Hierarchical adaptive sparse grids and Quasi Monte Carlo for option pricing under the rough Bergomi model

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Abstract

The rough Bergomi (rBergomi) model, introduced in [1], is a promising rough volatility model in quantitative finance. In the absence of analytical European option pricing formulas for the model, and due to the non-Markovian nature of the fractional driver, the prevalent option is to use Monte Carlo (MC) simulation for pricing. Despite recent advances in the MC method in this context, pricing under the rBergomi model is still a time-consuming task. To overcome this issue, we design a novel, alternative, hierarchical approach, based on i) adaptive sparse grids quadrature (ASGQ), specifically using the same construction in [6]; ii) Monte Carlo (QMC) and Richardson extrapolation. Both techniques are coupled with Brownian bridge construction and Richardson extrapolation. By uncovering the available regularity, our hierarchical methods demonstrate substantial computational gains with respect to the standard MC method, when reaching a sufficiently small error tolerance in the price estimates across different parameter constellations, even for very small values of the Hurst parameter.

Rough Volatility

The rough Bergomi Model: Analytic Smoothing

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\begin{equation}
\frac{dS}{S} = \frac{\eta(\tau_k)}{\tau_k} \cdot \frac{dW_{\tau_k}}{\tau_k}, \quad \tau_k = \tau_{k-1} + \Delta \tau, \quad \tau_0 = 0,
\end{equation}

where \( \eta(\tau_k) \) is the rough volatility, \( \tau_k \) is the volatility increments, and \( \Delta \tau \) is the volatility step.

Challenges

Numerically:

• The model is non-affine and non-Markovian.
• Standard numerical methods (PDEs, characteristic functions) seem inapplicable.
• The only prevalent pricing method for vanilla options is Monte Carlo [1, 2, 7], still a time-consuming task.
• Discretization methods have poor behavior of the strong order, that is, the convergence rate is of order \( N^{-1/2} \) [1].

Theoretical:

• No proper weak error analysis done in the rough volatility context.

Contributions

1. We design an alternative hierarchical efficient pricing method based on:
   i) Analytic smoothing to uncover available regularity.
   ii) Approximating the option price using a deterministic quadrature method (ASGQ and QMC) coupled with Brownian bridge and Richardson Extrapolation.

2. Our hierarchical methods demonstrate substantial computational gains with respect to the standard MC method, assuming a sufficiently small relative error tolerance in the price estimates, even for small values of the Hurst parameter.

On the Choice of the Simulation Scheme

Figure 2: The convergence of the weak error \( \epsilon \) w.r.t. using MC with \( n \) = \( 10^k \) samples, for \( k \) = parameter in Table 2. The upper and lower bounds are confidence intervals. a) With the hybrid scheme b) With the exact scheme.

Error Comparison

\begin{equation}
\epsilon_{\text{MC}}(h) \leq \epsilon_{\text{MC}}(h) + \epsilon_{\text{ASGQ}}(h),
\end{equation}

where \( \epsilon_{\text{MC}}(h) \) is the error, \( \epsilon_{\text{ASGQ}}(h) \) is the bias, and \( \epsilon_{\text{MC}}(h) \) is the statistical error.

Numerical Experiments

Table 1: Reference solution, using MC with \( 10^3 \) time steps and \( 10^3 \) samples, \( \Delta = \sqrt{\frac{1}{2}} \), of call option price under the rough Bergomi model, for different parameter constellations. The surface between parentheses correspond to the statistical errors estimate.

Table 2: In this table, we highlight the computational gains achieved by ASGQ and QMC over MC method to meet a certain error tolerance. We note that the ratios are computed for the best configuration with Richardson extrapolation for each method.

The Hybrid Scheme [4]

Figure 3: Construction of the index set for ASGQ method. A posteriori adaptive construction: Given an index set \( \mathcal{I}_i \), compute the profile of the neighbor indices and select the most profitable one.

Comparing the Numerical Complexity of the Different Configurations

Figure 4: Comparing the numerical complexity of the different methods with the different configurations in terms of the level of Richardson extrapolation, for the case of parameter set in Table 1. a) MC methods. b) QMC methods. c) ASGQ method.

Comparing the Numerical Complexity of the Best Configurations

Figure 5: Computational work comparison for the different methods with the best configurations concluded from Figure 4, for the case of parameter set in Table 1.

References

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In this paper, we propose a novel hierarchical approach for pricing options under the rough Bergomi model. Our method combines adaptive sparse grids quadrature (ASGQ) and Monte Carlo (QMC) methods with Richardson extrapolation to achieve high accuracy with minimal computational effort. The rough Bergomi model, introduced in [1], is a promising model for rough volatility, allowing for stylized features of financial data that are not captured by standard models. However, the lack of analytical pricing formulas for this model makes Monte Carlo simulation a prevalent choice for pricing options.

We design a new hierarchical method, which leverages the available regularity in the model to achieve substantial computational gains compared to the standard Monte Carlo method. This is achieved through the use of adaptive sparse grids quadrature (ASGQ) for the deterministic part of the problem and Monte Carlo (QMC) for the stochastic part. Both techniques are coupled with Brownian bridge construction and Richardson extrapolation. Our method is validated through numerical experiments that demonstrate its effectiveness in a wide range of parameter settings, even for small values of the Hurst parameter.

The proposed method provides a significant improvement in computational efficiency, allowing for more accurate price estimates in less time. This is particularly important in financial markets where timely and accurate pricing information is crucial. The hierarchical approach is flexible and can be adapted to different parameter configurations, making it a versatile tool for practitioners and researchers.

Overall, the results presented in this paper highlight the potential of hierarchical methods in pricing complex financial derivatives, such as options under the rough Bergomi model, and open new avenues for further research in this area.