Performance Analysis of Satellite Communication Systems with Randomly Located Ground Users

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Abstract

Satellite communication (SatCom) is one of the emerging technologies that compensate for digital divide between urban and remote areas. The outage event of SatCom link connecting to a network is more critical in an infrastructure deficient remote area. In this paper, we analyze an outage probability (OP) and symbol error rate (SER) over SatCom downlink channels when the users are randomly located in a single beam area. The downlink beam will suffer from propagation loss and the shadowed-Rician fading depending on the user location which is assumed to follow a Poisson point process. For mathematically tractable, informative, and insightful interpretation, we obtain and verify the asymptotic OP and SER expressions of user link under several channel conditions in the high power regime.

Index Terms

Satellite systems, shadowed-Rician, stochastic geometry, Poisson point process, outage probability, symbol error rate.

I. Introduction

Satellite communications (SatCom) are an essential part of the wireless communication systems owing to their broad coverage and high throughput [1]. SatCom systems have attracted research attention to provide against the unprecedented demand on data and densification in 5G network [2]. Beyond 5G, SatCom systems have been also considered to apply to 6G network in order to guarantee communication fairness since SatCom can provide access for rural areas and achieve the worldwide connectivity [3], [4]. In addition, SatCom makes non-terrestrial communication (NTC) expected in 6G possible. For example, the authors in [5] investigated recent advances in NTS.

Since recent research on satellites targets aggressive frequency reuse for high throughput, interference mitigation techniques such as precoding at gateways and beam or cluster hopping methods are needed to reduce the inter-beam interference [6]–[9]. Moreover, due to obstacles between the satellite and users, hybrid satellite-terrestrial relay networks (HSTRNs) has been proposed as a way to improve the reliability of the link [10], [11]. Recently, in order to improve spectral efficiency, it is also proposed to employ non-orthogonal multiple access scheme in HSTRNs [12]. However, the aforementioned works do not consider either propagation loss or fading as a random variable because they fix user's location or ignore fading channel.

In this paper, we focus on a single beam on SatCom systems that propagates over atmospheric fading and assume that the multiple users in a beam are randomly located as a Poisson point process (PPP). We point out that the outage of SatCom link might be harsh for users located in a remote area where wireless network infrastructure is deficient. Therefore, we take into consideration the outage analysis over the randomness of all the elements in the SatCom channel model, which consists of beam pattern, path loss and shadowed-Rician (SR) fading. By using the property of stochastic geometry, we obtain the closed form result of the asymptotic outage probability (OP) in the high signal-to-noise ratio (SNR) region. We also have the asymptotic symbol error rate (SER) with derived the asymptotic OP. Finally, simulation results are provided to verify the asymptotic results.

II. System and Channel Model

We consider a multibeam SatCom downlink system as shown in Fig. 1 and suppose that inter-beam interference can be removed using the four-color frequency reuse or beam hopping. Thus, we can only see one beam among the beams which cover several users. Furthermore, we assume the users equipped with a single antenna are randomly distributed within the beam and the distribution follows a PPP Φ, which has intensity ρ in the closed disk of radius R. The transmitted signal from satellite undergoes rain attenuation, path loss and SR fading.

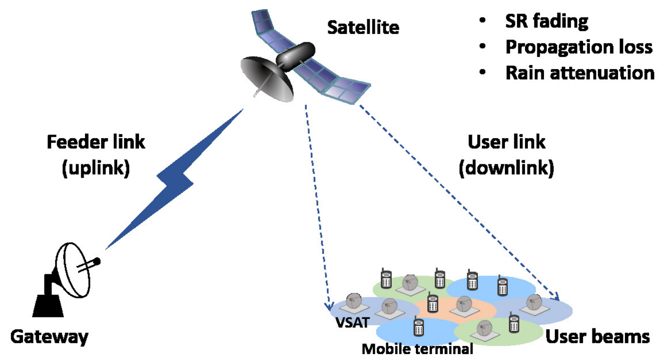


Fig. 1: System model of satellite communications

Then, the received SNR of the kth user can be expressed as

$$\gamma_k = \bar{P} \left| h_k \right|^2, \tag{1}$$

where $\bar{P}$ denotes the transmit power of the satellite. The channel coefficient of the kth user h_k can be decomposed as

$$h_k = A_k g_k, \tag{2}$$

where A_k and g_k denote propagation loss and SR fading, respectively. The propagation loss is given by [13]

$$A_k = s_k \frac{\lambda G_R q_k e^{j\xi_k}}{4\pi d_k \sqrt{\kappa T B}}, \tag{3}$$

where s_k denotes the rain attenuation coefficient, λ denotes the carrier wavelength, d_k is the distance between the satellite and the kth user, κ is the Boltzmann constant, T is the receiver noise temperature, and B is the bandwidth. Besides, G_R^2 denotes the user receiver antenna gain and ξ_k represents the phase due to the beam radiation pattern and the radio propagation. The satellite antenna pattern q_k which depends on the random user location can be approximated by [14]

$$q_k = G_{\max} \left[\frac{J_1(u_k)}{2u_k} + 36 \frac{J_3(u_k)}{u_k^3} \right], \tag{4}$$

where $G_{\max}^2$ is the maximum satellite antenna gain. In (4), J_1 and J_3 are the first-kind Bessel function of orders 1 and 3, respectively. The variable u_k is written as $u_k = 2.07123 \sin(\theta_k) / \sin(\theta_{3\mathrm{dB}})$, where θ_k denotes the angle between the center of beam and kth user, and $\theta_{3\mathrm{dB}}$ is the angle of the 3 dB power loss from the beam center.

The channel fading g_k is modeled as SR distribution denoted as $\mathcal{SR}\left(\Omega, b_0, m\right)$, where Ω denotes the average power of the line-of-sight component, $2b_0$ is the average power of the scatter component, and m is the Nakagami parameter. We note that the SR fading is the most widely employed distribution in SatCom systems, and cumulative distribution function (CDF) of the SR fading power gain g_k^2 can be expressed as [15]

$$F_{g_k^2}(t) = \left(\frac{2b_0 m}{2b_0 m + \Omega}\right)^m \sum_{n=0}^{\infty} \frac{(m)_n}{n!\Gamma(n+1)} \left(\frac{\Omega}{2b_0 m + \Omega}\right)^n \gamma\left(n+1, \frac{1}{2b_0}t\right), \tag{5}$$

where $\Gamma(\cdot)$ denotes the gamma function, $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function, and $(a)_n$ is the Pochhammer symbol [16].

III. Performance Analysis

A. Outage Probability

In this section, three cases of the user with the maximum received SNR, the user with the minimum received SNR, and the randomly selected user among the users within the beam are considered, and their OP expressions are obtained. The OP of the kth user is defined as the probability that γ_k falls below a predefined threshold γ_{th}, which can be expressed as

$$P_{out}\left(\gamma_{\mathrm{th}}\right) = \Pr\left(\gamma_k \leq \gamma_{\mathrm{th}}\right) = F_{\gamma_k}\left(\gamma_{\mathrm{th}}\right), \tag{6}$$

where $F_{\gamma_k}\left(\cdot\right)$ denotes the CDF of γ_{k}.

1) Best Link Selection: In order to obtain the best channel link in the beam, the maximum received SNR can be written as

$$\gamma_{\max} = \max_{k\in\Phi} \gamma_k. \tag{7}$$

For the simplicity, even though the power gain due to rain attenuation in dB follows lognormal distribution [17], we assume users in the same beam suffer from the same rain attenuation with average value since rain attenuation has spatial correlation in the tens of kilometers. In addition, we suppose the users suffer from the independent and identically distributed (i.i.d.) SR channel fading. When users are distributed in the beam with PPP, we excluded the case where 0 users are created and distributed. Otherwise, it is determined that an outage has occurred when there are zero users, and the cumulative distribution function (CDF) does not converge to zero or one.

Therefore, the CDF of the maximum received SNR can be calculated as

$$
\begin{aligned}
F_{\gamma_{\max}}(x) &= \Pr\left(\max_{k\in\Phi}\gamma_k \le x \mid \Phi > 0\right)\\
&= \mathrm{E}_{\Phi}\left[\prod_{k\in\Phi}\Pr\left(\gamma_k \le x \mid \Phi > 0\right)\right]\\
&= \frac{1}{\Pr\left(\Phi > 0\right)} \times \left\{\mathrm{E}_{\Phi}\left[\prod_{k\in\Phi}\Pr\left(\gamma_k \le x\right)\right] - \mathrm{E}_{\Phi}\left[\prod_{k\in\Phi}\Pr\left(\gamma_k \le x, \Phi = 0\right)\right]\right\}\\
&= \frac{1}{1-\Pr\left(\Phi = 0\right)} \times \left\{\mathrm{E}_{\Phi}\left[\prod_{k\in\Phi}\Pr\left(\gamma_k \le x\right)\right] - \Pr\left(\Phi = 0\right)\right\}\\
&= \frac{1}{1-\exp\left(-\rho\pi R^2\right)} \times \left\{\mathrm{E}_{\Phi}\left[\prod_{k\in\Phi}\Pr\left(\gamma_k \le x\right)\right] - \exp\left(-\rho\pi R^2\right)\right\}, \qquad (8)
\end{aligned}
$$

where E_{Φ} denotes expectation over Φ.

By using the property of stochastic geometry, the expectation term which remains in (8) can be obtained as

$$
\begin{aligned}
&\mathrm{E}_{\Phi}\left[\prod_{k\in\Phi}\Pr\left(\gamma_k \le x\right)\right]\\
&= \mathrm{E}_{\Phi}\left[\prod_{k\in\Phi}\Pr\left(\bar{P}\left|A_k\right|^2 g_k^2 \le x\right)\right]\\
&= \mathrm{E}_{\Phi}\left[\prod_{k\in\Phi}F_{g_k^2}\left(\frac{x}{\bar{P}\left|A_k\right|^2}\right)\right]\\
&= \mathrm{E}_{\Phi}\left[\prod_{k\in\Phi}\left\{\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \sum_{n=0}^{\infty}\frac{(m)_n}{n!\Gamma(n+1)}\left(\frac{\Omega}{2b_0 m+\Omega}\right)^n \gamma\left(n+1, \frac{1}{2b_0}\frac{x}{\bar{P}\left|A_k\right|^2}\right)\right\}\right]\\
&\overset{(a)}{=} \exp\left[-\rho\int_0^{2\pi}\int_0^R r\left\{1-\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \sum_{n=0}^{\infty}\frac{(m)_n}{n!\Gamma(n+1)}\left(\frac{\Omega}{2b_0 m+\Omega}\right)^n \gamma\left(n+1, \frac{1}{2b_0}\frac{x}{\bar{P}\left|A(r)\right|^2}\right)\right\} dr\, d\phi\right]\\
&= \exp\left(-\rho\pi R^2\right)\exp\left[\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \sum_{n=0}^{\infty}\frac{(m)_n}{n!\Gamma(n+1)}\left(\frac{\Omega}{2b_0 m+\Omega}\right)^n \int_0^R r\gamma\left(n+1, \frac{1}{2b_0}\frac{x}{\bar{P}\left|A(r)\right|^2}\right) dr\right]\\
&\overset{(b)}{\approx} \exp\left(-\rho\pi R^2\right)\exp\left[\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \sum_{n=0}^{\infty}\frac{(m)_n}{n!\Gamma(n+1)}\left(\frac{\Omega}{2b_0 m+\Omega}\right)^n \int_0^R r\frac{1}{n+1}\left(\frac{1}{2b_0}\frac{x}{\bar{P}\left|A(r)\right|^2}\right) dr\right]\\
&\overset{(c)}{=} \exp\left(-\rho\pi R^2\right)\exp\left[\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \int_0^R r\,{}_1F_1\left(m, 2; \frac{\Omega}{2b_0 m+\Omega}\frac{1}{2b_0}\frac{x}{\bar{P}\left|A_k(r)\right|^2}\right)\frac{1}{2b_0}\frac{x}{\bar{P}\left|A(r)\right|^2} dr\right]\\
&\overset{(d)}{\approx} \exp\left(-\rho\pi R^2\right)\exp\left[\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \frac{x}{2b_0\bar{P}}\int_0^R \frac{r}{\left|A(r)\right|^2} dr\right], \qquad (9)
\end{aligned}
$$

where propagation loss is a function of user's location that changes with PPP, and eq. (a) follows from the probability generating functional (PGFL) for PPP [18]; eq. (b) holds that incomplete gamma function can be approximated at high SNR $\left(\bar{P} \to \infty\right)$ [19]; in eq. (c), we employ the confluent hypergeometric function, ${}_1F_1\left(\cdot,\cdot;\cdot\right)$ [16]; eq. (d) holds that the confluent hypergeometric function goes to one at high SNR. The remaining integral term can be regarded as a constant if some parameters for propagation loss are set. Therefore, the asymptotic CDF of the maximum received SNR can be written as

$$F_{\gamma_{\max}}\left(x\right) \approx \frac{\exp\left(-\rho\pi R^2\right)}{1-\exp\left(-\rho\pi R^2\right)} \times \left\{\exp\left[\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \frac{x}{2b_0\bar{P}}\int_0^R \frac{r}{\left|A\left(r\right)\right|^2}\,dr\right]-1\right\}, \quad (10)$$

where we can approximate the asymptotic CDF by first order Taylor linearization at 0 as

$$F^{(1)}_{\gamma_{\max}}\left(x\right) \approx \frac{\exp\left(-\rho\pi R^2\right)}{1-\exp\left(-\rho\pi R^2\right)} \times \left\{\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \frac{x}{2b_0\bar{P}}\int_0^R \frac{r}{\left|A\left(r\right)\right|^2}\,dr\right\}. \quad (11)$$

2) Worst Link Selection: We choose the user who has the minimum received SNR within the beam and derive its OP in a similar way to maximum received SNR case. The minimum received SNR of users in the beam

$$\gamma_{\min} = \min_{k\in\Phi} \gamma_k. \quad (12)$$

The CDF of the minimum received SNR can be derived as

$$\begin{aligned}
F_{\gamma_{\min}}\left(x\right) &= \Pr\left(\min_{k\in\Phi}\gamma_k \le x \mid \Phi>0\right)\\
&= 1-\Pr\left(\min_{k\in\Phi}\gamma_k > x \mid \Phi>0\right)\\
&= 1-\mathrm{E}_\Phi\left[\prod_{k\in\Phi}\Pr\left(\gamma_k > x \mid \Phi>0\right)\right]\\
&= 1-\frac{1}{\Pr\left(\Phi>0\right)}\times\left\{\mathrm{E}_\Phi\left[\prod_{k\in\Phi}\Pr\left(\gamma_k>x\right)\right]-\mathrm{E}_\Phi\left[\prod_{k\in\Phi}\Pr\left(\gamma_k>x,\Phi=0\right)\right]\right\}\\
&= 1-\frac{1}{1-\Pr\left(\Phi=0\right)}\times\left\{\mathrm{E}_\Phi\left[\prod_{k\in\Phi}\left(1-\Pr\left(\gamma_k\le x\right)\right)\right]-\Pr\left(\Phi=0\right)\right\}\\
&= 1-\frac{1}{1-\exp\left(-\rho\pi R^2\right)}\times\left\{\mathrm{E}_\Phi\left[\prod_{k\in\Phi}\left(1-\Pr\left(\gamma_k>x\right)\right)\right]-\exp\left(-\rho\pi R^2\right)\right\}. \quad (13)
\end{aligned}$$

As in (9), with PGFL for PPP and high SNR approximation, the expectation term in (13) can be calculated as

$$\begin{aligned}
&\mathrm{E}_{\Phi}\left[\prod_{k\in\Phi}\left(1-\Pr\left(\gamma_k>x\right)\right)\right]\\
&=\exp\left[-\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m\sum_{n=0}^{\infty}\frac{(m)_n}{n!\Gamma(n+1)}\left(\frac{\Omega}{2b_0 m+\Omega}\right)^n\int_0^R r\gamma\left(n+1,\frac{1}{2b_0}\frac{x}{\bar{P}\left|A_k\left(r\right)\right|^2}\right)dr\right]\\
&\approx\exp\left[-\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m\frac{x}{2b_0\bar{P}}\int_0^R\frac{r}{\left|A\left(r\right)\right|^2}dr\right],
\end{aligned}\tag{14}$$

where we can also view the integral term as a constant when the propagation loss parameters are set. Hence, the asymptotic CDF of the minimum received SNR can be expressed as

$$F_{\gamma_{\min}}\left(x\right)\approx 1-\frac{1}{1-\exp\left(-\rho\pi R^2\right)}\times\left\{\exp\left[-\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m\frac{x}{2b_0\bar{P}}\int_0^R\frac{r}{\left|A\left(r\right)\right|^2}dr\right]-\exp\left(-\rho\pi R^2\right)\right\}.\tag{15}$$

This asymptotic CDF can be also approximated using first order Taylor linearlization at 0 as the case of the maximum received SNR. It can be written as

$$F^{(1)}_{\gamma_{\min}}\left(x\right)\approx\frac{1}{1-\exp\left(-\rho\pi R^2\right)}\times\left\{\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m\frac{x}{2b_0\bar{P}}\int_0^R\frac{r}{\left|A\left(r\right)\right|^2}dr\right\}.\tag{16}$$

3) Random Link Selection: To compare the performance to the other cases, we obtain the CDF of user which is randomly selected. Since selecting one user randomly among the several users within the beam is equivalent to the circumstance where only one user is randomly located in the beam, we use the latter for the mathematical tractability. Thus, the SNR of the user randomly chosen can be expressed as

$$\gamma_{\text{random}}=\gamma_1,\tag{17}$$

where γ_1 denotes the SNR of the one user.

Since we just consider one user is uniformly distributed within the beam, so the asymptotic expression of the CDF can be calculated as

$$\begin{aligned}
&F_{\gamma_{\text{random}}}(x) = \Pr\left(\gamma_1 \le x \mid \Phi = 1\right) \\
&= \int_0^{2\pi}\int_0^R \frac{r}{\pi R^2} F_{g_k^2}\left(\frac{x}{\bar{P}\left|A(r)\right|^2}\right) dr\, d\phi \\
&= \int_0^{2\pi}\int_0^R \frac{r}{\pi R^2}\left\{\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \sum_{n=0}^{\infty}\frac{(m)_n}{n!\Gamma(n+1)}\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \gamma\left(n+1, \frac{1}{2b_0}\frac{x}{\bar{P}\left|A(r)\right|^2}\right)\right\} dr\, d\phi \\
&= \frac{2}{R^2}\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \sum_{n=0}^{\infty}\frac{(m)_n}{n!\Gamma(n+1)}\left(\frac{\Omega}{2b_0 m+\Omega}\right)^n \int_0^R r\gamma\left(n+1, \frac{1}{2b_0}\frac{x}{\bar{P}\left|A(r)\right|^2}\right) dr \\
&\approx \frac{x}{b_0 R^2 \bar{P}}\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \int_0^R \frac{r}{\left|A(r)\right|^2}\, dr, \qquad (18)
\end{aligned}$$

where the approximation is used in the same way as the case of the maximum received SNR.

B. Symbol Error Rate

The symbol error rate (SER) of M-ary phase shift keying (M-PSK) modulation can be written as [20]

$$P_{M-\text{PSK}} = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} M_\gamma\left(-\frac{\sin^2\left(\pi/M\right)}{\sin^2\theta}\right) d\theta, \qquad (19)$$

where $M_\gamma(s) = \mathrm{E}\left[e^{s\gamma}\right]$ is the moment generating function (MGF) of γ. Since (19) is intractable, alternatively, the approximate expression can be used [10], [21], [22]

$$\begin{aligned}
P_{M-\text{PSK}} \approx &\left(\frac{\theta_M}{2\pi}-\frac{1}{6}\right) M_\gamma\left(-\sin^2\left(\frac{\pi}{M}\right)\right) + \frac{1}{4} M_\gamma\left(-\frac{4}{3}\sin^2\left(\frac{\pi}{M}\right)\right) \\
&+ \left(\frac{\theta_M}{2\pi}-\frac{1}{4}\right) M_\gamma\left(-\frac{\sin^2\left(\pi/M\right)}{\sin^2\theta_M}\right), \qquad (20)
\end{aligned}$$

where $\theta_M = \pi - \pi/M$. To analyze the SER of user for each case, we derive the asymptotic MGF for each case with the asymptotic CDF calculated above.

1) Best Link Selection: By taking the derivative of (10), the probability density function (PDF) of the user with the maximum SNR can be expressed as

$$\begin{aligned}
f_{\gamma_{\max}}(x) \approx &\overbrace{\frac{\exp\left(-\rho\pi R^2\right)}{1-\exp\left(-\rho\pi R^2\right)}}^{I_1} \times \overbrace{\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \frac{1}{2b_0\bar{P}}\int_0^R \frac{r}{\left|A_k(r)\right|^2}\, dr}^{I_2} \\
&\times \exp\left[\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \frac{x}{2b_0\bar{P}}\int_0^R \frac{r}{\left|A_k(r)\right|^2}\, dr\right]. \qquad (21)
\end{aligned}$$

For the sake of convenience, we substitute $\exp\left(-\rho\pi R^2\right)$ and $\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \frac{1}{2b_0\bar{P}}\int_0^R \frac{r}{|A_k(r)|^2}\,dr$ into I_1 and I_2, respectively. The MGF of user can be calculated as

$$\begin{aligned} M_{\gamma_{max}}(s) &= \mathrm{E}\left[e^{s\gamma_{max}}\right] \\ &= \int_0^\infty e^{sx} f_{\gamma_{\max}}(x)\,dx \\ &\approx \int_0^\infty e^{sx}\frac{I_1 I_2}{1-I_1}e^{I_2 x}\,dx \\ &\overset{(a)}{=} -\frac{I_1 I_2}{\left(1-I_1\right)\left(s+I_2\right)}, \end{aligned} \tag{22}$$

where eq. (a) follows from $s+I_2<0$ which is satisfied because s is negative for SER of M-PSK and I_2 goes to 0 at high SNR.

2) Worst Link Selection: From (15), the pdf of user with the minimum SNR can be expressed as

$$f_{\gamma_{\min}}(x) \approx \frac{\overbrace{\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \frac{1}{2b_0\bar{P}}\int_0^R \frac{r}{\left|A_k(r)\right|^2}\,dr}^{I_2} \times \exp\left[-\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \frac{x}{2b_0\bar{P}}\int_0^R \frac{r}{|A_k(r)|^2}\,dr\right]}{1-\underbrace{\exp\left(-\rho\pi R^2\right)}_{I_1}}, \tag{23}$$

where we substitute $\exp\left(-\rho\pi R^2\right)$ and $\rho 2\pi\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \frac{1}{2b_0\bar{P}}\int_0^R \frac{r}{|A_k(r)|^2}\,dr$ into I_1 and I_2, as the case of maximum SNR user selection, respectively. The MGF of user can be computed as

$$\begin{aligned} M_{\gamma_{min}}(s) &= \mathrm{E}\left[e^{s\gamma_{min}}\right] \\ &= \int_0^\infty e^{sx} f_{\gamma_{\min}}(x)\,dx \\ &\approx \int_0^\infty e^{sx}\frac{I_2}{1-I_1}e^{-I_2 x}\,dx \\ &\overset{(a)}{=} -\frac{I_2}{\left(1-I_1\right)\left(s-I_2\right)}, \end{aligned} \tag{24}$$

where eq. (a) holds for $s-I_2<0$ which is also satisfied for the same reason as the case of the best link selection.

3) Random Link Selection: From (18), the pdf of user selected randomly can be written as

$$\begin{aligned} f_{\gamma_{random}}(x) &\approx \frac{1}{b_0 R^2\bar{P}}\left(\frac{2b_0 m}{2b_0 m+\Omega}\right)^m \int_0^R \frac{r}{\left|A(r)\right|^2}\,dr \\ &= I_3, \end{aligned} \tag{25}$$

where the pdf is not a function of x. The MGF of the randomly selected user can be obtained as

$$\begin{aligned} M_{\gamma_{random}}(s) &= \mathrm{E}\left[e^{s\gamma_{random}}\right] \\ &= \int_0^\infty e^{sx} f_{\gamma_{\mathrm{random}}}(x)\, dx \\ &\approx \int_0^\infty e^{sx} I_3\, dx \\ &\overset{(a)}{=} -\frac{I_3}{s}, \end{aligned} \tag{26}$$

where eq. (a) holds for $s < 0$ which is always satisfied for SER of the M-PSK.

IV. Numerical Results

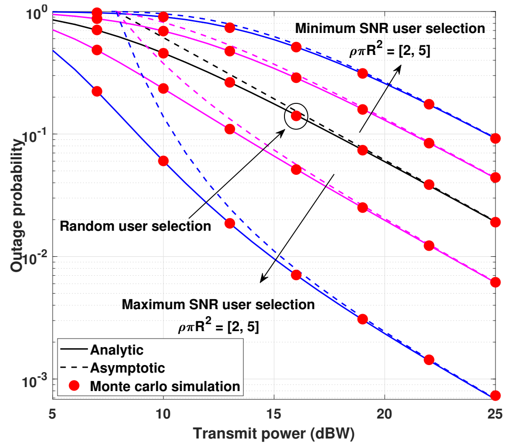
TABLE I: Parameters of the SR fading [23]

Shadowing scenario	Ω	b_0	m
Frequent heavy shadowing	8.97×10^{-4}	0.063	0.793
Infrequent light shadowing	1.29	0.158	19.4
Average shadowing	0.835	0.126	10.1

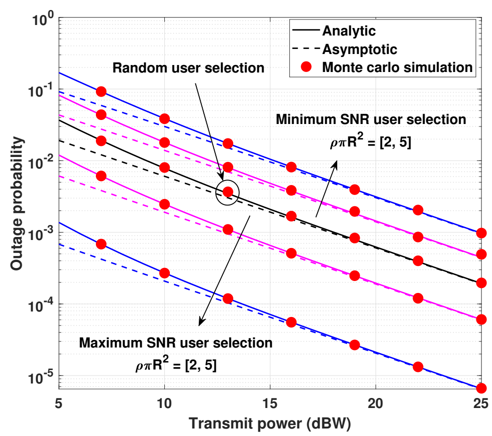
In this section, we present numerical results to confirm the validity of the analytic expression and demonstrate the accuracy of the obtained asymptotic expressions (10), (15), (18) at high SNR regime. In the simulation, we exploit SR fading parameter given in Table I [23], and set the intensity of the PPP with $\rho = [4.0744 \times 10^{-11}, 1.0186 \times 10^{-10}]$ which means there are two users and five users on average within a beam for each value of ρ, respectively. The system parameters used in the simulations are listed in Table II.

Fig. 2 illustrates the OP versus transmit power for different SR fading scenarios. We obtain the analytic expression and calculate its simulation results by truncating the infinite series, i.e., $n = 20$. As we expect, the analytic results agree well with Monte carlo simulation results, and the asymptotic curves match well with them at high SNR, which implies that the theoretical analysis accurately evaluates the OP. Moreover, we can confirm the gap in power gain between different user densities, while there is no change in diversity order.

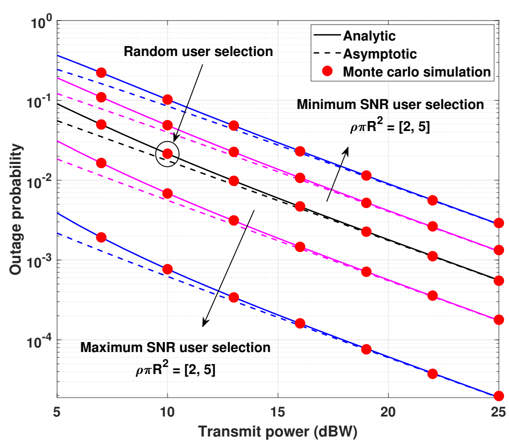
Fig. 3 depicts the SER performances with M-PSK modulation for different SR fading scenarios. As in the simulation of the OP, we obtain the simulation results and confirm that the



(a)

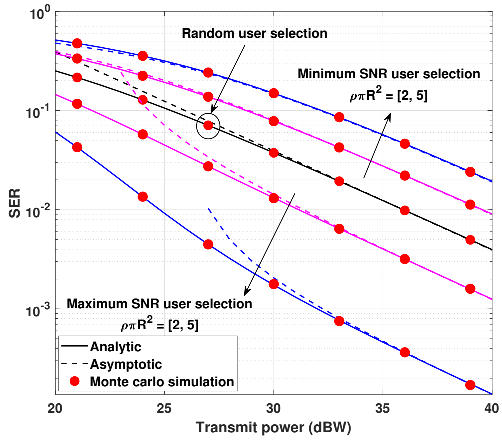


(b)

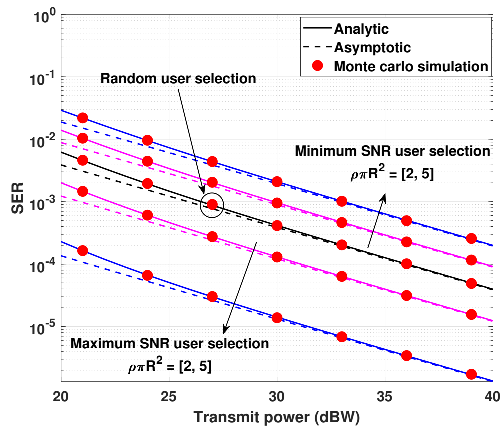


(c)

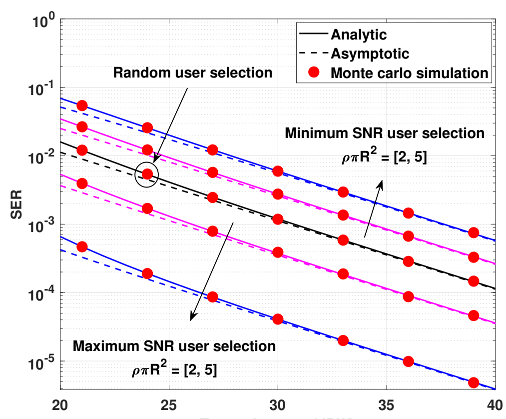
Fig. 2: OP versus the transmit power under different shadowing models: (a) frequent heavy shadowing, (b) infrequent light shadowing, (c) average shadowing



(a)



(b)



(c)

Fig. 3: SER versus the transmit power under different shadowing models: (a) frequent heavy shadowing, (b) infrequent light shadowing, (c) average shadowing

TABLE II: System Parameter

PARAMETER	VALUE
Satellite height	35786 km (GEO)
Link frequency band	f_c = 20 GHz (Ka)
Beam bandwidth	B = 500 MHz
Noise temperature	T = 517 K
Boltzmann constant	$\kappa = 1.3807 \times 10^{-23}$
Beam diameter	250 km
User antenna gain	41.7 dBi
Satellite antenna gain	52 dBi
3 dB angle	$0.4°$
Average rain attenuation	-3.125 dB
Outage threshold	$\gamma_{th} = -2.85$ dB

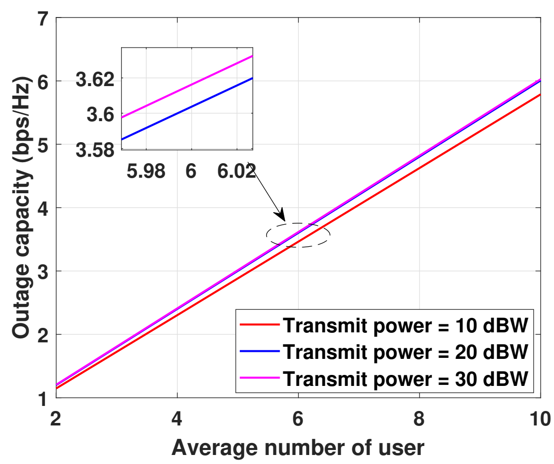


Fig. 4: Outage capacity versus the average number of beam

asymptotic results fit well with the analytic result at high SNR. Similar to the case of the OP, It can be seen that the better the channel scenario, the closer asymptotic curves are to the analytic curves at lower SNR.

Fig. 4 plots the outage capacity of users having SNR over $\gamma_{\min}$ in the beam according versus the average number of users. As shown in Fig. 2, in the case of the minimum SNR user selection, the OP increases as the average number of users increases. This means that as the number of users in the beam increases, the number of users that outage occurs also increases. Therefore, we calculate the total outage capacity of users in the beam. In the case of the minimum SNR user selection, although the number of users having outage increases according to the average number of users, it can be seen that the satellite can cover more users, so that the total outage capacity increases.

V. Conclusion

In this paper, we analyzed the performance of downlink users which are distributed as PPP within each beam in a SatCom system. By taking advantage of stochastic geometry, we derived the analytic expressions and asymptotic closed forms of the outage probabilities and SER. Numerical results demonstrated that the analytic and simulation results show reasonably good match and the asymptotic results converge to the analytic results at high SNR.

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