A symmetrical bistable mechanism from combination of pre-shaped microbeams

H. Hussein, F. Khan, M. I. Younis

Physical Science and Engineering Division, King Abdullah University of Science and Technology, Thuwal, 23955-6900, Saudi Arabia. Mohammad.Younis@kaust.edu.sa.

ABSTRACT

A bistable mechanism is proposed with symmetrical snap-through behavior between the two equilibrium positions. The mechanism is monolithic and does not require external axial load, as buckled beams. It consists of two pre-shaped beams of opposite curvature with similar properties and dimensions, which are connected after fabrication. The combination of the opposite pre-shaped beams, each having non-symmetrical bistability, results in total symmetric bistability. This symmetrical bistable mechanism provides the advantage of having the same switching force in the forward and backward directions. The concept of the new mechanism is presented, analyzed based on the snapping force curves, and tested experimentally on microfabricated prototypes.

1. Introduction

A pre-shaped beam bistable mechanism is considered an attractive alternative to buckled beams; since it is more compatible with monolithic microfabrication and does not require external axial load to induce buckling [1,2]. Pre-shaped beams are fabricated in a specific initial shape; most commonly curved (as the first buckling mode shape) and inclined (V-shaped). These mechanisms are widely used at the small and large scale in many applications (switches, relays, valves, sensors, resonators, multistable states, logics, biomedical and space applications, large stroke displacement with low restoring forces, constant-force and negative stiffness behavior, energy harvesting, etc.) [3].

Compared to buckled beams, the bistability of pre-shaped beams is conditional and the snap-through behavior is not symmetric between the two sides of equilibrium positions. A key issue is the stability margin of pre-shaped beams, which is more critical in the as-fabricated position considering the shifted initial state. Several works have been presented to improve the symmetry of the snap-through behavior of pre-shaped beams between the two stable sides, for example using stop blocks [4], by connecting opposite pre-shaped beams and tuning with axial loads [5], or by optimizing hybrid shapes with variable cross-section [6–9].

A symmetrical bistable mechanism is presented in this paper. It consists of a combination of pre-shaped beams, curved opposite to each other and connected after fabrication. The proposed mechanism has a simple and monolithic structure. It combines the advantages of both pre-shaped and buckled beams; i.e. it has a symmetrical snap-through behavior and bistability, as buckled beams, while it does not require the application of external axial loads or moveable boundaries to induce buckling, as pre-shaped beams. This simplifies the design, fabrication, and integration of the symmetrical bistable mechanism at the micro-scale.
The symmetrical snap-through behavior indicates that the snapping force is symmetric (similar magnitude and opposite direction) with respect to the middle position in-between forward and backward lateral directions. The symmetry is desirable for many applications (switches, relays, valves, bistable mechanisms, etc.), where same conditions are required for triggering snap-through from either of the stable sides [4–9]. The bistability in this case is robust, even with very small distance between the stable positions. The concept of the symmetrical bistable mechanism proposed in this paper is analyzed based on the snapping force curves, and validated experimentally on microfabricated prototypes.

2. Symmetrical bistable mechanism

Fig. 1 shows the symmetrical bistable mechanism, consisting of two opposite pre-shaped beams with the same properties and dimensions. Fig. 1a shows the directions of the snapping force \( f_1, f_2, \) and deflection \( d_1, d_2, \) and \( d \) for the top beam, bottom beam, and their combination, respectively. Fig. 1b shows the locking mechanism in-between the two opposite beams, allowing to connect them together after fabrication (Fig. 1c and d).

![Fig. 1: (a) Schematic of the symmetric bistable mechanism. (b) Enlarged view of the locking mechanism in the as-fabricated configuration. After activation, shuttles are connected and the mechanism has two stable position in the (c) top and (d) bottom sides.](image)

The locking mechanism can be activated manually using probe needles, or using actuators. It constraints the same lateral displacement in the two directions after moving the two beams toward each other a distance \( \delta \) (Fig. 1b-d).

\[
d = d_1 = \delta - d_2
\] (1)

As the opposite beams have the same material and dimensions, their snapping force behavior is equivalent in their range of deflection (i.e. \( f_1 \) with respect to \( d_1 \) and \( f_2 \) with respect to \( d_2 \)). This makes the following constraint:

\[
f_1(d_1) = f_2(d_2)
\] (2)

The connection of the two beams combines \( f_1 \) and \( f_2 \) in the total snapping force \( f \) as follows:

\[
f(d) = f_1(d) - f_2(d)
\] (3)

The purpose of connecting the two opposite and similar pre-shaped beams is to obtain a symmetric total snapping force curve. The total snapping force after connection of pre-shaped beams is symmetric with respect to the middle...
position \( (d = \frac{\delta}{2}) \). This can be proven by substituting \((d = \frac{\delta}{2} - x)\) and \((d = \frac{\delta}{2} + x)\) into (1), (2), and (3) separately, where we obtain the following relation \( f \left( \frac{\delta}{2} - x \right) = -f \left( \frac{\delta}{2} + x \right) \).

The snapping force is equally opposite to the static restoring force at the mid-point of the pre-shaped beam. The snapping force curve of a pre-shaped beam is extensively analyzed in the literature [1–4]. Fig. 2a shows the snapping force curves for each single pre-shaped beam in the mechanism \((f_1(d)\) and \(-f_2(d)\)) and the corresponding total snapping force curve after activation \((f(d))\). Fig. 2b shows their corresponding lateral stiffness curves \(\frac{\partial f}{\partial d}\).

![Fig. 2: (a) Snapping force curves of the pre-shaped beams and the mechanism, and (b) their corresponding lateral stiffness curves.](image)

The curves of \(f_1\) and \(f_2\) in Fig. 2a are the snapping force curves for the top and bottom pre-shaped curved beam, respectively. These curves are calculated analytically considering the following dimensions and properties: length: 1.96mm, thickness: 6μm, height: 26μm, width: 25μm, and Young’s modulus: 169GPa [3]. The width and material property (Young’s modulus) are chosen to be compatible with SOI-MUMPs\(^\text{TM}\) fabrication process [10]. The thickness is usually the smallest dimension of the beam. It is limited by the minimum feature size in the fabrication process and it should be thick enough along the beam length to avoid fragility. Further, the thickness is small enough and the length is large enough to avoid exceeding the stress limits inside the beam during lateral deflection [4]. A minimum height-to-thickness ratio (~2.31) should be considered to ensure bistability of the pre-shaped curved beam. The distance between the two stable positions is mainly dependent on the height and thickness. We choose these dimensions to obtain a distance of 50μm between the two stable positions. This distance should be high enough to enable integrating the locking mechanism in-between the opposite pre-shaped beams of the symmetrical bistable mechanism. Further design elements of the pre-shaped beam are investigated in [4].

The connection between the opposite beams in the symmetrical bistable mechanism, using the locking mechanism, forms a constraint of parallel configuration, where several pre-shaped beams are connected to the same shuttle at their mid-length. This constraints a linear displacement of the shuttle, avoids its rotation, constraints non-symmetrical modes of buckling, and improves bistability [3]. The total snapping force curve \(f\) in Fig. 2a after connection of the opposite beams is then calculated from (3).
The bistability of a mechanism means the existence of two stable positions, characterized by zero-force ($f = 0$) and positive stiffness behavior ($\frac{\partial f}{\partial d} > 0$). The stiffness refers to the reaction force variation of the beam shuttle after a slight displacement around its position; positive stiffness means that the force is increasing, while negative stiffness means that the force is decreasing with further displacement. Fig. 2b shows the lateral stiffness curves for each pre-shaped beam and for the mechanism after activation. As shown in Figs. 2a and b, each bistable pre-shaped beam has three zero-force positions, two are stables with positive stiffness (at $d=0$ and $d=50\mu$m) and one in-between is unstable with negative stiffness (at $d=34.67\mu$m). From (3), the total snapping force of the new mechanism has a zero-force position at the mid-position between the two pre-shaped beams ($f\left(\frac{\delta}{2}\right) = 0$). Thereby, the condition to have bistability in the new mechanism is to have a negative stiffness behavior around the mid-position ($\frac{\partial f}{\partial d}\left(\frac{\delta}{2}\right) < 0$).

Accordingly, the distance $\delta$ between the opposite pre-shaped beams can be chosen to ensure a negative stiffness behavior around the mid-position. In the example considered in this paper, we choose $\delta = 50\mu$m equivalent to the distance between the two stable positions. In this way, in addition to the negative stiffness at the mid-position, the stable positions after combination remain at the as-fabricated positions of both beams ($f(0)= f(\delta)=0, \frac{\partial f}{\partial d}(0)=\frac{\partial f}{\partial d}(\delta)>0$, see Fig. 1e).

Prototypes with the same dimensions of the mechanism were fabricated using SOI-MUMPs™ fabrication process through MEMSCAP [10]. Fig. 3 shows a bistable mechanism prototype as-fabricated and after activation at the top and bottom stable positions. The activation and switching are made manually using probe needles from both sides of the mechanism shuttle. The prototypes are tested on-chip using Cascade Microtech’s Summit 12000 AP semi-automatic probe station, as shown in Fig. 3a.
The activation is made manually using probe needles as shown in Fig. 3b. After activation, the switching between stable positions is made manually in the experiment by pushing the mid-shuttle with the needles. The experimental measurements (Fig. 3.c-d) show that the fabricated mechanism is bistable and the two stable positions are at the as-fabricated positions of both beams, as expected analytically. The horizontal red lines in Fig. 3c-d show this fact, where the top and bottom stable positions are at the same level of initial positions of top and bottom pre-shaped beams, respectively. This validates the concept of the new mechanism with symmetrical bistability and shows its simplicity.

The initial distance $\delta$ between the opposite pre-shaped beams can be set differently in the design. In the previous example, $\delta$ was set equivalent to the distance between stable positions for a single pre-shaped beam (parallel configuration). The mechanism after activation have two stable positions at the as-fabricated position for each single pre-shaped beam, as discussed previously and shown in the experiments. The as-fabricated positions are also the stable positions of the mechanism after activation for $\delta$ equivalent to the distance to the zero-force negative-stiffness position for a single pre-shaped beam (i.e. $\delta = 34.67\mu m$).

For other values of $\delta$, Fig. 4 shows variation of the distance between stable positions $d_{stable}$ and the top snapping force $f_{top}$ (equivalent in magnitude to $f_{bot}$ due to symmetry). $d_{top}$, $f_{top}$, $d_{bot}$, $f_{bot}$, and $d_{stable}$ are the snapping points and distance between stable position for the symmetric bistable mechanism, as clarified in Fig. 4c. The parameters $d_{top1}$, $f_{top1}$, $d_{bot1}$, $f_{bot1}$, and $d_{stable1}$ are for a single pre-shaped beam.

![Diagram](image)

Fig. 4: (a) Variation of the distance between stable positions, (b) and of the top snapping force with respect to the distance between the opposite beams before activation. (c) Snapping points ($d_{top}$, $f_{top}$, $d_{bot}$, $f_{bot}$), and distance between stable positions ($d_{stable}$) in the snapping force curve.

The curves in Fig. 4a and b are normalized with respect to $d_{stable1}$ and $f_{top1}$ ($d_{stable1} = 50\mu m$, $f_{top1} = 0.176mN$). Fig. 4a shows that $d_{stable}$ can reach maximum around $d_{stable1}$ in the range of $\delta$ around $d_{stable1}$. Fig. 4b shows that $f_{top}$ reaches maximum for $\delta$ higher than $d_{stable1}$ (more precisely for $\delta = d_{top1} + d_{bot1}$). Further, $\delta$ can be varied roughly in the range $[2d_{top1}, 2d_{bot1}]$ while keeping bistability. Thereby, $\delta$ is a design parameter that can be set to calibrate the stable positions and the total snapping forces. Further, if small distance between stable positions is required, Fig. 4 shows that this can be obtained with large value of $\delta$ relatively, instead of considering a small value. Smaller values of $\delta$ may be constrained by the fabrication limits and the locking mechanism dimensions.
A multistable mechanism in general needs two functions for operating in any device or any application: holding function and switching function. For the buckled and pre-shaped beams and the bistable mechanism proposed in this paper, the holding function relies on the elastic stability of the structure in two different positions. The switching function is satisfied by applying forces on the moving part to switch between the stable positions. In the experiments presented in this paper, the switching was made manually using probe needles. However, practically this can be achieved in a MEMS device using a suitable actuator, such as electrothermal [11] and electrostatic [12] actuators. The activation phase of the mechanism can be also achieved using the actuator, if it is sufficiently powerful. The symmetrical bistability of the proposed mechanism is attractive for many applications where the same conditions are required in both sides of bistability. This is beneficial for many current bistable mechanisms applications, where no need to deal with different holding/switching conditions at each stable position. This is the case for discrete positioning [13,14] for example. Also, this mechanism can be useful for other applications; it can be combined to a positive stiffness mechanism for example forming a near zero force and zero stiffness mechanism [15].

3. Conclusions

A symmetrical monolithic bistable mechanism was proposed in this paper. The mechanism consists of opposite pre-shaped beams, where their interaction after connection results in a symmetrical snap-through behavior. Many elements can be varied in the design of the mechanism, including the initial shape of pre-shaped beams, their dimensions, and the distance of activation. Experiments on microfabricated prototypes were conducted and validated the concept, allowing the mechanism to be introduced in more complicated systems for more advantageous functions.

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5. References


