Subsurface wavefields based on the generalized internal multiple imaging

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SUMMARY

Full Green’s functions between image points and the recording surface are crucial to constructing accurate subsurface wavefields and images beyond the single-scattering assumption. A direct approach to do so is offered by utilizing the recorded data combined with a background imaging velocity. The process includes extrapolating the recorded data back in time followed by a simple interferometric cross-correlation of the back-propagated wavefield with the recorded data. This interferometric step offers the opportunity to extract subsurface Green’s functions with first-order scattering forming the transmission component, and the second-order scattering becoming the leading scattering term. A cross-correlation of the resulting, assumed upgoing, wavefield with a forward modelled down going wavefield highlights the double-scattered reflectivity in a process referred to as the generalized internal multiple imaging (GIMI). The resulting image is vulnerable to cross-talk between different order multiples interacting with each other. Thus, we develop the adjoint GIMI operation that takes us from the image to the data, and use it to formulate a least-squares optimization problem to fit the image to the data. The result is reduced cross-talk and cleaner higher resolution multiscattered images. We also extract space extensions of the image, which offers the opportunity to evaluate the focusing capability of the velocity model, and formulate updates for that model based on double scattering. We show the features of this approach on the modified Marmousi model.

Key words: Image processing; Numerical modelling; Numerical solutions; Waveform inversion; Seismic interferometry; Wave scattering and diffraction.

1 INTRODUCTION

Data we acquire on the Earth’s surface include recorded waves that are scattered in the subsurface single, and often multiple times. Multiscattered energy can provide invaluable information in illuminating interfaces not captured by single-scattered waves, which is often ignored in our conventional imaging assumption of single scattering (Claerbout 1985). As a result, multiples appear in our images as artifacts and noise that we strive to mitigate (Berkhout & Verschuur 1994). Thus, we end up with reflections illuminated by the single-scattering assumption. Multiscattered energy not only provides additional scattering information, but also it can provide additional wave velocity information along its unique path (Alkhalifah & Wu 2016).

Imaging multiscattered energy has always required additional algorithmic and computational effort. Part of the problem is that to identify multiples, we need to first identify the primaries, as well as their image locations. In other words, we need to identify the components of the scattering series (Weglein et al. 1997; Ikelle et al. 2009). For example, imaging reflections that include surface multiples require extracting part of the scattering series, those corresponding to scattering from the often known surface of the Earth. These imaging methods, which include imaging multiscattered energy, admitted an increase in illumination and accuracy of our images (Berkhout & Verschuur 1994; Youn & Zhou 2001; Guitton 2002; Shan 2003; Zuberi & Alkhalifah 2013). Internal multiples, which require more components of the scattering series to be known, are far more complicated to image (Malcolm et al. 2009a). Additional developments in imaging multiscattered energy are given by Malcolm et al. (2009a, 2011), for imaging prismatic waves and other higher-order internal scatterings. However, these methods rely on having knowledge of one of the scattering surfaces. A combination of inversion for velocity perturbations and modeling between these perturbations is used to complete the loop of imaging internal scattering (Berkhout 2017), that is, this type of approach does not make any assumptions about the geometry of the velocity perturbations. Such wave equation methods, however, require additional extrapolations (or obtaining Green’s functions) corresponding to the additional wave paths.

Interferometry provides a way to extract such Green’s functions for the additional wave paths directly from the data. Using the Marchenko algorithm, an iterative imaging procedure has been suggested by Behura et al. (2012) to image all internal scatterings including the single-scattering energy. This iterative procedure
does not allow for separate imaging of different orders of multiples, and it also requires manual muting of anticausal events after cross-correlation. This makes the method very involved (van der Neut et al. 2015). Zuberi & Alkhalifah (2013, 2014b) proposed an interferometric approach that utilizes directly the Born series, that is, they do not make any assumptions about the geometry of the velocity perturbations. Their approach, referred to as the generalized internal multiple imaging (GIMI) process, images any order internal scattering, separately. The GIMI process relies solely on the background Green’s function based on a smooth (migration) velocity model, which we often use in conventional imaging. The additional computational cost in GIMI is an interferometric cross-correlation of the surface seismic data with the back-propagated data. The number of cross-correlations required is one less than the order of the term in the Born scattering series (or equal to the order of the internal multiple) we intend to image. However, GIMI is hampered by crosstalk, and beholden to the weak scattering assumption. The crosstalk is the result of cross-correlations of events that are not related to each other. The small velocity perturbation assumption was, however, necessary for the convergence of the Born scattering series. Aldawood et al. (2015) attempted to modify parts of GIMI with a least-squares optimization to handle the cost. Though the approach provided clean double and higher order scattered images, it was prohibitively expensive, as the least-squares optimization involved the subsurface wavefield not the image. Considering the interferometric implementation, GIMI, like the Marchenko approach, also requires the sources and receivers cover the same surface. A lack of good coverage is expected to degrade the interferometric implementation and yield artifacts, similar to those faced by conventional imaging when we are at the edge of the acquisition, and the stationary solution necessary to build the reflector is not captured.

In this paper, we try to mitigate some of GIMI’s limitations by formulating a GIMI process based least-squares optimization in which we fit the different order multiple images to the data. To do so, we derive the adjoint GIMI operation that takes us from the image to the data. In the process of doing so, we analyse the different steps of the GIMI procedure, and suggest additional applications for some of the features we highlight. Such analysis will be demonstrated on a simple model to help us understand the process. We will also develop the adjoint process necessary to formulate a least-squares implementation of GIMI. A more complicated application using a modified Marmousi model follows.

2 THEORY

Our objective in this section is to review the GIMI process and highlight some of its features with a simple model example. The model is made up of two homogeneous layers, as shown in Fig. 1(a), with a vertical reflector in which single-scattered waves from that reflector will not be captured along our recording surface. This model is representative of a fault or a Salt flank, and is sampled at 0.01524 km in both directions requiring 200 samples laterally and 100 vertically. The velocity of the first layer is 2.4 km s$^{-1}$ and the second layer 2.7 km s$^{-1}$. Using a Ricker wavelet with a peak frequency of 18 Hz, we generate synthetic data using a finite-difference scheme for an acoustic isotropic constant-density medium. The data correspond to 21 shots and 100 receivers both equally sampled to cover the surface area of the model. This amounts to a receiver spacing of 30 m and a coarse source spacing of 150 m. Fig. 1(b) shows a representative shot gather for a source located at 2.286 km, highlighted in Fig. 1(a).

The later reflection prominent on the right-hand side of the section corresponds to double scattering from the vertical reflector.

The GIMI process is capable of imaging internal multiples of any order. The key is an interferometric step that translates the energy from higher order scattering to become the leading scattering term. The main requirement is that the sources and receivers cover the same surface, preferably sampled well. Thus, as introduced by Zuberi & Alkhalifah (2014a), the GIMI process to image double-scattered energy includes three steps:

(i) Back propagate the recorded data from the surface to the model points.
(ii) An interferometric cross-correlation of the back-propagated wavefield with the recorded data over sources.
(iii) Zero-lag cross-correlation of the interferometric data with modelled data from the surface.

The result is a function of the medium containing reflectivity corresponding mostly to double-bounce events. It also contains artifacts caused by higher-order scattering (cross-talk) and low-frequency energy corresponding to the first-order scattering. The key step here is the middle interferometric step. The other steps are similar to what we do for the single-scattering imaging process, such as reverse time migration (RTM). A repeat of the interferometric cross-correlation between the output from step 2 and the data on the recording surface over receivers highlights triple scattering as the leading scattering term.

To demonstrate the physical meaning of these operations, we track the evolution of wavefields after every step of GIMI for the model shown in Fig. 1(a). In the imaging process, we use the velocity of the first layer (2.4 km s$^{-1}$). In mathematical terms, we can write the three steps, constituting GIMI, in three equations starting with the conventional back propagation of the recorded reflections corresponding to upgoing waves, $R_s^-$ (a shot representation is given in Fig. 1b), to obtain the mainly upgoing wavefield (superscript $-$) from the source, $s$, to the image point, $x$:

$$G_{ss} = G_{rs}^* R_s,$$  

(1)

with the Einstein summation notation used over receivers $r$, and $G_{rs}^*$ is the complex conjugate (superscript $*$) of the background Green’s function from $x$ to $r$. This is equivalent to the first step of RTM. Fig. 2(a) shows such wavefields for the image point shown in Fig. 1(a) as a function of sources and time. The resulting wavefield $G_{ss}$ is not purely upgoing as it contains even down going waves and other events, but what we care for and try to isolate, the double scattering, is upgoing. In the Marchenko process, with an opposite side implementation (source side), the directionality of the waves is isolated using the causal and anticausal parts of the correlation process along with muting in an iterative fashion (van der Neut et al. 2015). Here, we will try to make things simple. Thus, we next perform the interferometric step given by

$$G_{rs} = R_s G_{rs}^*,$$  

(2)

with the Einstein summation notation, which constitutes here a summation over sources. This step, as described by Zuberi & Alkhalifah (2014a), moves the second-order scattering term to be the leading scattering term heading in the upward direction (thus, the negative sign in $G_{rs}$). Meanwhile, the first-order scattering term will be propagating downwards, such as those we experience with diving waves. Fig. 2(b) shows the result of this step for the same image point shown in Fig. 1(a), but now as a function of receivers and time. The corresponding background down going wavefield, $(G_{rs})^+$, is shown...
Figure 1. (a) The velocity model with an example double-bounce ray path that could image the vertical reflector. The image point location will be used to analyse the GIMI procedure. (b) A shot gather corresponding to the shot point location in (a).

Figure 2. (a) The function $G_{xs}$, (b) The function $G_{rx}$ and (c) The function $(G_{rx})^+$. All three functions plotted for the image point in Fig. 1(a).

in Fig. 2(c). Finally, the imaging condition for double scattering

$$I_x = (G_{rx})^+ G_{rx},$$

which includes a summation only over receivers. The Einstein summation notation in this paper applies only on sources and receivers. With respect to image parameter $x$, the repeated notation constitutes a dot product. The result is an image of the second-order scattering as a leading term of the scattering series, marred with low-frequency wave path energy between the recording surface and the reflectors, as shown in Fig. 3(a). Such wave path energy often has more intensity than the second-order reflectivity image, as they come from the primary reflections. These wave path low-wavenumber components of the image can be removed in many ways, including using a Laplace filter, separating up and down going wavefields prior to cross-correlation, or some innovative imaging conditions, such as the inverse scattering imaging condition (Whitmore & Crawley 2012). We use a simple 2-D low-cut filter to obtain Fig. 3(b), which highlights the double-scattering reflectivity.

To obtain higher order scattering images, the middle step in GIMI is replaced by

$$G_{rx} = [R_{x} G_{rx}^+ R_{x}] R_{rx},$$

which will highlight triple scattering. Here, $e$ is a dummy subscript variable representing either sources or receivers, and thus, adheres to the Einstein summation notation. The same holds for $v$ for fourth-order scattering:

$$G_{rx} = [R_{x} G_{rx}^+ R_{x}] R_{rx}.$$

and so on. Fig. 3(c) shows the image for triple scattering, and mainly it highlights the double-scattering wave paths. A similar filter to that used for double scattering is applied here to obtain the image in Fig. 3(d). The sharp edges of the model includes triple scattering.
3 THE ADJOINT

We, next, formulate the adjoint of the GIMI to use in a least-squares implementation to reduce the cross-talk artifacts associated with the process. As GIMI transforms the full recorded data to an image corresponding to double scattering (or higher order scattering). The adjoint operation should produce the approximate full recorded upgoing data from the double-scattered image.

An adjoint operation can be formulated by recognizing that the interferometric step remains the same, with replacing the summation over sources with a summation over receivers. Since the other two steps of GIMI constitute single-scattering imaging, the adjoint is mainly given by Born scattering (Pratt & Shipp 1999). Thus, for the adjoint, we have the following three steps:

\[ G_{rx} = (G_{rx}^b)^+ I_x, \]  

which isolates the source wavefield at the image locations, and, with the principle of reciprocity, we care mainly for the part heading upwards as it is scattered. In classic Born scattering (or more accurately demigration), \( G_{rx}^b \) is the source of the scattered wavefield so we convolve it with a Green’s function from the image point to the surface, \( G_{xs}^b (R_{xs} = G_{xs}^b (G_{xs}^b)^+). \) Since the image pertains to double scattering, we first pass \( G_{rx}^b \) through the interferometric step:

\[ G_{xs} = G_{rx}^b R_{rx}. \]  

Finally, we convolve it with the background Green’s function from the image point to the receiver:

\[ R_{mx} = (G_{mx}^b)^+ G_{xs}. \]  

which is the forward propagation of \( G_{xs} \) from \( x \) to \( r \). Fig. 4(a) shows the modelled data from the double-scattering image shown in Fig. 3(a). Note that the adjoint will approximately reproduce the full data, even though the image corresponds to mainly the double scattering, as the single scattering resides in the wave path parts of the image. We will only miss the direct arrivals (zero scattering). If we apply the adjoint on the filtered image in Fig. 3(b), we obtain data corresponding mainly to the double-scattering term.

4 THE LEAST-SQUARES IMPLEMENTATION

Since we have the imaging operation and its adjoint, we can formulate a least-squares optimization problem to find the second-order
scattering image that fits the data. We can also utilize additional regularization terms to improve such an image (i.e. in resolution; Aldawood et al. 2015, 2016). The least-squares implementation, however, is also based on a weak scattering assumption, similar to the conventional GIMI.

An optimization problem can be formulated to improve the image with an objective function:

$$J(I_x) = \frac{1}{2}|\Delta R - R_{m}(I_x)|^2,$$

such that $R_{m}(I_x)$ is the modelled (adjoint GIMI) reflections from the double-scattering image $I_x$ using eqs (6)–(8). The operation $| |^2$ stands for the $l_2$ norm squared. The gradient is given by GIMI applied to the residual $\Delta R = R_{m} - R_{m}(I_x)$ using eqs (1)–(3). Initially, we set $I_x = 0$, and thus, $\Delta R = R_x$, and the image is given by GIMI, and shown in Fig. 3(b). We, then, use this image to compute $R_{m}(I_x)$ shown in Fig. 4(b). The difference between $R_{m}(I_x)$ and the data forms the residuals used as the adjoint source to update the image. Like conventional least-squares migration, we continue the iterative process until we meet a convergence criterion. After 20 iterations, we obtain the image shown Fig. 5. It is free of artifacts and has higher resolution compared to Fig. 3(b).

5 SPACE EXTENSIONS

The multiscattered image can, such as in the single-scattering case, be used to evaluate the accuracy of the velocity model by assessing the focusing of energy to zero lag when we introduce an extension. Introducing an extension to GIMI implies that we need to compare up and down going wavefields at points that are not coincident, and specifically located $2h$ apart, where $h$ is the subsurface space extension vector, with components $\{h_x, h_y, h_z\}$. The offset between these points formulates the subsurface offset, and provides data (or images) corresponding to subsurface sources and receivers that are not coincident (Vasconcelos et al. 2010). Conveniently, the extensions are introduced to GIMI in the last step:

$$I_{xh} = G^h_{rxh} +^* G^h_{rx}.$$

Alternatively,

$$I_{r'x'} = G^h_{r'x'} +^* G^h_{rr}.$$

where $s$ and $r'$ are indexes corresponding to the locations of the subsurface source and receiver, respectively. In both cases, if the velocity is accurate, we expect the energy to be focused at $h = 0$ or $s' = r'$. Applying a multidimensional extension is costly. Since horizontal layering dominates our data, we often utilize a horizontal extension. Such an extension provides the necessary information to evaluate the velocity model for a layered medium. If the reflector of
interest is vertical, we will need a vertical extension as horizontal extensions are, in this case, not dependent on the velocity. For the example model shown in Fig. 1(a), the reflector of interest is vertical, and thus, velocity sensitivity is expected to show in a vertical extension. Fig. 6 shows the extension vertically, $h_z$, for the area containing the vertical reflector. We repeat the GIMI with an inaccurate velocity of the first layer, and as a result, some energy is not focused at zero lag. Such non-focusing has distinct features for lower and higher velocities. Using the adjoint, we can utilize such energy to update the velocity along the path of double scattering. We will leave this potential application for future study.

**Figure 7.** (a) The modified Marmousi model used to model the recorded data, (b) the region (white dashed lines) demonstrating schematically where the double scattering may occur, (c) a linearly increasing velocity model and (d) a smoothed version of the Marmousi, which is kinematically accurate as it is used for imaging.

**Figure 8.** The RTM image calculated using (a) the smoothed velocity and (b) the linearly increasing velocity.

6 THE MARMOUSI MODEL

Now, we test the least-squares GIMI on part of a modified Marmousi model. We will focus on the shallow part of the original Marmousi model where the multiscattering tends to be more evident. Fig. 7(a) shows the Marmousi inspired model in which data were simulated. We expect reflections from the sloping faults to be a source for double-scattered wavefields, as demonstrated in Fig. 7(b). Fig. 7(c) shows a linearly increasing with depth velocity model often used as an initial model for full waveform inversion (FWI). Meanwhile, Fig. 7(d) shows a smoothed version of the Marmousi model, which is a model we often strive to use in imaging, and we expect a
successful FWI implementation to provide us with such a model. Unlike the linearly increasing model, this model is expected to be kinematically accurate resulting in a focused image. The linearly increasing model is used to highlight some of the features of the zero-scattering wave path image and the image with subsurface space extension in the case of an inaccurate velocity. We, initially, show the GIMI process for an 18 Hz peak frequency data, as data can be better displayed and analysed for such low frequencies. Later, we show final results for the 30 Hz peak frequency data, which are frequencies closer to what we often image in practical applications.

For comparison, we apply a conventional RTM using the smooth velocity model on the 18 Hz peak frequency data. The resulting image is shown in Fig. 8(a). Most of the reflections are accurately positioned reflecting the kinematically accurate nature of the velocity model in Fig. 7(d). Contrast that with the RTM result shown in Fig. 8(b), using the linearly increasing velocity model in Fig. 7(c). Clearly, the smoothed Marmousi velocity model admitted an overall accurate image reflecting its accurate kinematic representation of the model, whereas the linearly increasing velocity model admitted a poorly focused and positioned image. Nevertheless, though some of the fault reflections were illuminated using the smooth Marmousi velocity model, many of the fault reflection energy were missing with this single scattering imaging algorithm, even with a kinematically accurate velocity model.

In the application of GIMI, as we stated earlier, the single-scattering reflections transform to zero scattering, while the double-scattered reflections become the leading scattering term. As a result, Fig. 9(a) shows the unfiltered GIMI image (first iteration) using the smoothed Marmousi velocity model. Fig. 9(b) shows the result after the application of a Laplacian filter to mitigate the wave path updates (similar to what we apply to remove the low-frequency artifacts from diving waves in conventional imaging). The fault reflections are dominant in this image as they correspond to double scattering. Also, as discussed before, we see some artifacts courtesy of the cross-talk. The least-squares optimization of the image yields unfiltered images in Fig. 10(a), with the Laplacian filtered version shown in Fig. 10(b). We will use colour plots for unfiltered images to highlight the background energy, and the grey scale for the filtered images to highlight reflectivity. The energy in the unfiltered version illuminates most of the model, which bodes well for any wave path based inversion update. As mentioned earlier, they also constitute most of the energy in the data as they correspond to primaries. The filtered image shows higher resolution and more double-scattering energy, which includes other components of the double scattering. In both the single iteration GIMI and the least-squares version, the filtered image displays the regions in which double reflections occur as demonstrated schematically in Fig. 7(b). To assess the quality of the least-squares optimization, we compare a shot gather from the original data (Fig. 11a) with that modeled from the unfiltered inverted image (Fig. 11b). As expected, the image reproduced most of the data elements including the single- and double-scattering reflections. If we model the filtered image, which corresponds to the double scattering, we obtain the data in Fig. 11(c). This feature separates GIMI from the Marchenko method, where scattering
order separation is not possible. We also display the objective function, shown in Fig. 11(d) corresponding to the difference between the original data and the ones modeled from the inverted unfiltered image, as it decreases with iterations. Convergence to 30 percent misfit is achieved around the 13th iteration. A closer look, by extracting near offset traces from Figs 11(b) and (c), and comparing them to the original data at the same location, which is shown, respectively, in Figs 12(a) and (b), reveal the amount of fitting we managed to achieve. Modelling from the filtered image reproduces events mainly corresponding to double scattering, but also events corresponding to single scattering that were not properly filtered. As a result, the amplitude is much lower.

For potential wave path update applications, such as in reflection waveform inversion, we also show the inverted image corresponding to the linearly increasing velocity model unfiltered (Fig. 13a) and filtered (Fig. 13b). The energy and the resolution of the wave path update are indicative of the inaccurate velocity model used in the imaging. The filtered image reveals the mispositioning and reduced focusing of the double-scattered events. For velocity inversion applications, the residuals between the observed and predicted data are fed into GIMI with proper scaling to obtain velocity gradients. In fact, if we filter the image to maintain only the smooth part, which is an inverse filter to the one used to highlight the image, we obtain the smooth update shown in Fig. 13(c).

For the 30 Hz data, we show the RTM image in Fig. 14(a). The higher resolution image is courtesy of the higher frequencies involved in the imaging. The least-squares image for single scattering is shown in Fig. 14(b). A mild increase in resolution and energy can be observed throughout. However, like before, the reflections from some of the faults are missing. The least-squares second-order inverted image after a Laplacian filter is shown in Fig. 14(c). This image managed to capture the reflectivity corresponding to two bounces. However, the resolution of the double-scattered image is expected to be lower than that in the single-scattered image due to the larger average opening angles involved in double-scattered reflections as demonstrated in Fig. 7(b). Nevertheless, we directly sum the single- and double-scattered images to obtain a more complete image shown in Fig. 14(d). The new image shows an overall increase in illumination, especially for those reflectors that would have not been illuminated from single scattering. Specifically, the right dipping faults throughout show a pattern of continuous energy along the fault (the magnitude of derivative of the image along the faults is comparably small), indicative of their reflection energy being imaged, as opposed to relying on the breaks in sediments to identify faults.

Finally, we show the extended image for the smoothed Marmousi velocity model (kinematically accurate) and for the linearly increasing model. The subsurface offset extension, such as that for the
Figure 12. A trace, given in solid blue, from the near offset part of Fig. 11(a); (a) The same location trace, in dashed red, from Fig. 11(b) corresponding to modeling from the full image shown in Fig. 10(a); (b) the same location trace, in dashed black, from Fig. 11(c) corresponding to modeling from the filtered image shown in Fig. 10(b).

Figure 13. (a) The image corresponding to least-squares GIMI using the linearly increasing velocity in Fig. 7(c), (b) The Laplacian filtered version on the image and (c) the wave path version by filtering out high wavenumbers.
Figure 14. (a) The 30 Hz RTM image using the accurate velocity model shown in Fig. 7(d), (b) the least-squares image using the same velocity model, (c) the least-squares first-order GIMI image and (d) the sum of the least-squares RTM and GIMI images.

Figure 15. The extended image for an extension in the vertical subsurface offset for the image obtained using the (a) smoothed Marmousi model, and using the (b) linearly increasing velocity model. The three slices correspond to the part of the image corresponding to the fault located at 2.4 km (bottom left), to depth versus extension in vertical subsurface offset (bottom right), and to position versus vertical subsurface offset (top).

simple model above, is vertical. We focus on the fault under location 2.4 km, which was poorly illuminated with the single scattering. Fig. 15(a) shows the extended image. The slices are focused at the fault as indicated by the crossing lines in the bottom left section. The extension shows reasonable focusing at zero lag, though the dip of the reflector is about 30 deg. Meanwhile, for the linearly increasing velocity model, the extension demonstrated poorly focused energy of the fault in Fig. 15(b).

7 A SALT MODEL EXAMPLE

To illustrate the ability of GIMI to improve the imaging of a complex top-of-the-salt structure, we use a piece of the Sigsbee model, shown in Fig. 16(a), which includes a steep valley. We expect considerable multiscattering from the flanks of the valley. The dashed line highlights the location of the salt boundary, which we will use to evaluate the RTM and GIMI imaging accuracy. Of course, the focus here is the double scattering from these flanks and their impact on
Figure 16. (a) A part of the Sigsbee model that includes a valley. The dashed line maps the top of the salt. (b) A smoothed version of the model for imaging.

Figure 17. (a) The least-squares RTM image, (b) the least-squares GIMI full image, (c) after Laplace filtering and (d) the direct summation of the RTM and GIMI images. The dashed blue line shows the location of the top of the salt.

the image. For imaging, we use a smoothed version of the velocity model, as shown in Fig. 16(b), which do not induce any scattering of its own.

Fig. 17(a) shows the single-scattering image, obtained using a least-squares RTM. This image is focused on the primary data. The valley though clearly imaged, the image does not reflect the true shape of the valley, as most of the recorded energy corresponding to the valley is captured by sensors above the valley and corresponds to double scattering. On the other hand, the least-squares GIMI image, shown in Fig. 17(b), contains the transformed to zero-order primary reflections and the double-scattering part as reflectivity. A Laplacian filter highlights the reflectivity part, as shown in Fig. 17(c). The position of the reflectors of the top of the Salt in the valley is more accurate with higher resolution in the double-scattered image. The conventional RTM also captures that reflectivity, but it is relatively weak compared to the mispositioning of the stronger double-scattered events. The direct summation of the two images will enhance the accuracy of the reflectivity of the Salt top, but will not remove some of the mispositioned salt valley reflectivity coming from the single-scattering image, as demonstrated in Fig. 17(d). The bottom of the valley would most probably require triple or higher order scattering to perfect.

8 DISCUSSION

Unlike in Zuberi & Alkhalifah (2014b) in which the Kirchhoff (integral) approach were used for wavefield reconstruction, here we use finite-difference solutions of the wave equation. In this case, we expect to gain some of the advantages provided by the finite-difference
solutions of the wave equation as compared to the high-frequency asymptotic solutions used in Kirchhoff approaches (Zhu & Lines 1998). We do not simply apply the three-step GIMI, we use the adjoint GIMI to fit the double- and triple-scattered images to the data. Thus, we obtain optimized multis scattered GIMI images, in which each order image is obtained separately. We can then add them to obtain a more complete image. These optimized images are cleaner with higher resolution and less artefacts than the conventional GIMI image.

GIMI has an additional feature pointed out by Zuberi & Alkhalifah (2013) in which its first-order implementation transforms first-order scattering to zero-order admitting wave path energy between the reflector and acquisition surface. With a least-squares optimization, this zero-order scattering information fits the single-scattering data. To obtain second-order scattering images, which is transformed to first-order courtesy of GIMI, we filter out the low-frequency wave path information using any available filter. One of the most commonly used filters for such applications is the Laplacian filter (Zhang & Sun 2011). We can also deploy an imaging condition such as the inverse scattering imaging condition to remove such energy (Whitmore & Crawley 2012). On the other hand, if we apply the inverse of any of the aforementioned filters, we can, otherwise, isolate the wave path energy, which can be used in an inversion procedure to update the background. Second-order GIMI can admit double-scattering wave path energy updates that can be isolated as well. In this case, the data misfit can also be used to provide background velocity information. The ability to handle the different-order scattering separately may allow us to scale the updates from these components differently, and thus, optimize them. Applications in inversion will be the topic of future investigations.

The cost of the approach, such as in the case of the Marchenko method, is moderately higher than conventional methods. The bulk of the additional cost is attributed to the interferometric step, which depends on the number of sources and receivers. Specifically, we need to extrapolate all the data to reconstruct the wavefields corresponding to all shots prior to the interferometric step. The other two steps are similar to conventional RTM. For higher order multiples, we will have to apply the interferometric step additional times, equal to the order of the multiple we intend to image. In the least-squares sense, this operation will be repeated numerous times proportional to the number of iterations required to converge. From the memory side, we will have to store an additional dimension of data, either sources or receivers, as compared with the conventional approach. A frequency domain implementation, which reduces the dimensions of the problem, as thus, provides more efficient correlation and convolution operations, can reduce the imaging cost considerably. However, for imaging purposes, the process must be repeated for all frequencies. On the other hand, for inversion applications, the frequency domain version can provide efficient wave path updates between the reflector and the acquisition surface. For a time domain approach, to reduce the cost, we can use the method for local imaging, or even datuming. In this case, the interferometric step is applied for limited image points and possibly we can use part of the data. A frequency domain implementation, as well as applications in datuming, will reduce the memory requirements considerably, as opposed to imaging with a time domain GIMI.

A limitation of the approach, also shared by the Marchenko method, is the need for a full coverage of sources and receivers on the recording surface. Missing sources or receivers will mainly affect the interferometric step of the imaging. However, unlike the Marchenko approach, the effect is isolated to events in which the stationary solution of the the wave path is not captured by the acquisition. This is also common in conventional imaging towards the edge of the image. Luckily, in the least-squares implementation, which acts as a Hessian, it will also try to balance the energy due to the missing data, though some artifacts may remain regardless.

All imaging methods, due to the limited acquisition aperture, are vulnerable to artifacts. The artifacts can also be enhanced by numerical errors. Since the energy associated with double scattering is an order weaker than single scattering, these artifacts appear larger in magnitude in double-scattered images. Despite that the double-scattered reflectivity using least-squares fitting is expected to include comparable magnitude energy to its single-scattering counterpart, the weak amplitudes of the corresponding reflections in the data imply that artifacts and noise will be enhanced to a similar degree. In other words, the doubly scattered reflectivity, obtained from the weaker multiple reflections, corresponds to effectively noiser data. This limitation is inherent to the nature of double-scattered reflections.

9 CONCLUSIONS

We developed the adjoint operation to the GIMI. Since GIMI is susceptible to cross-talk and resolution issues, using the adjoint we formulated a least-squares optimization problem to invert for the image. The interferometric cross-correlation step in GIMI converts the leading scattering term to a transmission component, which can be suppressed using a Laplacian filter or any advanced imaging condition. More importantly, it transforms second-order scattering to be the leading scattering term, which results in images corresponding to double scattering. Higher order scattering is attained by additional interferometric correlations. Space extensions of the double-scattering image can be used to evaluate the focusing and the velocity model used in GIMI. The examples clearly showed the advantages of imaging doubly scattered energy, especially in Salt valley regimes.

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