Modeling and Assessment of Dynamic Charging for Electric Vehicles in Metropolitan Cities

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ABSTRACT

Modeling and Assessment of Dynamic Charging for Electric Vehicles in Metropolitan Cities

Duc Minh Nguyen

Electric vehicles (EVs) have emerged to be the future of transportation as the world observes its rising demand and usage across continents. However, currently, one of the biggest bottlenecks of EVs is the battery. Small batteries limit the EVs driving range, while big batteries are expensive and not environmentally friendly. One potential solution to this challenge is the deployment of *charging roads*, i.e., dynamic wireless charging systems installed under the roads that enable EVs to be charged while driving. In this thesis, we establish a framework using stochastic geometry to study the performance of deploying charging roads in metropolitan cities. We first present the course of actions that a driver may take when driving from a random source to a random destination, and then analyze the distribution of the distance to the nearest charging road and the probability that the trip passes through at least one charging road. These probability distributions assist not only urban planners and policy makers in designing deployment plans of dynamic wireless charging systems, but also drivers and automobile manufacturers in choosing the best driving routes given the road conditions and level of energy of EVs.

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LIST OF ABBREVIATIONS

N-VC	Nearest Vertical Charging Road from Source
N-HC	Nearest Horizontal Charging Road from Source
N-VNC	Nearest Vertical Non-Charging Road from Source
N-HNC	Nearest Horizontal Non-Charging Road from Source
EV	Electric Vehicle

LIST OF SYMBOLS

p	probability of a road being a charging road
d_h	horizontal distance between a source and a destination
d_v	vertical distance between a source and a destination
D_n	distance from a source to the nearest charging road
$D_{\mathrm{N-HC}}$	distance from a source to the nearest horizontal charging road
$D_{\mathrm{N-VC}}$	distance from a source to the nearest vertical charging road
$D_{\mathrm{N-HNC}}$	distance from a source to the nearest horizontal non-charging road
$D_{\mathrm{N-VNC}}$	distance from a source to the nearest vertical non-charging road
T_c	event that any given trip passes through at least one charging road
$\overline{T_c}$	event that any given trip passes through no charging road

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Chapter 1

Introduction

1.1 Background

As the global trend towards sustainable energy has significantly transformed several industries, automobile manufacturing is not an exception. Almost every major car manufacturer now has models that run entirely on electric batteries. It is expected that in the next decades, electric vehicles (EVs) will account for a large portion of total car produced [1].

Although there are several challenges with EVs, e.g., engines, sensors, one of the biggest bottlenecks of EVs is the battery [2]. Ideally, batteries for EVs should last for a comparable distance compared to gasoline tanks. The battery should also be quickly charged and remain in good condition after thousands of charging cycles. Moreover, it should be affordable and finally, be environmentally friendly. However, there are a couple of important trade-offs with the current EVs batteries that need to be considered [3]. For example, large-capacity batteries, which are optimized for driving distance, are expensive to make, slow to charge, and not friendly to the environment. On the other hand, small-capacity batteries are more affordable but require more frequent charging.

There have been steady advances in producing batteries that fit the demand [4]. However, as batteries require charging, we still need solutions on how to charge them effectively and efficiently. In the literature, several works have been presented about optimally deploying charging stations [5, 6, 7]. Although this solution seems to extend the driving range of EVs, one major drawback of charging stations is the waiting time, especially in metropolitan cities. For example, during rush hours, charging stations may not meet the charging demand if each EV needs half an hour or more to charge. Moreover, as the frequency of people using EVs in their daily commute increases, a better solution that benefits all commuters is dynamic wireless vehicle charing systems [8, 9], i.e, roads that are able to charge EVs while driving without the need to stop. Charging roads are equipped with Wireless Power Transfer technology that enables complete wireless charging without any wired accessories [10, 11]. This technology has been thoroughly researched in several research institutes around the world such as KAIST (Korea), University of Auckland (UoA - New Zealand), and Oak Ridge National Laboratory (ORNL - United States) [12]. At KAIST, since 2009, researchers have introduced six generations of dynamic wireless charging systems for both driving vehicles and stationary EVs with improved charging efficiency in each new generation [13]. UoA has been active in designing coil structure and layout for dynamic charging systems [14, 15]. ORNL has focused on integrated wireless charging systems for vehicles. They have tested their systems in models such as Toyota Prius [16], Toyota RAV4 [17] and achieved good power transfer efficiency [18]. Samples of dynamic electric vehicular charging systems have also been demonstrated by QualComm Halo [19], which was later acquired by a company named WiTricity [20].

1.2 Motivation and Main Contributions

Motivated by the great demand forecast for charging roads, we would like to study its deployment plan in a metropolitan city setting. Our goal is to provide a useful analytical framework to urban planners, city policy makers, car manufacturers, and drivers about the deployment of charging roads. To be more precise, we consider a random deployment scheme, in which each road has an equal probability of being a charging road. The random deployment scheme allows our framework to be applicable to a generic city, serving as a baseline model compared to other well-planned deployment plans. Then, given a random source and a random destination, we analyze the two performance metrics: (i) the probability that any given trip passes through at least one charging road, and (ii) the distribution of the distance from the source to the nearest charging road. These two metrics are crucial for city planners and policymakers to evaluate how effective and efficient charging roads are deployed, as demonstrated in more details in Section 3.3. We make use of stochastic geometry as the main tool for our analysis, as it has been used extensively to model vehicular networks and has been proven useful to study several network-related problems [21, 22]. A more comprehensive overview of stochastic geometry and its application in vehicular network modeling is discussed in Section 2.2 and Section 3.1. In this thesis, our contribution are three-fold and summarized as follows:

- We introduce a routing policy that a driver would take for all situations given a random source and a random destination, assuming that the driver will always choose the shortest route and maximize the time spent on charging roads throughout the trip.
- We present the distribution of the distance from a random source to the nearest charging road.
- We analyze the probability that a given trip passes through at least one charging road.
- We rigorously verify our analytical results for the two performance metrics through simulation.

To the best of our knowledge, this thesis is the first one to incorporate stochastic geometry into the performance analysis of charging road deployment in metropolitan cities. Thus, it sheds light on how analytical tools such as stochastic geometry can be used to capture the randomness of the road systems to assess the performance of charging roads deployment in a generic urban city.

Chapter 2

Related Work

2.1 Wireless Charging for Electric Vehicles

The wireless charging technology for EVs can be categorized into two major branches: capacitive power transfer and inductive power transfer. Capacitive power transfer utilizes the electric field interaction between coupled capacitor. Hence, it is only viable to transfer energy through a short air gap between 10^{-4} and 10^{-3} meters [23], which is not suitable for charging EVs while running. Thus, we mainly focus on the inductive wireless charging system, in which there can be an air gap up to a few meters between a power transmitter in the roads and a receiver in the vehicles [12]. It can transfer power electro-magnetically to EVs while driving and its main components consist of long primary windings (installed under the road) and secondary pick-up windings (installed in the EV). There are several other components that go into the wireless charging system. Optimizing the design of those components is an active research field on its own [24]. For example, we refer the readers to resources about optimizing track length [25], power pads [26], and coil design [12].

Since an inductive wireless power transfer system can power EVs while driving, it significantly increase the driving range of EVs without the need to stop and charge at stationary charging stations [27]. Furthermore, the current design and cost of deployment of charging roads suggest that it is most suitable to deploy charging roads in metropolitan cities. Since the total energy transferred to an EV is the power of the charging system multiplied by the time that the vehicle spends on the charging road, it is desirable to maximize the time vehicles spend on charging roads, given a fixed power of the charging system. However, longer charging roads directly increase the cost of deployment. Also, it is preferred to have a high density of traffic travelled on the charging roads to fully utilize the charging system and reduce waste of energy. Thus, the suitable place to install a charging road system is in an urban setting since it has a high density of transportation, slower vehicle driving speed compared to highways, and shorter driving trajectory compared to highways. Thus, we can maximize charging performance while minimizing deployment cost.

2.2 Vehicular Network Modeling

In the literature, several models have been proposed for vehicular networks [28]. The classic Erdos-Renyo (ER) graph model proposes that a graph of n nodes is constructed by connecting those n nodes randomly, i.e., each edge has an equal probability p of being included in the graph [29]. However, ER graphs do not closely represent several real-world networks since they have low clustering coefficients and do not account for the formation of hubs. Watts-Strogatz small-world network models [30] address the first limitation of ER graph by accounting for clustering while maintaining the average path length as the ER graphs. Hammersley graphs [31] define a vertex with exactly four edges, while all vertices in the network follows an infinite Poisson Point Process. However, all of these network models do not correctly reflect the road systems in metropolitan cities since they they do not capture the continuity of streets. To alleviate this problem, a good alternative is to model the streets in vehicular networks as a set of random lines, which collectively forms a *line process* [32, 33]. A well-known model for line processes is the Poisson Line Process (PLP) [34]. Especially, several modern cities in the world, e.g., New York, have a grid-like street network that can be closely modeled with a special case of PLP named *Manhattan Poisson Line Process (MPLP)*. For example, several properties of PLP and MPLP that are useful for modeling vehicular networks is discussed in [35]. A method to analyze the coverage of wireless signals propagating through the streets modeled with MPLP is introduced in [36]. Hence, in this thesis, given the goal to assess the deployment of dynamic charging roads, we choose to model vehicular networks in a metropolitan settings using a MPLP. Details about MPLP and our network model are elaborated in Section 3.1 and Section 3.2, respectively.

2.3 Charging Lanes Deployment

As the importance of charging roads are realized by researchers and companies around the world, some researches have been presented on the deployment of charging lanes for EVs. For example, a plan to support electric buses running on a pre-defined route to minimize cost of deployment is introduced in [13]. A categorization and clustering method to choose the landmarks to deploy charging lanes in metropolitan cities is presented in [37]. Unlike those studies, our work aims to provide a general analytical framework to assess the deployment of charging road in metropolitan cities, and thus can be applied to several big cities and benefit various groups from city planners to EV manufacturers.

Chapter 3

Analytical Framework

3.1 Poisson Point Process and Poisson Line Process Preliminaries

Since our vehicular network model in this thesis is based on Poisson point process and Poisson line process, we briefly review the essence of those processes in this section. For a more details discussion regarding this topic, we refer the reader to sources such as [38, 39] and [34, 40].

Poisson Point Process. Intuitively, a point process is a random collection of points in some spaces. Let $N(B)$ denote the number of points in a Borel set B. A point process is a homogeneous Poisson Point Process (PPP) with intensity parameter $\beta > 0$ if:

- $N(B) \sim \text{Poisson}(\beta * m(B))$, where $m(B)$ is the measure of set B, i.e., $P(N(B) = v) = \frac{(\beta * m(B))^v}{v!} e^{-\beta * m(B)}$.
- For Borel sets B_1 and B_2 such that B_1 and B_2 are mutually exclusive, $N(B_1)$ and $N(B_2)$ are independent.

One important property of PPP is that given $N(B) = v$, the locations of those v points are independent and identically distributed and uniform in B.

Poisson Line Process. Similar to a point process, a line process is a random collection of lines in a 2D plane. A random undirected line in this plane can be fully characterized by a set of two parameters (ρ, θ), where $\rho \in \mathbb{R}$ is a real number denoting the perpendicular distance from the origin $o \equiv (0, 0)$ and θ is the angle between the positive x-axis and the line, i.e., $\theta \in [0, \pi)$. It is worth noting that ρ is positive if the line is above or to the right of the origin, and negative otherwise. We can represent all the possible values of (ρ, θ) in a plane denoting $\mathcal{C} \equiv [0, \pi) \times \mathbb{R}$. Since the mapping between the set of points in $\mathcal{C}$ and the set of lines in $\mathbb{R}^2$ is one-to-one, one can generate a line process in $\mathbb{R}^2$ by generating a point process in $\mathcal{C}$. For example, a set of lines generated by a PPP on $\mathcal{C}$ is a Poisson Line Process (PLP).

In this thesis, we will focus on a special instance of PLP, namely the Manhattan Possion line processes (MPLP) [36]. A MPLP has $\rho \in \mathbb{R}$ and $\theta \in \{0, \frac{\pi}{2}\}$, i.e., the set of lines has a grid-like shape.

3.2 System Model

We model the system of roads in metropolitan cities by an MPLP in $\mathbb{R}^2$, where an overview about line process and MPLP are given in Section 3.1. Then, we consider random positions for the source and the destination of a trip in this road system. There are two main possibilities. One is when the source and the destination are on two parallel roads, the other is when the source and the destination are on two perpendicular roads.

3.3 Performance Metrics

In this thesis, our goal is to assess the deployment of charging roads in a metropolitan city. We propose two performance metrics whose definitions are given as follows:

Metric 1. Probability distribution of the distance from a random source to the nearest charging road: It is the probability that the travel distance, i.e. Manhattan distance, from a random source to the nearest charging road is less than a positive real number x.

Metric 2. Probability that a trip passes through at least one charging road: It is the probability that a driver travels on at least one charging road while driving from a random source to a random destination.

These metrics have practical significance in understanding how charging roads can serve the needs of commuters. For example, urban planners and city policy makers can use the metric to answer questions such as how densely charging roads should be deployed so that 80% of the time a driver will pass through at least one charging road in his or her trip. Another example would be for car manufacturers to see based on the distance to the nearest charging road, how big the battery should be designed to fit with traffic condition in a particular city. Given that electric vehicles are the future of transportation and the need for charging roads as illustrated in Chapter 1, our metrics will provide useful insights for the deployment of charging roads.

Chapter 4

Routing Policy & Distribution of the distance to the nearest charging road

We denote the probability that a road being charging is p. The horizontal and vertical distances between source and destination are d_h and d_v, respectively. We analyze the probability that the distance to the nearest charging road, denoted D_n, is less than a positive real number x. To calculate $P(D_n < x)$, we break it down into eight sub-cases, i.e.,

$$P(D_n < x) = \sum_{i=1}^{8} P(D_n < x|E_i)P(E_i),$$

each of which will be discussed in sections 4.1.1-4.2.4. In each case, D_n is calculated based on an assumption that a driver always chooses the shortest route from the current position to the destination. If there are multiple routes with the same minimum distance, priority is given to the routes containing the largest portion of charging roads.

4.1 Case a: When source (S) and destinations (D) are on two parallel roads

Let A denote the case when S and D are on two parallel roads. The probability of case A is $P(A) = \frac{1}{2}$. We consider four scenarios:

1. Both source and destination roads are charging,

2. Only source road is charging,

3. Only destination road is charging,

4. Both source and destination roads are not charging.

In each scenario, we first describe its structure, i.e., the sub-events in that scenario expressed in the form of probability trees. A probability tree starts from one root. Then, it is split into levels such that events in one level are mutually exclusive and are independent from events on the other levels. Next, we discuss each event in details together with corresponding a graph of the event. Note that we provide graphs for only the events that an action needs to be made in choosing the driving route. All proofs for $P(D_n < x|E_{k,i,j})$ are available in Appendix A.

4.1.1 Both source and destination roads are charging

Let E_1 denote the case when both source and destination roads are on two parallel roads and are charging. The probability of event E_1 is p^2. We hereby denote subevents as $E_{1,i,j}$, in which i is the level of depth of the event in the probability tree and j is the index of the event at that level. A graph of E_1 is given as follows.

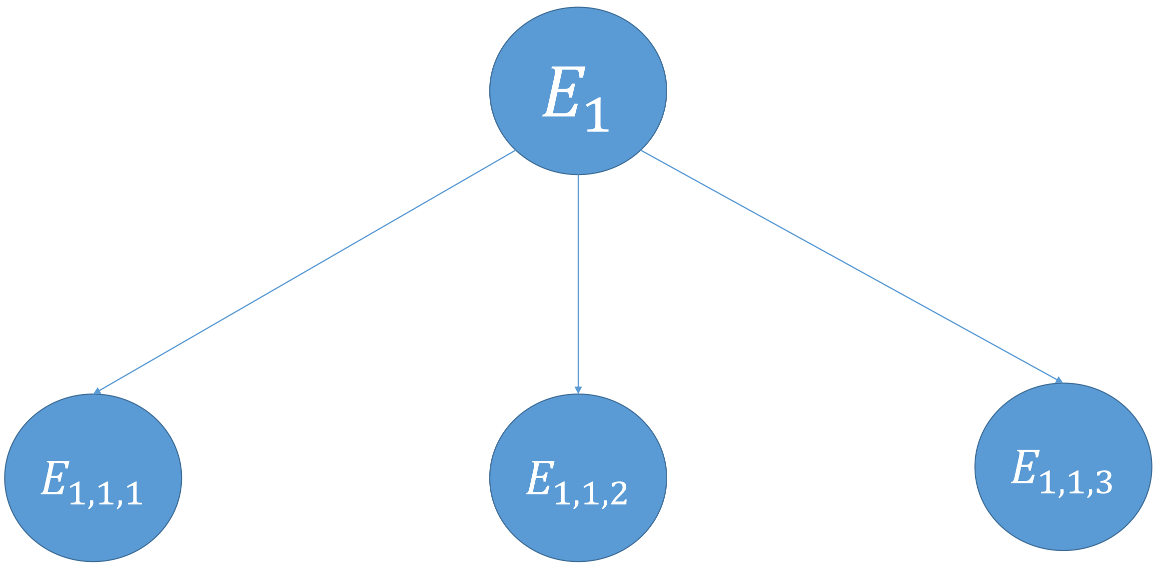

Figure 4.1: Tree E_1

We have $P(D_n < x|E_1)P(E_1) = \sum_{i=1}^{1}\sum_{j=1}^{3} P(D_n < x|E_{1,i,j})P(E_{1,i,j})$.

- Event $E_{1,1,1}$:

  Description: If there are no horizontal roads between S and D, as shown in Figure 4.2;

  Probability: $P(E_{1,1,1}|E_1) = e^{-\lambda d_v}$;

  Action: we simply take the shortest path from S to D (either upper path or lower path).

- Event $E_{1,1,2}$:

  Description: If there is at least one horizontal charging road between S and D, as shown in Figure 4.3;

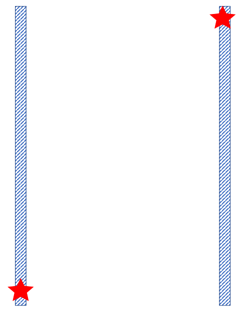

Figure 4.2: Event $E_{1,1,1}$

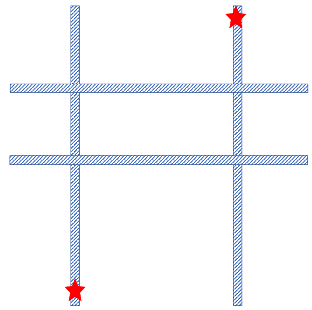

Figure 4.3: Event $E_{1,1,2}$

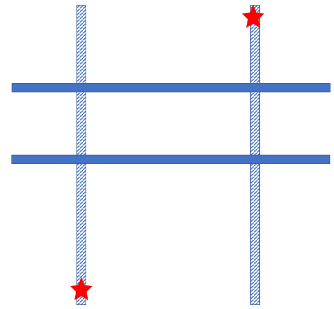

Figure 4.4: Event $E_{1,1,3}$

Probability: $P(E_{1,1,2}|E_1) = 1 - e^{-\lambda p d_v}$;

Action: we can take any horizontal charging road between S and D.

- Event $E_{1,1,3}$:

  Description: If there is no horizontal charging road but at least one horizontal non-charging road between S and D, as shown in Figure 4.4;

  Probability: $P(E_{1,1,3}|E_1) = e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v})$;

  Action: we take any horizontal non-charging road between S and D.

Since the source road is already a charging road, $P(D_n < x|E_{1,i,j}) = 1$ for all i, j.

4.1.2 Only source road is charging

Let E_2 denote the case when both source and destination roads are on two parallel roads and the source road is charging. The probability of event E_2 is $p(1-p)$. We denote subevents as $E_{2,i,j}$, in which i is the level of depth of the event in the probability tree and j is the index of the event at that level.

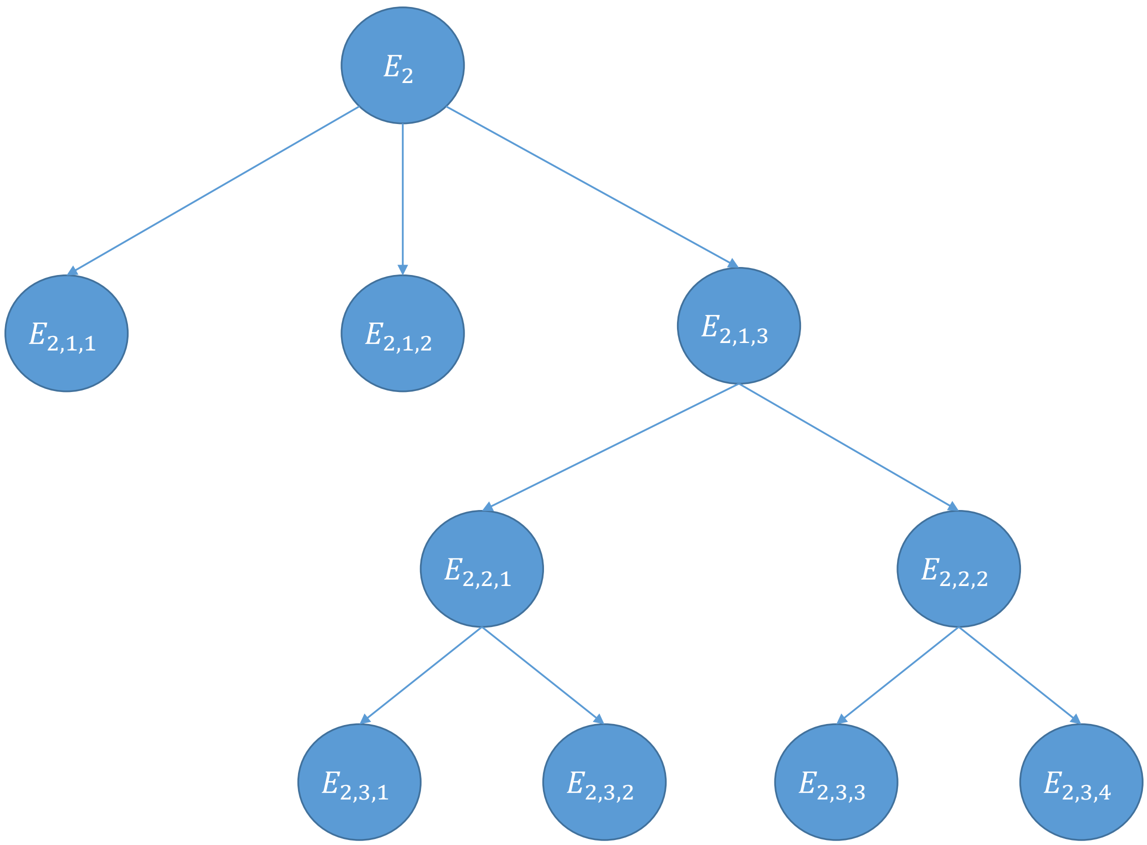

Figure 4.5: Tree E_2

We have $P(D_n < x|E_2)P(E_2) = \sum_{i=1}^{3}\sum_{j=1}^{4} P(D_n < x|E_{2,i,j})P(E_{2,i,j})$.

- Event $E_{2,1,1}$:

  Description: If there are no horizontal roads between S and D, as shown in Figure 4.6;

  Probability: $P(E_{2,1,1}|E_2) = e^{-\lambda d_v}$;

  Action: we simply take the shortest path from S to D (either upper path or

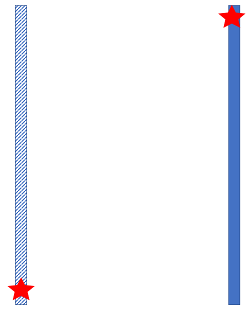

Figure 4.6: Event $E_{2,1,1}$

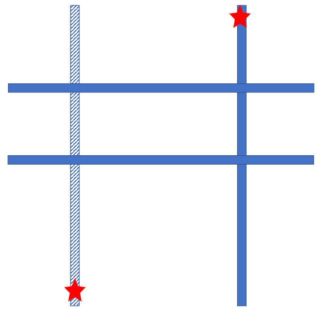

Figure 4.7: Event $E_{2,1,2}$

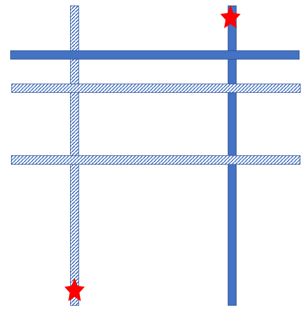

Figure 4.8: Event $E_{2,3,1}$

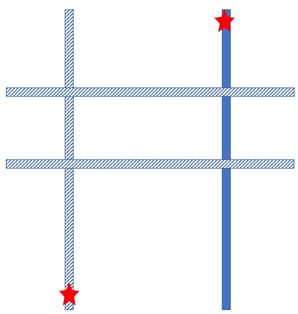

Figure 4.9: Event $E_{2,3,2}$

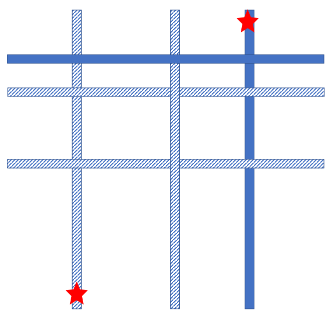

Figure 4.10: Event $E_{2,3,3}$

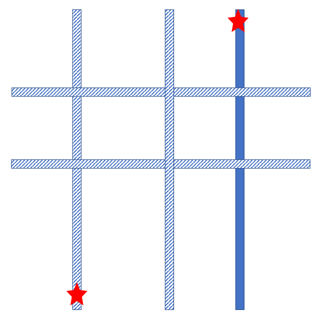

Figure 4.11: Event $E_{2,3,4}$

lower path).

- Event $E_{2,1,2}$:

  Description: If there is no horizontal charging road but at least one horizontal non-charging road between S and D, as shown in Figure 4.7;

  Probability: $P(E_{2,1,2}|E_2) = e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v})$;

  Action: we take the furthest horizontal non-charging road from S.

- Event $E_{2,1,3}$:

  Description: If there is at least one horizontal charging road between S and D;

  Probability: $P(E_{2,1,3}|E_2) = 1 - e^{-\lambda p d_v}$:

  – Event $E_{2,2,1}$:

    Description: If there are no vertical charging roads between S and D;

    Probability: $P(E_{2,2,1}|E_2) = e^{-\lambda p d_h}$

    * Event $E_{2,3,1}$:

Description: If there exists at least one horizontal non-charging road above the furthest horizontal charging road from S, as shown in Figure 4.8;

Probability: $P(E_{2,3,1}|E_2) = 1 - \frac{p-p*e^{-\lambda d_v}}{1-e^{-\lambda p d_v}}$;

Action: we compare the vertical distance between the furthest horizontal charging road and the furthest horizontal non charging road, and dh, to take the longer one.

* Event $E_{2,3,2}$:

  Description: If there does not exist horizontal non-charging roads above the furthest horizontal charging road from S, as shown in Figure 4.9;

  Probability: $P(E_{2,3,}|E_2) = \frac{p-p*e^{-\lambda d_v}}{1-e^{-\lambda p d_v}}$;

  Action: we simply take the furthest horizontal charging road.

- Event $E_{2,2,2}$:

  Description: If there is at least one vertical charging road between S and D;

  Probability: $P(E_{2,2,2}|E_2) = 1 - e^{-\lambda p d_h}$

  * Event $E_{2,3,3}$:

    Description: If there exists at least one horizontal non-charging road above the furthest horizontal charging road from S, as shown in Figure 4.10;

    Probability: $P(E_{2,3,3}|E_2) = 1 - \frac{p-p*e^{-\lambda d_v}}{1-e^{-\lambda p d_v}}$;

    Action: we compare the distance between the farthest horizontal charging road and the farthest horizontal non-charging road, to the horizontal distance between the farthest vertical charging road and destination, to take the longer one.

  * Event $E_{2,3,4}$:

Description: If there does not exist horizontal non-charging roads above the furthest horizontal charging road from S, as shown in Figure 4.11;

Probability: $P(E_{2,3,4}|E_2) = \frac{p - p * e^{-\lambda d_v}}{1 - e^{-\lambda p d_v}}$;

Action: we simply go with the farthest horizontal charging road.

Since the source road is already a charging road, $P(D_n < x|E_{2,i,j}) = 1$ for all i, j.

4.1.3 Only destination road is charging

Let E_3 denote the case when both source and destination roads are on two parallel roads and the destination road is charging. The probability of event E_3 is $p(1-p)$. We denote subevents as $E_{3,i,j}$, in which i is the level of depth of the event in the probability tree and j is the index of the event at that level.

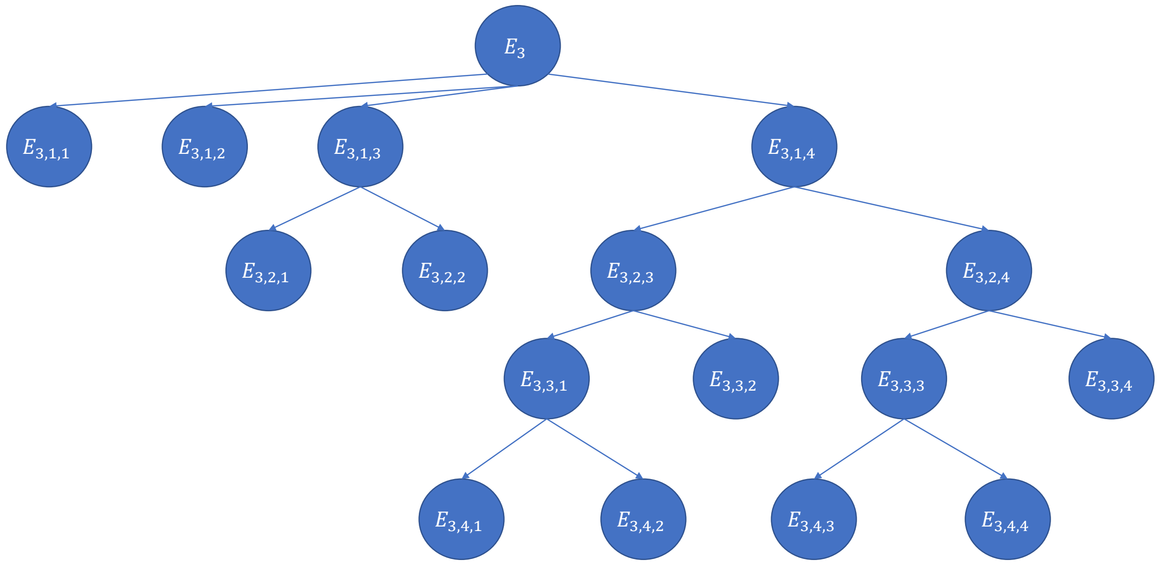

Figure 4.12: Tree E_3

We have $P(D_n < x|E_3)P(E_3) = \sum_{i=1}^{4}\sum_{j=1}^{4} P(D_n < x|E_{3,i,j})P(E_{3,i,j})$.

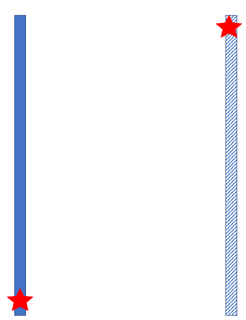

Figure 4.13: Event $E_{3,1,1}$

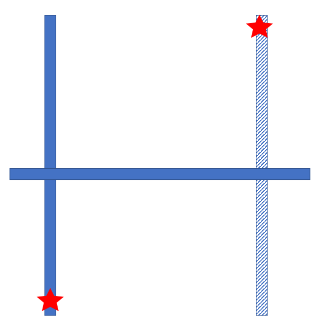

Figure 4.14: Event $E_{3,1,2}$

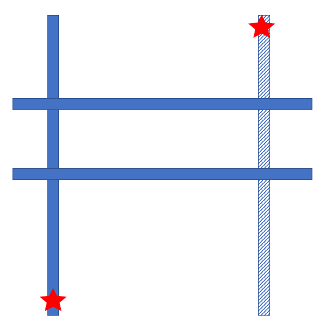

Figure 4.15: Event $E_{3,2,1}$

- Event $E_{3,1,1}$:

  Description: If there are no horizontal roads between S and D, as shown in Figure 4.13;

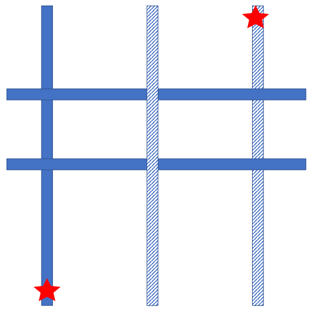

Figure 4.16: Event $E_{3,2,2}$

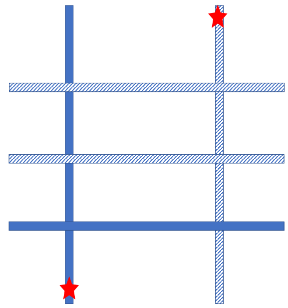

Figure 4.17: Event $E_{3,3,1}$

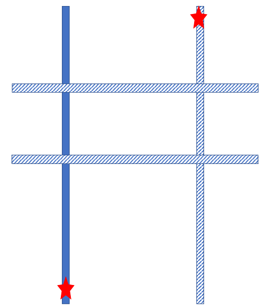

Figure 4.18: Event $E_{3,3,2}$

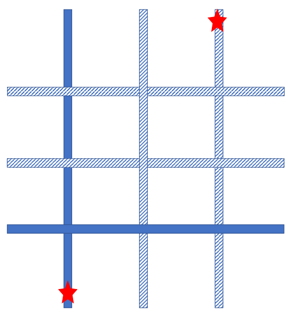

Figure 4.19: Event $E_{3,3,3}$

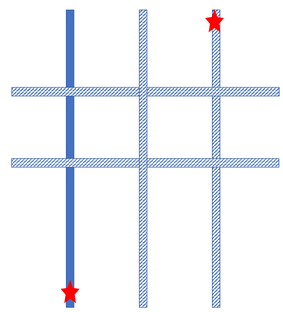

Figure 4.20: Event $E_{3,3,4}$

Probability: $P(E_{3,1,1}|E_3) = e^{-\lambda d_v}$;

Action: we simply take the shortest path from S to D.

$P(D_n < x|E_{3,1,1})$ is given in equation A.1 in Appendix A.

- Event $E_{3,1,2}$:

  Description: If there is no horizontal charging road but only one horizontal non-charging road between S and D, as shown in Figure 4.14;

  Probability: $P(E_{3,1,2}|E_3) = e^{-\lambda p d_v}\lambda(1-p)d_v e^{-\lambda(1-p)d_v}$;

  Action: we take the nearest horizontal non-charging road from S.

  $P(D_n < x|E_{3,1,2})$ is given in equation A.2 in Appendix A.

- Event $E_{3,1,3}$:

  Description: If there is no horizontal charging road but at least two horizontal non-charging road between S and D;

  Probability: $P(E_{3,1,2}|E_3) = e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v} - \lambda(1-p)d_v e^{-\lambda(1-p)d_v})$:

- Event $E_{3,2,1}$:

  Description: If there are no vertical charging roads between S and D, as shown in Figure 4.15;

  Probability: $P(E_{3,2,1}|E_3) = e^{-\lambda p d_h}$;

  Action: we take the nearest horizontal non-charging road from S.

  $P(D_n < x|E_{3,2,1}, E_{3,1,3})$ is identical to $P(D_n < x|E_{3,1,2})$.

- Event $E_{3,2,2}$:

  Description: If there is at least one vertical charging road between S and D, as shown in Figure 4.16;

  Probability: $P(E_{3,2,2}|E_3) = 1 - e^{-\lambda p d_h}$;

  Action: we take the nearest horizontal non-charging road from S, then switch to the nearest vertical charging road.

  $P(D_n < x|E_{3,2,2}, E_{3,1,3})$ is given in equation A.3 in Appendix A.

• Event $E_{3,1,4}$:

Description: If there is at least one horizontal charging road between S and D;

Probability: $P(E_{3,1,4}|E_3) = 1 - e^{-\lambda p d_v}$:

- Event $E_{3,2,3}$:

  Description: If there are no vertical charging roads between S and D;

  Probability: $P(E_{3,2,3}|E_3) = e^{-\lambda p d_h}$

  * Event $E_{3,3,1}$:

    Description: If there exists at least one horizontal non-charging road below the nearest horizontal charging road from S, as shown in Figure 4.17;

    Probability: $P(E_{3,3,1}) = 1 - \frac{p - p * e^{-\lambda d_v}}{1 - e^{-\lambda p d_v}}$;

    Action: we compare the vertical distance between the nearest horizontal charging road and the nearest horizontal non charging road, to

dh, to take the longer one.

· Event $E_{3,4,1}$:

Description: If we decide to take the nearest horizontal charging road;

Probability: $P(E_{3,4,1}|E_3) = F_{X_2}(d_h)$.

$P(D_n < x|E_{3,4,1}, E_{3,3,1}, E_{3,2,3}, E_{3,1,4})$ is given in equation A.4 in Appendix A.

· Event $E_{3,4,2}$:

Description: If we take the nearest horizontal non-charging road;

Probability: $P(E_{3,4,2}|E_3) = 1 - F_{X_2}(d_h)$.

$P(D_n < x|E_{3,4,2}, E_{3,3,1}, E_{3,2,3}, E_{3,1,4})$ is given in equation A.5 in Appendix A.

∗ Event $E_{3,3,2}$:

Description: If there does not exist horizontal non-charging roads below the nearest horizontal charging road from S, as shown in Figure 4.18;

Probability: $P(E_{3,3,2}|E_3) = \frac{p-p*e^{-\lambda d_v}}{1-e^{-\lambda p d_v}}$;

Action: we simply take the nearest horizontal charging road.

$P(D_n < x|E_{3,3,2}, E_{3,2,3}, E_{3,1,4})$ is given in equation A.6 in Appendix A.

– Event $E_{3,2,4}$:

Description: If there is at least one vertical charging road between S and D;

Probability: $P(E_{3,2,4}|E_3) = 1 - e^{-\lambda p d_h}$

∗ Event $E_{3,3,3}$:

Description: If there exists at least one horizontal non-charging road below the nearest horizontal charging road from S, as shown in Figure 4.19;

Probability: $P(E_{3,3,3}|E_3) = 1 - \frac{p - p * e^{-\lambda d_v}}{1 - e^{-\lambda p d_v}}$;

Action: we compare the distance between the nearest horizontal charging road and the nearest horizontal non-charging road, to the horizontal distance between the nearest vertical charging road and source, to take the longer one.

- Event $E_{3,4,3}$:

  Description: If we take the nearest vertical charging road;

  Probability: $P(E_{3,4,3}|E_3) = 1 - \int_0^{d_h} F_{X_2}(D_{\text{N}-\text{VC}}) f_{D_{\text{N}-\text{VC}}}(x) \mathrm{d}x)$

  $P(D_n < x|E_{3,4,3}, E_{3,3,3}, E_{3,2,4}, E_{3,1,4})$ is given in equation A.7 in Appendix A.

- Event $E_{3,4,4}$:

  Description: If we take the nearest horizontal charging road;

  Probability: $P(E_{3,4,4}|E_3) = \int_0^{d_h} F_{X_2}(D_{\text{N}-\text{VC}}) f_{D_{\text{N}-\text{VC}}}(x) \mathrm{d}x)$

  $P(D_n < x|E_{3,4,4}, E_{3,3,3}, E_{3,2,4}, E_{3,1,4})$ is given in equation A.8 in Appendix A.

* Event $E_{3,3,4}$:

  Description: If there does not exist horizontal non-charging roads below the nearest horizontal charging road from S, as shown in Figure 4.20;

  Probability: $P(E_{3,3,4}|E_3) = \frac{p - p * e^{-\lambda d_v}}{1 - e^{-\lambda p d_v}}$;

  Action: we simply take the nearest horizontal charging road.

  $P(D_n < x|E_{3,3,4}, E_{3,2,4}, E_{3,1,4})$ is given in equation A.9 in Appendix A.

4.1.4 Both source and destination roads are not charging

Let E_4 denote the case when both source and destination roads are on two parallel roads and the destination road is charging. The probability of event E_4 is $(1-p)^2$. We denote subevents as $E_{4,i,j}$, in which i is the level of depth of the event in the probability tree and j is the index of the event at that level.

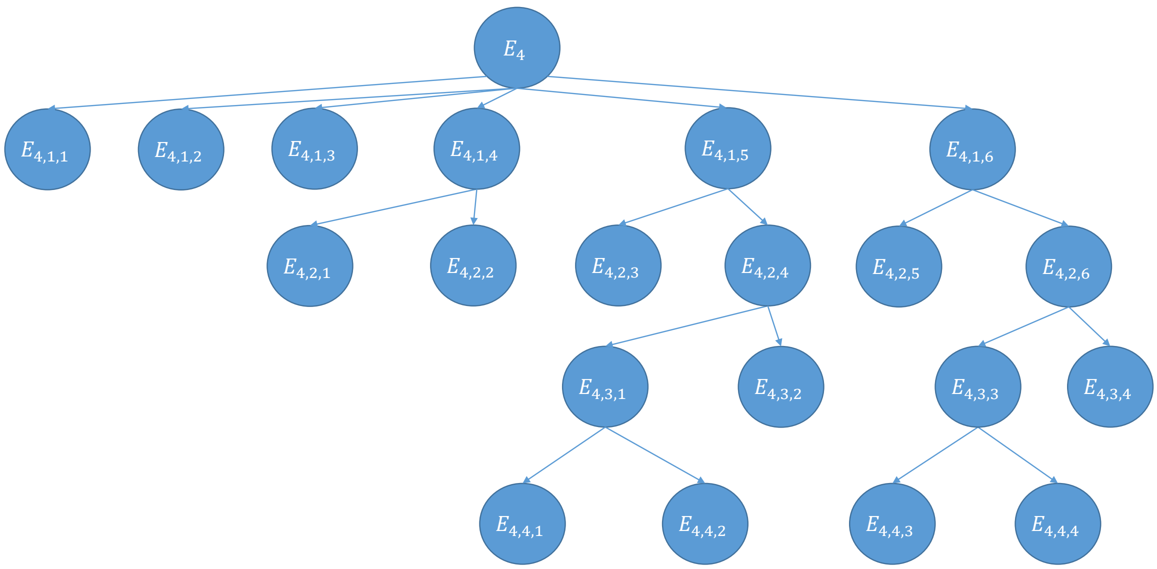

Figure 4.21: Tree E_4

We have $P(D_n < x|E_4)P(E_4) = \sum_{i=1}^{4}\sum_{j=1}^{6} P(D_n < x|E_{4,i,j})P(E_{4,i,j})$.

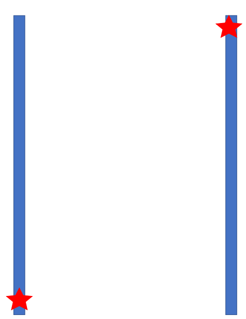

Figure 4.22: Event $E_{4,1,1}$

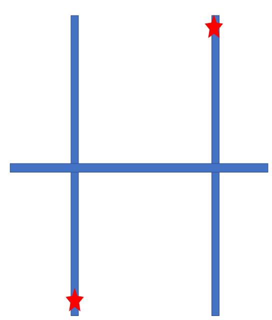

Figure 4.23: Event $E_{4,1,2}$

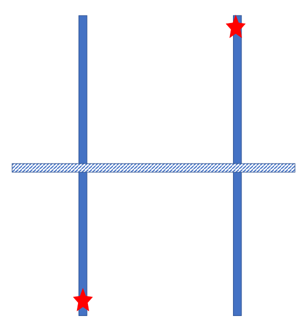

Figure 4.24: Event $E_{4,1,3}$

- Event $E_{4,1,1}$:

  Description: If there are no horizontal roads between S and D, as shown in Figure 4.22;

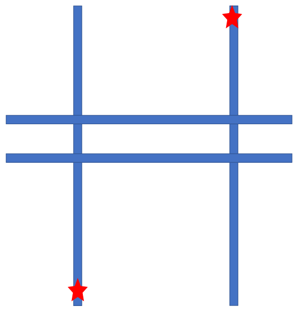

Figure 4.25: Event $E_{4,2,1}$

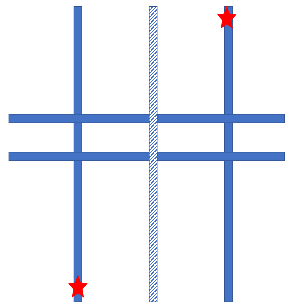

Figure 4.26: Event $E_{4,2,2}$

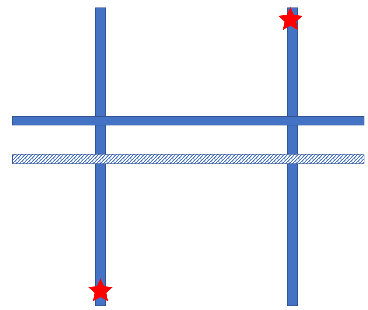

Figure 4.27: Event $E_{4,2,3}$

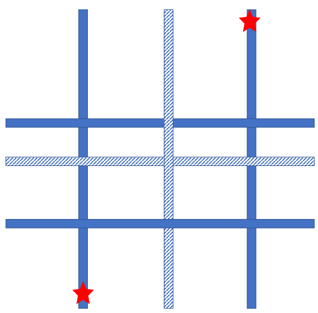

Figure 4.28: Event $E_{4,3,1}$

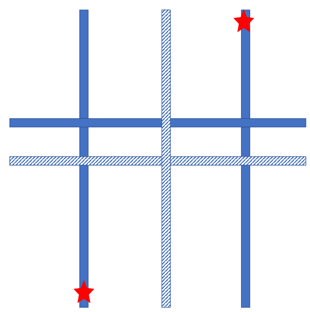

Figure 4.29: Event $E_{4,3,2}$

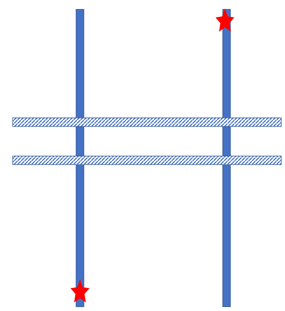

Figure 4.30: Event $E_{4,2,5}$

Probability: $P(E_{4,1,1}|E_4) = e^{-\lambda d_v}$;

Action: We simply take the shortest path from S to D.

$P(D_n < x|E_{4,1,1})$ is given in equation A.10 in Appendix A.

- Event $E_{4,1,2}$:

  Description: If there is only one horizontal non-charging road and no horizontal charging road between S and D, as shown in Figure 4.23;

  Probability: $P(E_{4,1,2}|E_4) = \lambda(1-p)d_v e^{-\lambda(1-p)d_v} e^{-\lambda p d_v}$;

  Action: We take that horizontal non-charging road.

$$P(D_n < x|E_{4,1,2}) = 0$$

- Event $E_{4,1,3}$:

  Description: If there is only one horizontal charging road and no horizontal non-charging road between S and D, as shown in Figure 4.24;

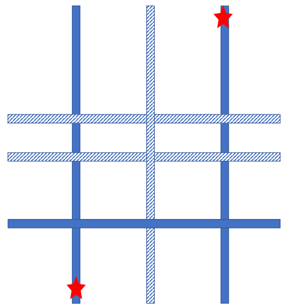

Figure 4.31: Event $E_{4,3,3}$

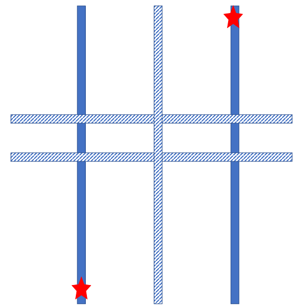

Figure 4.32: Event $E_{4,3,4}$

Probability: $P(E_{4,1,3}|E_4) = \lambda p d_v e^{-\lambda p d_v} e^{-\lambda(1-p)d_v}$;

Action: We take that horizontal charging road.

$P(D_n < x|E_{4,1,3})$ is given in equation A.11 in Appendix A.

- Event $E_{4,1,4}$:

  Description: If there are no horizontal charging roads but at least two horizontal non-charging road between S and D;

  Probability: $P(E_{4,1,4}|E_4) = e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v} - \lambda(1-p)d_v e^{-\lambda(1-p)d_v})$:

  - Event $E_{4,2,1}$:

    Description: If there are no vertical charging roads between S and D, as shown in Figure 4.25;

    Probability: $P(E_{4,2,1}|E_4) = e^{-\lambda p d_h}$;

    Action: We take any horizontal non-charging road between S and D.

$$P(D_n < x|E_{4,2,1}, E_{4,1,4}) = 0$$

  - Event $E_{4,2,2}$:

    Description: If there is at least one vertical charging road between S and D, as shown in Figure 4.26;

    Probability: $P(E_{4,2,2}|E_4) = 1 - e^{-\lambda p d_h}$;

    Action: We first go to the nearest horizontal non-charging road, then switch to the nearest vertical charging road, then switch to the furthest horizontal non-charging road.

$P(D_n < x|E_{4,2,2}, E_{4,1,4})$ is identical to $P(D_n < x|E_{3,2,2}, E_{3,1,3})$.

- Event $E_{4,1,5}$:

  Description: If there is one horizontal charging road and at least one horizontal non-charging road between S and D;

  Probability: $P(E_{4,1,5}|E_4) = \lambda p d_v e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v})$:

  - Event $E_{4,2,3}$:

    Description: If there are no vertical charging roads between S and D, as shown in Figure 4.27;

    Probability: $P(E_{4,2,3}|E_4) = e^{-\lambda p d_h}$;

    Action: We take the horizontal charging road between S and D.

    $P(D_n < x|E_{4,2,3}, E_{4,1,5})$ is given in equation A.12 in Appendix A.

  - Event $E_{4,2,4}$:

    Description: If there is at least one vertical charging road between S and D;

    Probability: $P(E_{4,2,4}|E_4) = 1 - e^{-\lambda p d_h}$:

    * Event $E_{4,3,1}$:

      Description: If there exists at least one horizontal non-charging road below the nearest horizontal charging road from S, as shown in Figure 4.28;

      Probability: $P(E_{4,3,1}|E_4) = 1 - \frac{p - p * e^{-\lambda d_v}}{1 - e^{-\lambda p d_v}}$;

      Action: we compare the distance between the nearest horizontal charging road and the nearest horizontal non-charging road, to the horizontal distance between the nearest vertical charging road and source, to take the longer one.

· Event $E_{4,4,1}$:

Description: If we take the nearest vertical charging road;

Probability: $P(E_{4,4,1}|E_4) = 1 - \int_0^{d_h} F_{X_2}(D_{\text{N}-\text{VC}}) f_{D_{\text{N}-\text{VC}}}(x)\text{d}x)$

$P(D_n < x|E_{4,4,1}, E_{4,3,1}, E_{4,2,4}, E_{4,1,5})$ is given in Appendix A.

· Event $E_{4,4,2}$:

Description: If we take the nearest horizontal charging road;

Probability: $P(E_{4,4,2}|E_4) = \int_0^{d_h} F_{X_2}(D_{\text{N}-\text{VC}}) f_{D_{\text{N}-\text{VC}}}(x)\text{d}x)$

$P(D_n < x|E_{4,4,2}, E_{4,3,1}, E_{4,2,4}, E_{4,1,5})$ is given in Appendix A.

* Event $E_{4,3,2}$:

Description: If there does not exist horizontal non-charging roads below the nearest horizontal charging road from S, as shown in Figure 4.29;

Probability: $P(E_{4,3,2}|E_4) = \frac{p - p * e^{-\lambda d_v}}{1 - e^{-\lambda p d_v}}$;

Action: we simply take the nearest horizontal charging road.

$P(D_n < x|E_{4,3,2}, E_{4,2,4}, E_{4,1,5})$ is given in Appendix A.

$$\begin{aligned} &P(D_n < x|E_{4,3,2}, E_{4,2,4}, E_{4,1,5}) \\ &= \frac{1 - e^{-\lambda p x}}{1 - e^{-\lambda p d_v}} \mathbb{1}\{x < d_v\} + \mathbb{1}\{x > d_v\}. \end{aligned}$$

- Event $E_{4,1,6}$:

  Description: If there are at least two horizontal charging roads between S and D;

  Probability: $P(E_{4,1,6}|E_4) = 1 - e^{-\lambda p d_v} - \lambda p d_v e^{-\lambda p d_v}$:

  – Event $E_{4,2,5}$:

  Description: If there are no vertical charging roads between S and D, as shown in Figure 4.30;

  Probability: $P(E_{4,2,5}|E_4) = e^{-\lambda p d_h}$;

Action: We take any horizontal charging road between S and D.

$P(D_n < x|E_{4,2,5}, E_{4,1,6})$ is identical to $P(D_n < x|E_{4,1,3})$.

- Event $E_{4,2,6}$:

  Description: If there is at least one vertical charging road between S and D;

  Probability: $P(E_{4,2,6}|E_4) = 1 - e^{-\lambda p d_h}$:

  * Event $E_{4,3,3}$:

    Description: If there exists at least one horizontal non-charging road below the nearest horizontal charging road from S, as shown in Figure 4.31;

    Probability: $P(E_{4,3,3}|E_4) = 1 - \frac{p - p * e^{-\lambda d_v}}{1 - e^{-\lambda p d_v}}$;

    Action: we compare the distance between the nearest horizontal charging road and the nearest horizontal non-charging road, to the horizontal distance between the nearest vertical charging road and source, to take the longer one.

    · Event $E_{4,4,3}$:

      Description: If we take the nearest vertical charging road;

      Probability: $P(E_{4,4,3}|E_4) = 1 - \int_0^{d_h} F_{X_2}(D_{\text{N-VC}}) f_{D_{\text{N-VC}}}(x)\text{d}x)$

      $P(D_n < x|E_{4,4,3}, E_{4,3,3}, E_{4,2,6}, E_{4,1,6})$ is given in Appendix A.

    · Event $E_{4,4,4}$:

      Description: If we take the nearest horizontal charging road;

      Probability: $P(E_{4,4,4}|E_4) = \int_0^{d_h} F_{X_2}(D_{\text{N-VC}}) f_{D_{\text{N-VC}}}(x)\text{d}x)$

      $P(D_n < x|E_{4,4,4}, E_{4,3,3}, E_{4,2,6}, E_{4,1,6})$ is given in Appendix A.

  * Event $E_{4,3,4}$:

    Description: If there does not exist horizontal non-charging roads below the nearest horizontal charging road from S, as shown in Figure 4.32;

Probability: $P(E_{4,3,4}|E_4) = \frac{p-p*e^{-\lambda d_v}}{1-e^{-\lambda p d_v}}$;

Action: we simply take the nearest horizontal charging road.

$P(D_n < x|E_{4,3,4}, E_{4,2,6}, E_{4,1,6})$ is given in Appendix A.

4.2 Case b: When source (S) and destinations (D) are on two perpendicular roads

Let B denote the case when S and D are on two parallel roads. $P(B) = \frac{1}{2}$. We consider four scenarios:

1. Both source and destination roads are charging,

2. Only source road is charging,

3. Only destination road is charging,

4. both source and destination roads are not charging.

In each scenario, we first describe its structure, i.e., the sub-events in that scenario expressed in the form of probability trees. A probability tree starts from one root. Then, it is split into levels such that events in one level are mutually exclusive and are independent from events on the other levels. Next, we discuss each event in details together with corresponding a graph of the event. Note that we provide graphs for only the events that an action needs to be made in choosing the driving route. All proofs for $P(D_n < x|E_{k,i,j})$ are available in Appendix A.

4.2.1 Both source and destination roads are charging

Let E_5 denote the case when both source and destination roads are on two perpendicular roads and are charging. The probability of event E_5 is p^2. Since both the source road and the destination road are charging, the optimal driving route is always taking the source road and the destination road. Hence, $P(D_n < x|E_5)$ is always 1.

4.2.2 Only source road is charging

Let E_6 denote the case when both source and destination roads are on two perpendicular roads and the source road is charging. The probability of event E_6 is $p(1-p)$. We denote subevents as $E_{6,i,j}$, in which i is the level of depth of the event in the probability tree and j is the index of the event at that level.

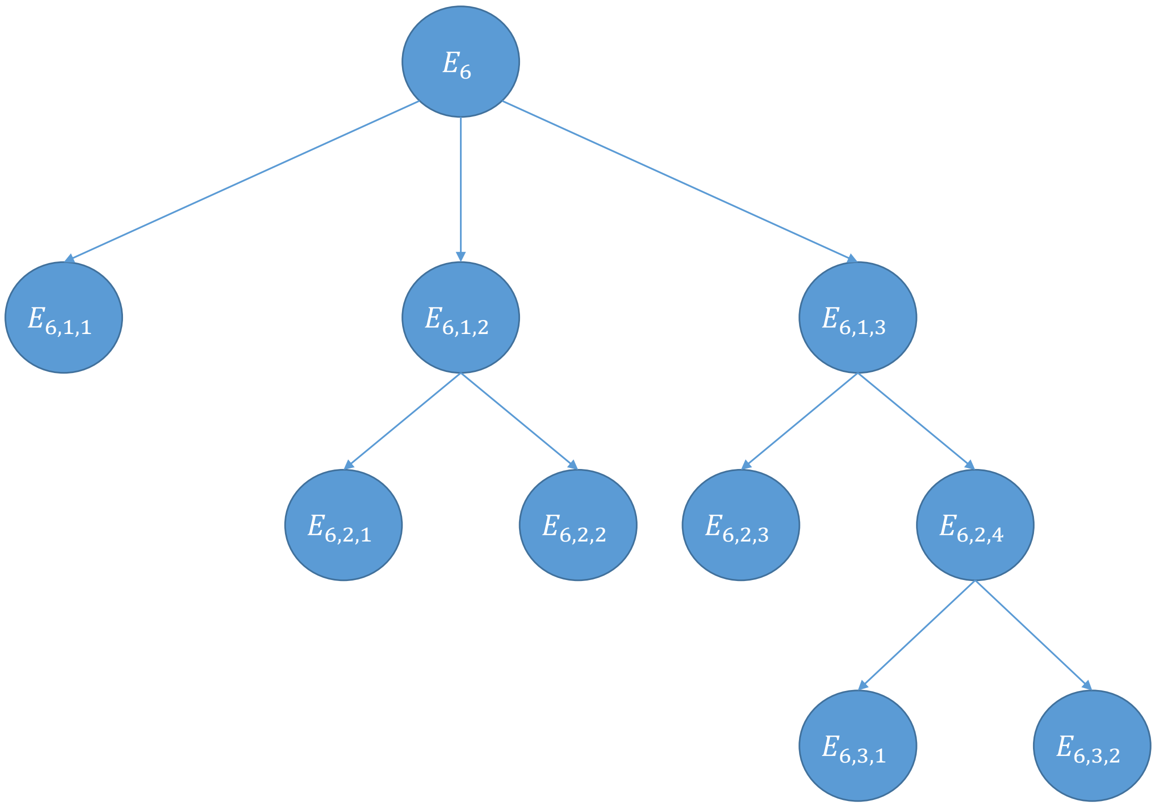

Figure 4.33: Tree E_6

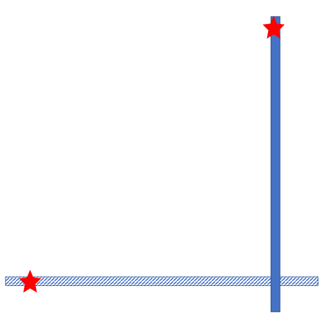

Figure 4.34: Event $E_{6,1,1}$

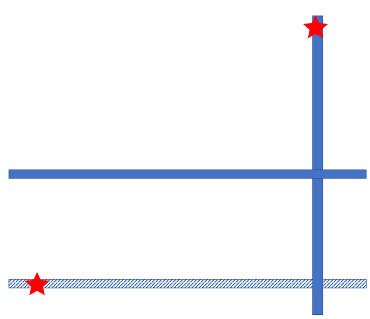

Figure 4.35: Event $E_{6,2,1}$

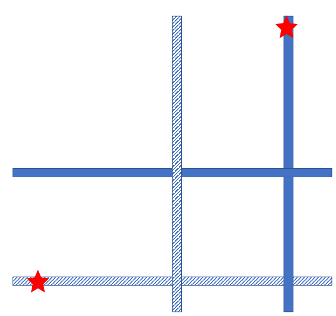

Figure 4.36: Event $E_{6,2,2}$

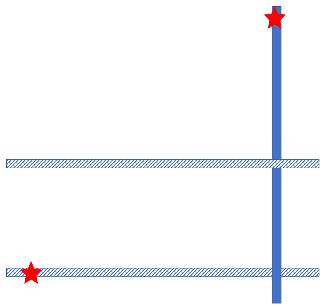

Figure 4.37: Event $E_{6,2,3}$

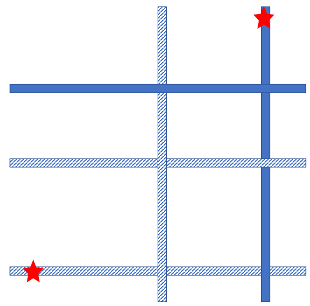

Figure 4.38: Event $E_{6,3,1}$

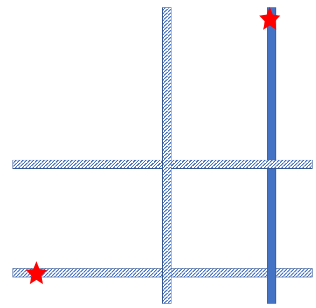

Figure 4.39: Event $E_{6,3,2}$

- Event $E_{6,1,1}$:

  Description: If there are no horizontal roads between S and D, as shown in Figure 4.34;

  Probability: $P(E_{6,1,1}|E_6) = e^{-\lambda d_v}$;

  Action: We simply take the source road then the destination road.

- Event $E_{6,1,2}$:

  Description: If there is no horizontal charging road but at least one horizontal non-charging road between S and D;

  Probability: $P(E_{6,1,2}|E_6) = e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v})$:

  - Event $E_{6,2,1}$:

    Description: If there are no vertical charging roads between S and D, as shown in Figure 4.35;

    Probability: $P(E_{6,2,1}|E_6) = e^{-\lambda p d_h}$;

    Action: We take the source road then the destination road.

  - Event $E_{6,2,2}$:

    Description: If there is at least one vertical charging road between S and D, as shown in Figure 4.36;

    Probability: $P(E_{6,2,2}|E_6) = 1 - e^{-\lambda p d_h}$;

    Action: we compare the distance between source and the farthest horizontal non-charging road to the distance between the farthest vertical charging road to destination, to take the longer one.

- Event $E_{6,1,3}$:

  Description: If there is at least one horizontal charging road between S and D;

  Probability: $P(E_{6,1,3}|E_6) = 1 - e^{-\lambda p d_v}$:

  - Event $E_{6,2,3}$:

    Description: If there are no vertical charging roads between S and D, as shown in Figure 4.37;

    Probability: $P(E_{6,2,3}|E_6) = e^{-\lambda p d_h}$;

    Action: We take the source road then the destination road.

  - Event $E_{6,2,4}$:

    Description: If there is at least one vertical charging road between S and D;

    Probability: $P(E_{6,2,4}|E_6) = 1 - e^{-\lambda p d_h}$:

    * Event $E_{6,3,1}$:

      Description: If there exists at least one horizontal non-charging road above the furthest horizontal charging road from S, as shown in Figure 4.38;

      Probability: $P(E_{6,3,1}|E_6) = 1 - \frac{p - p * e^{-\lambda d_v}}{1 - e^{-\lambda p d_v}}$;

      Action: we compare the distance between the farthest horizontal charging road and the farthest horizontal non-charging road, to the horizontal distance between the farthest vertical charging road and destination, to take the longer one.

    * Event $E_{6,3,2}$:

      Description: If there does not exist one horizontal non-charging road above the furthest horizontal charging road from S, as shown in Figure

4.39;

Probability: $P(E_{6,3,2}|E_6) = \frac{p-p*e^{-\lambda d_v}}{1-e^{-\lambda p d_v}}$;

Action: we simply go with the farthest horizontal charging road.

Since the source road is already a charging road, $P(D_n < x|E_{6,i,j}) = 1$ for all i, j.

4.2.3 Only destination road is charging

Let E_7 denote the case when both source and destination roads are on two perpendicular roads and the destination road is charging. The probability of event E_7 is $p(1-p)$. We denote subevents as $E_{7,i,j}$, in which i is the level of depth of the event in the probability tree and j is the index of the event at that level.

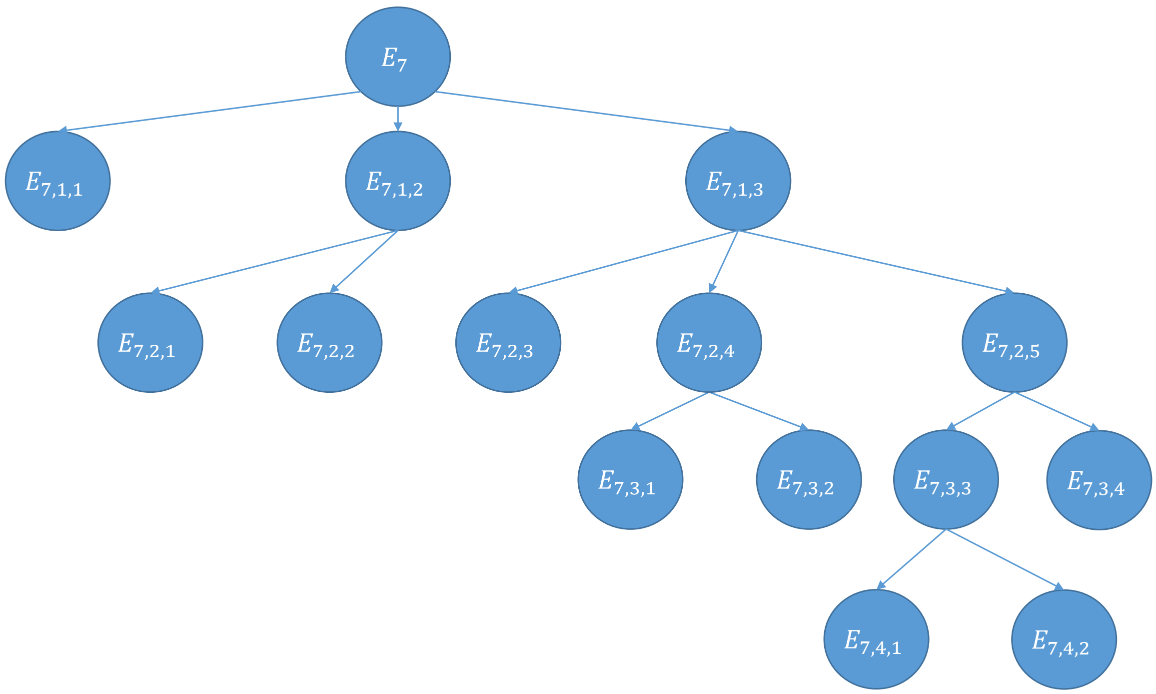

Figure 4.40: Tree E_7

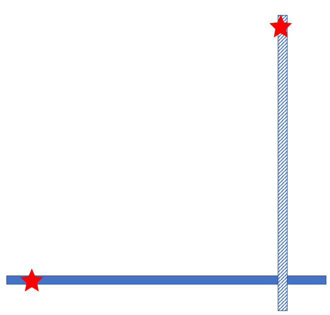

Figure 4.41: Event $E_{7,1,1}$

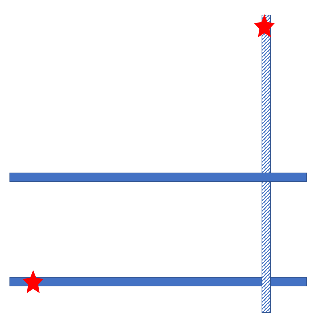

Figure 4.42: Event $E_{7,2,1}$

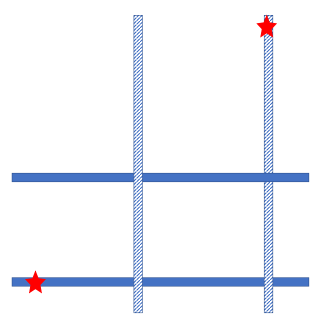

Figure 4.43: Event $E_{7,2,2}$

- Event $E_{7,1,1}$:

  Description: If there are no horizontal roads between S and D, as shown in

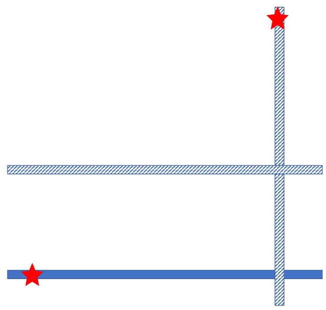

Figure 4.44: Event $E_{7,2,3}$

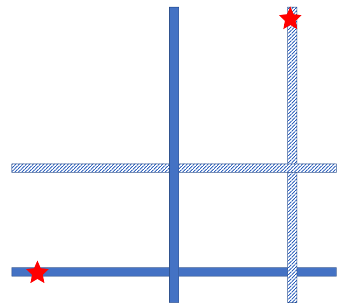

Figure 4.45: Event $E_{7,2,4}$

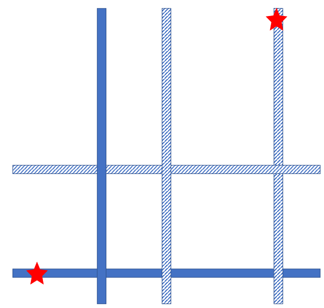

Figure 4.46: Event $E_{7,3,3}$

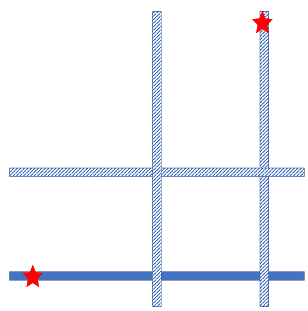

Figure 4.47: Event $E_{7,3,4}$

Figure 4.41;

Probability: $P(E_{7,1,1}|E_7) = e^{-\lambda d_v}$;

Action: We simply take the source road then the destination road.

$$P(D_n < x|E_{7,1,1}) = \mathbb{1}\{x > d_h\}.$$

- Event $E_{7,1,2}$:

  Description: If there is no horizontal charging road but at least one horizontal non-charging road between S and D;

  Probability: $P(E_{7,1,2}|E_7) = e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v})$;

  – Event $E_{7,2,1}$:

    Description: If there are no vertical charging roads between S and D, as shown in Figure 4.42;

    Probability: $P(E_{7,2,1}|E_7) = e^{-\lambda p d_h}$;

Action: We simply take the source road then the destination road.

$$P(D_n < x | E_{7,2,1}, E_{7,1,2}) = \mathbb{1}\{x > d_h\}.$$

- Event $E_{7,2,2}$:

  Description: If there is at least one vertical charging road between S and D, as shown in Figure 4.43;

  Probability: $P(E_{7,2,1}|E_7) = 1 - e^{-\lambda p d_h}$;

  Action: We take the nearest vertical charging road.

  $P(D_n < x|E_{7,2,2}, E_{7,1,2})$ is given in equation A.13 in Appendix A.

- Event $E_{7,1,3}$:

  Description: If there is at least one horizontal charging road between S and D;

  Probability: $P(E_{7,1,3}|E_7) = 1 - e^{-\lambda p d_v}$:

  - Event $E_{7,2,3}$:

    Description: If there are no vertical roads between S and D, as shown in Figure 4.44;

    Probability: $P(E_{7,2,3}|E_7) = e^{-\lambda d_h}$;

    Action: We simply take the source road then the destination road.

$$P(D_n < x | E_{7,2,3}, E_{7,1,3}) = \mathbb{1}\{x > d_h\}.$$

  - Event $E_{7,2,4}$:

    Description: If there is no vertical charging road but at least one vertical non-charging road between S and D, as shown in Figure 4.45;

    Probability: $P(E_{7,2,4}|E_7) = e^{-\lambda p d_h}(1 - e^{-\lambda(1-p)d_h})$;

    Action: We compare the distance from the nearest vertical non-charging road to destination to the distance from source to the nearest horizontal charging road.

- Event $E_{7,3,1}$:

  Description: If we take the horizontal charging road;

  Probability: $P(E_{7,3,1}|E_7) = \int_0^{d_h} \frac{1-e^{-\lambda p(dh-w)}}{1-e^{-\lambda p d_v}} * \frac{\lambda(1-p)e^{-\lambda(1-p)w}}{1-e^{-\lambda p d_v}} dw)$

  $P(D_n < x|E_{7,3,1}, E_{7,2,4}, E_{7,1,3})$ is given in equation A.14 in Appendix A.

- Event $E_{7,3,2}$:

  Description: If we take the destination vertical road;

  Probability: $P(E_{7,3,2}|E_7) = 1 - \int_0^{d_h} \frac{1-e^{-\lambda p(dh-w)}}{1-e^{-\lambda p d_v}} * \frac{\lambda(1-p)e^{-\lambda(1-p)w}}{1-e^{-\lambda p d_v}} dw)$,

$$P(D_n < x|E_{7,3,2}, E_{7,2,4}, E_{7,1,3}) = \mathbb{1}\{x > d_h\}.$$

– Event $E_{7,2,5}$:

Description: If there is at least one vertical charging road between S and D;

Probability: $P(E_{7,2,5}|E_7) = 1 - e^{-\lambda p d_h}$:

- Event $E_{7,3,3}$:

  Description: If there exists at least one vertical non-charging road before the nearest vertical charging road from source, as shown in Figure 4.46;

  Probability: $P(E_{7,3,3}|E_7) = 1 - \frac{p - p * e^{-\lambda d_h}}{1-e^{-\lambda p d_h}}$;

  Action: we compare the distance between the nearest vertical charging road and the nearest vertical non-charging road, to the vertical distance between the nearest horizontal charging road and source, to take the longer one.

  · Event $E_{7,4,1}$:

  Description: If we take the nearest horizontal charging road;

Probability: $P(E_{7,4,1}|E_7) = 1 - \int_0^{d_v} F_{X_1}(D_{\mathrm{N-HC}}) f_{D_{\mathrm{N-HC}(x)}} \mathrm{d}x)$

$P(D_n < x|E_{7,4,1}, E_{7,3,3}, E_{7,2,5}, E_{7,1,3})$ is given in equation A.15 in Appendix A.

· Event $E_{7,4,2}$:

Description: If we take the nearest vertical charging road;

Probability: $P(E_{7,4,2}|E_7) = \int_0^{d_v} F_{X_1}(D_{\mathrm{N-HC}}) f_{D_{\mathrm{N-HC}(x)}} \mathrm{d}x)$

$P(D_n < x|E_{7,4,2}, E_{7,3,3}, E_{7,2,5}, E_{7,1,3})$ is given in equation A.16 in Appendix A.

∗ Event $E_{7,3,4}$:

Description: If there exists no vertical non-charging road before the nearest vertical charging road from source, as shown in Figure 4.47;

Probability: $P(E_{7,3,4}|E_7) = \frac{p - p * e^{-\lambda d_h}}{1 - e^{-\lambda p d_v}}$;

Action: we simply go with the nearest vertical charging road.

$P(D_n < x|E_{7,3,4}, E_{7,2,5}, E_{7,1,3})$ is given in equation A.17 in Appendix A.

4.2.4 Both source and destination roads are not charging.

Let E_8 denote the case when both source and destination roads are on two perpendicular roads and both are not charging. The probability of event E_8 is $(1-p)^2$. We denote subevents as $E_{8,i,j}$, in which i is the level of depth of the event in the probability tree and j is the index of the event at that level.

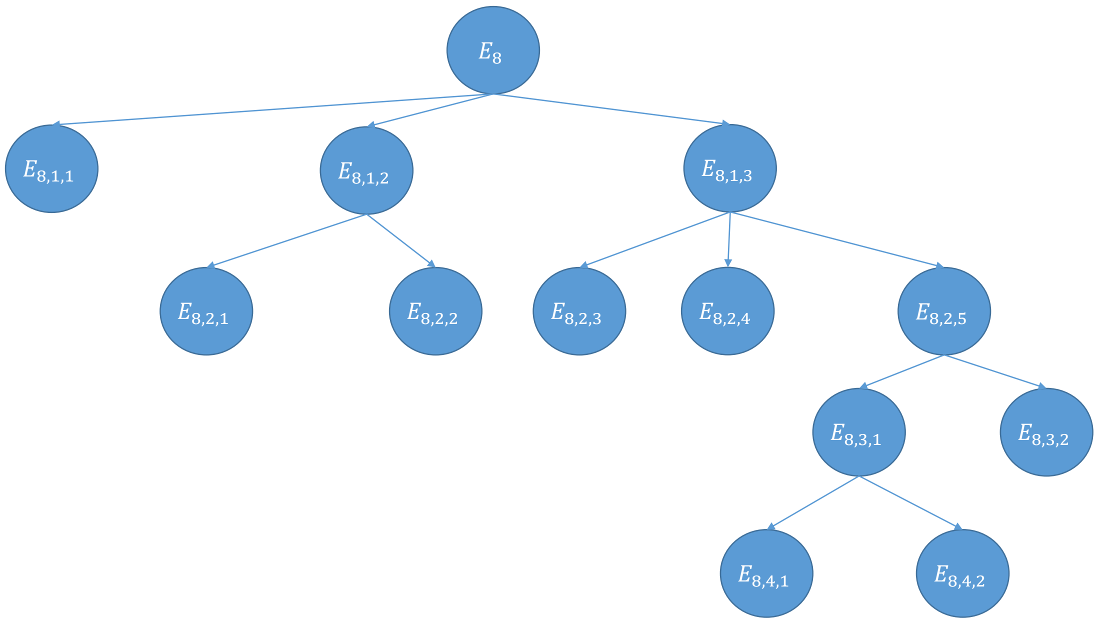

Figure 4.48: Tree E_8

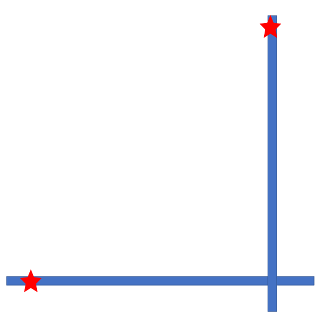

Figure 4.49: Event $E_{8,1,1}$

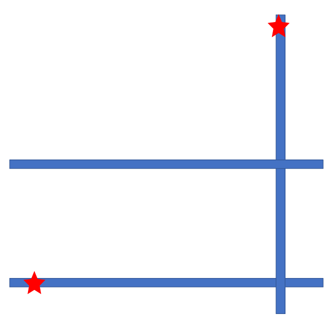

Figure 4.50: Event $E_{8,2,1}$

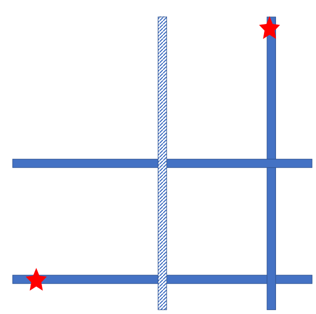

Figure 4.51: Event $E_{8,2,2}$

- Event $E_{8,1,1}$:

  Description: If there are no horizontal roads between S and D, as shown in Figure 4.49;

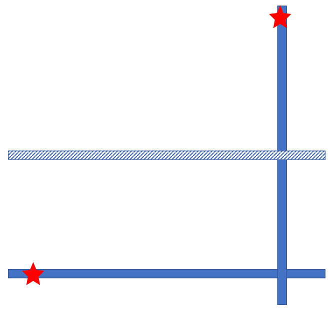

Figure 4.52: Event $E_{8,2,3}$

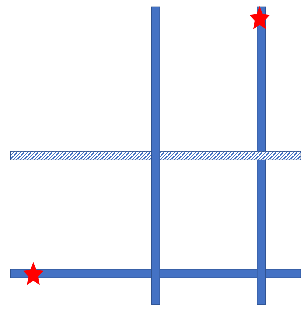

Figure 4.53: Event $E_{8,2,4}$

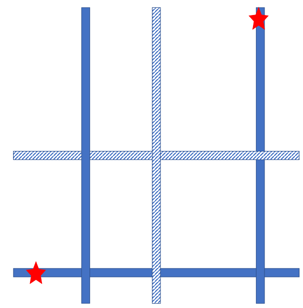

Figure 4.54: Event $E_{8,3,1}$

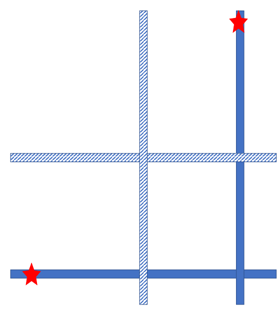

Figure 4.55: Event $E_{8,3,2}$

Probability: $P(E_{8,1,1}|E_8) = e^{-\lambda d_v}$;

Action: We simply take the source road then the destination road.

$$P(D_n < x|E_{8,1,1}) = 0.$$

- Event $E_{8,1,2}$:

  Description: If there is no horizontal charging road but at least one horizontal non-charging road between S and D;

  Probability: $P(E_{8,1,2}|E_8) = e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v})$:

  - Event $E_{8,2,1}$:

    Description: If there are no vertical charging roads between S and D, as shown in Figure 4.50;

    Probability: $P(E_{8,2,1}|E_8) = e^{-\lambda p d_h}$;

Action: We take the source road then the destination road.

$$P(D_n < x | E_{8,2,1}, E_{8,1,2}) = 0.$$

- Event $E_{8,2,2}$:

  Description: If there is at least one vertical charging road between S and D, as shown in Figure 4.51;

  Probability: $P(E_{8,2,2}|E_8) = 1 - e^{-\lambda p d_h}$;

  Action: We go to the nearest vertical charging road from source, then switch to the furthest horizontal non-charging road from source.

  $P(D_n < x|E_{8,2,2}, E_{8,1,2})$ is identical to $P(D_n < x|E_{7,2,2}, E_{7,1,2})$.

- Event $E_{8,1,3}$:

  Description: If there is at least one horizontal charging road between S and D;

  Probability: $P(E_{8,1,3}|E_8) = 1 - e^{-\lambda p d_v}$:

  - Event $E_{8,2,3}$:

    Description: If there are no vertical roads between S and D, as shown in Figure 4.52;

    Probability: $P(E_{8,2,3}|E_8) = e^{-\lambda d_h}$;

    Action: We take the source road then the destination road.

$$P(D_n < x | E_{8,2,3}, E_{8,1,3}) = 0.$$

  - Event $E_{8,2,4}$:

    Description: If there is no vertical charging road but at least one vertical non-charging road between S and D, as shown in Figure 4.53;

    Probability: $P(E_{8,2,4}|E_8) = e^{-\lambda p d_h}(1 - e^{-\lambda(1-p)d_h})$;

    Action: We go to the nearest vertical non-charging road, then switch to any horizontal charging road between S and D.

$P(D_n < x|E_{8,2,4}, E_{8,1,3})$ is given in equation A.1 in Appendix A.

- Event $E_{8,2,5}$:

  Description: If there is at least one vertical charging road between S and D;

  Probability: $P(E_{8,2,5}|E_8) = 1 - e^{-\lambda p d_h}$:

  * Event $E_{8,3,1}$:

    Description: If there exists at least one vertical non-charging road before the nearest vertical charging road from S, as shown in Figure 4.54;

    Probability: $P(E_{8,3,1}|E_8) = 1 - \frac{p - p * e^{-\lambda d_h}}{1 - e^{-\lambda p d_h}}$;

    Action: we compare the distance between the nearest vertical charging road and the nearest vertical non-charging road, to the vertical distance between the nearest horizontal charging road and source, to take the longer one.

    · Event $E_{8,4,1}$:

    Description: If we take the nearest horizontal charging road;

    Probability: $P(E_{8,4,1}|E_8) = 1 - \int_0^{d_v} F_{X_1}(D_{\mathrm{N-HC}}) f_{D_{\mathrm{N-HC}}(x)} \mathrm{d}x)$

    $P(D_n < x|E_{8,4,1}, E_{8,3,1}, E_{8,2,5}, E_{8,1,3})$ is given in Appendix A.

    · Event $E_{8,4,2}$:

    Description: If we take the nearest vertical charging road;

    Probability: $P(E_{8,4,2}|E_8) = \int_0^{d_v} F_{X_1}(D_{\mathrm{N-HC}}) f_{D_{\mathrm{N-HC}}(x)} \mathrm{d}x)$

    $P(D_n < x|E_{8,4,2}, E_{8,3,1}, E_{8,2,5}, E_{8,1,3})$ is given in Appendix A.

  * Event $E_{8,3,2}$:

    Description: If there exists no vertical non-charging road before the nearest vertical charging road from source, as shown in Figure 4.55;

    Probability: $P(E_{8,3,2}|E_8) = \frac{p - p * e^{-\lambda d_h}}{1 - e^{-\lambda p d_h}}$;

Action: we simply go with the nearest vertical charging road.

$P(D_n < x | E_{8,3,2}, E_{8,2,5}, E_{8,1,3})$ is given in Appendix A.

4.3 Extension

In this section, we briefly discuss two possible extensions that can be derived from the distribution of the distance to the nearest charging road. The first one is the distribution of the lowerbound on the portion of distance traveled on non-charging road over the trip, i.e., the distribution of $\frac{D_n}{d_h+d_v}$. The second one is the distribution of the upperbound on the portion of distance traveled on charging road throughout the trip, i.e., the distribution of $1 - \frac{D_n}{d_h+d_v}$. These two extensions provide a relative view on the utilization of charging roads with respective to the total distance traveled in a trip. Hence, they serve as useful companions with the distance to the nearest charging road, which yields the absolute values. In Section 4.1 and Section 4.2, since we already discussed the distribution of D_n, the following distributions of $\frac{D_n}{d_h+d_v}$ and $1 - \frac{D_n}{d_h+d_v}$ can be obtained using simple random variable transformation.

Chapter 5

Probability that any given trip passes through at least one charging road

We denote the event that any given trip passes through at least one charging road T_c, the event that any given trip passes through no charging road $\overline{T_c}$. In this section, we calculate the probability $P(T_c)$ based on the routing policy we made in section 4.

$$P(T_c) = 1 - P(\overline{T_c}) = 1 - \sum_{i=1}^{8} P(\overline{T_c}|E_i)P(E_i),$$

where E_i's are defined in section 4. It is apparent that $P(\overline{T_c}|E_i) = 0$ for $i \in \{1, 2, 3, 5, 6, 7\}$ since at least one of the source and destination roads is already a charging road in those cases. Hence, the probability of interest is reduced as follows:

$$P(T_c) = 1 - \sum_{i=4,8} P(\overline{T_c}|E_i)P(E_i).$$

Following the routing policy in each case, we have

$$P(\overline{T_c}|E_4)P(E_4) = [e^{-\lambda d_v}(1-p) + \lambda(1-p)d_v e^{-\lambda(1-p)d_v} e^{-\lambda p d_v}$$
$$+ e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v} - \lambda(1-p)d_v e^{-\lambda(1-p)d_v})e^{-\lambda p d_h}] * (1-p)^2,$$

$$P(\overline{T_c}|E_8)P(E_8)$$
$$= [e^{-\lambda d_v} + e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v})e^{-\lambda p d_h} + (1 - e^{-\lambda p d_v})e^{-\lambda d_h}] * (1-p)^2.$$

Proof. See Appendix A. □

Chapter 6

Numerical Results

In this section, we present the analytical and simulation results of the two performance metrics with various values of $p > 0$.

6.1 Distribution of the distance to the nearest charging road

Since our analysis focuses on the deployment of charging roads in a metropolitan setting, we select d_h and d_v to be 10 kilometers (km) and 12km, respectively, so that it represents a typical trip in a metropolitan city. We also set $\lambda = 0.005$ (road/meter), which corresponds to one road every 200 meters. It is apparent that we can choose other values for those parameters and our analysis will still hold. The distribution of the distance to the nearest charging road, i.e., $P(D_n < x)$, for this parameter set is shown in Figure 6.1. We observe the overall trend that as the density of charging road increases, i.e., higher values of p, the quicker $P(D_n < x)$ goes to one, or in other words, the closer the nearest charging road is from the source. In addition, for all the curves of $P(D_n < x)$ with different values of p, for a small value of x, $P(D_n < x)$ is p, which is intuitive since p is exactly the probability that the source road is a charging road. Another interesting observation is that even though the trip length d_h, d_v are 10km and 12km, respectively, after about only 1km, we are almost surely will come across a charging road. These insightful findings may benefit urban planers and policy makers to design how densely it needs to deploy charging roads. Car manufacturers can also refer to this metric to customize the battery size for electric vehicles sale in a specific city, considering the traffic volume of that city.

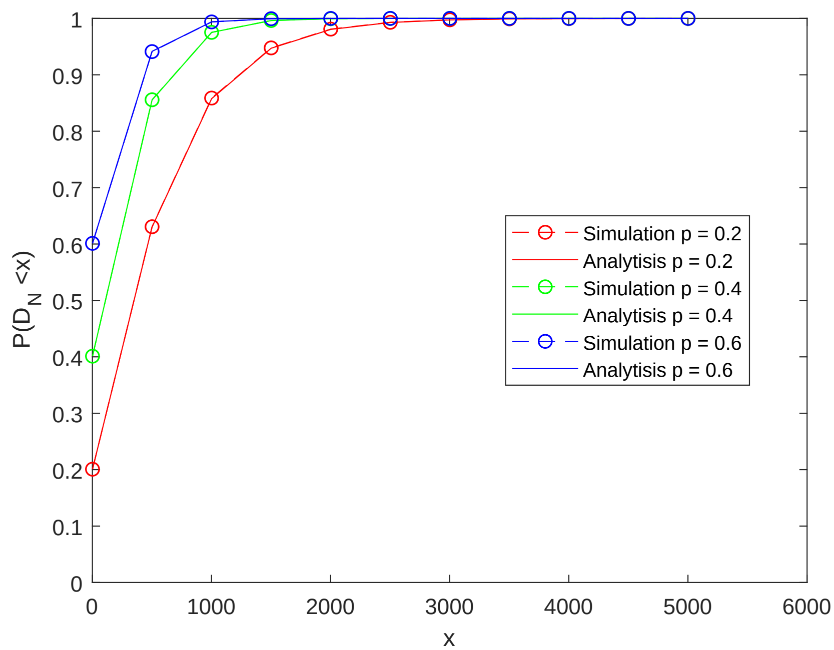

Figure 6.1: $P(D_n < x)$ when $d_h < d_v$

6.2 Probability that any given trip passes through at least one charging road

In this subsection we demonstrate the result for the probability that any given trip passes through at least one charging road, i.e., $P(T_c)$. In Figure 6.2, we plot $P(T_c)$ as a function of the Manhattan distance distance between the source and the destination. The trend is that as the distance between source and destination increases, it is more certain that the trip passes through at least one charging road.

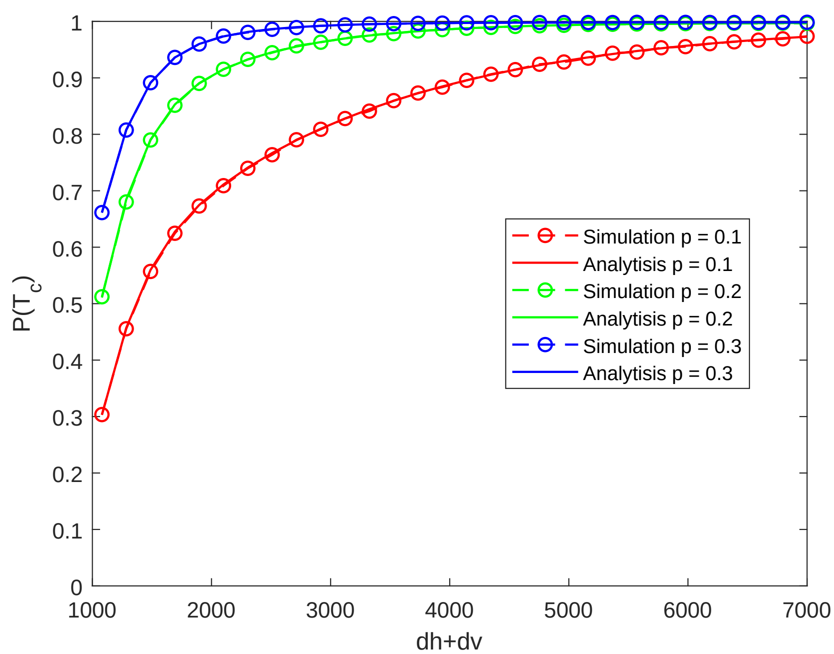

Figure 6.2: $P(T_c)$

Chapter 7

Conclusion & Future Work

In this thesis, we introduced a framework using stochastic geometry to assess the deployment of charging roads in a metropolitan setting. We provided a routing policy for drivers such that the shortest route is always selected and the time spent on charging roads throughout the trip is maximized. Then, we proposed analytical solutions to the two performance metrics: (i) the distribution of the distance from the source to the nearest charging road, and (ii) the probability that any given trip passes through at least one charging road. This analytical framework takes an important steps towards a better understanding of the charging roads deployment in metropolitan cities and provides insights for various groups such as city planners, policy makers, car manufacturers, and drivers.

Further extension to this thesis may include spatial and temporal information about the traffic condition so that practical issues such as traffic congestion, latency, and scheduling can be taken into account to formulate a more accurate routing policy and update the two performance metrics.

REFERENCES

[1] D. Sperling, *Future drive: Electric vehicles and sustainable transportation.* Island Press, 2013.

[2] M. Catenacci, E. Verdolini, V. Bosetti, and G. Fiorese, "Going electric: Expert survey on the future of battery technologies for electric vehicles," *Energy Policy*, vol. 61, pp. 403 – 413, 2013. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0301421513005855

[3] L. Lu, X. Han, J. Li, J. Hua, and M. Ouyang, "A review on the key issues for lithium-ion battery management in electric vehicles," *Journal of Power Sources*, vol. 226, pp. 272 – 288, 2013. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0378775312016163

[4] B. Nykvist and M. Nilsson, "Rapidly falling costs of battery packs for electric vehicles," *Nature climate change*, vol. 5, no. 4, pp. 329–332, 2015.

[5] A. Abdalrahman and W. Zhuang, "PEV charging infrastructure siting based on spatial-temporal traffic flow distribution," *IEEE Transactions on Smart Grid*, vol. PP, pp. 1–1, 01 2019.

[6] ——, "A survey on pev charging infrastructure: Impact assessment and planning," *Energies*, vol. 10, 10 2017.

[7] A. Abdalrahman and W. Zhuang, "QoS-aware capacity planning of networked PEV charging infrastructure," *IEEE Open Journal of Vehicular Technology*, pp. 1–1, 2020.

[8] C. Panchal, S. Stegen, and J. Lu, "Review of static and dynamic wireless electric vehicle charging system," *Engineering Science and Technology, an International Journal*, vol. 21, no. 5, pp. 922 – 937, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S221509861830154X

[9] H. Wu, A. Gilchrist, K. Sealy, P. Israelsen, and J. Muhs, "A review on inductive charging for electric vehicles," 06 2011, pp. 143 – 147.

[10] T. Stamati and P. Bauer, "On-road charging of electric vehicles," in *2013 IEEE Transportation Electrification Conference and Expo (ITEC)*, June 2013, pp. 1–8.

[11] C. C. Mi, G. Buja, S. Y. Choi, and C. T. Rim, "Modern advances in wireless power transfer systems for roadway powered electric vehicles," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 10, pp. 6533–6545, Oct 2016.

[12] P. Machura and Q. Li, "A critical review on wireless charging for electric vehicles," *Renewable and Sustainable Energy Reviews*, vol. 104, pp. 209 – 234, 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1364032119300383

[13] Y. J. Jang, E. S. Suh, and J. Kim, "System architecture and mathematical models of electric transit bus system utilizing wireless power transfer technology," *IEEE Systems Journal*, vol. 99, pp. 1–12, 01 2015.

[14] J. T. Boys and G. A. Covic, "The inductive power transfer story at the university of auckland," *IEEE Circuits and Systems Magazine*, vol. 15, no. 2, pp. 6–27, Secondquarter 2015.

[15] M. Budhia, J. T. Boys, G. A. Covic, and C. Huang, "Development of a single-sided flux magnetic coupler for electric vehicle IPT charging systems," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 1, pp. 318–328, Jan 2013.

[16] O. C. Onar, J. M. Miller, S. L. Campbell, C. Coomer, C. P. White, and L. E. Seiber, "Oak Ridge national laboratory wireless power transfer development for sustainable campus initiative," in *2013 IEEE Transportation Electrification Conference and Expo (ITEC)*, June 2013, pp. 1–8.

[17] O. C. Onar, S. L. Campbell, L. E. Seiber, C. P. White, and M. Chinthavali, "A high-power wireless charging system development and integration for a toyota rav4 electric vehicle," in *2016 IEEE Transportation Electrification Conference and Expo (ITEC)*, June 2016, pp. 1–8.

[18] O. C. Onar, M. Chinthavali, S. L. Campbell, L. E. Seiber, C. P. White, and V. P. Galigekere, "Modeling, simulation, and experimental verification of a 20-kw series-series wireless power transfer system for a toyota rav4 electric vehicle," in *2018 IEEE Transportation Electrification Conference and Expo (ITEC)*, June 2018, pp. 874–880.

[19] "Qualcomm demonstrates dynamic electric vehicle charging," https://www.qualcomm.com/news/releases/2017/05/18/qualcomm-demonstrates-dynamic-electric-vehicle-charging, accessed: 2020-03-18.

[20] "Witricity acquires qualcomm halo," https://witricity.com/witricity-acquires-qualcomm-halo, accessed: 2020-03-18.

[21] V. V. Chetlur and H. Dhillon, "Coverage analysis of a vehicular network modeled as cox process driven by poisson line process," *IEEE Transactions on Wireless Communications*, vol. PP, 09 2017.

[22] V. V. C. Ravi, H. S. Dhillon, and C. P. Dettmann, "Characterizing shortest paths in road systems modeled as manhattan poisson line processes," *CoRR*, vol. abs/1811.11332, 2018. [Online]. Available: http://arxiv.org/abs/1811.11332

[23] J. Dai and D. C. Ludois, "A survey of wireless power transfer and a critical comparison of inductive and capacitive coupling for small gap applications," *IEEE Transactions on Power Electronics*, vol. 30, no. 11, pp. 6017–6029, 2015.

[24] M. Yilmaz, V. T. Buyukdegirmenci, and P. T. Krein, "General design requirements and analysis of roadbed inductive power transfer system for dynamic electric vehicle charging," in *2012 IEEE Transportation Electrification Conference and Expo (ITEC)*, June 2012, pp. 1–6.

[25] W. Zhang, S. Wong, C. K. Tse, and Q. Chen, "An optimized track length in roadway inductive power transfer systems," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 2, no. 3, pp. 598–608, Sep. 2014.

[26] G. R. Nagendra, G. A. Covic, and J. T. Boys, "Sizing of inductive power pads for dynamic charging of EVs on IPT highways," *IEEE Transactions on Transportation Electrification*, vol. 3, no. 2, pp. 405–417, June 2017.

[27] S. Chopra and P. Bauer, "Driving range extension of ev with on-road contactless power transfer-a case study," *Industrial Electronics, IEEE Transactions on*, vol. 60, 11 2011.

[28] S. P. Hoogendoorn and P. H. L. Bovy, "State-of-the-art of vehicular traffic flow modelling," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 215, no. 4, pp. 283–303, 2001. [Online]. Available: https://doi.org/10.1177/095965180121500402

[29] P. Erdős and A. Rényi, "On the evolution of random graphs," *Publ. Math. Inst. Hung. Acad. Sci*, vol. 5, no. 1, pp. 17–60, 1960.

[30] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," *nature*, vol. 393, no. 6684, p. 440, 1998.

[31] D. J. Aldous, J. Shun *et al.*, "Connected spatial networks over random points and a route-length statistic," *Statistical Science*, vol. 25, no. 3, pp. 275–288, 2010.

[32] F. Baccelli, M. Klein, M. Lebourges, and S. Zuyev, "Stochastic geometry and architecture of communication networks," *Telecommunication Systems*, vol. 7, pp. 209–227, 06 1997.

[33] C. Gloaguen, F. Fleischer, H. Schmidt, and V. Schmidt, "Analysis of shortest paths and subscriber line lengths in telecommunication access networks," *Networks and Spatial Economics*, vol. 10, pp. 15–47, 02 2010.

[34] S. Chiu, D. Stoyan, W. Kendall, and J. Mecke, *Stochastic Geometry and Its Applications*, 09 2013.

[35] C.-S. Choi and F. Baccelli, "Poisson cox point processes for vehicular networks," *IEEE Transactions on Vehicular Technology*, 01 2018.

[36] Y. Wang, K. Venugopal, A. Molisch, and R. Heath, "Mmwave vehicle-to-infrastructure communication: Analysis of urban microcellular networks," *IEEE Transactions on Vehicular Technology*, vol. PP, 02 2017.

[37] L. Yan, H. Shen, J. Zhao, C. Xu, F. Luo, and C. Qiu, "Catcharger: Deploying wireless charging lanes in a metropolitan road network through categorization and clustering of vehicle traffic," *IEEE INFOCOM 2017 - IEEE Conference on Computer Communications*, pp. 1–9, 2017.

[38] M. Haenggi, *Stochastic Geometry for Wireless Networks.* Cambridge University Press, 2012.

[39] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Modeling and analysis of k-tier downlink heterogeneous cellular networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 3, pp. 550–560, April 2012.

[40] J. G. Andrews, A. K. Gupta, and H. S. Dhillon, "A primer on cellular network analysis using stochastic geometry," *CoRR*, vol. abs/1604.03183, 2016. [Online]. Available: http://arxiv.org/abs/1604.03183

APPENDICES

A Appendix A

In this appendix, we first provide the distribution of some entities that appear frequently in the later proofs. The proof for each cases in Chapter 4 and Chapter 5 are then enumerated.

Let X_1 be the distance between the nearest vertical non-charging road and the nearest vertical charging road from source. The CDF of X_1 is given by

$$F_{X_1}(w) = P(X_1 < w) = 1 - \int_x^{d_h} \frac{1 - e^{-\lambda(1-p)(t-x)}}{1 - e^{-\lambda(1-p)t}} * \frac{\lambda p e^{-\lambda p t}}{1 - e^{-\lambda p d_h}} \mathrm{d}t.$$

The PDF of X_1 is given by

$$\int_x^{d_h} \frac{\lambda^2 (1-p) p e^{-\lambda p t - \lambda(1-p)(t-x)}}{(1 - e^{-\lambda p d_h}) * (1 - e^{-\lambda(1-p)t})} \mathrm{d}t.$$

Let X_2 be the distance between the nearest horizontal non-charging road and the nearest horizontal charging road from source. The CDF of X_2 is given by

$$F_{X_2}(w) = P(X_2 < w) = 1 - \int_x^{d_v} \frac{1 - e^{-\lambda(1-p)(t-x)}}{1 - e^{-\lambda(1-p)t}} * \frac{\lambda p e^{-\lambda p t}}{1 - e^{-\lambda p d_v}} \mathrm{d}t.$$

The PDF of X_2 is given by

$$\int_{x}^{d_v} \frac{\lambda^2(1-p)pe^{-\lambda pt-\lambda(1-p)(t-x)}}{(1-e^{-\lambda pd_v}) * (1-e^{-\lambda(1-p)t})}\mathrm{d}t.$$

Let $D_{\mathrm{N-HC}}$ be the distance from source to the nearest horizontal charging road. The CDF of $D_{\mathrm{N-HC}}$ is

$$P(D_{\mathrm{N-HC}} < x) = 1 - e^{-\lambda px}.$$

The PDF of $D_{\mathrm{N-HC}}$ is

$$f_{D_{\mathrm{N-HC}}}(x) = \lambda pe^{-\lambda px}.$$

Let $D_{\mathrm{N-VC}}$ be the distance from source to the nearest vertical charging road. The CDF of $D_{\mathrm{N-VC}}$ is

$$P(D_{\mathrm{N-VC}} < x) = 1 - e^{-\lambda px}.$$

The PDF of $D_{\mathrm{N-VC}}$ is

$$f_{D_{\mathrm{N-VC}}}(x) = \lambda pe^{-\lambda px}.$$

Let $D_{\mathrm{N-HNC}}$ be the distance from source to the nearest horizontal charging road. The CDF of $D_{\mathrm{N-HNC}}$ is

$$P(D_{\mathrm{N-HNC}} < x) = 1 - e^{-\lambda(1-p)x}.$$

The PDF of $D_{\mathrm{N-HNC}}$ is

$$f_{D_{\mathrm{N-HNC}}}(x) = \lambda(1-p)e^{-\lambda(1-p)x}.$$

Let $D_{\mathrm{N-VNC}}$ be the distance from source to the nearest vertical charging road. The CDF of $D_{\mathrm{N-VNC}}$ is

$$P(D_{\mathrm{N-VNC}} < x) = 1 - e^{-\lambda(1-p)x}.$$

The PDF of $D_{\mathrm{N-VNC}}$ is

$$f_{D_{\mathrm{N-VNC}}}(x) = \lambda(1-p)e^{-\lambda(1-p)x}.$$

Let d_L be the distance from source to the nearest horizontal road in the opposite direction of the destination. The CDF of d_L is

$$P(d_L < x) = 1 - e^{-\lambda x}.$$

The PDF of d_L is

$$f_{d_L}(x) = \lambda e^{-\lambda x}.$$

Event $E_{3,1,1}$:

$$
\begin{aligned}
&P(D_n < x|E_{3,1,1})\\
&= P(D_n < x|D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v)\\
&= p * P(D_{\text{N-HC}} < x|D_{\text{N-HC}} < D_{\text{N-HNC}}, D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v, D_{\text{N-HC}} < d_L + d_v)\\
&+ (1-p) * P(D_{\text{N-HNC}} + d_h < x|D_{\text{N-HC}} > D_{\text{N-HNC}}, D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v,\\
&D_{\text{N-HNC}} < d_L + d_v)\\
&+ p * P(d_L < x|D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v, D_{\text{N-HC}} < D_{\text{N-HNC}}, d_L < D_{\text{N-HC}} - d_v)\\
&+ p * P(d_L < x|D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v, D_{\text{N-HC}} > D_{\text{N-HNC}}, d_L < D_{\text{N-HNC}} - d_v)\\
&+ (1-p) * P(d_L + d_h < x|D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v, D_{\text{N-HC}} < D_{\text{N-HNC}}, d_L < D_{\text{N-HC}} - d_v)\\
&+ (1-p) * P(d_L + d_h < x|D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v, D_{\text{N-HC}} > D_{\text{N-HNC}}, d_L < D_{\text{N-HNC}} - d_v)\\
&= p * \frac{\int_{x-d_v}^{\infty}[f_1 + f_2 + f_3] f_{d_L}(b)\mathrm{d}b}{\int_0^{\infty}[f_4 + f_5] f_{d_L}(b)\mathrm{d}b}\\
&+ (1-p) * \frac{f_6 + f_7 + f_8}{\int_0^{\infty}[f_9 + f_{10}] f_{d_L}(b)\mathrm{d}b}\\
&+ p * \frac{\int_{d_v}^{\infty}[\int_0^{\min(x, w-d_v)}(F_{D_{\text{N-HC}}}(w) - F_{D_{\text{N-HC}}}(b + d_v)) f_{d_L}(b)\mathrm{d}b] f_{D_{\text{N-HNC}}}(w)\mathrm{d}w}{\int_{d_v}^{\infty}[\int_0^{w-d_v}(F_{D_{\text{N-HC}}}(w) - F_{D_{\text{N-HC}}}(b + d_v)) f_{d_L}(b)\mathrm{d}b] f_{D_{\text{N-HNC}}}(w)\mathrm{d}w}\\
&+ p * \frac{\int_{d_v}^{\infty}[\int_0^{\min(x, a-d_v)}(F_{D_{\text{N-HNC}}}(a) - F_{D_{\text{N-HNC}}}(b + d_v)) f_{d_L}(b)\mathrm{d}b] f_{D_{\text{N-HC}}}(a)\mathrm{d}a}{\int_{d_v}^{\infty}[\int_0^{a-d_v}(F_{D_{\text{N-HNC}}}(a) - F_{D_{\text{N-HNC}}}(b + d_v)) f_{d_L}(b)\mathrm{d}b] f_{D_{\text{N-HC}}}(a)\mathrm{d}a}\\
&+ (1-p) * \frac{\int_{d_v}^{\infty}[\int_0^{\min(x-d_h, w-d_v)}(F_{D_{\text{N-HC}}}(w) - F_{D_{\text{N-HC}}}(b + d_v)) \mathbb{1}\{x > d_h\} f_{d_L}(b)\mathrm{d}b] f_{D_{\text{N-HNC}}}(w)\mathrm{d}w}{\int_{d_v}^{\infty}[\int_0^{w-d_v}(F_{D_{\text{N-HC}}}(w) - F_{D_{\text{N-HC}}}(b + d_v)) f_{d_L}(b)\mathrm{d}b] f_{D_{\text{N-HNC}}}(w)\mathrm{d}w}\\
&+ (1-p) * \frac{f_{11} + f_{12}}{\int_0^{\infty}[f_{13} + f_{14}] f_{d_L}(b)\mathrm{d}b},
\end{aligned}
\tag{A.1}
$$

where

$$f_1 = \int_x^{\infty} (F_{D_{\text{N-HC}}}(x) - F_{D_{\text{N-HC}}}(d_v)) \mathbb{1}\{x > d_v\} f_{D_{\text{N-HNC}}}(w)\mathrm{d}w,$$

$$f_2 = \int_{d_v}^{\min(x, b+d_v)} (F_{D_{\text{N-HC}}}(w) - F_{D_{\text{N-HC}}}(d_v)) f_{D_{\text{N-HNC}}}(w)\mathrm{d}w,$$

$$f_3 = \int_{b+dv}^{\infty} (F_{D_{\mathrm{N-HC}}}(b+d_v) - F_{D_{\mathrm{N-HC}}}(d_v))\mathbb{1}\{b+d_v < x\} f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w,$$

$$f_4 = \int_{d_v}^{\infty} (F_{D_{\mathrm{N-HC}}}(w) - F_{D_{\mathrm{N-HC}}}(d_v))\mathbb{1}\{b+d_v > w > d_v\} f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w,$$

$$f_5 = \int_{b+d_v}^{\infty} (F_{D_{\mathrm{N-HC}}}(b+d_v) - F_{D_{\mathrm{N-HC}}}(d_v))\mathbb{1}\{w > b+d_v\} f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w,$$

$$f_6 = \int_{x-d_h-d_v}^{\infty} \int_{x-d_h}^{\infty} (F_{D_{\mathrm{N-HNC}}}(x-d_h) - F_{D_{\mathrm{N-HNC}}}(d_v))\mathbb{1}\{x-d_h > d_v\} f_{D_{\mathrm{N-HC}}}(a)\mathrm{d}a f_{d_L}(b)\mathrm{d}b,$$

$$f_7 = \int_{0}^{\infty} \int_{d_v}^{\min(x-d_h, b+d_v)} (F_{D_{\mathrm{N-HNC}}}(a) - F_{D_{\mathrm{N-HNC}}}(d_v))\mathbb{1}\{x-d_h > d_v\} f_{D_{\mathrm{N-HC}}}(a)\mathrm{d}a f_{d_L}(b)\mathrm{d}b,$$

$$f_8 = \int_{0}^{x-d_h-d_v} \int_{0}^{b+d_v} (F_{D_{\mathrm{N-HNC}}}(b+d_v) - F_{D_{\mathrm{N-HNC}}}(d_v))\mathbb{1}\{x-d_h > b+d_v\} f_{D_{\mathrm{N-HC}}}(a)\mathrm{d}a f_{d_L}(b)\mathrm{d}b,$$

$$f_9 = \int_{d_v}^{b+d_v} (F_{D_{\mathrm{N-HNC}}}(a) - F_{D_{\mathrm{N-HNC}}}(d_v)) f_{D_{\mathrm{N-HC}}}(a)\mathrm{d}a,$$

$$f_{10} \int_{0}^{b+d_v} (F_{D_{\mathrm{N-HNC}}}(b+d_v) - F_{D_{\mathrm{N-HNC}}}(d_v)) f_{D_{\mathrm{N-HC}}}(a)\mathrm{d}a,$$

$$f_{11} = \int_{0}^{x-d_h} \int_{d_v}^{b+d_v} (1 - F_{D_{\mathrm{N-HC}}}(w))\mathbb{1}\{x > d_h\} f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w f_{d_L}(b)\mathrm{d}b,$$

$$f_{12} = \int_{0}^{x-d_h} \int_{b+d_v}^{\infty} (1 - F_{D_{\mathrm{N-HC}}}(b+d_v))\mathbb{1}\{x > d_h\} f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w f_{d_L}(b)\mathrm{d}b,$$

$$f_{13} = \int_{d_v}^{b+d_v} (1 - F_{D_{\mathrm{N-HC}}}(w)) f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w,$$

$$f_{14} = \int_{b+d_v}^{\infty} (1 - F_{D_{\mathrm{N-HC}}}(b+d_v)) f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w.$$

Event $E_{3,1,2}$:

$$
\begin{aligned}
&P(D_n < x | E_{3,1,2}) = \\
&P(D_{\mathrm{N-HNC}} + d_h < x | D_{\mathrm{N-HNC}} < d_v, D_{\mathrm{N-HC}} > d_v)\mathbb{1}\{d_h < x < d_h + d_v\} + \mathbb{1}\{x > d_h + d_v\} \\
&= \frac{P(D_{\mathrm{N-HNC}} + d_h < \min(x, d_v), D_{\mathrm{N-HC}} > d_v)}{P(D_{\mathrm{N-HNC}} < d_v, D_{\mathrm{N-HC}} > d_v)}\mathbb{1}\{d_h < x < d_h + d_v\} + \mathbb{1}\{x > d_h + d_v\} \\
&= \frac{(P(D_{\mathrm{N-HNC}} + d_h < x)\mathbb{1}\{x < d_v\} + P(D_{\mathrm{N-HNC}} + d_h < d_v)\mathbb{1}\{x > d_v\})P(D_{\mathrm{N-HC}} > d_v)}{P(D_{\mathrm{N-HNC}} < d_v)P(D_{\mathrm{N-HC}} > d_v)} \\
&* \mathbb{1}\{d_h < x < d_h + d_v\} + \mathbb{1}\{x > d_h + d_v\} \\
&= \frac{(F_{D_{\mathrm{N-HNC}}}(x - d_h)\mathbb{1}\{d_h < x < d_v\} + F_{D_{\mathrm{N-HNC}}}(d_v - d_h)\mathbb{1}\{d_h < d_v < x\})(1 - F_{D_{\mathrm{N-HC}}}(d_v))}{F_{D_{\mathrm{N-HNC}}}(d_v)(1 - F_{D_{\mathrm{N-HC}}}(d_v))} \\
&* \mathbb{1}\{d_h < x < d_h + d_v\} + \mathbb{1}\{x > d_h + d_v\} \\
&= \frac{((1 - e^{-\lambda(1-p)(x-d_h)})\mathbb{1}\{d_h < x < d_v\} + (1 - e^{-\lambda(1-p)(d_v-d_h)})\mathbb{1}\{d_h < d_v < x\})e^{-\lambda p d_v}}{(1 - e^{-\lambda(1-p)d_v})e^{-\lambda p d_v}} \\
&* \mathbb{1}\{d_h < x < d_h + d_v\} + \mathbb{1}\{x > d_h + d_v\}
\end{aligned}
\tag{A.2}
$$

Event $E_{3,2,2}$:

$$
\begin{aligned}
&P(D_n < x | E_{3,2,2}, E_{3,1,3}) = \\
&P(D_{\mathrm{N-HNC}} + D_{\mathrm{N-VC}} < x | D_{\mathrm{N-HNC}} < d_v, D_{\mathrm{N-VC}} < d_h, D_{\mathrm{N-HC}} > d_v)\mathbb{1}\{d_h < x < d_h + d_v\} \\
&+ \mathbb{1}\{x > d_h + d_v\} \\
&= \frac{P(D_{\mathrm{N-HNC}} + D_{\mathrm{N-VC}} < x, D_{\mathrm{N-HNC}} < d_v, D_{\mathrm{N-VC}} < d_h, D_{\mathrm{N-HC}} > d_v)}{P(D_{\mathrm{N-HNC}} < d_v, D_{\mathrm{N-VC}} < d_h, D_{\mathrm{N-HC}} > d_v)}\mathbb{1}\{d_h < x < d_h + d_v\} \\
&+ \mathbb{1}\{x > d_h + d_v\} \\
&= \frac{\int_{\max(x-dv,0)}^{\min(d_h,x)} F_{D_{\mathrm{N-HNC}}}(x - r)\mathbb{1}\{x > r\}(1 - F_{D_{\mathrm{N-HC}}}(d_v))f_{D_{\mathrm{N-VC}}}(r)\mathrm{d}r}{F_{D_{\mathrm{N-HNC}}}(d_v) * F_{D_{\mathrm{N-VC}}}(d_h) * (1 - F_{D_{\mathrm{N-HC}}}(d_v))} \\
&+ \frac{\int_{0}^{\min(d_h,x-d_v)} F_{D_{\mathrm{N-HNC}}}(d_v)\mathbb{1}\{x > d_v\}(1 - F_{D_{\mathrm{N-HC}}}(d_v))f_{D_{\mathrm{N-VC}}}(r)\mathrm{d}r}{F_{D_{\mathrm{N-HNC}}}(d_v) * F_{D_{\mathrm{N-VC}}}(d_h) * (1 - F_{D_{\mathrm{N-HC}}}(d_v))}
\end{aligned}
\tag{A.3}
$$

Event $E_{3,4,1}$:

$$P(D_n < x | E_{3,4,1}, E_{3,3,1}, E_{3,2,3}, E_{3,1,4})$$

$$= P(D_{\mathrm{N-HC}} < x | D_{\mathrm{N-HC}} < d_v, D_{\mathrm{N-HNC}} < D_{\mathrm{N-HC}}, D_{\mathrm{N-HNC}} + d_h > D_{\mathrm{N-HC}}) \mathbb{1}\{x < d_v\}$$

$$+ \mathbb{1}\{x > d_v\}$$

$$= \frac{\int_0^{\max(x-d_h,0)} (F_{D_{\mathrm{N-HC}}}(d_h + w) - F_{D_{\mathrm{N-HC}}}(w)) f_{D_{\mathrm{N-HNC}}}(w) \mathrm{d}w + f_{15}}{\int_0^{\max(d_v-d_h,0)} (F_{D_{\mathrm{N-HC}}}(d_h + w) - F_{D_{\mathrm{N-HC}}}(w)) f_{D_{\mathrm{N-HNC}}}(w) \mathrm{d}w + f_{16}}, \tag{A.4}$$

where

$$f_{15} = \int_{\max(x-d_h,0)}^{x} (F_{D_{\mathrm{N-HC}}}(x) - F_{D_{\mathrm{N-HC}}}(w) \mathbb{1}\{x > w\}) f_{D_{\mathrm{N-HNC}}}(w) \mathrm{d}w,$$

$$f_{16} = \int_{\max(d_v-d_h,0)}^{d_v} (F_{D_{\mathrm{N-HC}}}(d_v) - F_{D_{\mathrm{N-HC}}}(w) \mathbb{1}\{d_v > w\}) f_{D_{\mathrm{N-HNC}}}(w) \mathrm{d}w.$$

Event $E_{3,4,2}$:

$$P(D_n < x | E_{3,4,2}, E_{3,3,1}, E_{3,2,3}, E_{3,1,4})$$

$$= P(D_{\mathrm{N-HNC}} + d_h < x | D_{\mathrm{N-HC}} < d_v, D_{\mathrm{N-HNC}} < D_{\mathrm{N-HC}}, d_h < D_{\mathrm{N-HC}} - D_{\mathrm{N-HNC}}) \mathbb{1}\{x < d_v\}$$

$$+ \mathbb{1}\{x > d_v\}$$

$$= \frac{P(D_{\mathrm{N-HNC}} + d_h < x, D_{\mathrm{N-HC}} < d_v, D_{\mathrm{N-HNC}} < D_{\mathrm{N-HC}}, d_h < D_{\mathrm{N-HC}} - D_{\mathrm{N-HNC}})}{P(D_{\mathrm{N-HC}} < d_v, D_{\mathrm{N-HNC}} < D_{\mathrm{N-HC}}, d_h < D_{\mathrm{N-HC}} - D_{\mathrm{N-HNC}})} \mathbb{1}\{x < d_v\}$$

$$+ \mathbb{1}\{x > d_v\}$$

$$= \frac{P(D_{\mathrm{N-HNC}} + d_h < \min(x, D_{\mathrm{N-HC}}), D_{\mathrm{N-HC}} < d_v)}{P(D_{\mathrm{N-HNC}} + d_h < D_{\mathrm{N-HC}} < d_v)} \mathbb{1}\{x < d_v\}$$

$$+ \mathbb{1}\{x > d_v\}$$

$$= \frac{\int_x^{d_v} (F_{D_{\mathrm{N-HNC}}}(x - d_h) \mathbb{1}\{x > d_h\}) f_{D_{\mathrm{N-HC}}}(a) \mathrm{d}a + \int_{d_h}^{x} (F_{D_{\mathrm{N-HNC}}}(a - d_h) \mathbb{1}\{a > d_h\}) f_{D_{\mathrm{N-HC}}}(a) \mathrm{d}a}{\int_0^{d_v-d_h} (F_{D_{\mathrm{N-HC}}}(d_v) - F_{D_{\mathrm{N-HC}}}(d_h + w)) \mathbb{1}\{d_v > d_h + w\} f_{D_{\mathrm{N-HNC}}}(w) \mathrm{d}w} \tag{A.5}$$

Event $E_{3,3,2}$:

$$\begin{aligned}
&P(D_n < x | E_{3,3,2}, E_{3,2,3}, E_{3,1,4}) \\
&= P(D_{\mathrm{N-HC}} < x | D_{\mathrm{N-HC}} < D_{\mathrm{N-HNC}}, D_{\mathrm{N-HC}} < d_v) \\
&= \frac{P(D_{\mathrm{N-HC}} < \min(x, D_{\mathrm{N-HNC}}, d_v))}{P(D_{\mathrm{N-HC}} < \min(D_{\mathrm{N-HNC}}, d_v))} \\
&= \frac{\int_x^\infty F_{D_{\mathrm{N-HC}}}(x) f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w + \int_0^x F_{D_{\mathrm{N-HC}}}(w) f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w}{\int_{d_v}^\infty F_{D_{\mathrm{N-HC}}}(d_v) f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w + \int_0^{d_v} F_{D_{\mathrm{N-HC}}}(w) f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w}
\end{aligned} \tag{A.6}$$

Event $E_{3,4,3}$:

$$\begin{aligned}
&P(D_n < x | E_{3,4,3}, E_{3,3,3}, E_{3,2,4}, E_{3,1,4}) \\
&= P(D_{\mathrm{N-HNC}} + D_{\mathrm{N-VC}} < x | D_{\mathrm{N-HC}} < d_v, D_{\mathrm{N-HNC}} < D_{\mathrm{N-HC}}, D_{\mathrm{N-HC}} > D_{\mathrm{N-VC}} + D_{\mathrm{N-HNC}}) \\
&* \mathbb{1}\{x < d_v\} + \mathbb{1}\{x > d_v\} \\
&= \frac{P(D_{\mathrm{N-HNC}} + D_{\mathrm{N-VC}} < \min(x, D_{\mathrm{N-HC}}), D_{\mathrm{N-HC}} < d_v)}{P(D_{\mathrm{N-HNC}} + D_{\mathrm{N-VC}} < D_{\mathrm{N-HC}} < d_v)} \\
&= \frac{\int_0^x \int_x^{dv} F_{D_{\mathrm{N-VC}}}(x-w) f_{D_{\mathrm{N-HC}}}(a)\mathrm{d}a f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w}{\int_0^{d_v} \int_0^{\max(dv-r,0)} (F_{D_{\mathrm{N-HC}}}(dv) - F_{D_{\mathrm{N-HC}}}(r+w)) f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w f_{D_{\mathrm{N-VC}}}(r)\mathrm{d}r} \\
&+ \frac{\int_0^x \int_w^x F_{D_{\mathrm{N-VC}}}(a-w) f_{D_{\mathrm{N-HC}}}(a)\mathrm{d}a f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w}{\int_0^{d_v} \int_0^{\max(dv-r,0)} (F_{D_{\mathrm{N-HC}}}(dv) - F_{D_{\mathrm{N-HC}}}(r+w)) f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w f_{D_{\mathrm{N-VC}}}(r)\mathrm{d}r}
\end{aligned} \tag{A.7}$$

Event $E_{3,4,4}$:

$$\begin{aligned}
&P(D_n < x | E_{3,4,4}, E_{3,3,3}, E_{3,2,4}, E_{3,1,4}) \\
&= P(D_{\mathrm{N-HC}} < x | D_{\mathrm{N-HC}} < d_v, D_{\mathrm{N-HNC}} < D_{\mathrm{N-HC}}, D_{\mathrm{N-HC}} - D_{\mathrm{N-HNC}} < D_{\mathrm{N-VC}}) \\
&= \frac{P(D_{\mathrm{N-HNC}} < D_{\mathrm{N-HC}} < \min(x, d_v, D_{\mathrm{N-HNC}} + D_{\mathrm{N-VC}}))}{P(D_{\mathrm{N-HNC}} < D_{\mathrm{N-HC}} < \min(d_v, D_{\mathrm{N-HNC}} + D_{\mathrm{N-VC}}))} \\
&= \frac{\int_0^\infty [\int_0^{\max(x-r,0)} (F_{D_{\mathrm{N-HC}}}(w+r) - F_{D_{\mathrm{N-HC}}}(w)) f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w + f_{17}] f_{D_{\mathrm{N-VC}}}(r)\mathrm{d}r}{\int_0^\infty [\int_0^{\max(d_v-r,0)} (F_{D_{\mathrm{N-HC}}}(w+r) - F_{D_{\mathrm{N-HC}}}(w)) f_{D_{\mathrm{N-HNC}}}(w)\mathrm{d}w + f_{18}] f_{D_{\mathrm{N-VC}}}(r)\mathrm{d}r},
\end{aligned} \tag{A.8}$$

where

$$f_{17} = \int_{\max(x-r,0)}^{x} (F_{D_{\text{N-HC}}}(x) - F_{D_{\text{N-HC}}}(w)) f_{D_{\text{N-HNC}}}(w) \mathrm{d}w,$$

$$f_{18} = \int_{\max(d_v-r,0)}^{d_v} (F_{D_{\text{N-HC}}}(x) - F_{D_{\text{N-HC}}}(w)) f_{D_{\text{N-HNC}}}(w) \mathrm{d}w.$$

Event $E_{3,3,4}$:

$$\begin{aligned}
&P(D_n < x | E_{3,3,4}, E_{3,2,4}, E_{3,1,4}) \\
&= P(D_{\text{N-HC}} < x | D_{\text{N-HC}} < D_{\text{N-HNC}}, D_{\text{N-HC}} < d_v) \\
&= \frac{P(D_{\text{N-HC}} < \min(x, D_{\text{N-HNC}})}{P(D_{\text{N-HC}} < \min(D_{\text{N-HNC}}, d_v))} \\
&= \frac{\int_0^x F_{D_{\text{N-HNC}}}(r) F_{D_{\text{N-HC}}}(r) \mathrm{d}r + \int_x^\infty F_{D_{\text{N-HNC}}}(r) F_{D_{\text{N-HC}}}(x) \mathrm{d}r}{\int_0^{d_v} F_{D_{\text{N-HNC}}}(r) F_{D_{\text{N-HC}}}(r) \mathrm{d}r + \int_{d_v}^\infty F_{D_{\text{N-HNC}}}(r) F_{D_{\text{N-HC}}}(d_v) \mathrm{d}r}
\end{aligned} \tag{A.9}$$

Event $E_{4,1,1}$:

$$\begin{aligned}
&P(D_n < x | E_{4,1,1}) \\
&= P(D_n < x | D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v) \\
&= p * P(D_{\text{N-HC}} < x | D_{\text{N-HC}} < D_{\text{N-HNC}}, D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v, D_{\text{N-HC}} < d_L + d_v) \\
&+ p * P(d_L < x | D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v, D_{\text{N-HC}} < D_{\text{N-HNC}}, d_L < D_{\text{N-HC}} - d_v) \\
&+ p * P(d_L < x | D_{\text{N-HC}} > d_v, D_{\text{N-HNC}} > d_v, D_{\text{N-HC}} > D_{\text{N-HNC}}, d_L < D_{\text{N-HNC}} - d_v) \\
&= p * \frac{\int_{x-d_v}^\infty [f_1 + f_2 + f_3] f_{d_L}(b) \mathrm{d}b}{\int_0^\infty [f_4 + f_5] f_{d_L}(b) \mathrm{d}b} \\
&+ p * \frac{\int_{d_v}^\infty [\int_0^{\min(x,w-d_v)} (F_{D_{\text{N-HC}}}(w) - F_{D_{\text{N-HC}}}(b+d_v)) f_{d_L}(b) \mathrm{d}b] f_{D_{\text{N-HNC}}}(w) \mathrm{d}w}{\int_{d_v}^\infty [\int_0^{w-d_v} (F_{D_{\text{N-HC}}}(w) - F_{D_{\text{N-HC}}}(b+d_v)) f_{d_L}(b) \mathrm{d}b] f_{D_{\text{N-HNC}}}(w) \mathrm{d}w} \\
&+ p * \frac{\int_{d_v}^\infty [\int_0^{\min(x,a-d_v)} (F_{D_{\text{N-HNC}}}(a) - F_{D_{\text{N-HNC}}}(b+d_v)) f_{d_L}(b) \mathrm{d}b] f_{D_{\text{N-HC}}}(a) \mathrm{d}a}{\int_{d_v}^\infty [\int_0^{a-d_v} (F_{D_{\text{N-HNC}}}(a) - F_{D_{\text{N-HNC}}}(b+d_v)) f_{d_L}(b) \mathrm{d}b] f_{D_{\text{N-HC}}}(a) \mathrm{d}a}
\end{aligned}$$

(A.10)

Event $E_{4,1,3}$:

$$
\begin{aligned}
&P(D_n < x|E_{4,1,3}) \\
&= P(D_n < x|D_{\text{N-HC}} < d_v, D_{\text{N-HNC}} > d_v) \\
&= \frac{P(D_{\text{N-HC}} < \min(x, d_v), D_{\text{N-HNC}} > d_v)}{P(D_{\text{N-HC}} < d_v < D_{\text{N-HNC}})} \\
&= \frac{[F_{D_{\text{N-HC}}}(x)\mathbb{1}\{x < d_v\} + F_{D_{\text{N-HC}}}(d_v)\mathbb{1}\{x > d_v\}] * (1 - F_{D_{\text{N-HNC}}}(dv))}{F_{D_{\text{N-HC}}}(dv)(1 - F_{D_{\text{N-HNC}}}(d_v))}
\end{aligned}
\tag{A.11}
$$

Event $E_{4,2,2}$: The proof for $P(D_n < x|E_{4,2,2}, E_{4,1,4})$ is similar to that of $P(D_n < x|E_{3,2,2}, E_{3,1,3})$ given above.

Event $E_{4,2,3}$:

$$
\begin{aligned}
&P(D_n < x|E_{4,2,3}, E_{4,1,5}) \\
&= P(D_n < x|D_{\text{N-HC}} < D_{\text{N-HNC}} < d_v) \\
&= \frac{P(D_{\text{N-HC}} < \min(x, D_{\text{N-HNC}}), D_{\text{N-HNC}} < d_v)}{P(D_{\text{N-HC}} < D_{\text{N-HNC}} < d_v)} \\
&= \frac{\int_x^{d_v} F_{D_{\text{N-HC}}}(x) f_{D_{\text{N-HNC}}}(w) \mathrm{d}w + \int_0^x F_{D_{\text{N-HC}}}(w) f_{D_{\text{N-HNC}}}(w) \mathrm{d}w}{\int_0^{d_v} F_{D_{\text{N-HC}}}(w) f_{D_{\text{N-HNC}}}(w) \mathrm{d}w}
\end{aligned}
\tag{A.12}
$$

Event $E_{4,4,1}$ and Event $E_{4,4,3}$: The proof for $P(D_n < x|E_{4,4,1}, E_{4,3,1}, E_{4,2,4}, E_{4,1,5})$ and $P(D_n < x|E_{4,4,3}, E_{4,3,3}, E_{4,2,6}, E_{4,1,6})$ is similar to that of $P(D_n < x|E_{3,4,3}, E_{3,3,3}, E_{3,2,4}, E_{3,1,4})$ given above.

Event $E_{4,4,2}$ and Event $E_{4,4,4}$: The proof for $P(D_n < x|E_{4,4,2}, E_{4,3,1}, E_{4,2,4}, E_{4,1,5})$ and $P(D_n < x|E_{4,4,4}, E_{4,3,3}, E_{4,2,6}, E_{4,1,6})$ is similar to that of $P(D_n < x|E_{3,4,4}, E_{3,3,3}, E_{3,2,4}, E_{3,1,4})$ given above.

Event $E_{4,3,2}$ and Event $E_{4,3,4}$: The proof for $P(D_n < x|E_{4,3,2}, E_{4,2,4}, E_{4,1,5})$ and $P(D_n < x|E_{4,3,4}, E_{4,2,6}, E_{4,1,6})$ is similar to that of $P(D_n < x|E_{3,3,4}, E_{3,2,4}, E_{3,1,4})$ given above.

Event $E_{7,2,2}$:

$$\begin{aligned}
&P(D_n < x | E_{7,2,2}, E_{7,1,2}) \\
&= P(D_n < x | D_{\text{N}-\text{HNC}} < d_v, D_{\text{N}-\text{HC}} > d_v, D_{\text{N}-\text{VC}} < d_h) \\
&= P(D_{\text{N}-\text{VC}} < x | D_{\text{N}-\text{HNC}} < d_v, D_{\text{N}-\text{HC}} > d_v, D_{\text{N}-\text{VC}} < d_h) \\
&= \frac{F_{D_{\text{N}-\text{VC}}}(x)\mathbb{1}\{x < d_h\} + F_{D_{\text{N}-\text{VC}}}(d_h)\mathbb{1}\{x > d_h\}}{F_{D_{\text{N}-\text{VC}}}(d_h)}
\end{aligned} \tag{A.13}$$

Event $E_{7,3,1}$:

$$\begin{aligned}
&P(D_n < x | E_{7,3,1}, E_{7,2,4} E_{7,1,3}) \\
&= P(D_n < x | D_{\text{N}-\text{HC}} < d_v, D_{\text{N}-\text{VNC}} < d_h, D_{\text{N}-\text{VC}} > d_h, d_h - D_{\text{N}-\text{VNC}} > D_{\text{N}-\text{HC}}) \\
&= P(D_{\text{N}-\text{VNC}} + D_{\text{N}-\text{HC}} < x | D_{\text{N}-\text{HC}} < d_v, D_{\text{N}-\text{VNC}} < d_h, D_{\text{N}-\text{VC}} > d_h, d_h - D_{\text{N}-\text{VNC}} > D_{\text{N}-\text{HC}}) \\
&= \frac{P(D_{\text{N}-\text{HC}} < \min(x - D_{\text{N}-\text{VNC}}, d_v, d_h - D_{\text{N}-\text{VNC}}), D_{\text{N}-\text{VNC}} < d_h, D_{\text{N}-\text{VC}} > d_h)}{P(D_{\text{N}-\text{HC}} < \min(d_v, d_h - D_{\text{N}-\text{VNC}}), D_{\text{N}-\text{VNC}} < d_h, D_{\text{N}-\text{VC}} > d_h)} \\
&= \frac{\int_0^x F_{D_{\text{N}-\text{HC}}}(x-v)\mathbb{1}\{x < d_h\} f_{D_{\text{N}-\text{VNC}}}(o)\mathrm{d}o + \int_0^{d_h} F_{D_{\text{N}-\text{HC}}}(d_h - v)\mathbb{1}\{x > d_h\} f_{D_{\text{N}-\text{VNC}}}(o)\mathrm{d}o}{\int_0^{\min(dh-dv,0)} F_{D_{\text{N}-\text{HC}}}(d_v) f_{D_{\text{N}-\text{VNC}}}(o)\mathrm{d}o + \int_{\max(dh-dv,0)}^{d_h} F_{D_{\text{N}-\text{HC}}}(d_h - v) f_{D_{\text{N}-\text{VNC}}}(o)\mathrm{d}o}
\end{aligned} \tag{A.14}$$

Event $E_{7,4,1}$:

$$\begin{aligned}
&P(D_n < x | E_{7,4,1}, E_{7,3,3}, E_{7,2,5}, E_{7,1,3}) \\
&= P(D_{\text{N}-\text{VNC}} + D_{\text{N}-\text{HC}} < x | D_{\text{N}-\text{VC}} < d_h, D_{\text{N}-\text{VNC}} < D_{\text{N}-\text{VC}}, D_{\text{N}-\text{VC}} - D_{\text{N}-\text{VNC}} > D_{\text{N}-\text{HC}}) \\
&* \mathbb{1}\{x < d_h\} + \mathbb{1}\{x > d_h\} \\
&= \frac{\int_0^x \int_x^{d_h} F_{D_{\text{N}-\text{HC}}}(x-w) f_{D_{\text{N}-\text{VC}}}(r)\mathrm{d}r f_{D_{\text{N}-\text{VNC}}}(o)\mathrm{d}o}{\int_0^{d_h}\int_0^{\max(d_h - r,0)} (F_{D_{\text{N}-\text{VC}}}(d_h) - F_{D_{\text{N}-\text{VC}}}(a+o)) f_{D_{\text{N}-\text{VNC}}}(o)\mathrm{d}o f_{D_{\text{N}-\text{HC}}}(a)\mathrm{d}a} \\
&+ \frac{\int_0^x \int_w^x F_{D_{\text{N}-\text{HC}}}(a-w) f_{D_{\text{N}-\text{VC}}}(r)\mathrm{d}r f_{D_{\text{N}-\text{VNC}}}(o)\mathrm{d}o}{\int_0^{d_h}\int_0^{\max(d_h - r,0)} (F_{D_{\text{N}-\text{VC}}}(d_h) - F_{D_{\text{N}-\text{VC}}}(a+o)) f_{D_{\text{N}-\text{VNC}}}(o)\mathrm{d}o f_{D_{\text{N}-\text{HC}}}(a)\mathrm{d}a}
\end{aligned} \tag{A.15}$$

The proof for $P(D_n < x | E_{7,4,1}, E_{7,3,3}, E_{7,2,5}, E_{7,1,3})$ is similar to that of $P(D_n <$

$x|E_{3,4,3}, E_{3,3,3}, E_{3,2,4}, E_{3,1,4})$ given above.

Event $E_{7,4,2}$:

$$\begin{aligned} & P(D_n < x|E_{7,4,2}, E_{7,3,3}, E_{7,2,5}, E_{7,1,3}) \\ & = P(D_{\mathrm{N-VC}} < x|D_{\mathrm{N-VC}} < d_h, D_{\mathrm{N-VNC}} < D_{\mathrm{N-VC}}, D_{\mathrm{N-VC}} - D_{\mathrm{N-VNC}} < D_{\mathrm{N-HC}}) \\ & = \frac{\int_0^\infty [\int_0^{\max(x-a,0)} (F_{D_{\mathrm{N-VC}}}(o+a) - F_{D_{\mathrm{N-VC}}}(o)) f_{D_{\mathrm{N-VNC}}}(o)\mathrm{d}o + f_{19}] f_{D_{\mathrm{N-HC}}}(a)\mathrm{d}a}{\int_0^\infty [\int_0^{\max(d_h-a,0)} (F_{D_{\mathrm{N-VC}}}(o+a) - F_{D_{\mathrm{N-VC}}}(o)) f_{D_{\mathrm{N-VNC}}}(o)\mathrm{d}o + f_{20}] f_{D_{\mathrm{N-HC}}}(a)\mathrm{d}a}, \end{aligned} \tag{A.16}$$

where

$$f_{19} = \int_{\max(x-a,0)}^{x} (F_{D_{\mathrm{N-VC}}}(x) - F_{D_{\mathrm{N-VC}}}(o)) f_{D_{\mathrm{N-VNC}}}(o)\mathrm{d}o,$$

$$f_{20} = \int_{\max(d_h-a,0)}^{d_h} (F_{D_{\mathrm{N-VC}}}(x) - F_{D_{\mathrm{N-VC}}}(o)) f_{D_{\mathrm{N-VNC}}}(o)\mathrm{d}o.$$

The proof for $P(D_n < x|E_{7,4,2}, E_{7,3,3}, E_{7,2,5}, E_{7,1,3})$ is similar to that of $P(D_n < x|E_{3,4,4}, E_{3,3,3}, E_{3,2,4}, E_{3,1,4})$ given above.

Event $E_{7,3,4}$:

$$\begin{aligned} & P(D_n < x|E_{7,3,4}, E_{7,2,5}, E_{7,1,3}) \\ & = P(D_{\mathrm{N-VC}} < x|D_{\mathrm{N-VC}} < D_{\mathrm{N-VNC}}, D_{\mathrm{N-VC}} < d_h) \\ & = \frac{P(D_{\mathrm{N-VC}} < \min(x, D_{\mathrm{N-VNC}})}{P(D_{\mathrm{N-VC}} < \min(D_{\mathrm{N-VNC}}, d_h))} \\ & = \frac{\int_0^x F_{D_{\mathrm{N-VNC}}}(a) F_{D_{\mathrm{N-VC}}}(a)\mathrm{d}a + \int_x^\infty F_{D_{\mathrm{N-VNC}}}(a) F_{D_{\mathrm{N-VC}}}(x)\mathrm{d}a}{\int_0^{d_h} F_{D_{\mathrm{N-VNC}}}(a) F_{D_{\mathrm{N-VC}}}(a)\mathrm{d}a + \int_{d_h}^\infty F_{D_{\mathrm{N-VNC}}}(a) F_{D_{\mathrm{N-VC}}}(d_h)\mathrm{d}a} \end{aligned} \tag{A.17}$$

The proof for $P(D_n < x|E_{7,3,4}, E_{7,2,5}, E_{7,1,3})$ is similar to that of $P(D_n < x|E_{3,3,4}, E_{3,2,4}, E_{3,1,4})$ given above.

Event $E_{8,2,4}$:

$$
\begin{aligned}
&P(D_n < x|E_{8,2,4}, E_{8,1,3}) \\
&= P(D_{\mathrm{N-VNC}} + D_{\mathrm{N-HC}} < x|D_{\mathrm{N-HC}} < d_v, D_{\mathrm{N-VC}} > d_h, D_{\mathrm{N-VNC}} < d_h) \\
&= \frac{P(D_{\mathrm{N-HC}} < \min(x - D_{\mathrm{N-VNC}}, d_v), D_{\mathrm{N-VC}} > d_h, D_{\mathrm{N-VNC}} < d_h)}{P(D_{\mathrm{N-HC}} < d_v, D_{\mathrm{N-VNC}} < d_h < D_{\mathrm{N-VC}})} \\
&= \frac{\int_{\max(x-d_v,0)}^{\infty} F_{D_{\mathrm{N-HC}}}(x-v)\mathbb{1}\{x > v\} f_{D_{\mathrm{N-VNC}}}(o)\mathrm{d}o}{F_{D_{\mathrm{N-HC}}}(dv)} \\
&+ \frac{\int_{0}^{\min(x-d_v,0)} F_{D_{\mathrm{N-HC}}}(d_v)\mathbb{1}\{x > v\} f_{D_{\mathrm{N-VNC}}}(o)\mathrm{d}o}{F_{D_{\mathrm{N-HC}}}(dv)}
\end{aligned} \tag{A.18}
$$

Event $E_{8,4,1}$: The proof for $P(D_n < x|E_{8,4,1}, E_{8,3,1}, E_{8,2,5}, E_{8,1,3})$ is similar to that of $P(D_n < x|E_{7,4,1}, E_{7,3,3}, E_{7,2,5}, E_{7,1,3})$ given above.

Event $E_{8,4,2}$: The proof for $P(D_n < x|E_{8,4,2}, E_{8,3,1}, E_{8,2,5}, E_{8,1,3})$ is similar to that of $P(D_n < x|E_{7,4,2}, E_{7,3,3}, E_{7,2,5}, E_{7,1,3})$ given above.

Event $E_{8,3,2}$: The proof for $P(D_n < x|E_{8,3,2}, E_{8,2,5}, E_{8,1,3})$ is similar to that of $P(D_n < x|E_{7,3,4}, E_{7,2,5}, E_{7,1,3})$ given above.

$P(\overline{T_c}|E_4)P(E_4)$:

$$
\begin{aligned}
&P(\overline{T_c}|E_4)P(E_4) = [P(E_{4,1,1}) * (1-p) + P(E_{4,1,2}) + P(E_{4,1,4}) * P(E_{4,2,1})] * P(E_4) \\
&= [e^{-\lambda d_v}(1-p) + \lambda(1-p)d_v e^{-\lambda(1-p)d_v} e^{-\lambda p d_v} \\
&+ e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v} - \lambda(1-p)d_v e^{-\lambda(1-p)d_v})e^{-\lambda p d_h}] * (1-p)^2
\end{aligned} \tag{A.19}
$$

$P(\overline{T_c}|E_8)P(E_8)$:

$$
\begin{aligned}
&P(\overline{T_c}|E_8)P(E_8) = [P(E_{8,1,1}) + P(E_{8,1,2}) * P(E_{8,2,1}) + P(E_{8,1,3}) * P(E_{8,2,3})] * P(E_8) \\
&= [e^{-\lambda d_v} + e^{-\lambda p d_v}(1 - e^{-\lambda(1-p)d_v})e^{-\lambda p d_h} + (1 - e^{-\lambda p d_v})e^{-\lambda d_h}] * (1-p)^2
\end{aligned} \tag{A.20}
$$