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CDF-Based Multiuser Scheduling for Downlink Non-Orthogonal Multiple Access (NOMA)

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ABSTRACT In this paper, we propose cumulative distribution function (CDF)-based multiuser scheduling (CMS) for downlink non-orthogonal multiple access (NOMA) to accommodate a massive number of IoT devices over limited radio resources and statistically analyze the ergodic capacity of devices. With the proposed CMS algorithm, multiple devices having relatively better channel gain to exploit multiuser diversity are selected while meeting QoS requirements by controlling the amount of resources. We first provide the user selection probability in terms of the weight of each device, which determines the amount of time resources. Based on some selected results, it is shown that equal weights ensure fair resource allocation. Further, for fair resource allocation scenario, the distribution of signal-to-interference-plus-noise ratio (SINR) when a specific device is selected is derived. With this derived statistical result of SINR, we analyze the ergodic capacity of the specific device. Some selected results show that the proposed CMS algorithm allocates the time resource more fairly than proportional fair scheduling. Further, the average throughput and amount of resources can be controlled by adjusting the weights of each device, so different QoS requirements can be guaranteed.

INDEX TERMS NOMA, scheduling, CDF, fair resource allocation, ergodic capacity.

I. INTRODUCTION

Machine type communications (MTC) as one of the scenarios for 5G networks have received tremendous attention in recent years and have regarded as one of the technologies to realize the internet of things (IoT) [1]. The main challenge of MTC is to provide massive connectivity to tens of billion of low-cost and low-power machine type devices. In current cellular networks, however, there are not enough resources to accommodate a massive number of devices for the orthogonal resource allocation. To achieve massive connectivity which is a key feature of IoT and better spectrum efficiency, non-orthogonal multiple access (NOMA) has been considered as a promising radio access technique [2]–[5]. By contrast with orthogonal multiple access (OMA) schemes such as time division multiple access (TDMA) and orthogonal frequency division multiple access (OFDMA), NOMA serves multiple users who are simultaneously sharing the same time and frequency resources by exploiting power domain [6]. With these attractive features, NOMA is the potential multiple access technology which enhances the connectivity by allowing a massive number of devices to simultaneously transmit in the same radio resources [7].

Although non-orthogonal resource allocation can support a large number of devices, the number of supported devices is limited due to practical issues of NOMA such as complexity of successive interference cancellation (SIC). Thus, all the devices in networks cannot be simultaneously supported and the quality of services (QoS) of all the devices may not be satisfied. Accordingly, multiuser scheduling plays crucial role to guarantee different QoS requirements and connect a large number of devices over the limited radio resources. Furthermore, NOMA has significant gain over OMA in asymmetric channel [8], [9] but its unbalanced channel qualities lead to challenging problem of multiuser scheduling due to different throughput of each link [10]. Therefore, a multiuser scheduling policy that affects the system throughput, fairness and connectivity is an important issue and the appropriate multiuser scheduling needs to be designed for NOMA.

A. RELATED WORKS AND MOTIVATIONS

Several scheduling schemes have been proposed considering both fairness and system throughput [11]–[14]. These works adopt throughput-based fairness scheduling, which maximizes the system throughput while satisfying its own fairness criteria. In particular, proportional fairness (PF) scheduling is well known to achieve a good trade-off between fairness and system throughput, which maximizes the product of the user throughput [14]. The PF metric is widely used for user selection and power allocation in NOMA systems [15]–[18]. In NOMA with PF scheduling, the optimal user set can be found by selecting one of all the possible combinations of user sets whose PF metric is the highest (i.e., an exhaustive search) but this method is very complex [19]. Thus, it is difficult to apply PF scheduling to massive connectivity scenario. Also, devices have diverse service requirements and it leads to different QoS requirements of each IoT device [7]. Although each device requires different throughput owing to different QoS requirements, PF scheduling allocates the most resources to users at cell edge resulting in degradation of the throughput of other users. Therefore, PF may not guarantee the throughput requirement of each user since it is not able to control the amount of resources allocated to each user. Furthermore, because of the high complexity of PF, it is not suitable as a multiuser scheduling for the IoT.

Resource-based fairness scheduling can allocate the required amount of resources to each user [20]–[25]. However, the normalized signal-to-noise ratio (SNR)-based scheduling in [20] does not provide fair resource allocation under different SNR distribution while the round-robin scheduling in [21] does not exploit the multiuser diversity. To resolve these shortcomings, the cumulative distribution function (CDF)-based scheduling (CS) algorithm was proposed to provide fair resource allocation and exploit multiuser diversity [26]. The CS algorithm assigns a time slot to the user that has relatively better channel gain. It has attractive properties in that the exact average rate of each user can be obtained independently and the time fraction and average rate of each user can be controlled unlike the other scheduling schemes. Thus, CS algorithm guarantees the different QoS requirements by adjusting the amount of time slots allocated to IoT devices. However, the conventional CS algorithm selects only one user in each time slot, which cannot be directly applied to NOMA and is not suitable for massive connectivity. Therefore, a multiuser scheduling while satisfying the different QoS requirements is needed and to the best our knowledge, the effect of scheduling scheme has not been well studied for NOMA.

B. CONTRIBUTIONS

In this paper, we propose the CDF-based multiuser scheduling (CMS) algorithm to provide fair resource allocation in two-user NOMA systems, which can support massive connectivity guaranteeing QoS requirements and reducing the complexity compared with PF scheduling. We derive the user selection probability in each time slot to control the amount of time fraction of each user. It is shown that fair resource allocation can be achieved by the equal weight case. For that case, we analyze the distribution of signal-to-interference-plus-noise ratio (SINR) and ergodic capacity of each user. Our contributions are summarized as follows:

1) To support massive connectivity, we propose CDF-based multiuser scheduling (CMS) scheme for downlink NOMA that can select the multiple devices in each time slot based on the channel gain CDF. Furthermore, it exploits multiuser diversity and offers flexible selection by adjusting the weight.
2) The user selection probability is analyzed in terms of the weight of each device when two devices are selected. It can control the amount of resources by adjusting the weight, which affects the outage and ergodic capacity. Therefore, different QoS requirements of each device can be satisfied by adjusting the weight. Also, we show that it guarantees fair resource allocation for the equal weight of each device.
3) The SINR distribution is derived when a specific device is selected using CMS algorithm. It is shown that high SINR can be achieved compared with other scheduling schemes since the devices with relatively better channel gain are selected.
4) Previous works analyze the ergodic capacity or outage probability of the user with the k-th maximum channel gain using order statistics but they did not show the performance of a specific user located in the cell. To show the performance of a specific user, we analyze the ergodic capacity of the k-th device far from the BS for the equal weight case.
5) We present the simulation results to evaluate the proposed scheduling schemes. It shows that system throughput or fairness in terms of throughput can be improved depending on the weight allocation method. Therefore, CMS algorithm can flexibly select the user set in terms of system throughput and fairness by adjusting the weight.

C. ORGANIZATION

The rest of paper is organized as follows. Section II presents the NOMA system model. In Section III, we propose the CMS algorithm for NOMA and analyze the user selection probability. In Section IV, we analyze the distribution of SINR and the ergodic capacity. We show some selected simulation and numerical results to demonstrate the performance of the proposed CMS algorithm in Section V and conclusion is provided in Section VI.

II. SYSTEM MODEL

We consider cellular downlink communication with one BS and K IoT devices and the frequency bandwidth is divided into M sub-bands [27], [28]. Each sub-band supports multiple devices simultaneously sharing the same bandwidth and time slot with different power levels as illustrated in Fig. 1. Suppose that only two devices are multiplexed with different

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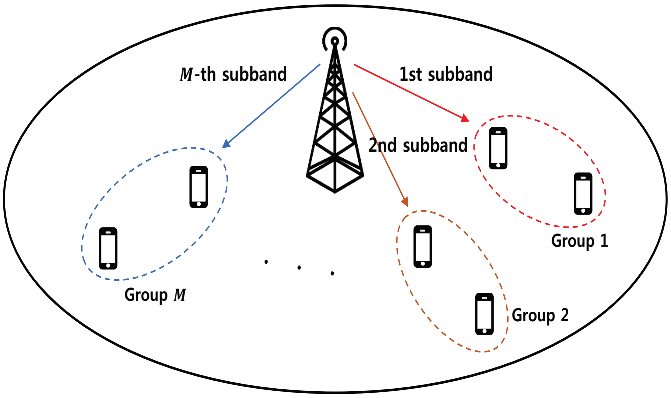

FIGURE 1: Multiple access systems combining NOMA with conventional FDMA.

power in each sub-band owing to the complexity of SIC and practical issues [29]. Thus, two devices are paired in each sub-band for NOMA and the total number of devices connected by BS is $2M$ ($K \geq 2M$) to support as many devices as possible for massive connectivity. Also, we consider frequency flat Rayleigh fading environment and equal power allocation over sub-bands denoted by P_t. Then, the received signal of the i-th device when the i-th and j-th devices are paired for NOMA is given by

$$y_i = \sqrt{P_i} h_i x_i + \sqrt{P_j} h_i x_j + n_i, \tag{1}$$

where P_i is the power allocated to the i-th device with $P_i + P_j \leq P_t$, x_i is the transmit signal to the i-th device with $E[|x_i|^2] = 1$, n_i is AWGN with $n_i \sim \mathcal{CN}(0, N_0)$, and h_i is the channel coefficient between BS and the i-th device. In particular, h_i consists of large and small scale fading components given by $h_i = \omega d_i^{-\beta} g_i$ where ω is a constant related to the carrier frequency and antenna gain, d_i is the distance between BS and the i-th device, β is the path loss exponent, and g_i is an independent and identically distributed (i.i.d) complex Gaussian random variable (RV) denoted by $g_i \sim \mathcal{CN}(0, 1)$. Without loss of generality, $\sigma_1^2 \geq \sigma_2^2 \geq \cdots \geq \sigma_K^2$ is assumed where $\sigma_i^2 = E[|h_i|^2]$ since each device is randomly deployed and thus it has different distance, d_i. To detect the signal of the i-th device, the channel gains of the i-th and j-th devices are first compared. If $|h_i|^2 \leq |h_j|^2$ (i.e., $P_i \geq P_j$), the signal of the i-th device can be directly decoded regarding the signal of the j-th device as interference. Conversely, in the case of $|h_i|^2 \geq |h_j|^2$, the i-th device employs SIC to eliminate the interference caused by the j-th device. Thus, the signal of the j-th device is first decoded and then the decoded data is subtracted from the received signal, y_i, before the signal of the i-th device is decoded. Assuming perfect SIC for each device, the instantaneous rate of the i-th device can be written as

$$R_i = \begin{cases} \log_2\left(1 + \frac{|h_i|^2 P_i}{N_0 + |h_i|^2 P_j}\right), & \text{for } |h_i|^2 \leq |h_j|^2 \\ \log_2\left(1 + \frac{|h_i|^2 P_i}{N_0}\right), & \text{for } |h_i|^2 \geq |h_j|^2. \end{cases} \tag{2}$$

In (2), the instantaneous rate is affected by the power allocation scheme. For instance, the fixed power allocation (FPA) scheme assigns a constant power to each user regardless of their channel condition. Then, the rate of each user is only affected by its own channel quality. Fractional transmit power allocation (FTPA) dynamically adjusts the transmit power depending on the channel gain of the paired users [30]. Thus, users having the lower channel gain tend to get paired with users having the higher channel gain to increase the data rate. Therefore, it is important which users are selected in the cell and paired in the selected users. In the following section, we propose CDF-based multiuser scheduling method, which guarantees fairness user selection.

III. CDF-BASED MULTIUSER SCHEDULING FOR NOMA

Conventional proposed CS selects only one user with the highest CDF value in each time slot. Since NOMA supports multiple users simultaneously in each time slot, the conventional CS algorithm cannot be applied directly to the NOMA scenario.

For the FPA scheme, the instantaneous rate is only affected by the user's own channel gain. To increase system throughput, users with the higher channel gain need to be selected but in this case, the QoS of users at cell edge cannot be guaranteed. Thus, multiple user selection algorithm guaranteeing fair selection is needed while increasing the system throughput. CS algorithm can provide fairness in user selection, by selecting the users having relatively better channel gain.

Let $F_i(\cdot)$ denote the channel gain CDF of the i-th user. In the channel estimation phase, BS transmits training or pilot signals to estimate the channel of each device. Then, a massive number of static devices for IoT can periodically send the estimated channel information to BS. Thus, each device can estimate the channel information and feed it back to BS. Therefore, the channel gain CDF can be obtained from long-term observations by BS. Conventional CS selects a single user in time slot t given by

$$i^*(t) = \arg \max_{i=1,\cdots,K} F_i(\alpha_i(t))^{\frac{1}{w_i}}, \tag{3}$$

where $\alpha_i = |h_i|^2$ is the instantaneous channel gain, $w_i \geq 0$ denotes the weight of the i-th user, which determines that user's channel access ratio. It selects only a single user having relatively better channel gain in each time slot. Thus, some modification is needed to select multiple users for NOMA. Now, we propose CMS scheme for IoT that guarantees fair resource allocation among K devices . The BS selects the user set whose channel gain CDF is the highest, which implies the user set having relatively better channel condition. Then, the scheduled user set that maximizes channel gain CDF is given by

$$\mathcal{S}^*(t) = \arg \max_{\mathcal{S}} \sum_{i \in \mathcal{S}} F_i(\alpha_i(t))^{\frac{1}{w_i}}, \tag{4}$$

where $\mathcal{S}$ is the candidate user set with size $2M$. The user set that maximizes the sum of the CDFs can be obtained by finding the users having first $2M$ maximum CDF value.[1] In other words, if the users' CDFs are ordered as $F_1(\alpha_1(t))^{\frac{1}{w_1}} \geq \cdots \geq F_K(\alpha_K(t))^{\frac{1}{w_K}}$, the user set that satisfies the condition of (4) is $\mathcal{S}^* = \{1, 2, \ldots, 2M\}$. In addition, the proposed CMS scheme can be extended to MIMO systems when substituting the instantaneous channel gain in (4) into MIMO effective channel gain as $\alpha_{n,i}(t) = |\mathbf{v}_{n,i}\mathbf{H}_{n,i}\mathbf{p}_n|^2$ where $\mathbf{v}_{n,i}$ and $\mathbf{H}_{n,i}$ are the combining vector and channel matrix of the i-th user in the n-th cluster, respectively, and $\mathbf{p}_n$ is the precoding vector for the n-th cluster [31].

For the simple case of only two selected devices ($M = 1$), the user selection probability of the i-th user can be written as

$$\begin{aligned} &\Pr\left(\text{the } i\text{-th user is selected}\right) \\ &= \Pr\left(F_i(\alpha_i)^{\frac{1}{w_i}} \text{ is the 1st maximum}\right) \\ &\quad + \Pr\left(F_i(\alpha_i)^{\frac{1}{w_i}} \text{ is the 2nd maximum}\right). \end{aligned} \tag{5}$$

Note that the CDF of an arbitrary RV has a uniform distribution with $[0, 1]$. Thus, $\Pr\left(F_i(\alpha_i)^{\frac{1}{w_i}} \text{ is the 1st maximum}\right)$ in (5) can be calculated as [24]

$$\begin{aligned} &\Pr\left(F_i(\alpha_i)^{\frac{1}{w_i}} \text{ is the 1st maximum}\right) \\ &= \Pr\left(F_k(\alpha_k) < F_i(\alpha_i)^{\frac{w_k}{w_i}} \text{ for } k = 1, \ldots i-1, i+1, \ldots, K\right) \\ &= \int_0^1 \prod_{\substack{j=1 \\ j\neq i}}^{K} \Pr\left(U_j < u_i^{\frac{w_j}{w_i}}\right) f_{U_i}(u_i) du_i = \frac{w_i}{\sum_{j=1}^{K} w_j}. \end{aligned} \tag{6}$$

where $U_i = F_i(\alpha_i)$ is the uniform RV. The second term on the right-hand side in (5) can be also calculated as

$$\begin{aligned} &\Pr\left(F_i(\alpha_i)^{\frac{1}{w_i}} \text{ is the 2nd maximum}\right) \\ &= \int_0^1 \sum_{\substack{k=1 \\ k\neq i}}^{K} \prod_{\substack{j=1 \\ j\neq i,k}}^{K} \Pr\left(U_j < u_i^{\frac{w_j}{w_i}}\right) \Pr\left(U_k > u_i^{\frac{w_k}{w_i}}\right) du_i \\ &= \int_0^1 \sum_{\substack{k=1 \\ k\neq i}}^{K} u_i^{\sum_{j=1,\ j\neq i,k}^{K} \frac{w_j}{w_i}} \left(1 - u_i^{\frac{w_k}{w_i}}\right) du_i \\ &= \sum_{\substack{k=1 \\ k\neq i}}^{K} \left(\frac{w_i}{\sum_{\substack{j=1 \\ j\neq k}}^{K} w_j} - \frac{w_i}{\sum_{j=1}^{K} w_j} \right). \end{aligned} \tag{7}$$

Therefore, the selection probability of the i-th user can be obtained by adding the results of (6) and (7). Note that for the case of $w_1 = w_2 = \cdots = w_K$, the results of (6) and (7) are reduced to $\frac{1}{K}$, respectively. Thus, the user selection probability for the case of $M = 1$ and equal weights is given as

$$\Pr\left(\text{the } i\text{-th user is selected}\right) = \frac{2}{K}. \tag{8}$$

[1] We note that CMS scheme can be applied to correlated fading channels since CDF value is independently calculated using the channel gain of each user.

We here point out that fair user selection can be achieved by equal weights in the multiple users selection scenario. If w_i for any i is the same, all the users have the same time fraction. With out loss of generality, we assume $w_i = 1$ for $i = 1, 2, \ldots, K$ to guarantee fair resource allocation. In this case, the selection probability of the i-th user in each time slot can be represented as

$$\begin{aligned} &\Pr\left(\text{the } i\text{-th user is selected}\right) \\ &= \sum_{k=1}^{2M} \Pr\left(F_i(\alpha_i) \text{ is the } k\text{-th maximum}\right). \end{aligned} \tag{9}$$

Furthermore, the probability that $F_i(\alpha_i)$ is the k-th maximum in (9) can be calculated as

$$\begin{aligned} &\Pr\left(F_i(\alpha_i) \text{ is the } k\text{-th maximum}\right) \\ &= \Pr\left(U_a \leq U_i \leq U_b \text{ for } a \in \mathcal{S}_{K-k}, b \in \mathcal{S}_{k-1}\right) \\ &= \int_0^1 \binom{K-1}{k-1} \prod_{\substack{a\in\mathcal{S}_{K-k} \\ b\in\mathcal{S}_{k-1}}} \Pr(U_a \leq u_i) \Pr(U_b \geq u_i) f_{U_i}(u_i) du_i \\ &= \int_0^1 \binom{K-1}{k-1} u_i^{K-k}(1-u_i)^{k-1} du_i \\ &= \binom{K-1}{k-1} \mathrm{B}(K-k+1, k) \\ &= \frac{(K-1)!}{(k-1)!(K-k)!} \frac{(K-k)!(k-1)!}{K!} = \frac{1}{K}, \end{aligned} \tag{10}$$

where $\mathcal{S}_{K-k}$ and $\mathcal{S}_{k-1}$ are the user sets with size $K-k$ and $k-1$, respectively, and $\mathcal{S}_{K-k} \cup \mathcal{S}_{k-1} = \{1, 2, \ldots, i-1, i+1, \ldots, K\}$. Also, in the derivation of (10), $\mathrm{B}(x, y)$ is the Beta function defined by $\mathrm{B}(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} = \frac{(x-1)!(y-1)!}{(x+y-1)!}$ [32]. Therefore, the probability of the i-th user being selected can be written as

$$\Pr(\text{the } i\text{-th user is selected}) = \frac{2M}{K}. \tag{11}$$

This implies that each device has equal probability being selected in each time slot when the weights of each user are all the same. Note that To investigate the average throughput of the selected users, we analyze their ergodic capacity in the following section.

IV. ANALYSIS OF ERGODIC CAPACITY

Previous works commonly derived the outage probability or ergodic capacity using order statistics [33], [34]. Although the performance of the user with the i-th maximum channel gain was analyzed, the analysis did not show the performance of a specific user located in the cell. We here analyze the ergodic capacity of the specific device located in the i-th far from the BS.

A. SINR DISTRIBUTION

To derive the ergodic capacity of the i-th user, we first analyze the SINR distribution of the i-th user when the i-th user is selected in the current time slot by CMS [35]. We assume that the allocated power is constant (i.e., FPA). In other words, P_s

$$E\left[\log_2(1+\gamma_i)\mid \text{the } i\text{-th user is selected}\right]=\frac{K}{2M\ln 2}\sum_{k=1}^{2M}\sum_{n=0}^{k-1}\sum_{m=1}^{K-k+n+1}\binom{K-1}{k-1}\binom{k-1}{n}\binom{K-k+n+1}{m}(-1)^{n+m+1}\frac{y(m,\sigma_i^2)}{K-k+n+1}. \tag{22}$$

$$y(m,\sigma_i^2)=\begin{cases} e^{\frac{m}{\sigma_i^2\rho_s}}\operatorname{Ei}\left(-\frac{m}{\sigma_i^2\rho_s}\right)-e^{\frac{m}{\sigma_i^2(\rho_s+\rho_w)}}\operatorname{Ei}\left(-\frac{m}{\sigma_i^2(\rho_s+\rho_w)}\right), & \text{for } |h_i|^2\le|h_j|^2\\ -e^{\frac{m}{\sigma_i^2\rho_s}}\operatorname{Ei}\left(-\frac{m}{\sigma_i^2\rho_s}\right), & \text{for } |h_i|^2\ge|h_j|^2. \end{cases} \tag{23}$$

and P_w are always allocated to the strong and weak devices, respectively, with $P_s \le P_w$. Then, the SINR of the i-th user when the i-th user is selected can be written as

$$\gamma_i=\begin{cases}\frac{|h_i|^2\rho_w}{1+|h_i|^2\rho_s}, & \text{for } |h_i|^2\le|h_j|^2\\ |h_i|^2\rho_s, & \text{for } |h_i|^2\ge|h_j|^2,\end{cases} \tag{12}$$

where $\rho_s=\frac{P_s}{N_0}$ and $\rho_w=\frac{P_w}{N_0}$ are the transmit SNR for the strong and weak users, respectively. Furthermore, the CDF of the SINR of the i-th user can be expressed as

$$\begin{aligned}F_{\gamma_i}(\gamma)&=\Pr\left(\gamma_i\le\gamma\mid \text{the } i\text{-th user is selected}\right)\\ &=\frac{\Pr\left(\gamma_i\le\gamma,\ \text{the } i\text{-th user is selected}\right)}{\Pr\left(\text{the } i\text{-th user is selected}\right)}\\ &=\frac{K}{2M}\Pr\left(\gamma_i\le\gamma, \text{the } i\text{-th user is selected}\right)\\ &=\frac{K}{2M}\sum_{k=1}^{2M}\Pr\left(\gamma_i\le\gamma, F_i(\alpha_i) \text{ is the } k\text{-th maximum}\right).\end{aligned} \tag{13}$$

In the case of $|h_i|^2\le|h_j|^2$, $\gamma_i\le\gamma$ in (13) can be rewritten as $\alpha_i\le\frac{\gamma}{\rho_w-\gamma\rho_s}$ since $\gamma_i=\frac{\alpha_i\rho_w}{1+\alpha_i\rho_s}$. Thus, we can obtain the joint probability in (13) as

$$\begin{aligned}&\Pr\left(\gamma_i\le\gamma, F_i(\alpha_i) \text{ is the } k\text{-th maximum}\right)\\ &=\Pr\left(\alpha_i\le\frac{\gamma}{\rho_w-\gamma\rho_s},\ U_a\le F_i(\alpha_i)\le U_b\right)\\ &=\int_0^{\frac{\gamma}{\rho_w-\gamma\rho_s}}\binom{K-1}{k-1}F_i(\alpha_i)^{K-k}(1-F_i(\alpha_i))^{k-1}dF_i(\alpha_i)\\ &=\binom{K-1}{k-1}\int_0^{F_i\left(\frac{\gamma}{\rho_w-\gamma\rho_s}\right)}x^{K-k}(1-x)^{k-1}dx\\ &=\binom{K-1}{k-1}\mathrm{B}\left(F_i\left(\frac{\gamma}{\rho_w-\gamma\rho_s}\right);K-k+1,k\right),\end{aligned} \tag{14}$$

where $\mathrm{B}(x;a,b)\triangleq\int_0^x t^{a-1}(1-t)^{b-1}dt$ is the incomplete beta function.

In the case of $|h_i|^2\ge|h_j|^2$, $\gamma_i\le\gamma$ in (13) can be rewritten as $\alpha_i\le\frac{\gamma}{\rho_s}$. Thus, we can obtain the joint probability in (13) for $|h_i|^2\ge|h_j|^2$ as

$$\begin{aligned}&\Pr\left(\gamma_i\le\gamma, F_i(\alpha_i) \text{ is the } k\text{-th maximum}\right)\\ &=\binom{K-1}{k-1}\mathrm{B}\left(F_i\left(\frac{\gamma}{\rho_s}\right);K-k+1,k\right).\end{aligned} \tag{15}$$

Therefore, the CDF of the SINR of the i-th user when the i-th user is selected is given by

$$F_{\gamma_i}(\gamma)=\frac{K}{2M}\sum_{k=1}^{2M}\binom{K-1}{k-1}\mathrm{B}\left(F_i\left(g(\gamma)\right);K-k+1,k\right), \tag{16}$$

where

$$g(\gamma)=\begin{cases}\frac{\gamma}{\rho_w-\gamma\rho_s}, & \text{for } |h_i|^2\le|h_j|^2\\ \frac{\gamma}{\rho_s}, & \text{for } |h_i|^2\ge|h_j|^2.\end{cases} \tag{17}$$

Using (16), we can find the PDF of SINR as

$$\begin{aligned}f_{\gamma_i}(\gamma)&=\frac{dF_{\gamma_i}(\gamma)}{d\gamma}\\ &=\frac{K}{2M}\sum_{k=1}^{2M}\binom{K-1}{k-1}F_i\left(g(\gamma)\right)^{K-k}\left(1-F_i\left(g(\gamma)\right)\right)^{k-1}f_i\left(g(\gamma)\right)\frac{dg(\gamma)}{d\gamma}.\end{aligned} \tag{18}$$

B. CONDITIONAL ERGODIC CAPACITY

In previous subsection, we obtained the CDF and the PDF of the SINR of the i-th user when the i-th user is selected. Note that the i-th user is not selected in the current time slot, the instantaneous rate of the i-th device is zero. Thus, the ergodic capacity can be expressed as

$$\begin{aligned}E\left[\log_2(1+\gamma_i)\right]=&E\left[\log_2(1+\gamma_i)\mid \text{the } i\text{-th user is selected}\right]\\ &\times\Pr(\text{the } i\text{-th user is selected}).\end{aligned} \tag{19}$$

In (19), the user selection probability is obtained from (11). Let us first derive the conditional ergodic capacity, which can be obtained by

$$\begin{aligned}&E\left[\log_2(1+\gamma_i)\mid \text{the } i\text{-th user is selected}\right]\\ &=\int_0^\infty\log_2(1+\gamma)f_{\gamma_i}(\gamma)d\gamma\\ &=\frac{1}{\ln 2}\int_0^\infty\frac{1-F_{\gamma_i}(\gamma)}{1+\gamma}d\gamma.\end{aligned} \tag{20}$$

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Using the derived CDF of SINR in (16), we can calculate (20) as

$$\begin{aligned}
&\frac{1}{\ln 2}\int_0^\infty \frac{1-F_{\gamma_i}(\gamma)}{1+\gamma}d\gamma \\
&=\frac{K}{2M\ln 2}\sum_{k=1}^{2M}\binom{K-1}{k-1} \\
&\quad\times\int_0^\infty \frac{\mathrm{B}\left(K-k+1,k\right)-\mathrm{B}\left(F_i\left(g(\gamma)\right);K-k+1,k\right)}{1+\gamma}d\gamma \\
&=\frac{K}{2M\ln 2}\sum_{k=1}^{2M}\binom{K-1}{k-1} \\
&\quad\times\int_0^\infty \frac{1}{1+\gamma}\int_{F_i(g(\gamma))}^{1} t^{K-k}(1-t)^{k-1}dtd\gamma. \qquad (21)
\end{aligned}$$

Using (21), we can obtain the conditional ergodic capacity as (22). In (22), $y(m,\sigma_i^2)$ is given as (23) where $\mathrm{Ei}(x) = -\int_{-x}^{\infty}\frac{e^{-t}}{t}dt$ is the exponential integral function [32]. The detailed derivation of (22) is given in Appendix A.

In (22), the conditional ergodic capacity can be classified into two cases. As a result, it can be written as

$$\begin{aligned}
&E\left[\log_2(1+\gamma_i)\mid \text{the } i\text{-th user is selected}\right] \\
&=\sum_{\substack{j=1\\ j\neq i}}^{K}\Bigg(E\left[\log_2(1+\gamma_i)\mid \text{the } i\text{-th user is selected},\ |h_i|^2\le|h_j|^2\right] \\
&\quad\times\Pr\left(|h_i|^2\le|h_j|^2,\ (i,j)\text{ pair}\mid \text{the } i\text{-th user is selected}\right) \\
&\quad+E\left[\log_2(1+\gamma_i)\mid \text{the } i\text{-th user is selected},\ |h_i|^2\ge|h_j|^2\right] \\
&\quad\times\Pr\left(|h_i|^2\ge|h_j|^2,\ (i,j)\text{ pair}\mid \text{the } i\text{-th user is selected}\right)\Bigg). \qquad (24)
\end{aligned}$$

Note that in (24), the (i,j) pair means that the i-th and j-th users are selected and paired for NOMA. Then, we need to calculate the conditional probability of $|h_i|^2 \le |h_j|^2$ and $|h_i|^2 \ge |h_j|^2$ when the i-th user is selected to obtain the ergodic capacity.

C. CONDITIONAL PROBABILITY OF $|h_i|^2 \ge |h_j|^2$ AND $|h_i|^2 \le |h_j|^2$

First, we consider the case of $|h_i|^2 \ge |h_j|^2$. In (24), the (i,j) pair means that the i-th and j-th users are selected at the same time and are paired out of the selected $2M$ devices. Accordingly, $F_i(\alpha_i)$ and $F_j(\alpha_j)$ need to be the first $2M$ maximum and the event of the (i,j) pairing when the i-th and j-th users are selected is one of the possible ways of dividing $2M$ devices into M groups with size 2 when considering random pairing among the selected users. Then, we can write the conditional probability of $|h_i|^2 \ge |h_j|^2$ with (i,j) pair when the i-th user is selected as

$$\begin{aligned}
&\Pr\left(|h_i|^2\ge|h_j|^2,\ (i,j)\text{ pair}\mid \text{the } i\text{-th user is selected}\right) \\
&=\frac{K}{2M}\Pr\left(|h_i|^2\ge|h_j|^2,\ (i,j)\text{ pair},\ U_a\le F_i(\alpha_i),F_j(\alpha_j)\right), \qquad (25)
\end{aligned}$$

where $U_a \in S_{K-k}$ for $2 \le k \le 2M$. In (25), the conditional probability can be classified into two cases such that $F_i(\alpha_i) \le F_j(\alpha_j)$ and $F_i(\alpha_i) \ge F_j(\alpha_j)$. Therefore, the probability in (25) can be rewritten as

$$\begin{aligned}
&\Pr\left(|h_i|^2\ge|h_j|^2,\ (i,j)\text{ pair},\ U_a\le F_i(\alpha_i),F_j(\alpha_j)\right) \\
&=\Pr\left(|h_i|^2\ge|h_j|^2,\ (i,j)\text{ pair},\ U_a\le F_i(\alpha_i)\le F_j(\alpha_j)\right) \\
&\quad+\Pr\left(|h_i|^2\ge|h_j|^2,\ (i,j)\text{ pair},\ U_a\le F_j(\alpha_j)\le F_i(\alpha_i)\right). \qquad (26)
\end{aligned}$$

The closed form expression of (25) is given in (27) where

$$\phi(\sigma_i^2,\sigma_j^2,k,n)=\begin{cases}1-\frac{\sigma_i^2}{(k+n-1)\sigma_j^2+\sigma_i^2}-\frac{2}{k+n} & \text{for } \sigma_i^2\ge\sigma_j^2\\ 1-\frac{\sigma_j^2}{(k+n-1)\sigma_i^2+\sigma_j^2} & \text{for } \sigma_i^2\le\sigma_j^2.\end{cases} \qquad (28)$$

We provide the detailed derivation of (26) in Appendix B.

In the case of $|h_i|^2 \le |h_j|^2$, the joint probability, in a similar way to (25), can be also classified into two cases as in (26). Using $\Pr\left(|h_i|^2 \le |h_j|^2 \mid X\right) = 1 - \Pr\left(|h_i|^2 \ge |h_j|^2 \mid X\right)$ for any event X, the joint probability for $|h_i|^2 \le |h_j|^2$ can be represented as

$$\begin{aligned}
&\Pr\left(|h_i|^2\le|h_j|^2,\ (i,j)\text{ pair},\ U_a\le F_i(\alpha_i),F_j(\alpha_j)\right) \\
&=\Pr\left(\alpha_i\le\alpha_j\mid (i,j)\text{ pair},\ U_a\le F_i(\alpha_i)\le F_j(\alpha_j)\right) \\
&\quad\times\Pr\left((i,j)\text{ pair},\ U_a\le F_i(\alpha_i)\le F_j(\alpha_j)\right) \\
&\quad+\Pr\left(\alpha_i\le\alpha_j\mid (i,j)\text{ pair},\ U_a\le F_j(\alpha_j)\le F_i(\alpha_i)\right) \\
&\quad\times\Pr\left((i,j)\text{ pair},\ U_a\le F_j(\alpha_j)\le F_i(\alpha_i)\right) \\
&=\Pr\left((i,j)\text{ pair},\ U_a\le F_i(\alpha_i),F_j(\alpha_j)\right) \\
&\quad-\Pr\left(\alpha_i\ge\alpha_j,\ (i,j)\text{ pair},\ U_a\le F_i(\alpha_i),F_j(\alpha_j)\right). \qquad (29)
\end{aligned}$$

Since we obtained the second term on the right-hand side as in (27), we only need to calculate the first term on the right-hand side in (29) given as

$$\begin{aligned}
&\Pr\left((i,j)\text{ pair},\ U_a\le F_i(\alpha_i),F_j(\alpha_j)\right) \\
&=\Pr\left((i,j)\text{ pair},\ U_a\le F_i(\alpha_i)\le F_j(\alpha_j)\right) \\
&\quad+\Pr\left((i,j)\text{ pair},\ U_a\le F_j(\alpha_j)\le F_i(\alpha_i)\right) \\
&=\frac{(2!)^M M!}{2M!}\sum_{k=2}^{2M}\binom{K-2}{K-k}\Bigg(\int_0^\infty\int_0^{\frac{\sigma_i^2}{\sigma_j^2}\alpha_j}F_i(\alpha_i)^{K-k} \\
&\quad\times(1-F_i(\alpha_i))^{k-2}f_{\alpha_i}(\alpha_i)f_{\alpha_j}(\alpha_j)d\alpha_i d\alpha_j \\
&\quad+\int_0^\infty\int_0^{\frac{\sigma_j^2}{\sigma_i^2}\alpha_i}F_j(\alpha_j)^{K-k}(1-F_j(\alpha_j))^{k-2}f_{\alpha_i}(\alpha_i) \\
&\quad\times f_{\alpha_j}(\alpha_j)d\alpha_j d\alpha_i\Bigg) \\
&=\frac{(2!)^M M!}{2M!}\sum_{k=2}^{2M}\sum_{n=0}^{K-k}\binom{K-2}{K-k}\binom{K-k}{n}(-1)^n\frac{2}{k+n-1} \\
&\quad\times\left(1-\frac{1}{k+n}\right). \qquad (30)
\end{aligned}$$

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$$\Pr\left(|h_i|^2 \geq |h_j|^2,\ (i,j) \text{ pair} \mid \text{the } i\text{-th user is selected}\right) = \frac{K2^M(M!)}{2M(2M!)}\sum_{k=2}^{2M}\sum_{n=0}^{K-k}\binom{K-2}{K-k}\binom{K-k}{n}(-1)^n\frac{1}{k+n-1}\phi(\sigma_i^2,\sigma_j^2,k,n). \tag{27}$$

$$\Pr\left(|h_i|^2 \leq |h_j|^2,\ (i,j) \text{ pair} \mid \text{the } i\text{-th user is selected}\right) = \frac{K2^M(M!)}{2M(2M!)}\sum_{k=2}^{2M}\sum_{n=0}^{K-k}\binom{K-2}{K-k}\binom{K-k}{n}(-1)^n\frac{1}{k+n-1}\psi(\sigma_i^2,\sigma_j^2,k,n). \tag{31}$$

Therefore, the conditional probability of $|h_i|^2 \leq |h_j|^2$ can be written as (31) where

$$\psi(\sigma_i^2,\sigma_j^2,k,n) = 2 - \frac{2}{k+n} - \phi(\sigma_i^2,\sigma_j^2,k,n) \tag{32}$$

As a result, we obtain the ergodic capacity in (19) when the i-th user is selected.

V. SIMULATION AND NUMERICAL RESULTS

In this section, we present the performance of the proposed CMS algorithm for downlink NOMA systems. In our simulation, the total number of users in a cell is 10 ($K = 10$) and the total bandwidth is 20MHz which is divided into two sub-bands ($M = 2$). In other words, four out of ten users are selected to perform NOMA and two users are randomly paired in each sub-band. Also, we assume that the total transmit power is $P_t = 30$dBm as an example and the noise spectral density is -174dBm/Hz. The i-th user in a cell is located with distance $d_i = 0.05 \times i$ km for $i = 1, \ldots, 10$ away from the BS. For the channel model, the path loss model is given by $10\log_{10}(\omega d_i^{-\beta}) = 128.1 + 37.6\log_{10}(d_i)$ and small scale fading is assumed to be i.i.d frequency flat Rayleigh fading. We investigate the performance of CMS algorithm using the two power allocation methods of FPA and FTPA, which are simple fixed and dynamic power allocation schemes, respectively. For FPA, we allocate the power to the strong and weak users as $P_s = 0.25P_t$ and $P_w = 0.75P_t$, respectively. For FTPA, the ratio of power allocated to the i-th and j-th users is $P_i = \frac{\alpha_j}{\alpha_i+\alpha_j}P_t$ and $P_j = \frac{\alpha_i}{\alpha_i+\alpha_j}P_t$, respectively.

Fig. 2 shows the SINR distribution of users at cell center and cell edge and we compare our proposed CMS algorithm with PF and random selection scheduling schemes. We can see that the SINR of random selection is lower than that of the other scheduling schemes because it does not consider the channel condition. By contrast, CMS algorithm can achieve high SINR since it selects the users having relatively better channel gain. In particular, the SINR of user at cell edge can be significantly increased when using CMS algorithm, which implies that it can guarantee better QoS for the user at cell edge than the other schemes. Also, we confirm that our analytical SINR distribution results match the simulation results.

Fig. 3 represents the user selection probability and average rate of CMS algorithm compared with PF and random selection. In Fig. 3-(a), the selection probability of each user under CMS and random selection is all the same, which implies fair resource allocation. For FPA, the maximum rate of the weak user is limited to $\log_2(1+\frac{P_w}{P_s})$ owing to constant power allocation. The users at cell edge are often being the weak user when paired with another user due to low channel gain and their rates are limited. Therefore, when using PF scheduling, the user at cell edge is more frequently selected than other users since their average rate is lower than that of the other users. In Fig. 3-(b), we can see that the average rate of CMS is greater than that of random selection while both schemes guarantee fair resource allocation. Given that CMS selects the users with relatively better channel quality based on CDF, its average rate is greater than that of random selection. Although, CMS algorithm achieves high SINR of user at cell edge as in Fig. 2-(b), the average rate of user at cell edge when using PF is greater than that of the other schemes since most of resources are allocated to user at cell edge. To compare fairness with respect to throughput, Jain's fairness index is widely used as $J = \left(\sum_{k=1}^{K} R_k\right)^2 / K\sum_{k=1}^{K} R_k^2$ [36]. It has a value between 0 and 1 where $J = 1$ is achieved by equal user rates. Note that in Fig. 3-(b), CMS, PF, and random selection have the Jain's index as $J_{CMS} = 0.67$, $J_{PF} = 0.86$, and $J_{Rand} = 0.67$, respectively. This implies that PF achieves the best fairness in terms of throughput although it does not fairly allocate resources to the users. As a result, PF and CMS with equal weight achieve fairness in terms of throughput and resource allocation, respectively. Also, we confirm that our derived analytical ergodic capacity results match the simulation results for CMS algorithm.

Fig. 4 shows the comparison of three scheduling algorithms with respect to the ratio of allocated power to the strong user, ρ as $P_s = \rho P_t$. We can see that the average rate of user at cell center slightly increases and that of user at cell edge decreases as ρ increases since more power is allocated to the user at cell edge. When using PF scheduling, however, the average rate of the user at cell center decreases as ρ increases since most of resources are allocated to the user at cell edge. As the power allocated to the weak user decreases, their average rate decreases and most of resources

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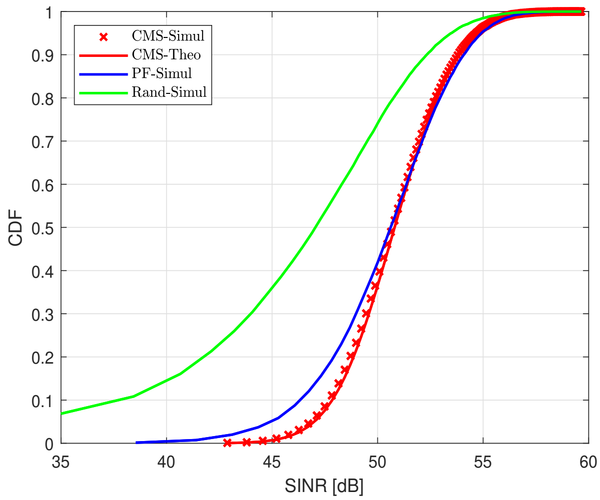
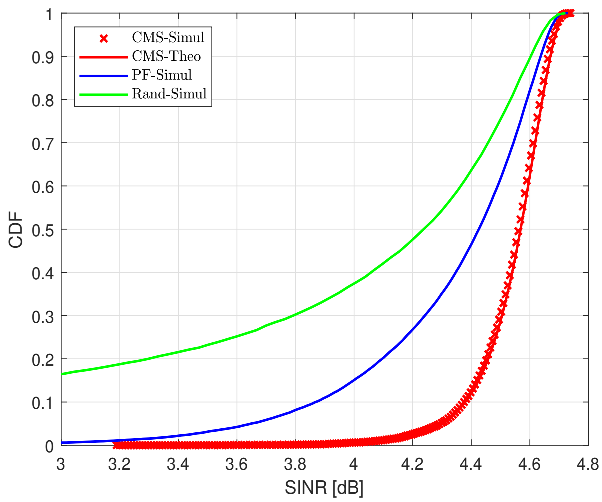
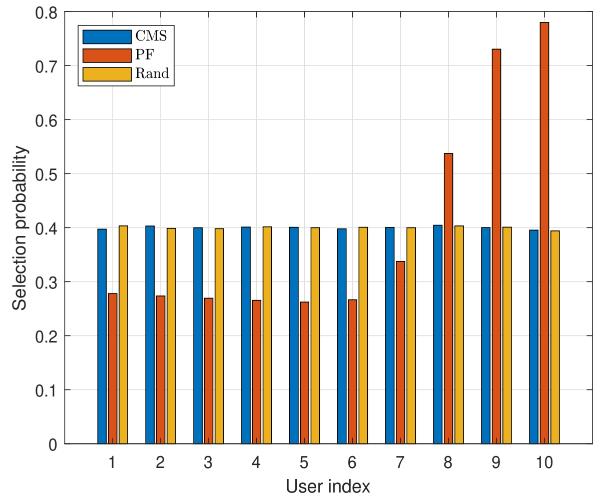


(a) SINR distribution of user at cell center when that user has the best channel quality

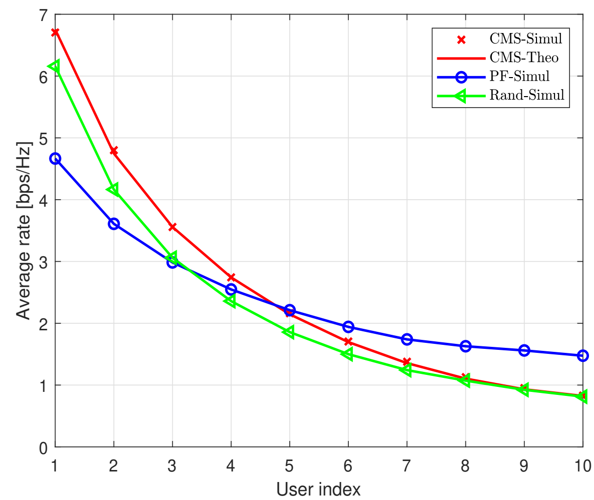


(b) SINR distribution of user at cell edge when that user has the worst channel quality

FIGURE 2: Comparison of SINR distribution between CMS, PF, and Random selection algorithm



(a) Selection probability of scheduling algorithms.



(b) Average rate of scheduling algorithms.

FIGURE 3: Comparison between various scheduling algorithms for fixed power allocation and equal weights.

are allocated to this user. Thus, the user at cell center is assigned a small time fraction that reduces the average rate although it can achieve fair throughput compared with the other schemes. We also observe that fair resource allocation can be achieved by CMS algorithm regardless of the allocated power ratio. Therefore, CMS algorithm with equal weight always guarantees fair resource allocation while improving the system throughput.

In previous simulations, we show that CMS algorithm provides fairness in terms of resource. As in (6) and (7), CMS algorithm can achieve throughput fairness by appropriately controlling the weight of each user since the amount of resources are determined by the weight. To show the effect of the weight on fairness in terms of throughput and resource, unequal weights with $\mathbf{w}_e = [1, 1, 1, 2, 2, 2, 3, 3, 3, 4]$ and $\mathbf{w}_c = [4, 3, 3, 3, 2, 2, 2, 1, 1, 1]$ are used in Fig. 5. As seen in Fig. 5, the weight can adjust the selection probability and average rate of each user. Also, the more weight is assigned to a user, the more the selection probability and average rate increase. Therefore, the different QoS requirements of each user can be satisfied by adjusting the weight compared to PF scheduling. To investigate the impact of the weight and power allocation method on the fairness, Jain's index of the FPA and FTPA schemes is calculated as $J_{FPA} = 0.911$ and $J_{FTPA} = 0.98$ with $\mathbf{w}_e$ and $J_{FPA} = 0.47$ and $J_{FTPA} = 0.71$ with $\mathbf{w}_c$. FTPA can achieve more fair throughput than FPA since the ratio of allocated power of FTPA depends on the channel gain of the paired user. Thus, the user at cell edge is allocated more power resulting in the increase of the average

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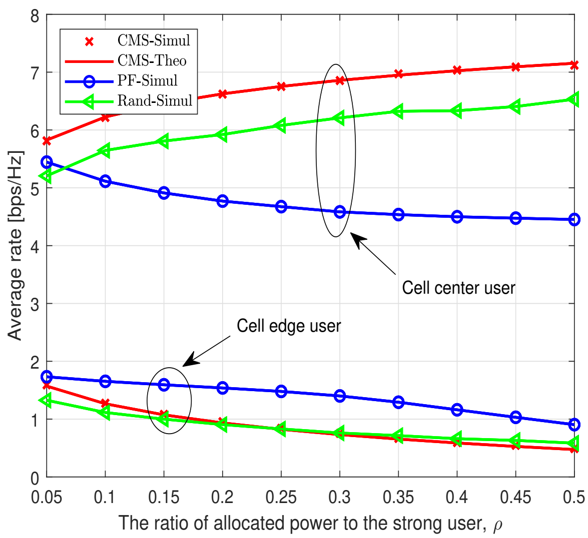

FIGURE 4: Performance of the average rate in terms of the ratio of allocated power to the strong user, ρ.

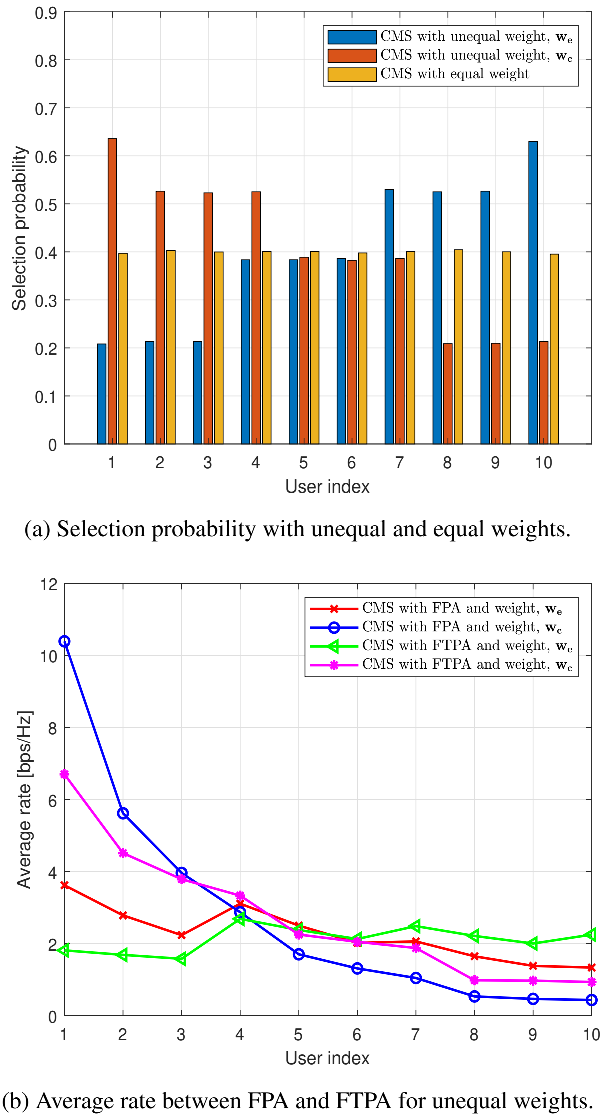

FIGURE 5: The performance of CMS algorithm for unequal weights with $\mathbf{w}_e = [1, 1, 1, 2, 2, 2, 3, 3, 3, 4]$ and $\mathbf{w}_c = [4, 3, 3, 3, 2, 2, 2, 1, 1, 1]$.

rate. When the high weight is assigned to users at cell edge such as $\mathbf{w}_e$, CMS algorithm can achieve more throughput fairness than PF ($J_{PF} = 0.86$) depending on the weight of each user. Therefore, CMS algorithm can flexibly select the user set to satisfy fairness in terms of throughput or resource by adjusting the weight of each user.

Fig. 6 shows the effect of the weight on the selection probability and average rate. In this simulation, the weight of users other than the user at cell edge is $w_i = 1$ for $i = 1, \ldots, 9$. Note that in Fig. 6-(a), the user at cell edge is frequently selected by increasing the weight, w_{10}. The average rate is also increased as in Fig. 6-(b) due to the high selection probability of user at cell edge and it can be further increased using FTPA. Given that FTPA has high Jain's index compared to FPA, the average rate of user at cell edge with FTPA is greater than that with FPA. The problem of weight value decision to satisfy the various requirements of different users is out of scope and kept as future work.

To show the effect of CMS algorithm on massive access scenario, Fig. 7 compares the sum rate and fairness with respect to the number of users. In Fig. 7, the users are uniform distributed within the cell with 0.5km radius and frequency bandwidth is divided into 20 sub-bands ($M = 20$) which support 40 users simultaneously. We observe that the sum rate of CMS and PF increases as the number of increases due to multiuser diversity. Also, the performance of CMS and PF is better than random selection since it does not exploit multiuser diversity and does not take into account the users' channel quality. For the case of CMS with $\mathbf{w}_2 = [K, K-1, \ldots, 1]$, it outperforms the other schemes in terms of sum rate while it cannot provide fairness as in Fig. 7-(b) since most of resources are allocated to users at cell center due to the high weight of them. In contrast, CMS with $\mathbf{w}_1 = [1, 2, \ldots, K]$ achieves the best fairness while it has the worst sum rate performance by allocating the most resources to users at cell edge. Therefore, in case that a massive number of devices exist in the cell, CMS algorithm can flexibly select the user set providing high sum rate performance or fairness by adjusting the weight of each user and it can satisfy the different QoS requirements of different users.

VI. CONCLUSION

In this paper, we propose CDF-based multiuser scheduling (CMS) algorithm for downlink NOMA. $2M$ users having relatively better channel gain are selected by using the channel gain CDF and selected users have the first $2M$ maximum CDF value. It has the advantage of being able to

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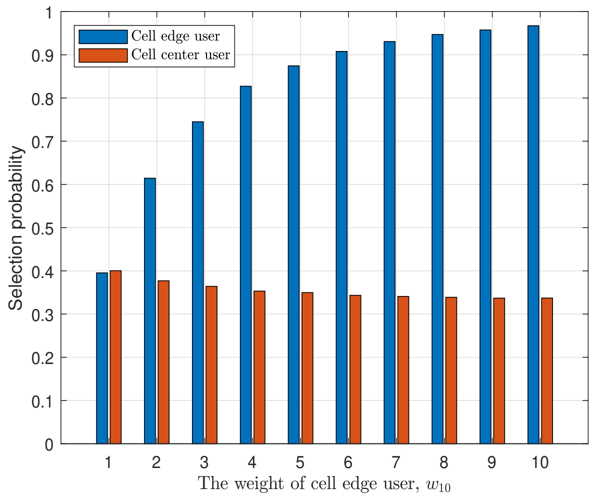
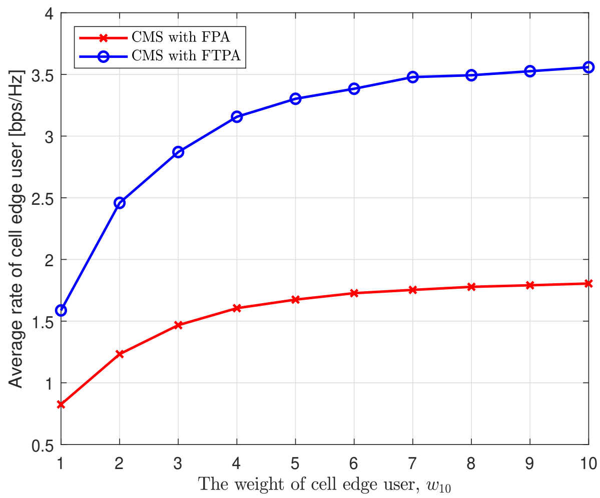
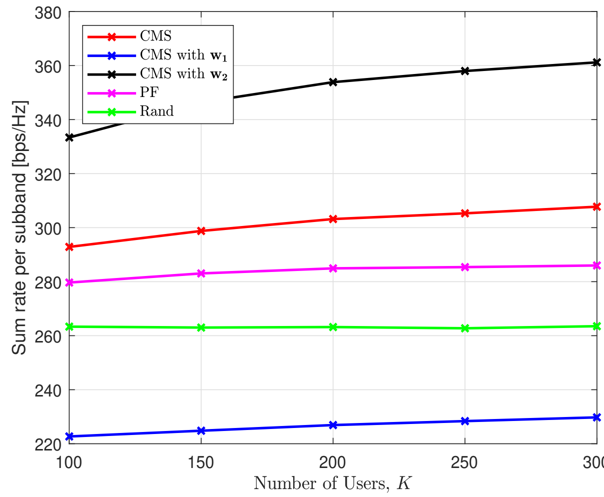


(a) Selection probability versus the weight of user at cell edge.

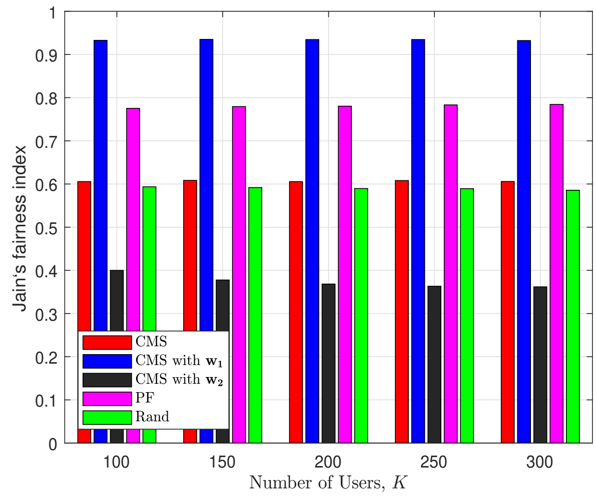


(b) Average rate versus the weight of user at cell edge with FPA and FTPA.

FIGURE 6: Selection probability and average rate for unequal weights with $w_i = 1$ for $i = 1, \dots, 9$.



(a) Sum rate of scheduling algorithms in terms of the number of users.



(b) Jain's fairness index of scheduling algorithms in terms of the number of users.

FIGURE 7: Comparison of different scheduling schemes in terms of the number of users where $\mathbf{w}_1 = [1, 2, \dots, K]$ and $\mathbf{w}_2 = [K, K-1, \dots, 1]$

control the amount of resources and average rate compared to other scheduling schemes. We first derived the selection probability when the different weight is assumed and showed that equal weights guarantee fair resource allocation. For the fair resource allocation case, we analyze the ergodic capacity of each user. Simulation results confirm that equal weight achieves fair resource allocation and that the selection probability and average rate of each user can be controlled by adjusting the weight. Specifically, for massive access scenario, proposed CMS algorithm improves the system throughput by exploiting multiuser diversity. Also, different QoS requirement can be satisfied and it guarantees fairness in terms of throughput or resource by controlling the weight of each user.

APPENDIX A

In the case of $|h_i|^2 \leq |h_j|^2$, $\lim_{\alpha_i \to \infty} \gamma_i \backsimeq \frac{\rho_w}{\rho_s}$. Thus, the integral interval for γ changes to $[0, \frac{\rho_w}{\rho_s}]$. To calculate the integral term over t in (21), $(1-t)^{k-1}$ needs to be expanded. Since $(1+x)^N$ can be expressed as $\sum_{n=0}^{N} \binom{N}{n} x^n$ using the binomial

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theorem, (21) can be calculated as

$$\begin{aligned}&\frac{K}{2M\ln 2}\sum_{k=1}^{2M}\binom{K-1}{k-1}\int_0^{\frac{\rho_w}{\rho_s}}\frac{1}{1+\gamma}\\&\quad\times\int_{F_i(g(\gamma))}^{1}\sum_{n=0}^{k-1}\binom{k-1}{n}(-1)^n t^{K-k+n}dtd\gamma\\&\quad=\frac{K}{2M\ln 2X}\sum_{k=1}^{2M}\sum_{n=0}^{k-1}\binom{K-1}{k-1}\binom{k-1}{n}(-1)^n\\&\quad\times\int_0^{\frac{\rho_w}{\rho_s}}\frac{1-F_i(g(\gamma))^{K-k+n+1}}{(K-k+n+1)(1+\gamma)}d\gamma. \quad (33)\end{aligned}$$

For Rayleigh channel with $E[|h_i|^2]=\sigma_i^2$, the channel gain CDF of the i-th user is $F_i(x)=1-e^{-\frac{x}{\sigma_i^2}}$. Thus, the integral term in (33) for the Rayleigh channel can be calculated as

$$\begin{aligned}&\int_0^{\frac{\rho_w}{\rho_s}}\frac{1-F_i(g(\gamma))^{K-k+n+1}}{1+\gamma}d\gamma\\&=\int_0^{\frac{\rho_w}{\rho_s}}\frac{1-(1-e^{-\frac{g(\gamma)}{\sigma_i^2}})^{K-k+n+1}}{1+\gamma}d\gamma\\&=\sum_{m=1}^{K-k+n+1}\binom{K-k+n+1}{m}(-1)^{m+1}\int_0^{\frac{\rho_w}{\rho_s}}\frac{e^{-\frac{g(\gamma)m}{\sigma_i^2}}}{1+\gamma}d\gamma. \quad (34)\end{aligned}$$

Let $t=g(\gamma)$ be the substitution function. Then, (34) can be rewritten using integration by substitution as

$$\begin{aligned}&\int_0^{\frac{\rho_w}{\rho_s}}\frac{e^{-\frac{g(\gamma)m}{\sigma_i^2}}}{1+\gamma}d\gamma\\&=\int_0^{\infty}\rho_w\frac{e^{-\frac{tm}{\sigma_i^2}}}{(1+t\rho_s)(1+t(\rho_s+\rho_w))}dt\\&=\int_0^{\infty}-\frac{1}{\frac{1}{\rho_s}+t}e^{-\frac{tm}{\sigma_i^2}}dt+\int_0^{\infty}\frac{1}{\frac{1}{\rho_s+\rho_w}+t}e^{-\frac{tm}{\sigma_i^2}}dt\\&=e^{\frac{m}{\sigma_i^2\rho_s}}\mathrm{Ei}\left(-\frac{m}{\sigma_i^2\rho_s}\right)-e^{\frac{m}{\sigma_i^2(\rho_s+\rho_w)}}\mathrm{Ei}\left(-\frac{m}{\sigma_i^2(\rho_s+\rho_w)}\right), \quad (35)\end{aligned}$$

where $\mathrm{Ei}(x)=-\int_{-x}^{\infty}\frac{e^{-t}}{t}dt$ is the exponential integral function. Similarly, the integral term in (33) in case of $|h_i|^2\geq|h_j|^2$ can be calculated as

$$\begin{aligned}&\int_0^{\infty}\frac{e^{-\frac{g(\gamma)m}{\sigma_i^2}}}{1+\gamma}d\gamma\\&=\int_0^{\infty}\rho_s\frac{e^{-\frac{tm}{\sigma_i^2}}}{1+t\rho_s}dt=-e^{\frac{m}{\sigma_i^2\rho_s}}\mathrm{Ei}\left(-\frac{m}{\sigma_i^2\rho_s}\right). \quad (36)\end{aligned}$$

Therefore, the conditional ergodic capacity for each case can be written as (22).

APPENDIX B

First, we calculate the first term on the right-hand side in (26) when the i,j-th users are randomly paired. In this equation, $F_i(\alpha_i)\leq F_j(\alpha_j)$ equals $\alpha_i\leq\frac{\sigma_i^2}{\sigma_j^2}\alpha_j$ for the Rayleigh channel. If $\sigma_i^2\leq\sigma_j^2$, the first term would be zero since $\Pr(\alpha_i\geq\alpha_j,\alpha_i\leq\frac{\sigma_i^2}{\sigma_j^2}\alpha_j)$ is zero. For $\sigma_i^2\geq\sigma_j^2$, the first term on the right-hand side in (26) can be calculated as

$$\begin{aligned}&\Pr\left(|h_i|^2\geq|h_j|^2,\ (i,j)\text{ pair},U_a\leq F_i(\alpha_i)\leq F_j(\alpha_j)\right)\\&=\frac{M!}{\binom{2M}{2,2,\ldots,2}}\Pr\left(\alpha_i\geq\alpha_j,U_a\leq F_i(\alpha_i)\leq U_b,F_i(\alpha_i)\leq F_j(\alpha_j)\right)\\&=\frac{(2!)^M M!}{2M!}\int_0^{\infty}\int_{\alpha_j}^{\frac{\sigma_i^2}{\sigma_j^2}\alpha_j}\sum_{k=2}^{2M}\binom{K-2}{K-k}\\&\quad\times F_i(\alpha_i)^{K-k}(1-F_i(\alpha_i))^{k-2}f_{\alpha_i}(\alpha_i)f_{\alpha_j}(\alpha_j)d\alpha_i d\alpha_j\\&=\frac{(2!)^M M!}{2M!}\sum_{k=2}^{2M}\binom{K-2}{K-k}\int_0^{\infty}\int_{\alpha_j}^{\frac{\sigma_i^2}{\sigma_j^2}\alpha_j}\sum_{n=0}^{K-k}\binom{K-k}{n}(-1)^n\\&\quad\times\frac{1}{\sigma_i^2}e^{-\frac{k+n-1}{\sigma_i^2}\alpha_i}d\alpha_i\frac{1}{\sigma_j^2}e^{-\frac{1}{\sigma_j^2}}d\alpha_j\\&=\frac{(2!)^M M!}{2M!}\sum_{k=2}^{2M}\binom{K-2}{K-k}\sum_{n=0}^{K-k}\binom{K-k}{n}(-1)^n\frac{1}{k+n-1}\\&\quad\times\left(\frac{\sigma_i^2}{(k+n-1)\sigma_j^2+\sigma_i^2}-\frac{1}{k+n}\right)\text{ for }\sigma_i^2\geq\sigma_j^2. \quad (37)\end{aligned}$$

Similarly, we can derive the second term on the right-hand side in (26) in the case of $\sigma_i^2\leq\sigma_j^2$ given as

$$\begin{aligned}&\Pr\left(|h_i|^2\geq|h_j|^2,\ (i,j)\text{ pair},U_a\leq F_j(\alpha_j)\leq F_i(\alpha_i)\right)\\&=\frac{(2!)^M M!}{2M!}\int_0^{\infty}\int_0^{\alpha_i}\sum_{k=2}^{2M}\binom{K-2}{K-k}\\&\quad\times F_j(\alpha_j)^{K-k}(1-F_j(\alpha_j))^{k-2}f_{\alpha_j}(\alpha_j)f_{\alpha_i}(\alpha_i)d\alpha_j d\alpha_i\\&=\frac{(2!)^M M!}{2M!}\sum_{k=2}^{2M}\binom{K-2}{K-k}\sum_{n=0}^{K-k}\binom{K-k}{n}(-1)^n\frac{1}{k+n-1}\\&\quad\times\left(1-\frac{\sigma_j^2}{(k+n-1)\sigma_i^2+\sigma_j^2}\right)\text{ for }\sigma_i^2\leq\sigma_j^2. \quad (38)\end{aligned}$$

For $\sigma_i^2\geq\sigma_j^2$, the integral interval over α_j is $[0,\frac{\sigma_j^2}{\sigma_i^2}\alpha_i]$. Thus, the second term on the right-hand side in (26) in the case of

$\sigma_i^2 \geq \sigma_j^2$ can be written as

$$\begin{aligned}
&\Pr\left(|h_i|^2 \geq |h_j|^2,\ (i,j)\text{ pair}, U_a \leq F_j(\alpha_j) \leq F_i(\alpha_i)\right)\\
&=\frac{(2!)^M M!}{2M!}\int_0^{\infty}\int_0^{\frac{\sigma_j^2}{\sigma_i^2}\alpha_i}\sum_{k=2}^{2M}\binom{K-2}{K-k}F_j(\alpha_j)^{K-k}\\
&\quad\times(1-F_j(\alpha_j))^{k-2}f_{\alpha_j}(\alpha_j)f_{\alpha_i}(\alpha_i)d\alpha_j d\alpha_i\\
&=\frac{(2!)^M M!}{2M!}\sum_{k=2}^{2M}\binom{K-2}{K-k}\sum_{n=0}^{K-k}\binom{K-k}{n}(-1)^n\frac{1}{k+n-1}\\
&\quad\times\left(1-\frac{1}{k+n}\right)\text{ for }\sigma_i^2 \geq \sigma_j^2. \qquad (39)
\end{aligned}$$

As a result, the conditional probability in (25) can be expressed as (31).

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