Individuality and Commonality based Multi-View Multi-Label Learning

Qiaoyu Tan, Guoxian Yu, Jun Wang, Carlotta Domeniconi, Xiangliang Zhang

***Abstract*—In multi-view multi-label learning, each object is represented by several heterogeneous feature representations and is also annotated with a set of discrete non-exclusive labels. Previous studies typically focus on capturing the shared latent patterns among multiple views, while do not sufficiently consider the diverse characteristics of individual views, which can cause performance degradation. In this paper, we propose a novel approach (ICM2L) to explicitly explore the individuality and commonality information of multi-label multiple view data in a unified model. Specifically, a common subspace is learned across different views to capture the shared patterns. Then, multiple individual classifiers are exploited to explore the characteristics of individual views. Next, an ensemble strategy is adopted to make prediction. Finally, we develop an alternative solution to jointly optimize our model, which can enhance the robustness of the proposed model towards rare labels and reinforce the reciprocal effects of individuality and commonality among heterogeneous views, and thus further improve the performance. Experiments on various real-word datasets validate the effectiveness of ICM2L against state-of-the-art solutions, and ICM2L can leverage the individuality and commonality information to achieve an improved performance as well as to enhance the robustness toward rare labels.**

***Index Terms*—Multi-view learning, Multi-label learning, Individuality, Commonality, Ensemble classification**

I. Introduction

In many real-world applications, data are often associated with several heterogeneous feature representations, each of which gives a different view of the data. For example, a news web page can be represented by two heterogeneous views, one is from the text (and image) information of the web page itself, the other is from the hyperlink to other pages; an image can be described using different features, such as texture descriptors, shape descriptors, color descriptors, surrounding texts, and so on. As a natural formulation for this type of data, multi-view learning has attracted a lot of attentions in machine learning and in various application domains [1], [2].

Although diverse multi-view learning methods have been proposed in the literature over the past years, they still have some limitations. On the one hand, most previous studies often assume that each sample is annotated with a single label [3], [4]. Nevertheless, in real-life applications, individual samples usually have more than one label. For instance, an image can be simultaneously annotated with several labels, such as sea, sky, and seagull; a web page could be tagged with multiple topics given as labels, such as economics, culture, sports and politics. On the other hand, the majority of existing studies are supervised approaches that require a large number of labeled samples [5], [6]. In practice, nevertheless, it is rather difficult and expensive to collect labeled samples, while unlabeled samples are easy to accumulate. Given this, a few semi-supervised multi-view multi-label learning approaches [7], [8] have been proposed to leverage limited labeled and abundant unlabeled samples. The key motivation behind them is to capture the complementary patterns among multiple views, which can boost the performance of multi-view learning [9].

Another limitation of the aforementioned methods is that they do not explicitly account for the distinctive information of individual views, which might degenerate their performance for a variety of reasons. Firstly, with respect to features, since multi-view samples have heterogeneous feature representations, in which each representation encodes different properties of the samples, they may fail to capture the global structure of multi-view data without exploring the distinctive information hidden in individual views. Secondly, with respect to labels, since each individual view captures a specific property of data, it is impossible for one view to comprehensively characterize all the relevant labels, especially when data is annotated with multiple labels. As a result, leveraging the *individuality* of each view, along with the *commonality* of multiple views may further improve the performance of the model, compared to focusing only on the individuality (or commonality) of the views.

Some researchers already explored the commonality and individuality of multi-view data for classification and clustering [10], [11], [12], [13], [14], [15]. It has been shown that the utilization of individual and shared patterns is beneficial for latent representation learning [11], [12], multi-output problem [15], and multi-label classification [10]. In multi-view multi-label classification scenarios, however, they may result in sub-optimal classifiers due to the isolated learning of hidden features and multi-label classifier [11], [12], or the lack of capacity to capture label correlations [10], [15]. More importantly, these methods mainly focus on learning a general classifier for all labels and treat them equally, which ignore the essential label imbalance problem in multi-label learning [16], [17]. To this end, the learned classifier may lose discriminant ability on rare labels, which are widely-witnessed in real-world applications. Motivated by this, in this paper, we propose to explore how individual and common patterns of multi-view

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data could be utilized to improve the performance of multi-label classification as well as in advancing the robustness of classifier towards rare labels.

To address the aforementioned issues, we propose a novel approach, called *I*ndividuality and *C*ommonality based *M*ulti-view *M*ulti-label *L*earning (ICM2L), to explicitly account for the individual and shared patterns hidden in different views. As shown in Figure 1, given multiple heterogeneous features of the input data, ICM2L seeks a shared subspace across heterogeneous views, which captures the commonality of different views, and adapts an ensemble classifier based on the shared subspace and on other individual feature spaces, as well as on the label information in a unified model. In this way, both the shared and view-specific information of different views could be used to boost the performance via a mutually beneficial effect, and thus further improve the performance of the model. The main contributions of this paper are summarized as follows:

- The proposed ICM2L can explicitly and jointly employ the individuality and commonality information of multi-view multi-label data. It learns a shared subspace from different views, label correlations, and an ensemble classifier based on individual and shared feature spaces in a unified model.
- ICM2L can explore the individuality of multiple views; as a result, it is superior to other methods in discovering rare labels.
- We develop an alternative optimization solution to iteratively optimize our model. Extensive empirical results on benchmark datasets demonstrate the superiority of ICM2L with respect to related and competitive methods lrMMC [7], LSML [8], CSMSC [13], MLAN [4], and SMMCL [18].

The rest of the paper is organized as follows. In Section 2, we briefly introduce the related work. Section 3 presents the proposed ICM2L. The experimental results and conclusions are discussed in Section 4 and Section 5, respectively.

II. Related Work

Our work is related to two branches of studies, multi-label learning and multi-view learning. In this section, we briefly review some related works in these two fields. For more details, please refer to [2] and [19].

A. Multi-Label Learning

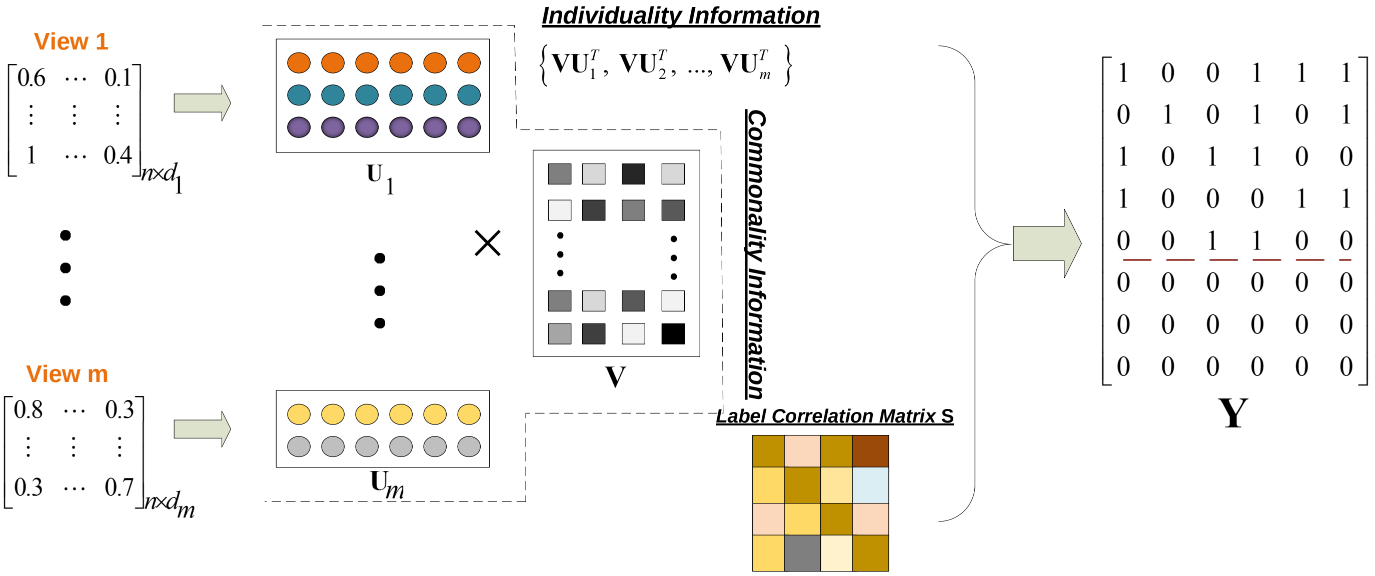
Different from binary classification scenarios, where each sample is associated with only one single semantic label, multi-label learning aims at assigning a set of discrete non-exclusive labels to a sample, and has received increasing interest in different machine learning tasks [20]. For instance, [21] and [22] assume that fully supervised signals are available and focus on learning multi-label classifiers under supervised setting. Such assumption, however, may not hold in real-world applications, because it requires exhaustive efforts to annotate multi-label samples. To avoid this limitation, researchers have resorted to develop semi-supervised multi-label classifiers [23], [24], [25], [26], [27], in which limited labeled samples as well as abundant unlabeled samples are jointly used for training. Besides, considering the fact that labeled data is tagged by human efforts, they might have some missing or noisly labels [28], [29], [30], [31], [32], [33], several approaches have been proposed to design multi-label classifiers under weak-label setting [28], [29], [34], [35] or with noisy labels [36], [32], [37], [38].

Although the aforementioned methods have achieved state-of-the-art performance for multi-label data, they mainly emphasize on single-view data and are not ready for multi-view data. In fact, it has been proved that directly applying existing multi-label algorithms to multi-view data by concatenating multiple feature vectors (views) together will result in a compromised performance [7], [2]. The reason is that such concatenation operation fails to explore the intra-view and inter-view relationships across heterogeneous views, which is very important for successful multi-view learning models [39]. In addition, given that label correlation is crucial for the success of multi-label learning [40], how to develop an effective algorithm that can jointly utilize the heterogeneous information as well as important label correlations among multi-view multi-label data still remains a challenge.

B. Multi-View Learning

Due to the ubiquity of multi-view data, multi-view learning has been an active research field in many real-world applications [2]. Several multi-view learning approaches were proposed to analyze multi-view multi-label data recently. For example, Nie *et al.,* [4] proposed a nearly parameter-free multi-view model MLAN by integrating semi-supervised classification and local structure learning simultaneously. Liu *et al.,* [7] proposed a matrix factorization based multi-view framework lrMMC, which firstly seeks a shared representation of multiple views and then conducts classification based on matrix completion on the shared feature space. Nevertheless, lrMMC models the fusion of multiple views and the follow-up prediction tasks as separate objectives, which may lead to suboptimal solution. To avoid such risk, some unified multi-view multi-label learning methods has been proposed [8], [41]. Specifically, Tan *et al.,* [41] aimed to improve multi-label prediction performance by seeking a shared subspace from incomplete views with weak labels, local label correlations, and a predictor in this subspace in a unified model. Zhang *et al.,* [8] sought a common feature representation and the corresponding projection model between the learned subspace and labels by simultaneously enforcing the consistence of latent semantic bases among different views in the kernel spaces. However, although such unified models can utilize the shared and complimentary information among different views to some extent, they do not explicitly take into account the individual patterns of heterogeneous views. As a result, they may have a compromised performance.

It has been recognized that exploring individuality and commonality of heterogeneous features can further boost the performance of multi-view data mining [10], [11], [12]. For example, [11] and [12] investigate the benefit of individual and common patterns in multi-view data to learn more discriminant low-dimensional representations. However, they both focus on representation learning and cannot be directly applied to


View 1
$\begin{bmatrix} 0.6 & \cdots & 0.1 \\ \vdots & \vdots & \vdots \\ 1 & \cdots & 0.4 \end{bmatrix}_{n \times d_1}$
U1
View m
$\begin{bmatrix} 0.8 & \cdots & 0.3 \\ \vdots & \vdots & \vdots \\ 0.3 & \cdots & 0.7 \end{bmatrix}_{n \times d_m}$
Um
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Individuality Information
$\{\mathbf{VU}_1^T, \mathbf{VU}_2^T, ..., \mathbf{VU}_m^T\}$
Commonality Information
Label Correlation Matrix S
$\begin{bmatrix} 1&0&0&1&1&1 \\ 0&1&0&1&0&1 \\ 1&0&1&1&0&0 \\ 1&0&0&0&1&1 \\ 0&0&1&1&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix}$
Y

Fig. 1. Method overview. ICM2L jointly leverages commonality and individuality information of multiple data views, as well as label correlations. The commonality information of heterogeneous views is captured by the shared low-dimensional representation $\mathbf{V}$; while the individuality information of multiple views is captured by the reconstructed data matrices $\{\mathbf{VU}_v\}_{v=1}^m$.

multi-label classification problem. Similarly, [13] and [14] separately learn the individual and common representations of multi-view data and then conduct clustering on the merged representations. They not only ignore the important correlations of multi-label data, but also suffer from a two-stage fashion that may result in sub-optimal problem. To bridge the gap, [10] introduces a unified model to jointly learn compact latent features and multi-label classifier. Specifically, it assumes that the final latent feature vector with size d is composed with multiple view-specific features with size d_s and one shared common feature vector with size d_c, such that $d = md_s + d_c$, where m denotes the number of views. With this assumption, this unified model is able to generate comprehensive feature representations that can capture both individual and common patterns of multi-view data. However, it still targets on developing a general multi-label classifier for all labels and may lose discriminant ability towards rare labels, which refers to label imbalance problem in multi-label classification. Moreover, it also cannot capture the crucial correlations between multiple labels.

One similar work with our ICM2L is SMMCL [18], which is a self-pace based label propagation method that model the common consensus and individuality of multiple teacher classifiers, each for one view. Although SMMCL can iteratively propagate labels to the most informative candidate unlabeled samples during training, it suffers from the scalable issue and can not work on a moderate(large)-scale data as shown in our experiments. Besides, SMMCL also ignores the label correlations of multi-labeled data, which is the cornerstone for successful multi-label learning algorithms [40]. Another core distinction between SMMCL and our model is that SMMCL uses individual information of multi-view data to discover the most informative unlabeled samples, while our method utilizes such information to improve the discriminant ability towards rare labels and the robustness. Extensive empirical results on benchmark datasets demonstrate the superiority of ICM2L to these related competitive methods [7], [42], [4], [18], [8].

III. The Proposed Approach

A. Problem Statement and Notations

Suppose $\mathcal{X} = \{\mathbf{X}_v\}_{v=1}^m$ represents a dataset with n samples and m views, where $\mathbf{X}_v = [\mathbf{x}_v^1, \mathbf{x}_v^2, \cdots, \mathbf{x}_v^n] \in \mathbb{R}^{n \times d_v}$ indicates the full feature space in view v. $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_n]^T \in \{-1, 1\}^{n \times c}$ is the corresponding label matrix, where $\mathbf{y}_i \in \{-1, 1\}^c$ is the label vector of $\mathbf{x}_i$ and c is the number of distinct labels. $\mathbf{y}_{ic'} = 1$ $(c' = 1, ..., c)$ means the c'-th label is relevant; otherwise, $\mathbf{y}_{ic'} = -1$. Without loss of generality, we assume that, out of n samples, the first l are labeled samples, while the remaining u are unlabeled samples. Our goal is to learn a predictive label matrix $\hat{\mathbf{Y}} \in \{1, -1\}^{n \times c}$ from heterogeneous feature representations $\{\mathbf{X}_v\}_{v=1}^m$ and partial label matrix $\mathbf{Y}$.

B. Problem Formulation

An intuitive strategy to deal with multi-view multi-label setting is to concatenate multi-view features into a single vector, and then conduct classification based on classical multi-label learning algorithms [19]. Such concatenation, however, ignores the fact that features are extracted from different spaces with different statistical properties, and directly applying multi-label methods on the concatenated data may suffer from the over-fitting problem, and often leads to high time complexity, which may be unacceptable in real-world applications. Besides, feature concatenation also ignores the useful diversity information across different views and might lead to a suboptimal problem [8].

In order to take advantage of consensus information, we advocate to seek the shared subspace from different views based on matrix factorization techniques [43]. Specifically, we resort to nonnegative matrix factorization (NMF) [44], [45] for its power in extracting representations of diverse data [46]. It is worth to note that the major difference between NMF and other matrix factorization methods, such as singular value decomposition [47], is the nonnegative constraints, which helps obtaining a part-based representation, as well as enhancing interpretability of the learned subspace. Another reason for the choice of NMF is due to the fact that most multi-view data are naturally nonnegative or can be easily transformed into nonnegative ones, without changing the proximity between the original data. Our model can also be adapted to mix-sign data by replacing NMF with other matrix factorization techniques (i.e., semi-nonnegative matrix factorization [48]). Besides, our method is flexible for basic matrix factorization techniques, thus more powerful algorithms, i.e., deep matrix factorization models [49], are expected to further improve the performance. Given heterogeneous representations of data $\{\mathbf{X}_v\}_{v=1}^m$ and the label matrix $\mathbf{Y}$, our unified objective function is given as follows:

$$\begin{aligned}\mathcal{L} =& \sum_{v=1}^{m} ||\mathbf{X}_v - \mathbf{V}\mathbf{U}_v^T||_F^2 + \alpha||\mathbf{V}\mathbf{W}_0 - \mathbf{Y}||_F^2 + \beta||\mathbf{W}_0||_F^2 \\ & s.t.\ \mathbf{U} \geq 0, \mathbf{V} \geq 0,\end{aligned} \tag{1}$$

where $\mathbf{U}_v \in \mathbb{R}^{d_v \times k}$ denotes the individual matrix for the v-th view, $\mathbf{V} \in \mathbb{R}^{n \times k}$ is the shared subspace, where k is the desired low-rank size, $||.||_F$ represents the Frobenius norm, $\mathbf{U}_v \geq 0$ and $\mathbf{V} \geq 0$ are the non-negative constraints for the matrices. $\mathbf{W}_0 \in \mathbb{R}^{k \times c}$ is the coefficient matrix corresponding to $\mathbf{V}$, α and β are two trade-off parameters. The first part of Eq. (1) is the consensus term, which aims to seek the common low-dimensional representation of multi-view data under the assumption that different views have distinct mapping matrices $\{\mathbf{U}_v\}_{v=1}^m$, but share the same latent feature space $\mathbf{V}$. For this reason, we can make use of the consensus information across multi-views to some extent. The second part of Eq. (1) is the standard empirical loss term. It measures the empirical loss on labeled samples and ensures that the predicted label matrix is consistent with the initial ground truth label assignment. By jointly optimizing the two terms, we can exploit the label information $\mathbf{Y}$ to induce the shared subspace towards a semantic label space. It not only helps to obtain a discriminate subspace but also may alleviate the widely spread semantic gap [50] between the input heterogeneous feature spaces and the semantic label space, since $\mathbf{V}$ can be viewed as a bridge between them. The last part is the widely used ℓ_2 norm regulation term, it is included to avoid the over-fitting problem and reduce the impact of noisy features.

By minimizing Eq. (1), we can learn a discriminate predictor by jointly using both shared information among views and label information of labeled samples. But the predictor won't be discriminant enough, as we completely ignore another important issue in multi-view multi-label learning, i.e., the specific characteristics of individual views (*individuality*), which may further improve the performance. As discussed before, individual patterns hidden in multiple views is useful in boosting the robustness of multi-label classifier towards rare labels. In addition, these individual patterns can also boost the robustness of the predictor, due to the known usefulness of diversity for ensemble learning. As such, we need to capture the individual information of different views to further improve the performance of our model.

Considering the fact that distinctive mapping matrices $\{\mathbf{U}_v\}_{v=1}^m$ are actually used to encode the corresponding unique properties of individual views, we define another set of coefficient matrices $\{\mathbf{W}_v\}_{v=1}^m$ to capture the distinctive characteristics of different views, where $\mathbf{W}_v \in \mathbb{R}^{d_v \times c}$ maps the reconstructed feature matrix $\mathbf{V}\mathbf{U}_v^T$ of the v-th view to the label space. In this way, we leverage the individual and common information of heterogeneous views and further extend Eq. (1) as follows:

$$\begin{aligned}\mathcal{L} =& \sum_{v=0}^{m} ||\mathbf{X}_v - \mathbf{V}\mathbf{U}_v^T||_F^2 + \alpha||\mathbf{V}\mathbf{W}_0 - \mathbf{Y}||_F^2 \\ &+ \frac{1-\alpha}{m}\sum_{v=1}^{m} ||\mathbf{V}\mathbf{U}_v^T\mathbf{W}_v - \mathbf{Y}||_F^2 \\ &+ \beta(||\mathbf{W}_0||_F^2 + ||\mathbf{W}_v||_F^2),\end{aligned} \tag{2}$$

where $\mathbf{X}_0 = \mathbf{V}$ and $\mathbf{U}_0$ is an identity matrix, $\alpha \in (0, 1)$ is a trade-off parameter which balances the contribution of individuality and commonality among multi-view data. Here $\mathbf{X}_0$ can be regarded as an additionally introduced common view spanned by the shared subspace $\mathbf{V}$. It is worth to note that $\mathbf{V}$ can be regarded as the dictionary matrix for all views, and $\mathbf{U}_v$ represents the view-specific coding coefficients. Therefore $\mathbf{V}\mathbf{U}_v^T$ targets to reserve the view-specific input signals, while $\mathbf{V}$ captures the shared information for all views. Hence, $\mathbf{V}$ and $\mathbf{V}\mathbf{U}_v^T$ could be regarded as different representations of multi-view data. By minimizing Eq. (2), we can achieve two goals. On the one hand, since both common and individual information of heterogeneous views are employed in an elegant way, the learned shared subspace is more discriminate. On the other hand, the final predictive model is also enhanced, because it absorbs both the public labels appeared in most views as well as the individual labels embedded in several specific views.

Another inherent property of learning from multi-label data is how to utilize label correlations, and this issue has not been addressed in Eq. (2). Label correlation has long been regarded as a fundamental challenge that can be used to improve the performance of multi-label learning [40], [19]. To address this limitation, we try to leverage label correlation among labels to estimate the predicted likelihood scores as follows:

$$\begin{aligned}\mathcal{L} =& \sum_{v=0}^{m} ||\mathbf{X}_v - \mathbf{V}\mathbf{U}_v^T||_F^2 + \alpha||\mathbf{V}\mathbf{W}_0\mathbf{S} - \mathbf{Y}||_F^2 \\ &+ \frac{1-\alpha}{m}\sum_{v=1}^{m} ||\mathbf{V}\mathbf{U}_v^T\mathbf{W}_v\mathbf{S} - \mathbf{Y}||_F^2 \\ &+ \beta(||\mathbf{W}_0||_F^2 + ||\mathbf{W}_v||_F^2),\end{aligned} \tag{3}$$

where $\mathbf{S} \in \mathbb{R}^{c \times c}$ represents the label correlations matrix, which can be estimated from the known $\mathbf{Y}$. For example, the correlation is often measured by the cosine similarity. However, since labeled samples may be not sufficient, we advocate to learn it by leveraging the features and already known labels of training data. In experiments, we randomly initialize $\mathbf{S}$ and treat it as a trainable parameter during optimization.

The final prediction label matrix $\hat{\mathbf{Y}}$ is given by majority voting $\hat{\mathbf{Y}} = \mathbf{V}(\sum_{v=1}^{m} \mathbf{U}_v \mathbf{W}_v + \mathbf{W}_0)\mathbf{S}$. Eq. (3) not only explicitly considers the common and individual information among multiple views in a principle way, but also absorbs label information to induce the shared subspace and enhance its discriminate power. Another advantage of our model is that it exploits ensemble learning to further derive robust results, especially for rare labels. Our following experiments will confirm these advantages.

C. Optimization

The objective function in Eq. (3) involves $\{\mathbf{U}_v\}_{v=1}^m$, $\mathbf{V}$, $\mathbf{S}$, $\{\mathbf{W}_v\}_{v=1}^m$ and $\mathbf{W}_0$, and it is not easy to optimize all the variables simultaneously. Given that, we adopt an alternating optimization technique to optimize the objective function by alternatively optimizing one variable while fixing other variables.

1) Update $\{\mathbf{U}_v\}_{v=1}^m$ with Fixed $\mathbf{V}$, $\{\mathbf{W}_v\}_{v=1}^m$, $\mathbf{W}_0$ and $\mathbf{S}$: When $\mathbf{V}$, $\mathbf{W}$, $\mathbf{W}_v$ and $\mathbf{S}$ are fixed, the computation of $\mathbf{U}_v$ is independent from $\mathbf{U}_{v'}, v' \neq v$. Thus, for each view v, we obtain the following equation by taking the derivative of Eq. (3) w.r.t. $\mathbf{U}_v$:

$$\begin{aligned}\mathcal{L}(\mathbf{U}_v) = -2\mathbf{X}_v^T\mathbf{V} + 2\mathbf{U}_v\mathbf{V}^T\mathbf{V} - 2(1-\alpha)/m\mathbf{W}_v\mathbf{S}\mathbf{Y}^T\mathbf{V} \\ + 2(1-\alpha)/m\mathbf{W}_v\mathbf{S}\mathbf{S}^T\mathbf{W}_v^T\mathbf{U}_v\mathbf{V}^T\mathbf{V}.\end{aligned} \tag{4}$$

Using the Karush-Kuhn-Tucker (KKT) condition [51], we can derive the following updating rule:

$$(\mathbf{U}_v)_{i,j} \leftarrow (\mathbf{U}_v)_{i,j} \frac{(\mathbf{X}_v^T\mathbf{V} + \frac{1-\alpha}{m}\mathbf{W}_v\mathbf{S}\mathbf{Y}_v^T\mathbf{V})_{i,j}}{(\mathbf{U}_v\mathbf{V}^T\mathbf{V} + \frac{1-\alpha}{m}\mathbf{W}_v\mathbf{S}\mathbf{S}^T\mathbf{W}_v^T\mathbf{U}_v\mathbf{V}^T\mathbf{V})_{i,j}}. \tag{5}$$

2) Update $\mathbf{V}$ with Fixed $\{\mathbf{U}_v\}_{v=1}^m$, $\{\mathbf{W}_v\}_{v=1}^m$, $\mathbf{W}_0$ and $\mathbf{S}$: When fixed $\{\mathbf{U}_v\}_{v=1}^m$, $\{\mathbf{W}_v\}_{v=1}^m$, $\mathbf{W}_0$ and $\mathbf{S}$, we obtain the following equation by taking the derivative of Eq. (3) w.r.t. $\mathbf{V}$ to zero:

$$\begin{aligned}2\sum_{v=0}^{m}(\mathbf{V}\mathbf{U}_v^T\mathbf{U}_v - \mathbf{X}_v\mathbf{U}_v) - 2\frac{1-\alpha}{m}\sum_{v=0}^{m}\mathbf{Y}\mathbf{S}^T\mathbf{W}_v^T\mathbf{U}_v \\ +2\frac{1-\alpha}{m}\sum_{v=0}^{m}\mathbf{V}\mathbf{U}_v^T\mathbf{W}_v\mathbf{S}\mathbf{S}^T\mathbf{W}_v^T\mathbf{U}_v \\ +2\alpha\mathbf{V}\mathbf{W}_0\mathbf{S}\mathbf{S}^T\mathbf{W}_0^T - 2\alpha\mathbf{Y}\mathbf{S}^T\mathbf{W}_0^T = 0.\end{aligned} \tag{6}$$

We then have the following closed-form solution for $\mathbf{V}$, which is updated efficiently,

$$\mathbf{V} = \frac{\sum_{v=0}^{m}\theta_v\mathbf{X}_v\mathbf{U}_v + \alpha\mathbf{Y}\mathbf{S}^T\mathbf{W}_v^T\mathbf{U}_v + \mathbf{Q}_1}{\sum_{v=1}^{m}\theta_v\mathbf{U}_v^T\mathbf{U}_v + \alpha\mathbf{V}\mathbf{W}_0\mathbf{S}\mathbf{S}^T\mathbf{W}_0^T + \mathbf{Q}_2}. \tag{7}$$

Where $\mathbf{Q}_1 = \frac{1-\alpha}{m}\sum_{v=0}^{m}\mathbf{Y}\mathbf{S}^T\mathbf{W}_v^T\mathbf{U}_v$ and $\mathbf{Q}_2 = \frac{1-\alpha}{m}\sum_{v=1}^{m}\mathbf{U}_v^T\mathbf{W}_v\mathbf{S}\mathbf{S}^T\mathbf{W}_v^T\mathbf{U}_v$.

3) Update $\mathbf{W}_v$ with Fixed $\mathbf{V}$, $\{\mathbf{U}_v\}_{v=1}^m$, $\mathbf{W}_0$ and $\mathbf{S}$: When $\mathbf{V}$, $\{\mathbf{U}_v\}_{v=1}^m$, $\mathbf{W}_0$ and $\mathbf{S}$ are fixed, similarly to the update of $\mathbf{U}_v$, we have the following equation by setting the derivative w.r.t. $\mathbf{W}_v$,

$$2\frac{1-\alpha}{m}(\mathbf{U}_v\mathbf{V}^T\mathbf{V}\mathbf{U}_v^T\mathbf{W}_v\mathbf{S}\mathbf{S}^T - \mathbf{U}_v\mathbf{V}^T\mathbf{Y}\mathbf{S}^T) + 2\beta\mathbf{W}_v. \tag{8}$$

Based on KKT condition, we can derive the following updating rule:

$$(\mathbf{W}_v)_{i,j} \leftarrow (\mathbf{W}_v)_{i,j}\frac{(\frac{1-\alpha}{m}\mathbf{U}_v\mathbf{V}^T\mathbf{Y}\mathbf{S}^T)_{i,j}}{(\frac{1-\alpha}{m}\mathbf{U}_v\mathbf{V}^T\mathbf{V}\mathbf{U}_v^T\mathbf{W}_v\mathbf{S}\mathbf{S}^T + \beta\mathbf{W}_v)_{i,j}}. \tag{9}$$

4) Update $\mathbf{W}_0$ with Fixed $\mathbf{V}$, $\{\mathbf{U}_v\}_{v=1}^m$, $\{\mathbf{W}_v\}_{v=1}^m$ and $\mathbf{S}$: When $\mathbf{V}$, $\{\mathbf{U}_v\}_{v=1}^m$, $\{\mathbf{W}_v\}_{v=1}^m$ and $\mathbf{S}$ are fixed, we obtain the following equation by taking the derivative of Eq. (3) w.r.t. $\mathbf{W}_0$ to zero:

$$\mathcal{L}(\mathbf{W}_0) = 2\alpha(\mathbf{V}^T\mathbf{V}\mathbf{W}_0\mathbf{S}\mathbf{S}^T - \mathbf{V}^T\mathbf{Y}\mathbf{S}^T) + 2\beta\mathbf{W}_0. \tag{10}$$

Based on KKT condition, we can derive the following update rule:

$$(\mathbf{W}_0)_{i,j} \leftarrow (\mathbf{W}_0)_{i,j}\frac{(\alpha\mathbf{V}^T\mathbf{Y}\mathbf{S}^T)_{i,j}}{(\alpha\mathbf{V}^T\mathbf{V}\mathbf{W}_0\mathbf{S}\mathbf{S}^T + \beta\mathbf{W}_0)_{i,j}}. \tag{11}$$

5) Update $\mathbf{S}$ with Fixed $\mathbf{V}$, $\{\mathbf{U}_v\}_{v=1}^m$, $\{\mathbf{W}_v\}_{v=1}^m$ and $\mathbf{W}_0$: When $\mathbf{V}$, $\{\mathbf{U}_v\}_{v=1}^m$, $\{\mathbf{W}_v\}_{v=0}^m$ and $\mathbf{W}_0$ are fixed, similarly to the update of $\mathbf{V}$, we have the following equation for $\mathbf{S}$ by setting the derivative w.r.t. $\mathbf{S}$ to zero,

$$\begin{aligned}2\sum_{v=0}^{m}\frac{1-\alpha}{m}(\mathbf{W}_v^T\mathbf{U}_v\mathbf{V}^T\mathbf{V}\mathbf{U}_v^T\mathbf{W}_v\mathbf{S} - \mathbf{W}_v^T\mathbf{U}_v\mathbf{V}^T\mathbf{Y}) \\ +2\alpha(\mathbf{W}^T\mathbf{V}^T\mathbf{V}\mathbf{W}_0\mathbf{S} - \mathbf{W}_0^T\mathbf{V}^T\mathbf{Y}) = 0.\end{aligned} \tag{12}$$

We therefore have the following closed-form solution for $\mathbf{S}$,

$$\begin{aligned}\mathbf{S} =&(\frac{1-\alpha}{m}\sum_{v=0}^{m}\mathbf{W}_v^T\mathbf{U}_v\mathbf{V}^T\mathbf{V}\mathbf{U}_v^T\mathbf{W}_v + \alpha\mathbf{W}^T\mathbf{V}^T\mathbf{V}\mathbf{W}_0\mathbf{S})^{-1} \\ &(\frac{1-\alpha}{m}\sum_{v=0}^{m}\mathbf{W}_v^T\mathbf{U}_v\mathbf{V}^T\mathbf{Y} + \alpha\mathbf{W}_0^T\mathbf{V}^T\mathbf{Y}).\end{aligned} \tag{13}$$

Given the above iterative optimization process, we summarize the main procedure of ICM2L in Algorithm 1.

D. Complexity analysis

The time complexity of ICM2L is dominated by the matrix multiplication and matrix inverse operations. The time complexity of matrix inverse for $\mathbf{V}$ and $\mathbf{S}$ is relatively small, so the time complexity of ICM2L is mainly driven by the computation for $\mathbf{U}_v$, $\mathbf{W}_v$ and $\mathbf{W}_0$. Concretely, the complexity for solving $\mathbf{U}_v$, $\mathbf{W}_v$ and $\mathbf{W}_0$ is $O(d_{max}nk + d_{max}k^2 + d_{max}ck)$, $O(d_{max}nk)$ and $O(nck)$, respectively, where d_{max} is the largest dimensionality of the views. Since $n \gg k$ and $n \gg c$, the overall time complexity of ICM2L is $O(d_{max}nkt)$, where t is the number of iterations to reach convergence. In practice, if $d_{max} \ll n$, the total complexity of ICM2L scales with the number of samples. In our experiments, we found that t usually does not exceed 80. In addition, some views have sparse feature matrices, so the actual time cost of the above operations can be further reduced.

IV. Experiments

In this section, we conduct extensive experiments over six real-world datasets to evaluate the efficiency and effectiveness of the proposed framework. There are four major questions

Algorithm 1 ICM2L: Individuality and Commonality based Multi-View Multi-Label Learning

```
Input:
    $\{\mathbf{X}_v\}_{v=1}^m$: $n$ training samples with $m$ views
    $\mathbf{Y}$: Initial label matrix for $n$ samples
    $k$: Dimensionality of the shared subspace
    $\alpha$ and $\beta$: Trade-off parameters used in Eq. (3)
    $\varepsilon$: Convergence threshold
    $t$: Number of iterations
Output:
    $\hat{\mathbf{Y}}$: Predicted label likelihood matrix for $n$ samples
Randomly initialize $\mathbf{U}_v$, $\mathbf{W}_v$, $\mathbf{V}$, $\mathbf{W}$, and $\mathbf{S}$;
Compute $\mathcal{L}_0$ by Eq. (3);
for $i = 1, 2, \cdots, t$ do
    for $v = 1, 2, \cdots, m$ do
        Update $\mathbf{U}_v$ by Eq. (5);
    end
    Update $\mathbf{V}$ by Eq. (7);
    for $v = 1, 2, \cdots, m$ do
        Update $\mathbf{W}_v$ by Eq. (9);
    end
    Update $\mathbf{W}_0$ by Eq. (11);
    Update $\mathbf{S}$ by Eq. (13);
    Update $\mathcal{L}_i$ by Eq. (3);
    If $|\mathcal{L}_i - \mathcal{L}_{i-1}| \leq \varepsilon$, Return.
end
```

we aim to answer. (1) How effective of ICM2L compared with other related methods in classifying multi-view multi-label data? (2) How robust is ICM2L in discovering rare labels compared with the state-of-the-arts methods? (3) What are the impacts of two parameters α and β on ICM2L? (4) How efficient is ICM2L in modeling multi-view multi-label learning problem?

A. Experimental Setup

Six multi-view datasets that we employed in the experiments are all publicly available. The statistics of them are summarized in Table I. Yeast is a biological data set with two views [52], one view is the genetic expression and the other is the phylogenetic profile of a gene. Core15k, Pascal07 and ESPGame are three widely used multi-view image datasets[1]. We collected the multiple features of these images from [53], where each image is represented by six representative feature views: HUE, SIFT, GIST, HSV, RGB and LAB. Each sample in Mirflicker[2] and Nus-wide[3] consists of an image and textual tags, we construct the two views (image and text) according to [54]. For each dataset, we randomly sample 30% data for training and use the remaining 70% data for testing (unlabeled data).

Baseline Methods: To study the performance of ICM2L, we compare it with six state-of-the-art methods. In addition, to investigate the contribution of encoding common information, individual information and using label correlations, we include three variants, i.e., ICM2L-c, ICM2L-i, ICM2L-lc.

TABLE I
STATISTICS OF FOUR MULTI-VIEW DATASETS: n IS THE NUMBER OF SAMPLES; m IS THE NUMBER OF VIEWS; c IS THE NUMBER OF DISTINCT LABELS; $\#$AVG IS THE AVERAGE NUMBER OF LABELS PER SAMPLE; d_{min} IS THE SMALLEST DIMENSION OF ALL VIEWS.

datasets	n	m	c	$\#$avg	d_{min}
Yeast	2417	2	14	4.237	24
Core15k	4999	6	260	3.396	100
Pascal07	9963	6	20	1.465	100
ESPGame	20770	6	268	4.686	100
Mirflicker	25000	2	24	3.794	512
Nus-wide	260648	2	81	2.783	500

- lrMMC [7] leverages a low-dimensional common representation of all views and matrix completion for multi-label classification.
- LSML [8] is a recent multi-view multi-label learning framework that learns the shared subpace among heterogeneous features as well the follow-up predictor in a unified objective function.
- MLAN [4] is another unified multi-view learning method and initially focuses on single label classification problem; we adapt it for multi-label scenario by assigning multiple labels instead of a single one to unlabeled data.
- CSMSC [13] is a multi-view subspace learning approach, which can jointly extract the consistency and specificity of heterogeneous features for subspace representation learning. We adopt the extracted individual and common representation features as inputs to our model to train the ensemble classifier.
- SMMCL [18] is a self-paced based multi-view multi-label learning method, which considers both the individuality and commonality characteristics among multi-view data.
- MVMC-LS [42] is a multi-view learning approach based on matrix completion, it combines the matrix completion outputs of different views with various weights.
- ICM2L-c is a variant of ICM2L by excluding individual information and makes prediction by $\mathbf{W}$ (commonality).
- ICM2L-i is a variant of ICM2L by excluding common information and makes prediction by integrating $\{\mathbf{W}_v\}_{v=1}^m$ (individuality).
- ICM2L-lc is a variant of ICML2 by excluding label correlations.

For comparing methods, five-fold cross validation on the training set is used to select the optimal parameter values from the range as suggested in the original papers. For our method, we selected the parameters α and β in the range of $\{0.1, 0.2, \cdots, 1\}$ and $\{0.1, 0.3, \cdots, 2\}$, respectively. To avoid random effects, all the experiments are independently repeated ten times, and both the mean and standard deviation are reported. For each comparing method, the code is released or provided by corresponding authors. The code of ICM2L is publicly available at http://mlda.swu.edu.cn/codes.php?name=ICM2L.

Evaluation: Four widely used metrics are adopted for performance comparisons: *Accuracy*, *Ranking Loss (RL)*, *Average Precision (AP)* and average *AUC*. Note that these metrics

[1] http://lear.inrialpes.fr/data/

[2] http://press.liacs.nl/mirflickr/mirdownload.html

[3] http://lms.comp.nus.edu.sg/research/NUS-WIDE.html

TABLE II
RESULTS ON FOUR DATASETS WITH $k = 0.5d_{min}$. d_{min} REPRESENTS THE MINIMUM DIMENSIONALITY OF MULTIPLE VIEWS.

Dataset	metric	lrMMC	MLAN	MVMC-LS	CSMSC	LSML	SMMCL	ICM2L
Yeast	Accuracy	$\mathbf{0.539 \pm 0.001}$	0.381 ± 0.001	0.517 ± 0.001	0.537 ± 0.001	0.535 ± 0.004	$\mathbf{0.542 \pm 0.002}$	0.536 ± 0.004
	1-RL	0.787 ± 0.001	$\mathbf{0.811 \pm 0.002}$	0.761 ± 0.000	0.793 ± 0.001	0.797 ± 0.003	$\mathbf{0.816 \pm 0.001}$	0.788 ± 0.005
	AP	0.703 ± 0.001	0.459 ± 0.001	0.662 ± 0.000	0.698 ± 0.002	0.702 ± 0.004	$\mathbf{0.717 \pm 0.004}$	0.702 ± 0.003
	AUC	0.798 ± 0.001	0.589 ± 0.001	0.778 ± 0.000	0.797 ± 0.001	0.799 ± 0.002	$\mathbf{0.812 \pm 0.003}$	0.799 ± 0.005
Core15k	Accuracy	$\mathbf{0.191 \pm 0.001}$	0.103 ± 0.001	0.172 ± 0.000	$\mathbf{0.193 \pm 0.001}$	$\mathbf{0.193 \pm 0.001}$	$\mathbf{0.192 \pm 0.002}$	$\mathbf{0.194 \pm 0.001}$
	1-RL	0.758 ± 0.001	0.521 ± 0.002	0.750 ± 0.000	0.762 ± 0.001	0.768 ± 0.001	0.771 ± 0.001	$\mathbf{0.795 \pm 0.003}$
	AP	0.236 ± 0.001	0.146 ± 0.001	0.215 ± 0.001	0.432 ± 0.001	0.256 ± 0.001	0.259 ± 0.002	$\mathbf{0.279 \pm 0.004}$
	AUC	0.760 ± 0.001	0.710 ± 0.001	0.752 ± 0.000	0.767 ± 0.001	0.774 ± 0.001	0.778 ± 0.003	$\mathbf{0.797 \pm 0.003}$
Pascal07	Accuracy	0.278 ± 0.000	0.205 ± 0.001	0.264 ± 0.001	0.281 ± 0.002	0.283 ± 0.001	0.285 ± 0.001	$\mathbf{0.296 \pm 0.003}$
	1-RL	0.697 ± 0.001	0.502 ± 0.001	0.692 ± 0.001	0.715 ± 0.001	0.725 ± 0.003	0.730 ± 0.002	$\mathbf{0.756 \pm 0.005}$
	AP	0.429 ± 0.000	0.350 ± 0.002	0.401 ± 0.002	0.424 ± 0.002	0.446 ± 0.001	0.451 ± 0.001	$\mathbf{0.452 \pm 0.001}$
	AUC	0.727 ± 0.000	0.646 ± 0.001	0.725 ± 0.001	0.739 ± 0.003	0.758 ± 0.002	0.763 ± 0.002	$\mathbf{0.785 \pm 0.005}$
ESPGame	Accuracy	0.170 ± 0.000	0.088 ± 0.000	0.134 ± 0.001	0.177 ± 0.000	0.189 ± 0.001	0.192 ± 0.001	$\mathbf{0.206 \pm 0.001}$
	1-RL	0.777 ± 0.000	0.521 ± 0.001	0.764 ± 0.001	0.784 ± 0.001	$\mathbf{0.796 \pm 0.001}$	$\mathbf{0.798 \pm 0.002}$	$\mathbf{0.796 \pm 0.001}$
	AP	0.189 ± 0.000	0.111 ± 0.000	0.167 ± 0.000	0.194 ± 0.001	0.205 ± 0.001	0.207 ± 0.001	$\mathbf{0.220 \pm 0.002}$
	AUC	0.783 ± 0.000	0.642 ± 0.000	0.770 ± 0.001	0.785 ± 0.002	0.789 ± 0.000	0.790 ± 0.002	$\mathbf{0.803 \pm 0.001}$
Mirflickr	Accuracy	0.376 ± 0.001	0.282 ± 0.004	0.355 ± 0.001	0.387 ± 0.002	0.394 ± 0.002	0.412 ± 0.001	$\mathbf{0.436 \pm 0.001}$
	1-RL	0.750 ± 0.002	0.675 ± 0.002	0.736 ± 0.002	0.758 ± 0.001	0.765 ± 0.003	0.773 ± 0.001	$\mathbf{0.796 \pm 0.001}$
	AP	0.466 ± 0.003	0.401 ± 0.001	0.419 ± 0.002	0.471 ± 0.002	0.485 ± 0.001	0.498 ± 0.002	$\mathbf{0.536 \pm 0.002}$
	AUC	0.757 ± 0.001	0.664 ± 0.003	0.737 ± 0.001	0.761 ± 0.001	0.769 ± 0.001	0.774 ± 0.001	$\mathbf{0.790 \pm 0.001}$
Nus-wide	Accuracy	0.249 ± 0.002	0.198 ± 0.001	0.231 ± 0.002	0.253 ± 0.001	0.268 ± 0.003	0.286 ± 0.004	$\mathbf{0.332 \pm 0.001}$
	1-RL	0.791 ± 0.003	0.714 ± 0.004	0.779 ± 0.002	0.804 ± 0.001	0.819 ± 0.002	0.835 ± 0.003	$\mathbf{0.923 \pm 0.002}$
	AP	0.311 ± 0.002	0.243 ± 0.001	0.304 ± 0.003	0.321 ± 0.001	0.338 ± 0.003	0.367 ± 0.004	$\mathbf{0.448 \pm 0.004}$
	AUC	0.812 ± 0.001	0.735 ± 0.002	0.798 ± 0.003	0.823 ± 0.004	0.841 ± 0.002	0.876 ± 0.003	$\mathbf{0.933 \pm 0.002}$

generally belong to two categories: example-based and label-based criteria [55]. RL, AP, and Accuracy are example based metrics, while AUC is a label-based criterion. They evaluate the performance from ranking and classification perspectives [19], in which RL, AP, and AUC are ranking based metrics, while Accuracy is an example-based classification criteria. Formal definition of the four metrics can be found in [19], [55]. *Accuracy* requires the predicted label likelihood vector to be a binary indicator vector. Here, we consider the labels corresponding to the r largest entries of the vector of the i-th sample as the predicted labels, where r is determined as the average number of labels (round to next integer) of labeled samples. To maintain consistency with other evaluation metrics, in our experiments, we report 1-RL instead of RL. Thus, as for other metrics, the higher the value of 1-RL, the better the performance is. These metrics evaluate multi-label classification from different points of view, and it is unlikely for a method outperforming all the other techniques across all the metrics.

B. Effectiveness of ICM2L

To investigate the first question stated at the beginning of this section, we compare the classification performance of all methods on the six datasets listed in Table II. Since SMMCL consumes a lot of memory in training, we can only obtain its results on Yeast dataset with a server (CentOS 6.9, 64GB RAM and MATLAB 2014a). For this reason, we independently sample 3000 instances from large datasets 20 times to construct new sampled datasets and report the best results of them. In Table II, the best (or comparable best) results are highlighted in **boldface** using the pairwise t-test at 95% significance level. Besides paired Student's t-test, we also apply Friedman's test [56] with a post-hoc Tukey's test [57] to assess the significant difference between ICM2L and other comparing methods, all the p-values are smaller than 10^{-4} and 0.04 for Friedman's and Tukey's tests, respectively. We implement the test based on *friedman* and *multcompare* functions in Matlab.

From the results reported in Table II, we can observe that ICM2L outperforms other comparing methods in most cases, especially on large-scale datasets. Although MVMC-LS and lrMMC are all designed for multi-view multi-label data, MVMC-LS is almost always outperformed by lrMMC. This is mainly because lrMMC exploits the commonality information among multiple views by assuming that different views are generated from a common low-dimensional subspace, while MVMC-LS just utilizes individual information by combining outputs of different views that cannot make full use of complementary information among views. Since MVMC-LS learns view combination coefficients by two-fold cross validation in training data, which may result in scarce labeled training samples in our semi-supervised setting and impact the combination coefficients learning. Both lrMMC and LSML aim to learn a shared subspace among heterogeneous views, but lrMMC loses to LSML many times. The possible reason is that lrMMC is a two-step method. lrMMC separates the learning process of shared subspace and multi-label classifier, which may result in a sub-optimal solution, while LSML can learn the subspace and the follow-up predictor based on the learned representation simultaneously. CSMSC is also a two-step approach, but it outperforms lrMMC in many cases. The main reason is that CSMSC takes advantage of multiple view-specific representations and common representation to train an ensemble classifier, while lrMMC learns a general multi-label classifier for all labels. This comparison justifies our motivation to explore individuality and commonality information to develop discriminant classifier. Both CSMSC and ICM2L develop an ensemble classifier for classification, but CSMSC loses to ICM2L in most cases. The crucial reason is that ICM2L jointly learn feature representations and ensemble classifier, while CSMSC separates the two learning processes. This comparison indicates the importance to learn feature representation and follow-up classifier jointly.

MLAN is another unified multi-view learning method, but it still loses to LSML almost in all cases. The possible reason is that MLAN is naturally designed for single-label problems and it cannot employ label correlations among multiple labels, which is very important in multi-label data as suggested in the literature. Both LSML and ICM2L can utilize label correlations and the commonality information to make prediction; LSML is outperformed by ICM2L in most cases. The principal reason is that ICM2L explicitly utilizes the individuality information of the views. These comparisons justify our motivation to jointly exploit both commonality and individual information of multiple data views. SMMCL is a recent state-of-the-art method that considers the individuality and commonality patterns of multi-view data. SMMCL can iteratively propagate labels to the most informative candidate unlabeled samples during training, and these samples are then augmented into the training set as labeled data for next iteration. For this reason, SMMCL in general consumes more labeled samples for training, and achieves better performance than other comparing methods in many cases. However, SMMCL is still outperformed by ICM2L in many cases especially over relative large-scale datasets, e.g., Core15k, Pascal07, and ESPGame, Mirflickr and Nus-wide. The crucial intuition behind is two sides: 1) the label imbalance problem in large-scale datasets is more serious than that in Yeast; 2) the bonus of augmenting training set is limited with the increased size of dataset, since a number of labeled data is ready to train an effective semi-supervised algorithm. These observations further validate our motivation to directly exploit individual information of various views for prediction instead of subspace learning. In addition, another bottleneck of SMMCL lies in its poor scalability, since it needs huge memory for training. These comparisons validate the effectiveness of ICM2L.

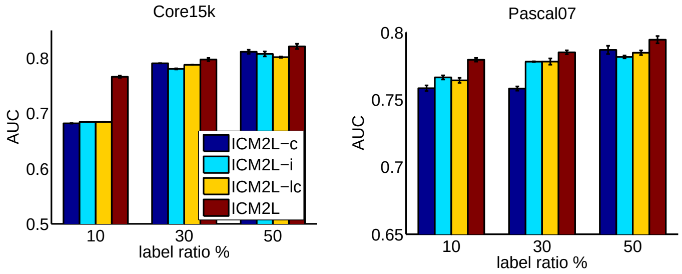

Fig. 2. Results of ICM2L variants under different ratios of labeled samples on Core15k and Pascal07.

C. Effectiveness of ICM2L via Component Analysis

To further justify the effectiveness of our model in capturing the commonality and individuality information of multiple data views, as well as of the label correlations, we conduct additional component analysis experiments on Core15k and Pascal07 datasets, and report the AUC values in Figure 2. In this figure, we set the ratios of labeled data equal to 10%, 30%, and 50%, respectively.

From the figure, we can see that the performance of all the variants of ICM2L increases as the increasing of labeled training data, and ICM2L outperforms its variants across all the settings. ICM2L-c and ICM2L-i disregard the individuality information and commonality information, respectively, and they are outperformed by ICM2L in many cases. This is mainly because ICM2L utilizes both types of information, and thus improves the final performance. These results corroborate our motivation to explicitly leverage the individual and common information of multiple data views. Both ICM2L and ICML2-lc take advantage of multi-view data from individual and common aspects, but ICM2L outperforms ICM2L-lc in many cases across two datasets. The inherent reason is that ICM2L captures label correlations, which are very important in multi-label learning. This fact validates the necessity of capturing label correlations and also proves the effectiveness of the learnt label correlation matrix $\mathbf{S}$.

D. Robustness of ICM2L towards Rare Labels

To answer the second proposed question, we conduct experiments to quantify the benefit of utilizing individual information towards rare labels. Let IR_c denote the imbalance ratio of label c, which is calculated by the ratio between the number of negative samples and that of positive samples for label c. We generate an imbalance dataset from Core15k by first discarding the samples that are annotated with few than three labels. Then we split the labels of the new dataset as general labels and rare labels based on IR_c. Specifically, we decide label c as a general label if $IR_c \leq 50$; otherwise, regarding the labels as rare label. In addition, to further investigate the performance of all methods in extreme cases, we divide the rare label into three levels: rare$_1$, rare$_2$ and rare$_3$. rare$_1$ includes the labels with $50 < IR_c \leq 100$, rare$_2$ includes the labels with $100 < IR_c \leq 150$, and rare$_3$ includes label with $IR_c \geq 150$. Finally, the new dataset has 4966 samples associated with 119 labels, 45 labels with $IR_C \leq 50$, and the other 74 rare labels. Specifically, 36 labels for rare$_1$, 27 labels for rare$_2$, and 11 labels for rare$_3$. Table III shows the performance of comparing methods on the general labels ($IR_C \leq 50$) and rare labels ($IR_C > 50$). Experimental configurations are the same with that in Subsection IV-B.

From the table, we can see that the performance of all methods decrease with IR_c increases, and ICM2L outperforms other comparing methods not only on the general labels but also on all rare label cases, which is consistent with the results in Table II. An interesting observation is that the differences between our model and other comparing methods in rare label cases are larger than those in general labels, especially in the most imbalance situations, e.g., rare$_2$ and rare$_3$. These comparisons validate the robustness of our model in classifying rare labels. In addition, although ICM2L, SMMCL and LSML aim to employ the complementary information, as well as individual information among different views, the last two methods still lose to ICM2L in most cases. The reason behind this fact is that ICM2L utilizes individual information of multiple views in order to capture rare labels hidden in specific views, while the other two approaches exploit individual patterns to enforce the subspace learning. These results intuitively justify our motivation to capture individual characteristics of multiple

TABLE III
EXPERIMENTAL RESULTS OF COMPARING METHODS ON THE PROCESSED CORE15K DATASET WITH DIFFERENT SCALES OF IMBALANCED LABELS.

IR		lrMMC	MLAN	MVMC-LS	CSMSC	LSML	SMMCL	ICM2L
≤ 50 (general)	1-RL	0.714 ± 0.001	0.497 ± 0.001	0.684 ± 0.002	0.724 ± 0.002	0.736 ± 0.001	0.742 ± 0.002	$\mathbf{0.778 \pm 0.002}$
	AP	0.298 ± 0.001	0.213 ± 0.002	0.243 ± 0.000	0.306 ± 0.003	0.312 ± 0.001	0.323 ± 0.001	$\mathbf{0.349 \pm 0.002}$
$50 < IR_c \leq 100$(rare$_1$)	1-RL	0.522 ± 0.001	0.394 ± 0.001	0.481 ± 0.001	0.618 ± 0.001	0.622 ± 0.001	0.633 ± 0.002	$\mathbf{0.681 \pm 0.001}$
	AP	0.246 ± 0.001	0.203 ± 0.001	0.219 ± 0.000	0.278 ± 0.001	0.283 ± 0.001	0.296 ± 0.001	$\mathbf{0.346 \pm 0.001}$
$100 < IR_c \leq 150$(rare$_2$)	1-RL	0.487 ± 0.001	0.326 ± 0.001	0.415 ± 0.001	0.516 ± 0.001	0.519 ± 0.001	0.535 ± 0.001	$\mathbf{0.598 \pm 0.002}$
	AP	0.141 ± 0.001	0.122 ± 0.001	0.137 ± 0.001	0.185 ± 0.001	0.187 ± 0.001	0.197 ± 0.001	$\mathbf{0.263 \pm 0.001}$
$IR_c > 150$(rare$_3$)	1-RL	0.458 ± 0.001	0.218 ± 0.001	0.398 ± 0.001	0.484 ± 0.002	0.486 ± 0.001	0.499 ± 0.002	$\mathbf{0.598 \pm 0.001}$
	AP	0.134 ± 0.001	0.109 ± 0.001	0.121 ± 0.001	0.151 ± 0.001	0.153 ± 0.001	0.168 ± 0.001	$\mathbf{0.225 \pm 0.002}$

data views. Another interesting observation is that CSMSC performs more similar to LSML in imbalance scenarios, in which the performance gap between them in rare$_2$ (or rare$_3$) is smaller than that in the general case. This result further validates the importance of directly exploring individual and common patterns to construct robust classifier.

E. Parameter Analysis

We now study the third proposed question. ICM2L has two parameters α and β, which control the importance of individual information and regularization terms, respectively. We test the sensitivity of ICM2L w.r.t. α and β in the range $\{0.1, 0.2, \cdots, 1\}$ and $\{0.1, 0.3, \cdots, 2\}$, respectively. We report Accuracy and AUC on Yeast in Figure 3; the results for the other datasets and evaluation metrics are similar and lead to similar conclusions.

From Figure 3, we can observe that ICM2L obtains relatively good performance when α is around 0.6 and β is around 0.7. In addition, when $\alpha \to 0$ or $\alpha \to 1$, the performance of ICM2L is reduced. These results further confirm the contribution of commonality information and individual information. Another interesting observation is that the performance of ICM2L decreases more sharply when $\alpha \approx 1$ than that when α is around 0. Since α controls the importance between individual and common patterns, and $\alpha \approx 1$ means we discard the individual information of multi-view data and only focus on common patterns. In contrast, $\alpha \approx 0$ means we only utilize individual patterns hidden in each view and discard the common patterns among them. These results again indicate the importance of individual information. When β is close to 0, the Accuracy and AUC values tend to decrease. This fact validates the effectiveness of the l_2 term. In our experiments, we set $\alpha = 0.6$ and $\beta = 0.7$.

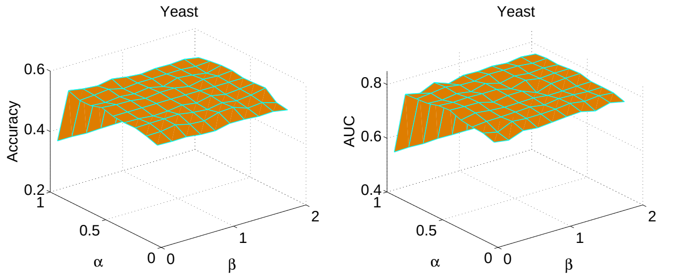

Fig. 3. The results of ICM2L under different input values of α and β.

In addition, we also conduct experiments to investigate the sensitivity of ICM2L w.r.t k. Figure 4 reports the 1-RL and AUC values of ICM2L on Yeast dataset with k varying from $0.1d_{min}$ to $0.8d_{min}$. As we can see, the performance of ICM2L first increases with k rising, then it decreases when $k > 0.5d_{min}$. For this reason, we set $k = 0.5d_{min}$ in experiments. For Mirflickr and Nus-wide, we set $d_{min} = 100$ for simplicity. For general multi-label datasets, we recommend setting the dimension of features ($k \approx$) $\#Avg \log_2(N)$, where $\#Avg$ indicates the number of associated labels per sample. The behind intuition is that the minimal number of bits to encode N data points in one class is $\log_2(N)$ in information theory. For multi-label data, it also needs to consider the statistical property of labels, which could be captured by $\#Avg$. As such, $\#Avg \log_2(N)$ bits are needed.

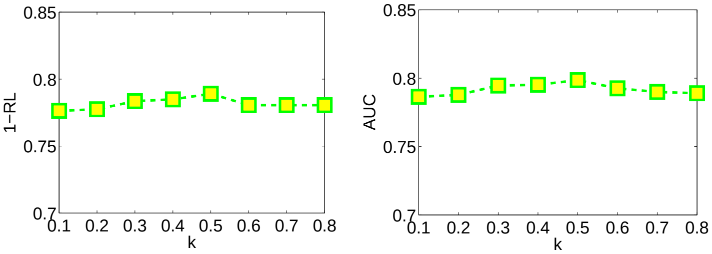

Fig. 4. The results of ICM2L under different input values of k

F. Efficiency of ICM2L

To investigate the proposed last question. We conduct experiments on all datasets with the same configuration in Section IV-B, and report the runtime costs of all methods and the convergence trend of ICM2L. We only report the runtime cost of comparing methods over datasets excluding Nus-wide since the run time cost on Nus-wide is generally much biger than other datasets. We observe similar run time cost trend on Nus-wide. Table IV reports the runtime costs of all the approaches on a server (CentOS 6.9 with Inter(R) Xeon E5-2678, 64GB RAM and MATLAB 2014a). From Table IV, we can see that ICM2L is much faster than MLAN, MVLC-LS, CSMSC and LSML in general. However, lrMMC runs much faster than ICM2L in most cases. This is because lrMMC is a two-step method that learns the shared subspace and the follow-up predictor in two separate steps, while ICM2L has to learn the low-dimensional representations and the multi-label classifier in each iteration. These comparisons corroborate the efficiency of our model.

Figure 4 shows the convergence curve of ICM2L on Yeast and Core15k datasets. We can see that ICM2L tends to converge after 70 iterations for Yeast data, and after 60 iterations

TABLE IV
RUNTIME COMPARISON (IN SECONDS).

	lrMMC	MLAN	MVLC-LS	CSMSC	LSML	SMMCL	ICM2L
Yeast	8.03	197.90	39.57	25.78	23.11	23.20	10.11
Core15k	290.56	357.92	13892.32	437.88	387.24	543.70	325.41
Pascal07	908.49	1604.04	8783.45	1378.44	1023.38	1588.92	768.29
ESPGame	1512.63	9785.80	21748.23	5436.28	4792.04	7332.42	3918.44
Mirflickr	1245.72	7752.33	18428.18	4493.65	3986.13	5856.18	3245.11
Total	3965.43	19697.99	62891.75	11772.03	10211.90	15344.42	8267.36

for Core15k. The convergence trends on the other datasets are the same as those reported in Figure 4. Overall, ICM2L converges at most in 80 iterations for the datasets used in the experiments.

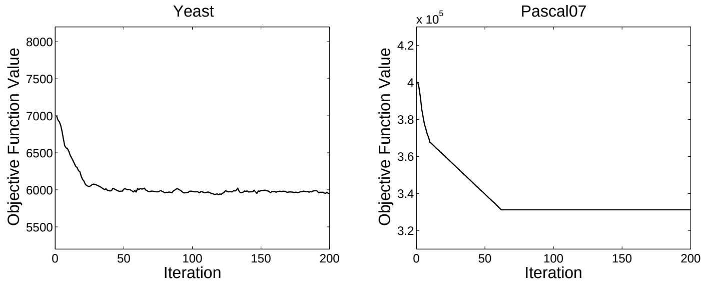

Fig. 5. Convergence trend analysis.

V. CONCLUSION

In this paper, we investigate how to explore the individuality and commonality of heterogeneous features for effective multi-view multi-label classification. To this end, a multi-view multi-label framework termed ICM2L is presented. ICM2L learns a shared subspace of heterogeneous views, label correlations, and an ensemble classifier that captures both individuality and commonality information of multiple views in a principled way. Different from previous works that focus on learning representative hidden representations by capturing the shared and individual patterns across multiple views, we utilize such information to improve the discriminant capacity of classifier towards rare labels. Experiments on several benchmark datasets demonstrate the superiority of the proposed model over related competitive solutions. In the future, we plan to further improve ICM2L by adapting non-linear mapping functions with deep models.

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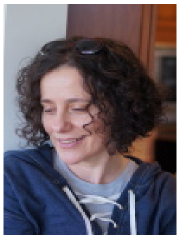

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