
<td>30.6–72.0</td>
<td>4.6</td>
<td>80,000-123,000</td>
<td>2.4</td>
<td>~7.8</td>
<td>9.7–23</td>
<td>20–21</td>
<td>257–270</td>
</tr>
<tr>
<td>29CO:21H₂:50N₂</td>
<td>1.9–12</td>
<td>3.5–50.6</td>
<td>4.6</td>
<td>10,000-101,000</td>
<td>1.5</td>
<td>~7.2</td>
<td>1.7–23</td>
<td>16–20</td>
<td>168–208</td>
</tr>
</tbody>
</table>

3. Results and discussion

Figure 1a shows the measured normalized flame heights as a function of the flame Froude number for conditions of Table 1. Note that pressure is not featured explicitly in Delichatsios’ model. If minute effects of pressure on the adiabatic flame temperature are neglected, pressure only modifies the Flame Froude number and, in turn, the flame height through the bulk jet velocity. Therefore, increasing pressure with a constant bulk jet velocity is not expected to modify flame Froude number or flame height. Also, because flame Froude Number and bulk jet velocity are proportional, increasing pressure with a constant Reynolds number decreases the flame Froude Number. Overall, Delichatsios’ correlation (solid black line) agrees well with the data herein and the plateauing for Fr_f > 5 is well captured. Consistent with Eq. (1), increasing pressure does not modify the height of momentum-controlled flames (Fr_f > 5). But increasing pressure with a constant Reynolds number decreases the height of buoyant flames or flames in the buoyancy-momentum transition (Fr_f < 1).

Some discrepancies between measured data and Delichatsios’ model are visible for ethane if Fr_f < 0.02 (P > 6 bar). This is perhaps because Delichatsios’ model was not designed for very buoyant jet flames or because radiative heat-losses should not be neglected. For momentum-controlled flames (Fr_f > 5), Delichatsios’ correlation slightly overestimates the flame heights for H₂:N₂ and CO:H₂:N₂. This was...
previously observed at atmospheric pressure for H$_2$ [17, 18] and CO:H$_2$ [18]. In the momentum-controlled region, Delichatsios [2] used only four fairly scattered data points from [1] to tune the two empirical constants of Eq. (1). Therefore, it is likely that these constants could be refined. This is what Hwang et al. [18] did with their H$_2$ and CO:H$_2$ data. However, they adjusted the constant that influences both buoyancy- and momentum-controlled regions (13.5 became 11.3 in Eq. (1)) even though their data did not extend to the buoyancy-controlled region. This is not optimal because agreement with Kalghatgi’s data [1, 2] as well as with data herein deteriorates in the buoyancy-controlled region. Instead, it is proposed here to update the other empirical constant in Eq. (1), from 0.07 to 0.11, which only affects the momentum-controlled region. The dashed red line in Fig. 1a shows that this update improves predictions for H$_2$:N$_2$ and CO:H$_2$:N$_2$ blends. Some disagreement still exists for 50H$_2$:50N$_2$ and 29CO:21H$_2$:50N$_2$. Interestingly, these are the two blends featuring the smallest air to fuel stoichiometric mass fractions (S = 2.3 and 1.5), well below the limit recommended by Delichatsios, S = 8. Blends featuring S > 2.3 are well predicted by the updated Delichatsios correlation.

Figure 1b shows normalized flame heights from the literature as a function of the flame Froude number for different fuels. The updated correlation preforms well as it predicts the vast majority of the data with a ±20% confidence. Some fraction of the data scatter is attributed to experimental uncertainties and, therefore, no correlation can perfectly collapse data from many different sources. Note that the original Delichatsios’ correlation also performs well, provided that H$_2$:N$_2$ and CO:H$_2$:N$_2$ blends featuring too small air to fuel stoichiometric mass fractions (S < 3) are excluded. Also note that, different from [13] with data recorded at 64 kPa for Fr$_f$ < 1, Delichatsios’ correlation does not need to be corrected for Burke-Schumann diffusion at elevated pressure. This is because, in the flames of interest here, transport is controlled by buoyancy and turbulence at atmospheric and elevated pressures [19].
4. Conclusions

The data herein and that from the literature show that Delichatsios’ model predicts well (within ±20%) the height of subsonic jet flames for different fuels, Reynolds numbers up to Re = 123,000, and pressures up to 13 bar as long as Fr_f > 0.02 and S > 6. Therefore, Delichatsios’ model is a valid engineering tool. However, it is advised to adjust one of the two empirical constants in Eq. (1), from 0.07 to 0.11, to improve predictions in the momentum-controlled region for H_2:N_2 and CO:H_2:N_2 blends featuring relatively small air to fuel stoichiometric mass fractions.

Declaration of Competing Interest

None.

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References


Fig. 1: Normalized flame heights measured in the HPCD (a) or from the literature (b) as a function of the flame Froude number. * For 64 kPa in [13], a corrected Froude number was used.