2019 IEEE 58th Conference on Decision and Control (CDC)
Palais des Congrès et des Expositions Nice Acropolis
Nice, France, December 11-13, 2019

State Estimation of LPV Discrete-Time Systems with Application to Output Feedback Stabilization

K. Chaib-Draa[1], A. Zemouche[2], R. Rajamani[3], Y. Wang[4], F. Bedouhene[5], H.R. Karimi[6], T.M. Laleg-Kirati[7]

Abstract— **This paper deals with new finite-time estimation algorithms for Linear Parameter Varying (LPV) discrete-time systems and their application to output feedback stabilization. Two exact finite-time estimation schemes are proposed. The first one provides a direct and explicit estimation algorithm based on the use of delayed outputs, while the second one uses two combined asymptotic observers, connected by a condition of invertibility of a certain time-varying matrix, to recover in a finite-time the solution of the LPV system. Furthermore, a stabilization strategy is proposed as an extension. This strategy, called Two Connected Observers Feedback (2-COF) stabilization method, is based on the use of the two combined observers based estimation algorithm.**

I. Introduction

State estimation has many applications in control system design. The estimates of state are needed for implementation of control laws and also for fault diagnosis [1], [2], [3], [4]. In nonlinear systems, the estimation problem is complicated and there is no general methodology to deal with observer design for all classes of nonlinear systems. Several methods have been proposed recently in the literature to improve observer design aiming to cover a wider class of nonlinear systems, but this issue still remains a challenge from nonlinear observer design point of view [5], [6], [7], [8].

One of the classes of nonlinear systems largely investigated in the literature, and often used to represent nonlinear systems, is the class of LPV systems [9], [10], [11], [12]. This particular structure of nonlinearity interests the automatic control community for two reasons:

- LPV systems represent a wide class of nonlinear real-world models, such as wind turbines models, vehicle models, biogas processes, and wastewater treatment models.
- Stabilizing LPV systems from an observer-based feedback control point of view is a hard task due to the difficulty in obtaining non conservative sufficient conditions ensuring the exponential convergence [13], [14], [9]. For instance, from LMI (Linear Matrix Inequality) point of view, all the available design methods for this class of systems provide conservative LMI conditions. Although a recent and new technique has been proposed in [15] to reduce conservativeness, the issue is far from being definitely solved.

In this paper, we focus first on exact and finite-time estimation of the state of a class of LPV systems in discrete-time, and then extension to output feedback stabilization is provided. Based on the ideas given in [16], [17], [18] for linear continuous-time systems, we propose a generalization to LPV systems, which is not obvious in the continuous-time case. Indeed, in continuous-time case, it is difficult to integrate the differential equations and get explicit solutions of the system. The computations will involve complications and the appearance of the derivatives of the time-varying parameters of the LPV system. Such derivatives require additional and conservative assumptions and constraints on the system (such as boundedness of the derivatives of the parameters, for instance). However, in discrete-time systems, only delayed values of the LPV parameters appear in the computation of the explicit solution. Two finite-time estimation algorithms are proposed:

- *Direct and explicit estimation:* This algorithm proposed for the three above mentioned classes of systems provides an explicit solution of the system in finite-time. This direct estimation approach is based on the use of delayed outputs to recover the solution of the system.
- *Two observers-based estimation:* This technique consists in combining two asymptotic state observers to reconstruct the solution of the system in finite-time.

To demonstrate the role of the proposed exact finite-time estimation algorithms, an extension to output feedback stabilization is provided. As a preliminary result, only one design technique is presented in this note. The stabilization DIOF procedure is based on the use of delayed inputs/outputs of the system. It is shown that this stabilization method avoids solving Bilinear Matrix Inequalities (BMIs), often encountered in control design of LPV systems.

[1] University of Luxembourg, Belval, Luxembourg (email: kh.chaibdraa@gmail.com).
[2] Université de Lorraine, CRAN CNRS UMR 7039, 54400 Cosnes et Romain, France (email: ali.zemouche@univ-lorraine.fr).
[3] Laboratory for Innovations in Sensing, Estimation, and Control, Department of Mechanical Engineering, University of Minnesota, Minneapolis, USA. (email: rajamani@umn.edu).
[4] Auris Health, Inc. 150 Shoreline Drive Redwood City, CA 94065, USA. (email: wangyan0731@outlook.com).
[5] Laboratoire de Mathématiques Pures et Appliquées, Université Mouloud Mammeri de Tizi-Ouzou, Algeria. (email: fbedouhene@yahoo.fr).
[6] Politecnico di Milano, Department of Mechanical Engineering, via La Masa 1, 20156 Milan, Italy. (email: hamidreza.karimi@polimi.it).
[7] Department of Computer, Electrical and Mathematical Science andEngineering, KAUST, Thuwal, Saudi Arabia. (email: taousmeriem.laleg@kaust.edu.sa).

II. Results on Exact Finite-Time Estimation of LPV Systems

This section is dedicated to the development of two exact finite-time estimation algorithms for a class of LPV systems. We consider the class of LPV systems defined by the following set of equations:

$$x_{k+1} = A(\rho_k)x_k + Bu_k \tag{1a}$$
$$y_k = Cx_k \tag{1b}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $y_k \in \mathbb{R}^p$ is the output measurement and $u_k \in \mathbb{R}^m$ is the control input vector, $\rho_k \in \Theta \subset \mathbb{R}^r$ is a bounded time-varying parameter. B, C and D are constant matrices of adequate dimensions.

We introduce the following assumptions:

- the parameter ρ_k is known and bounded;
- the matrix $A(\rho_k) \in \mathrm{Co}(A_1, ..., A_N)$, then it can be written under the form

$$A(\rho_k) = A_0 + \sum_{i=1}^{i=n_\rho} \xi^i(\rho_k)A_i \tag{2}$$

  where $\xi^i(\rho_k) \geq 0$ and $\sum_{i=1}^{i=n_\rho} \xi^i(\rho_k) = 1$;
- the pairs (A_i, C) are observable for all $i = 1, \ldots, n_\rho$.

A. Explicit Solutions Using Delayed Outputs

Before providing the first algorithm providing an exact finite-time estimation of the state x_k, we need to state the following lemma.

Lemma 1: Assume that the pairs (A_j, C) are observable for all $j = 0, \ldots, n_\rho$. Then there exist L_j, K_j, $j = 0, \ldots, n_\rho$, and $m \geq 1$ such that the matrix

$$\mathbb{E}_m(k) \triangleq \left[\prod_{i=1}^{m}\left(\sum_{j=0}^{n_\rho} \xi^j_{k-i}\big(A_j - L_jC\big)\right)\right]^{-1} - \left[\prod_{i=1}^{m}\left(\sum_{j=0}^{n_\rho} \xi^j_{k-i}\big(A_j - K_jC\big)\right)\right]^{-1} \tag{3}$$

exists and invertible for all $k \geq 1$, where $\xi^0(\rho_k) \triangleq 1$ and $\xi^i(\rho_k) \triangleq \xi^i_k$ for $i \geq 1$.

Proof: Since the parameters are bounded and (A_j, C) are observable, then it is always possible to find L_j, K_j, and $m \geq 1$ such that the matrix $\mathbb{E}_m(k)$ remains invertible for all $k \geq m$. Indeed, we can choose the eigenvalues of $\big(A_j - L_jC\big)$ inside the ball $\mathcal{B}(0,\delta)$ and those of $\big(A_j - K_jC\big)$ inside $\mathcal{B}(0,1)\backslash\mathcal{B}(0,\delta)$, where $0 < \delta < 1$. The detailed proof is omitted from this preliminary version of the paper. ■

Now, we provide a direct and exact estimation of the state x_k, presented in the following theorem.

Theorem 1: Assume that there exist

$$\mathbb{L}(\rho_k) = L_0 + \sum_{i=1}^{i=n_\rho} \xi^i(\rho_k)L_i,\ \mathbb{K}(\rho_k) = K_0 + \sum_{i=1}^{i=n_\rho} \xi^i(\rho_k)K_i$$

and $m \geq 1$ so that matrix $\mathbb{E}_m(k)$, defined in (4), exists and invertible for all $k \geq m$:

$$\mathbb{E}_m(k) \triangleq \left[\prod_{i=1}^{m}\left(\sum_{j=0}^{n_\rho} \xi^j_{k-i}\big(A_j - L_jC\big)\right)\right]^{-1} - \left[\prod_{i=1}^{m}\left(\sum_{j=0}^{n_\rho} \xi^j_{k-i}\big(A_j - K_jC\big)\right)\right]^{-1}. \tag{4}$$

Then a direct and exact estimation of the state x_k can be computed as in (5).

Proof: It is easy to show iteratively that x_k can be written under the forms (6) and (7). Then, by subtracting (7), after multiplication by $\left(\prod_{i=1}^{m}\Big(\mathbb{A}(\rho_{k-i}) - \mathbb{K}(\rho_{k-i})C\Big)\right)^{-1}$, from (6) multiplied by $\left(\prod_{i=1}^{m}\Big(\mathbb{A}(\rho_{k-i}) - \mathbb{L}(\rho_{k-i})C\Big)\right)^{-1}$, we get easily (5) by using the inverse of $\mathbb{E}_m(k)$. ■

B. Estimation By Using Two Combined Observers

Contrarily to the previous section where we used a sum of delayed outputs weighted by powers of $A - LC$ and $A - KC$, this section is devoted to estimate the solutions of the considered system by using two different asymptotic state observers. By using tools borrowed from the continuous-time results in [16] and [18], we get an exact estimation of the solution without using explicitly the delayed outputs. Indeed, the delayed output measurements are hidden and appear implicitly in the states of the intermediate observers. This way to provide an exact estimation of the state x_k is more suitable for practical implementation point of view.

Considering the class of systems (1), then an exact estimation of x_k may be obtained by using two combined asymptotic observers, instead of using directly an explicit solution. The result is summarized in the following theorem.

Theorem 2: Assume that the gain matrices L_i and K_i are selected such that:

i) all the eigenvalues of $\big(A_i - L_iC\big)$ and $\big(A_i - K_iC\big)$ are non-zero and less than one;
ii) there exists $m \geq 1$ so that the matrix $\mathbb{E}_m(k)$ exists and invertible.

Then the extended state dynamic system

$$\zeta_{k+1} = A(\rho_k)\zeta_k + Bu_k + \mathbb{L}(\rho_k)\Big(y_k - C\zeta_k\Big) \tag{8a}$$
$$\eta_{k+1} = A(\rho_k)\eta_k + Bu_k + \mathbb{K}(\rho_k)\Big(y_k - C\eta_k\Big) \tag{8b}$$
$$\hat{x}_k = \mathbb{E}_m^{-1}(k)\left[\left(\prod_{i=1}^{m}\left(\sum_{l=0}^{n_\rho} \xi^l_{k-i}\big(A_l - L_lC\big)\right)\right)^{-1}\zeta_k - \left(\prod_{i=1}^{m}\left(\sum_{l=0}^{n_\rho} \xi^l_{k-i}\big(A_l - K_lC\big)\right)\right)^{-1}\eta_k + \eta_{k-m} - \zeta_{k-m}\right] \tag{8c}$$

$$
\begin{aligned}
x_k = \mathbb{E}_m^{-1}(k)\sum_{j=1}^{m}\Bigg[&\left(\prod_{i=1}^{m}\left(\sum_{l=0}^{n_\rho}\xi_{k-i}^l\big(A_l-L_lC\big)\right)\right)^{-1}\left(\sum_{l=0}^{n_\rho}\xi_{k-j}^l\big(A_l-L_lC\big)\right)^{j-1}\left(\sum_{l=0}^{n_\rho}\xi_{k-j}^lL_l\right)\\
&-\left(\prod_{i=1}^{m}\left(\sum_{l=0}^{n_\rho}\xi_{k-i}^l\big(A_l-K_lC\big)\right)\right)^{-1}\left(\sum_{l=0}^{n_\rho}\xi_{k-j}^l\big(A_l-K_lC\big)\right)^{j-1}\left(\sum_{l=0}^{n_\rho}\xi_{k-j}^lK_l\right)\Bigg]y_{k-j}\\
+\mathbb{E}_m^{-1}(k)\sum_{j=1}^{m}\Bigg[&\left(\prod_{i=1}^{m}\left(\sum_{l=0}^{n_\rho}\xi_{k-i}^l\big(A_l-L_lC\big)\right)\right)^{-1}\left(\sum_{l=0}^{n_\rho}\xi_{k-j}^l\big(A_l-L_lC\big)\right)^{j-1}\\
&-\left(\prod_{i=1}^{m}\left(\sum_{l=0}^{n_\rho}\xi_{k-i}^l\big(A_l-K_lC\big)\right)\right)^{-1}\left(\sum_{l=0}^{n_\rho}\xi_{k-j}^l\big(A_l-K_lC\big)\right)^{j-1}\Bigg]Bu_{k-j}
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
x_k =& \left(\prod_{i=1}^{m}\left[\sum_{l=0}^{n_\rho}\xi_{k-j}^l\big(A_l-L_lC\big)\right]\right)x_{k-m}\\
&+\sum_{j=1}^{m}\left(\left[\sum_{l=0}^{n_\rho}\xi_{k-j}^l\big(A_l-L_lC\big)\right]^{j-1}\right)\left[\left(\sum_{l=0}^{n_\rho}\xi_{k-j}^lL_l\right)y_{k-j}+Bu_{k-j}\right]
\end{aligned}
\tag{6}
$$

and

$$
\begin{aligned}
x_k =& \left(\prod_{i=1}^{m}\left[\sum_{l=0}^{n_\rho}\xi_{k-j}^l\big(A_l-K_lC\big)\right]\right)x_{k-m}\\
&+\sum_{j=1}^{m}\left(\left[\sum_{l=0}^{n_\rho}\xi_{k-j}^l\big(A_l-K_lC\big)\right]^{j-1}\right)\left[\left(\sum_{l=0}^{n_\rho}\xi_{k-j}^lK_l\right)y_{k-j}+Bu_{k-j}\right]
\end{aligned}
\tag{7}
$$

is an observer for system (1), which converges in finite time $m \geq 1$.

Proof: The proof exploits the explicit solution technique. Indeed, by analogy to (6) and (7), we get (9) and (10). Hence, by substituting (9) and (10) in (5) and using the definition of $\mathbb{E}_m(k)$, we get from (8c) that $\hat{x}_k = x_k, \forall k \geq m$. ■

III. Results on Output Feedback Stabilization of LPV Systems

In this section, we propose an output feedback stabilization method based on the second exact finite-time estimation methodology proposed in this note. Due to the exact finite time estimation of the system state, we will propose necessary and sufficient LMI conditions ensuring poly-quadratic stabilization of the system state.

Theorem 3: Assume that the gain matrices L_i, K_i and F_i are selected such that:

i) there exists $m \geq 1$, $L_i, K_i, i = 1, \ldots, n_\rho$, so that the matrix $\mathbb{E}_m(k)$ exists and invertible;

ii) there exist matrices $\mathbb{P}_i = \mathbb{P}_i^{-1} > 0, i = 1, \ldots, n_\rho$ and matrices $\mathbb{X}_i, i = 1, \ldots, n_\rho$ of appropriate dimensions such that the following LMI conditions hold:

$$
\begin{bmatrix} -\mathbb{P}_j & A_i\mathbb{P}_i - B\mathbb{X}_i \\ \mathbb{P}_iA_i^\top - \mathbb{X}_i^\top B^\top & -\mathbb{P}_i \end{bmatrix} < 0, \forall i, j = 1, \ldots, n_\rho. \tag{11}
$$

Then the following observer-based controller

$$
\zeta_{k+1} = A(\rho_k)\zeta_k + Bu_k + \mathbb{L}(\rho_k)\Big(y_k - C\zeta_k\Big) \tag{12a}
$$

$$
\eta_{k+1} = A(\rho_k)\eta_k + Bu_k + \mathbb{K}(\rho_k)\Big(y_k - C\eta_k\Big) \tag{12b}
$$

$$
\begin{aligned}
\hat{x}_k = \mathbb{E}_m^{-1}(k)\Bigg[&\left(\prod_{i=1}^{m}\left(\sum_{l=0}^{n_\rho}\xi_{k-i}^l\big(A_l-L_lC\big)\right)\right)^{-1}\zeta_k\\
&-\left(\prod_{i=1}^{m}\left(\sum_{l=0}^{n_\rho}\xi_{k-i}^l\big(A_l-K_lC\big)\right)\right)^{-1}\eta_k\\
&+\eta_{k-m}-\zeta_{k-m}\Bigg]
\end{aligned}
\tag{12c}
$$

$$
u_k = -\mathbb{F}(\rho_k)\hat{x}_k \tag{12d}
$$

with

$$
\mathbb{F}(\rho_k) \triangleq F_0 + \sum_{i=1}^{i=n_\rho}\xi^i(\rho_k)F_i,\ F_i = \mathbb{X}_i\mathbb{P}_i^{-1} \tag{13}
$$

stabilizes globally asymptotically the system (1).

Proof: From Theorem 2, we know that if $i)$ and $ii)$ of Theorem 3 are satisfied, then (12c) provides an exact and finite-time estimation of x_k. that is $\hat{x}_k = x_k, \forall k \geq m$. It follows that for $k \geq m$, equation (12d) becomes

$$
u_k = -\mathbb{F}(\rho_k)x_k.
$$

$$\zeta_k = \left(\prod_{i=1}^{m}\left[\sum_{l=0}^{n_\rho}\xi_{k-j}^{l}\big(A_l - L_l C\big)\right]\right)\zeta_{k-m} + \sum_{j=1}^{m}\left(\left[\sum_{l=0}^{n_\rho}\xi_{k-j}^{l}\big(A_l - L_l C\big)\right]^{j-1}\right)\left[\left(\sum_{l=0}^{n_\rho}\xi_{k-j}^{l}L_l\right)y_{k-j} + Bu_{k-j}\right], \tag{9}$$

$$\eta_k = \left(\prod_{i=1}^{m}\left[\sum_{l=0}^{n_\rho}\xi_{k-j}^{l}\big(A_l - K_l C\big)\right]\right)\eta_{k-m} + \sum_{j=1}^{m}\left(\left[\sum_{l=0}^{n_\rho}\xi_{k-j}^{l}\big(A_l - K_l C\big)\right]^{j-1}\right)\left[\left(\sum_{l=0}^{n_\rho}\xi_{k-j}^{l}K_l\right)y_{k-j} + Bu_{k-j}\right]. \tag{10}$$

Consequently, for $k \geq m$, system (1) is rewritten after feedback as:

$$\begin{aligned} x_{k+1} &= \Big(A(\rho_k) - B\mathbb{F}(\rho_k)\Big)x_k \\ &= \sum_{j=0}^{n_\rho}\xi_k^j\big(A_j - BF_j\big)x_k \end{aligned} \tag{14}$$

which is globally asymptotically stable if there exists a Lyapunov function

$$\vartheta_k = \sum_{i=0}^{n_\rho}\xi_k^i x_k^\top \mathbb{P}_i^{-1}x_k$$

such that $\Delta\vartheta \triangleq \vartheta_{k+1} - \vartheta_k < 0, \forall x_k \neq 0$. By developing the $\Delta\vartheta$, we get

$$\Delta\vartheta = \sum_{i=0}^{n_\rho}\sum_{j=0}^{n_\rho}\xi_{k+1}^i\xi_k^j x_k^\top\left[\Big(A_j - BF_j\Big)^\top\mathbb{P}_i^{-1}\times \Big(A_j - BF_j\Big) - \mathbb{P}_j^{-1}\right]x_k. \tag{15}$$

It follows that $\Delta\vartheta < 0, \forall x_k \neq 0$ if the following inequalities hold, $\forall i, j = 1, \ldots, n_\rho$:

$$\Big(A_j - BF_j\Big)^\top\mathbb{P}_i^{-1}\Big(A_j - BF_j\Big) - \mathbb{P}_j^{-1} < 0. \tag{16}$$

On the other hand, (16) is equivalent to (11) by using the congruence principle and Schur lemma. This ends the proof. ■

Remark 1: It is worth to notice that LMIs (11) are necessary and sufficient conditions for the global poly-quadratic stabilization of system (1), however, they are only sufficient for its global asymptotic stabilization. Indeed, according to [19, Definition 2], the notion of poly-quadratic stability is stronger than asymptotic stability. Poly-quadratic stability is basically, by definition, a sufficient criterion to ensure asymptotic stability.

IV. Illustrative Example

This section is devoted to illustrate the theoretical contributions presented in the previous sections. However, only the estimation methodology based on the use of two combined observers will be illustrated in this note.

1) System description: As an example, consider the LPV system described by the following equations [20]:

$$x_{k+1} = \begin{bmatrix} 0.25 & 1 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.6+\rho_k \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u_k \tag{17a}$$

$$y_k = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} x_k \tag{17b}$$

with $\rho_k \in \begin{bmatrix} 0 & 0.5 \end{bmatrix}, k \in \mathbb{N}$. In this case, we can take the functions $\xi^1(\rho_k) = (0.5 - \rho_k)/0.5$ and $\xi^2(\rho_k) = \rho_k/0.5$ and $A_1 = A(0)$, $A_2 = A(0.5)$.

2) Estimation of the state: By using pole assignment algorithm, we obtain the following gains:

$$L_0 = -\begin{bmatrix} -0.0474 \\ 0.0007 \\ 0.3637 \end{bmatrix}, L_1 = -\begin{bmatrix} -0.0134 \\ -0.0009 \\ 0.6372 \end{bmatrix},$$

and

$$K_0 = -\begin{bmatrix} -0.0314 \\ -0.0010 \\ 0.3260 \end{bmatrix}, K_1 = -\begin{bmatrix} -0.0638 \\ 0.0033 \\ 0.7513 \end{bmatrix}.$$

for which the matrix $\mathbb{E}_m(k)$ in (4) exists and found invertible for any $k \geq 0$ with $m = 3$.

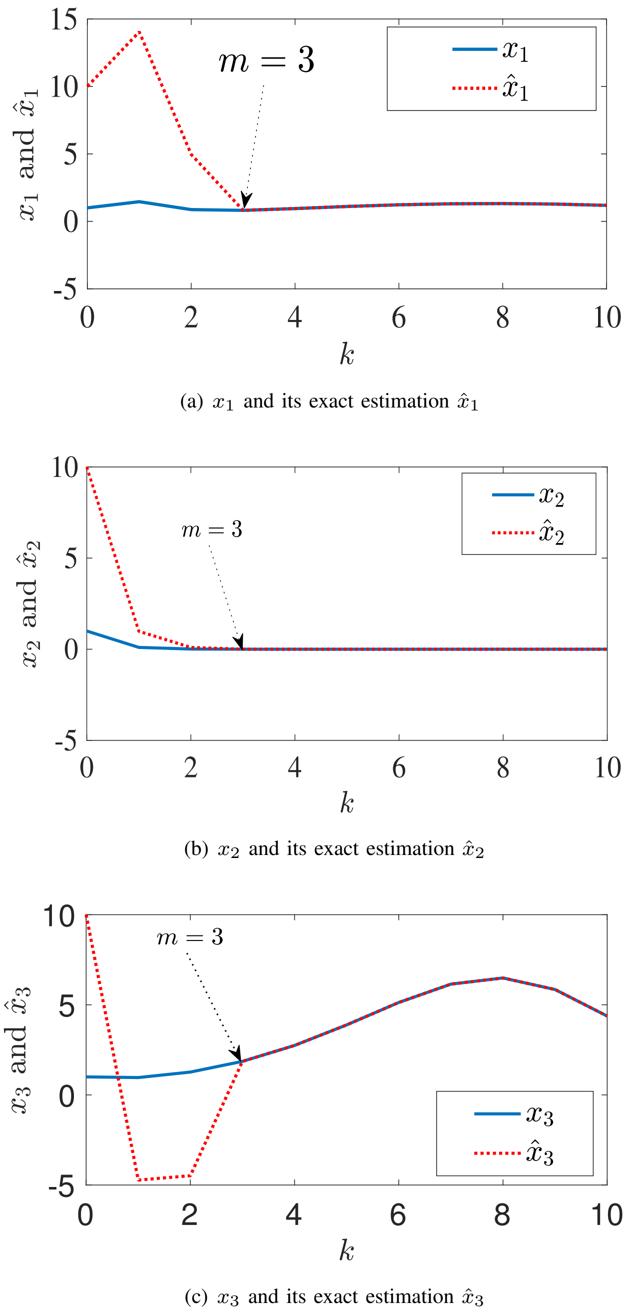
For simulations, we use

$$\rho_k = \frac{1}{2}\mid\sin(\frac{\pi}{10}k)\mid \text{ and } u_k = \sin(\frac{\pi}{15}k)).$$

The initial state of the system is $x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. As for ζ_0 and η_0 in (8a)-(8b) are given by $\begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$, respectively. We also use $\hat{x}_k = \zeta_k$ for $k = 0, \ldots, m-1$. It is quite clear from Figure 1 that the estimation $\hat{x}_k$ given by (8c) reaches exactly the solution x_k of (17a) in finite-time.

V. Conclusion

This paper provided two state estimation algorithms for LPV discrete-time systems. The first one allows computing explicitly the solution of the system through delayed outputs/inputs, while the second one uses the strategy of



(a) x_1 and its exact estimation $\hat{x}_1$

(b) x_2 and its exact estimation $\hat{x}_2$

(c) x_3 and its exact estimation $\hat{x}_3$

Fig. 1. Behavior of the states and their estimates.

two connected asymptotic observers. An extension to output feedback stabilization is proposed. Therefore, a novel control design strategy is proposed. A numerical example is provided to show the effectiveness of the second estimation algorithm.

As prospects, we aim to generalize the proposed methodology to systems with unknown parameters ($\rho_k = \rho_k^0 + \Delta\rho_k$) and disturbances. The goal is to provide robust and performant stabilization schemes. We also plan to investigate a systematic design procedure to compute the estimation parameters. Further extensions to output feedback stabilization and application to real-world models are in progress.

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