This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/LCOMM.2019.2945949, IEEE Communications Letters

End-to-end Performance Analysis of Delay-sensitive Multi-relay Networks

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***Abstract*—We study the end-to-end (E2E) performance of multi-relay networks in delay-constrained applications. The results are presented for both decode-and-forward (DF) and AF (A: amplify) relaying schemes. We use some fundamental results on the achievable rates of finite-length codes to analyze the system performance in the cases with short packets. Taking the message decoding delays and different numbers of hops into account, we derive closed-form expressions for the E2E packet transmission delay, the E2E error probability as well as the E2E throughput. Moreover, for different message decoding delays, we determine the appropriate codeword length and the relay power such that the same E2E error probability and packet transmission delay are achieved in the AF- and DF-relay networks. As we show, for different codeword lengths and numbers of hops, the E2E performance of multi-relay networks are affected by the message decoding delay of the nodes considerably.**

I. Introduction

Relay-assisted communication is one of the promising techniques that have been proposed for wireless networks [1], [2]. The main idea of a relay network is to improve the data transmission efficiency by implementing intermediate relay nodes that support the data transmission from a source to a destination.

Reviewing the literature, most of the results about relay networks assume infinite block-length codes where the achievable rates can be approximated by Shannon's capacity formula. However, finite block-length analysis of relay networks which is of interest in delay-constrained applications, has been rarely studied [3]–[8]. Different decode-and-forward (DF) [9], AF (A:amplify) [10], compute-and-forward [11] and compress-and-forward [12] methods have been proposed for relaying the messages, among which AF and DF are of particular interest due to their simplicity and appropriate data transmission efficiency.

With DF approach, the relay first decodes the message sent by a source node, and then re-encodes and transmits it to a destination. With AF approach, on the other hand, the relay acts as a repeater where, without message decoding, the relay scales the received signal to its maximum output power and forwards it to the destination. As opposed to the AF approach, the DF-relay node does not propagate the additive noise of its receiver to the destination. As a result, compared to the cases with AF relaying, higher signal-to-noise ratio (SNR) is observed by the destination receiving messages from an DF-relay node. However, the SNR improvement of the DF approach is at the cost of possible decoding failure in the DF-relay node(s) as well as increasing the end-to-end (E2E) packet transmission delay, because of the message decoding at the DF-relay node. Thus, there is a tradeoff and, depending on the channels quality and the message decoding delay, each of the AF- or DF-relaying approaches may lead to better E2E system performance.

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In this paper, we study the E2E performance of DF and AF multi-relay networks in delay-constrained applications. We use the fundamental results on the achievable rates of finite block-length codes [13] to analyze the system performance in the cases with short packets. With codewords of finite length and taking the message decoding delay into account, we derive closed-form expressions for the E2E packet transmission delay, the E2E error probability as well as the E2E throughput. Moreover, for different decoding delays, we determine the appropriate codeword length and the relay power such that the same E2E packet transmission delay and error probability are achieved in the DF- and AF-relay networks. Finally, we study the effect of different parameters such as the codeword length, the decoding delay and the number of hops on the system performance.

Our paper is different from [3]–[8] because 1) considering the message decoding delay and different numbers of hops, we study the E2E system performance and derive closed-form expressions for the E2E error probability/packet transmission delay/throught of both DF and AF multi-relaying schemes. Also, 2) we perform theoretical comparisons between the AF and DF methods and determine appropriate parameter settings such that they lead to the same system performance. The differences in the problem formulation and analysis make the problem solved in this paper completely different from the ones in the literature.

As we show, for different numbers of hops and codeword lengths, the E2E packet transmission delay and throughput are affected significantly by the message decoding delay in the relays and destination. Also, while with low decoding delays the DF-based approach leads to higher E2E throughput, compared to the AF-based method, the AF-based method outperforms the DF-based method, in terms of E2E throughput, as the decoding delay increases. Finally, as the number of hops increases, the DF-based approach outperforms the AF-based method, in terms of E2E throughput, for a broader range of decoding delays.

II. System model

We initially consider a relay-assisted communication setup consisting of a source, a relay and a destination. The extension of the results to the cases with multiple hops is presented in Section III.B. The channel coefficients in the source-relay and the relay-destination links are denoted by H_{sr} and H_{rd}, respectively, and we ignore the direct source-destination link. Then, we define $G_{\text{sr}} \doteq |H_{\text{sr}}|^2$ and $G_{\text{rd}} \doteq |H_{\text{rd}}|^2$ which are referred to as the channel gains in the following. Also, $Z_{\text{r}}^{\text{DF}}, Z_{\text{r}}^{\text{AF}}, Z_{\text{d}} \sim \mathcal{CN}(0,1)$ are the independent and identically distributed (iid) complex Gaussian noise added at the DF-relay, the AF-relay and the destination, respectively.

We study quasi-static conditions where the channel coefficients remain constant during multiple packet transmissions. In such systems, we can assume perfect channel state information (CSI) to be available at the receiver, because enough training sequences can be sent before data transmission, such that CSI is accurately estimated at the receiver. Also, we denote the probability density function (PDF) and the cumulative distribution function (CDF) of the random variable Ω by $f_\Omega(\cdot)$ and $F_\Omega(\cdot)$, respectively. We consider Rayleigh-fading conditions with $f_{G_{\text{A}}}(x) = \lambda_{\text{A}} e^{-\lambda_{\text{A}} x}, \text{A} = \{\text{sr}, \text{rd}\}$. To simplify the presentation of the analytical results, we set $\lambda_{\text{A}} = 1, \text{A} = \{\text{sr}, \text{rd}\}$, while the results can be rewritten for the cases with different values of $\lambda_{\text{A}}, \text{A} = \{\text{sr}, \text{rd}\}$.

Considering DF-based relaying, the source node encodes K information nats into a short codeword X^{DF} of length L^{DF} channel uses and rate $R^{\text{DF}} = \frac{K}{L^{\text{DF}}}$ nats-per-channel-uses (npcu). The codeword is sent to the relay in Slot 1 where, denoting the source transmit power by P_{s}, the received signal is given by

$$Y_{\text{r}}^{\text{DF}} = \sqrt{P_{\text{s}}} H_{\text{sr}} X^{\text{DF}} + Z_{\text{r}}^{\text{DF}}. \quad (1)$$

The DF-relay decodes the received signal and, with a successful message decoding, it re-encodes and sends the message to the destination receiving

$$Y_{\text{d}}^{\text{DF}} = \sqrt{P_{\text{r}}^{\text{DF}}} H_{\text{rd}} X^{\text{DF}} + Z_{\text{d}}. \quad (2)$$

Here, P_{r}^{DF} is the transmit power of the DF-relay node.

Using AF-relaying method, on the other hand, the source encodes the data into a codeword X^{AF} of length L^{AF} and rate $R^{\text{AF}} = \frac{K}{L^{\text{AF}}}$ (npcu). The relay only scales the received signal

$$Y_{\text{r}}^{\text{AF}} = \sqrt{P_{\text{s}}} H_{\text{sr}} X^{\text{AF}} + Z_{\text{r}}^{\text{AF}}, \quad (3)$$

to its maximum output power and sends the message $\tilde{X}^{\text{AF}} = \sqrt{\frac{P_{\text{r}}^{\text{AF}}}{P_{\text{s}} G_{\text{sr}} + 1}} Y_{\text{r}}^{\text{AF}}$, with P_{r}^{AF} being the maximum transmission power of the AF-relay node. Thus, the destination received signal is

$$Y_{\text{d}}^{\text{AF}} = \sqrt{\frac{P_{\text{r}}^{\text{AF}}}{P_{\text{s}} G_{\text{sr}} + 1}} H_{\text{rd}} \left(\sqrt{P_{\text{s}}} H_{\text{sr}} X^{\text{AF}} + Z_{\text{r}}^{\text{AF}} \right) + Z_{\text{d}}. \quad (4)$$

Comparing (2) and (4), DF-based relaying leads to higher received SNR at the destination, compared to the cases with AF-relaying. This is because by DF-relaying the additive noise of the relay node is not propagated to the relay-destination link. However, the SNR improvement is at the cost of 1) possible decoding failure at the DF-relay node and 2) additional decoding delay of the relay node. Therefore, depending on the channels condition and the decoding delay profile of the relay and destination, each of the DF- or AF-based approaches may lead to better E2E system performance. In the following, we study this tradeoff and determine the appropriate transmission parameters such that the same performance is achieved in these relaying methods.

III. Analytical results

A. Two-hop Networks

With DF-based approach, the E2E error probability is given by

$$\Gamma^{\text{DF}} = 1 - (1 - \gamma_{\text{sr}}^{\text{DF}})(1 - \gamma_{\text{rd}}^{\text{DF}}), \quad (5)$$

where $\gamma_{\text{sr}}^{\text{DF}}$ and $\gamma_{\text{rd}}^{\text{DF}}$ are the error probability at the DF-relay and destination, respectively. Let us define $\Delta(L)$ as the delay for decoding a message of length L. Then, the E2E packet transmission delay of the DF- and AF-based methods are, respectively, given by

$$\mathcal{L}^{\text{DF}} = L^{\text{DF}} + \Delta(L^{\text{DF}}) + (L^{\text{DF}} + \Delta(L^{\text{DF}}))(1 - \gamma_{\text{sr}}^{\text{DF}}), \quad (6)$$

and

$$\mathcal{L}^{\text{AF}} = 2L^{\text{AF}} + \Delta(L^{\text{AF}}). \quad (7)$$

Using the results of [13], [14] for the cases with codewords of finite length, the error terms $\gamma_{\text{A}}^{\text{DF}}, A = \{\text{sr}, \text{rd}\}$, are obtained as

$$\gamma_{\text{sr}}^{\text{DF}} = E\left\{ Q\left(\frac{\sqrt{L^{\text{DF}}}(\log(1 + G_{\text{sr}} P_{\text{s}}) - \frac{K}{L^{\text{DF}}})}{\sqrt{1 - \frac{1}{(1+G_{\text{sr}} P_{\text{s}})^2}}} \right) \right\}, \quad (8)$$

$$\gamma_{\text{rd}}^{\text{DF}} = E\left\{ Q\left(\frac{\sqrt{L^{\text{DF}}}(\log(1 + G_{\text{rd}} P_{\text{r}}^{\text{DF}}) - \frac{K}{L^{\text{DF}}})}{\sqrt{1 - \frac{1}{(1+G_{\text{rd}} P_{\text{r}}^{\text{DF}})^2}}} \right) \right\}, \quad (9)$$

where $E\{\cdot\}$ is the expectation with respect to the channel gain and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} \text{d}u$ is the Q-function.

Equations (8)-(9) have no closed-form expressions. For this reason, we use the linear approximation

$$Q\left(\frac{\sqrt{L}(\log(1 + ax) - \frac{K}{L})}{\sqrt{1 - \frac{1}{(1+ax)^2}}} \right) \simeq \mathcal{Y}(x, a, K, L)$$

$$\mathcal{Y}(x, a, K, L) = \begin{cases} 1 & x \leq \mu + \frac{1}{2\beta} \\ \frac{1}{2} + \beta(x - \mu) & x \in [\mu + \frac{1}{2\beta}, \mu - \frac{1}{2\beta}] \\ 0 & x \geq \mu - \frac{1}{2\beta} \end{cases}$$

$$\mu = \frac{e^{\frac{K}{L}} - 1}{a}, \beta = -a\sqrt{\frac{L}{2\pi(e^{\frac{2K}{L}} - 1)}}, \quad (10)$$

leading to

$$\gamma_{\text{sr}}^{\text{DF}} = \int_0^\infty e^{-x} Q\left(\frac{\sqrt{L^{\text{DF}}}(\log(1 + P_{\text{s}} x) - \frac{K}{L^{\text{DF}}})}{\sqrt{1 - \frac{1}{(1+P_{\text{s}} x)^2}}} \right) \text{d}x \simeq$$

$$\int_0^\infty e^{-x} \mathcal{Y}(x, P_{\text{s}}, K, L^{\text{DF}}) \text{d}x = 1 + \beta_{\text{sr}}^{\text{DF}} e^{-\mu_{\text{sr}}^{\text{DF}}} \left(e^{-\frac{1}{2\beta_{\text{sr}}^{\text{DF}}}} - e^{\frac{1}{2\beta_{\text{sr}}^{\text{DF}}}} \right),$$

$$\mu_{\text{sr}}^{\text{DF}} = \frac{e^{\frac{K}{L^{\text{DF}}}} - 1}{P_{\text{s}}}, \beta_{\text{sr}}^{\text{DF}} = -P_{\text{s}} \sqrt{\frac{L^{\text{DF}}}{2\pi(e^{\frac{2K}{L^{\text{DF}}}} - 1)}}, \quad (11)$$

and

$$\gamma_{\text{rd}}^{\text{DF}} \simeq 1 + \beta_{\text{rd}}^{\text{DF}} e^{-\mu_{\text{rd}}^{\text{DF}}} \left(e^{-\frac{1}{2\beta_{\text{rd}}^{\text{DF}}}} - e^{\frac{1}{2\beta_{\text{rd}}^{\text{DF}}}} \right),$$

$$\mu_{\text{rd}}^{\text{DF}} = \frac{e^{\frac{K}{L^{\text{DF}}}} - 1}{P_{\text{r}}^{\text{DF}}}, \beta_{\text{rd}}^{\text{DF}} = -P_{\text{r}}^{\text{DF}} \sqrt{\frac{L^{\text{DF}}}{2\pi(e^{\frac{2K}{L^{\text{DF}}}} - 1)}}, \quad (12)$$

where (12) is obtained with the same procedure as in (11).

Using (4), the SNR received by the destination from the data transmission of the AF-relay is found as

$$\theta^{\text{AF}} = \frac{P_{\text{s}} P_{\text{r}}^{\text{AF}} G_{\text{sr}} G_{\text{rd}}}{1 + P_{\text{s}} G_{\text{sr}} + P_{\text{r}}^{\text{AF}} G_{\text{rd}}}, \quad (13)$$

and the E2E error probability of the AF-based approach is

$$\Gamma^{\text{AF}} = E\left\{ Q\left(\frac{\sqrt{L^{\text{AF}}}(\log(1 + \theta^{\text{AF}}) - \frac{K}{L^{\text{AF}}})}{\sqrt{1 - \frac{1}{(1+\theta^{\text{AF}})^2}}} \right) \right\}, \quad (14)$$

with expectation over θ^{AF}. Considering Rayleigh-fading conditions, there is no closed-form expression for the PDF of θ^{AF}, which makes calculating (14) difficult. For this reason, Theorem 1 develops approximation schemes for calculating (14) as well as for the appropriate transmission parameters such that the AF- and DF-based schemes lead to the same E2E system performance.

Theorem 1. (I): The E2E error probability of the AF-based approach is approximately given by $\Gamma^{\text{AF}} = 1 - e^{-\left(\frac{1}{P_{\text{s}}}+\frac{1}{P_{\text{r}}^{\text{AF}}}\right)\left(e^{\frac{K}{L^{\text{AF}}}}-1\right)}$. Also, (II): consider a linear decoding delay function $\Delta(L) = \alpha L$ with α being a constant. Then, the AF- and the DF-based relaying schemes lead to the same E2E packet transmission delay if

$$L^{\text{AF}} = \frac{(1+\alpha)\left(2-\beta_{\text{sr}}^{\text{DF}}e^{-\mu_{\text{sr}}^{\text{DF}}}\left(e^{-\frac{1}{2\beta_{\text{sr}}^{\text{DF}}}}-e^{\frac{1}{2\beta_{\text{sr}}^{\text{DF}}}}\right)\right)}{(2+\alpha)}L^{\text{DF}}. \quad (15)$$

Finally, (III): for given values of $P_{\text{s}}, P_{\text{r}}^{AF}, L^{\text{DF}}, L^{\text{AF}}$ and K, the AF- and the DF-based relay network lead to the same E2E error probability if the transmission power of the DF-based relay node is determined by the numerical solution of (19).

Proof. To find (14), we use (10) leading to

$$\begin{aligned}
\Gamma^{\text{AF}} &\overset{(a)}{\simeq} \int_0^\infty f_{\theta^{\text{AF}}}(x)\mathcal{Y}(x,1,K,L^{\text{AF}})\mathrm{d}x \\
&\overset{(b)}{=} F_{\theta^{\text{AF}}}\left(\mu^{\text{AF}}+\frac{1}{2\beta^{\text{AF}}}\right)+\left(\frac{1}{2}-\beta^{\text{AF}}\mu^{\text{AF}}\right)\times \\
&\left(F_{\theta^{\text{AF}}}\left(\mu^{\text{AF}}-\frac{1}{2\beta^{\text{AF}}}\right)-F_{\theta^{\text{AF}}}\left(\mu^{\text{AF}}+\frac{1}{2\beta^{\text{AF}}}\right)\right) \\
&-\int_{\mu^{\text{AF}}+\frac{1}{2\beta^{\text{AF}}}}^{\mu^{\text{AF}}-\frac{1}{2\beta^{\text{AF}}}} x f_{\theta^{\text{AF}}}(x)\mathrm{d}x \overset{(c)}{\simeq} F_{\theta^{\text{AF}}}\left(\mu^{\text{AF}}\right) \overset{(d)}{\simeq} 1-e^{-\left(\frac{1}{P_{\text{s}}}+\frac{1}{P_{\text{r}}^{\text{AF}}}\right)\mu^{\text{AF}}}, \\
&\mu^{\text{AF}} = e^{\frac{K}{L^{\text{AF}}}}-1, \beta^{\text{AF}} = -\sqrt{\frac{L^{\text{AF}}}{2\pi(e^{\frac{2K}{L^{\text{AF}}}}-1)}}.
\end{aligned} \quad (16)$$

Here, (a) comes from (10) and (b) is based on partial integration. Then, (c) is obtained by the first order Riemann integral approximation $\int_{x_0}^{x_1} f(x)\mathrm{d}x = (x_1-x_0)f\left(\frac{x_0+x_1}{2}\right)$. Finally, (d) is based on the approximation $\theta^{\text{AF}} \simeq \mathcal{X}, \mathcal{X} = \min\left(P_{\text{s}}G_{\text{sr}}, P_{\text{r}}^{\text{AF}}G_{\text{rd}}\right)$ [15] following the CDF $F_{\mathcal{X}}(x) = 1-e^{-\left(\frac{1}{P_{\text{s}}}+\frac{1}{P_{\text{r}}^{\text{AF}}}\right)x}$.

To prove part (II), we note that with a linear decoding delay function $\Delta(L) = \alpha L$, which is an appropriate model for different coding schemes [16], the expected delay of the DF- and AF-based schemes, i.e., (6) and (7), are simplified as

$$\mathcal{L}^{\text{DF}} = L^{\text{DF}}(1+\alpha)(2-\gamma_{\text{sr}}^{\text{DF}}), \quad (17)$$

and

$$\mathcal{L}^{\text{AF}} = (2+\alpha)L^{\text{AF}}, \quad (18)$$

respectively. Then, the appropriate length of the codewords length for the AF-based approach leading to the same expected delay as in the DF-based approach is obtained by setting $\mathcal{L}^{\text{DF}} = \mathcal{L}^{\text{AF}}$ which, using (11), (17) and (18), results in (15).

Finally, the appropriate transmission power of the DF-relay node that, for given values of $P_{\text{s}}, P_{\text{r}}^{\text{AF}}, L^{\text{DF}}, L^{\text{AF}}$ and K, leads to the same E2E error probability in the AF- and DF-based schemes is obtained by setting $\Gamma^{\text{DF}} = \Gamma^{\text{AF}}$ which, using (11), (12) and (16), leads to

$$\begin{aligned}
P_{\text{r}}^{\text{DF}} = \arg_x \Bigg\{ & x\sqrt{\frac{L^{\text{DF}}}{2\pi(e^{\frac{2K}{L^{\text{DF}}}}-1)}}\beta_{\text{sr}}^{\text{DF}}e^{-\left(\frac{e^{\frac{K}{L^{\text{DF}}}}-1}{x}+\mu_{\text{sr}}^{DF}\right)}\times \\
&\left(e^{\frac{1}{x\sqrt{\frac{L^{\text{DF}}}{2\pi(e^{\frac{2K}{L^{\text{DF}}}}-1)}}}}-e^{\frac{-1}{x\sqrt{\frac{L^{\text{DF}}}{2\pi(e^{\frac{2K}{L^{\text{DF}}}}-1)}}}}\right)\left(e^{-\frac{1}{2\beta_{\text{sr}}^{\text{DF}}}}-e^{\frac{1}{2\beta_{\text{sr}}^{\text{DF}}}}\right) \\
&= e^{-\left(\frac{1}{P_{\text{s}}}+\frac{1}{P_{\text{r}}^{\text{AF}}}\right)\mu^{\text{AF}}}\Bigg\},
\end{aligned} \quad (19)$$

as stated in part III of the theorem. □

Note that letting $P_{\text{s}} \to \infty$ we have $\beta_{\text{sr}}^{\text{DF}} \to 0$ and (15) is rephrased as

$$L^{\text{AF}} \simeq \frac{2(1+\alpha)L^{\text{DF}}}{(2+\alpha)}, \quad (20)$$

for the case with moderate/high source powers. Also, (19) is a one-dimensional equation which can be effectively solved by, e.g., bisection method.

Finally, for different relaying methods, the E2E throughput, defined as the expected number of bits successfully received by the destination versus the expected E2E packet transmission delay, is given by

$$\begin{aligned}
\eta^{\text{A}} &= \frac{K\left(1-\Gamma^{\text{A}}\right)}{\mathcal{L}^{\text{A}}} = \\
&\begin{cases}
\dfrac{K(1-\varphi)}{L^{\text{DF}}(1+\alpha)\left(1-\beta_{\text{sr}}^{\text{DF}}e^{-\mu_{\text{sr}}^{\text{DF}}}\left(e^{-\frac{1}{2\beta_{\text{sr}}^{\text{DF}}}}-e^{\frac{1}{2\beta_{\text{sr}}^{\text{DF}}}}\right)\right)} & \text{if A} = \text{DF}, \\
\dfrac{Ke^{-\left(\frac{1}{P_{\text{s}}}+\frac{1}{P_{\text{r}}^{\text{AF}}}\right)\left(e^{\frac{K}{L^{\text{AF}}}}-1\right)}}{(2+\alpha)L^{\text{AF}}} & \text{if A} = \text{AF},
\end{cases} \\
\varphi &= \beta_{\text{sr}}^{\text{DF}}\beta_{\text{rd}}^{\text{DF}}e^{-\left(\mu_{\text{sr}}^{\text{DF}}+\mu_{\text{rd}}^{\text{DF}}\right)}\left(e^{-\frac{1}{2\beta_{\text{sr}}^{\text{DF}}}}-e^{\frac{1}{2\beta_{\text{sr}}^{\text{DF}}}}\right)\left(e^{-\frac{1}{2\beta_{\text{rd}}^{\text{DF}}}}-e^{\frac{1}{2\beta_{\text{rd}}^{\text{DF}}}}\right).
\end{aligned} \quad (21)$$

B. Extension to Multi-hop Networks

In Section III.A, we presented the results for two-hop networks. However, the same approaches can be applied in the cases with multi-hop networks. Particularly, considering an $(N+1)$-hop network with N DF-based relay nodes, the E2E packet transmission delay and error probability are given by

$$\mathcal{L}^{\text{DF}}(N) = \left(L^{\text{DF}}+\Delta(L^{\text{DF}})\right)\left(1+\sum_{i=1}^{N}\prod_{j=1}^{i}\left(1-\gamma_i^{\text{DF}}\right)\right), \quad (22)$$

and

$$\Gamma^{\text{DF}}(N) = 1-\prod_{i=1}^{N+1}\left(1-\gamma_i^{\text{DF}}\right), \quad (23)$$

respectively, where γ_i is the error probability in the i-th hop which is found with the same procedure as in (11)-(12).

On the other hand, with N AF-based relay nodes in an $(N+1)$-hop network, the E2E transmission delay is given by

$$\mathcal{L}^{\text{AF}}(N) = (N+1)L^{\text{AF}}+\Delta(L^{\text{AF}}). \quad (24)$$

Also, the E2E error probability is obtained by (14) where the SNR term is rephrased as

$$\theta^{\text{AF}}(N) = \frac{\prod_{i=1}^{N+1} \mathcal{S}_i}{1 + \sum_{\forall A \in \{1,\dots,N+1\}, A \neq \{1,\dots,N+1\}} \prod_{i \in A} \mathcal{S}_i}. \quad (25)$$

Here, $\mathcal{S}_i = P_i G_i$ is the SNR in the i-hop with G_i and P_i being the channel gain and the power of the transmit node in the i-th hop, respectively. In this way, one can use (22)-(25) to study the system performance for the cases with different numbers of hops and relaying methods.

IV. Simulation Results

The tightness of the approximations [13] increases with the codeword length. For this reason, we present the results for the cases with codewords of length ≥ 100 where, e.g., (8) gives a fairly accurate approximation of the error probability of short packets.

Setting $L^{\text{DF}} = 1000$ and considering a linear delay profile $\Delta(L) = \alpha L, \alpha = 3$ [16], Figs. 1 and 2 study the tightness of the analytical results (The simulations can be rerun for different decoding delay profiles). Particularly, Fig. 1 verifies the accuracy of the approximation (11), (12), (16) and shows the E2E error probability of the two-hop network. Here, the codeword length L^{AF} and the transmission power of the DF-based relay network are determined according to (15) and (19), respectively, such that the same E2E packet transmission delay and error probability are achieved by the DF- and AF-based relaying methods. That is, Fig. 1 shows the error probability of both AF- and DF-based schemes with different values of $K, P_{\text{r}}^{\text{AF}}$ and appropriate parameter settings for P_{r}^{DF} determined by Theorem 1 (the same approach can be applied by fixing L^{AF} and determining L^{DF} accordingly). Also, because the noise variances are set to 1, we define the source-relay link SNR as $10\log_{10} P_{\text{s}}$ dB.

Fig. 2a shows the required power of the DF-based scheme to guarantee the same E2E error probability in the DF- and AF-relay networks. Also, Fig. 2b studies the required length of the codeword length in the AF-based approach leading to the same E2E packet transmission delay in these methods, and verifies the accuracy of the high-SNR approximation (20). In these figures, the results are presented for $L^{\text{DF}} = 1000, \alpha = 3, P_{\text{r}}^{\text{AF}} = 0$ dB, and different values of K.

Considering different numbers of hops, Fig. 3 investigates the E2E throughput of the AF- and DF-based schemes for different values of the decoding delay factor α. Here, the results are obtained for the cases with $L^{\text{AF}} = L^{\text{DF}} = 500$, $K = 500$ and $P_i = 10$ dB $\forall i$. Finally, setting $\alpha = 3, K = 500$ nats, $P_i = 10$ dB$, \forall i$, Fig. 4 demonstrates the E2E throughput for different codewords length $L^{\text{AF}} = L^{\text{DF}} = L$.

From the figures, the following conclusions can be drawn:

- The theoretical results of (11), (12) and (16) properly approximate the E2E error probability of AF- and DF-based relay networks using short packets (Fig. 1). Also, the approximation schemes of Theorem 1 are tight for a broad range of parameter settings/SNRs (Figs. 1, 2). Thus, the results of Section III can be well utilized for the analytical performance evaluation of AF- and DF-based multi-hop networks in the cases with codewords of finite length and compare their E2E performance (Figs. 2a, 2b).
- For different numbers of bits per codeword K, the required power of the DF-relay node, to guarantee the same E2E error probability as in the AF-based scheme, is sensitive to source transmit power (Fig. 2a). However, its sensitivity decreases as the transmission power of the source increases. This is intuitively because with high SNR in the source-relay link the DF-relay node can correctly decode the message with high probability. Also, the required power of the DF-based scheme, to guarantee the same E2E error probability as the AF-based scheme, is not a monotonic function of the source power (Fig. 2a). Intuitively, this is because the E2E error probability of the AF- and DF-based schemes are affected by the source power in different ways.
- The required length of the codewords in the AF-based method, to have the same E2E transmission delay as in the DF-based approach, increases with the source power monotonically and converges to $L^{\text{AF}} = \frac{2(1+\alpha)L^{\text{DF}}}{(2+\alpha)}$ in harmony with (20) (Fig. 2b). Also, the required length of the codewords in the AF-relay node is sensitive to the number of bits per codeword at low SNRs, while its sensitivity decreases as the SNR increases (Fig. 2b).
- With low decoding delays, the DF-based approach leads to higher E2E throughput, compared to the AF-based method, and the relative performance gain of DF-based approach increases as the decoding delay decreases. However, as α increases, the AF-based method outperforms the DF-based method, in terms of E2E throughput. Also, for different relaying methods, the E2E throughput is sensitive to the decoding delay for small values of α while its sensitivity decreases as α increases (Fig. 3).
- For the parameter settings of Fig. 3, the DF-based approach leads to higher E2E throughput, compared to the AF-based method, for a broader range of values of α as the number of hops increases. Intuitively, this is because with a large number of hops propagating the additive noise to the next hops deteriorates the system performance considerably. Thus, to have high E2E throughput, it is better that the relays decode the received messages unless the decoding delay is large.
- For a given number of information nats per codeword K and different relaying methods, there is an optimal value for the codeword length maximizing the throughput (Fig. 4). This is because the codeword rate (resp. the error probability) increases (resp. decreases) with the length of the codewords. Thus, there is a tradeoff and the maximum throughput is achieved by a finite value of the codeword length. Then, the optimal value of the codeword length, in terms of E2E throughput, is higher in the AF-based approach, compared to the DF-based method. Finally, for both AF- and DF-based methods, the throughput-optimized codeword length increases with the number of hops (Fig. 4).

V. Conclusions

This paper studied the E2E performance of AF- and DF-based relaying methods in the cases with codewords of finite length. We showed that the E2E throughput of relaying methods

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/LCOMM.2019.2945949, IEEE Communications Letters

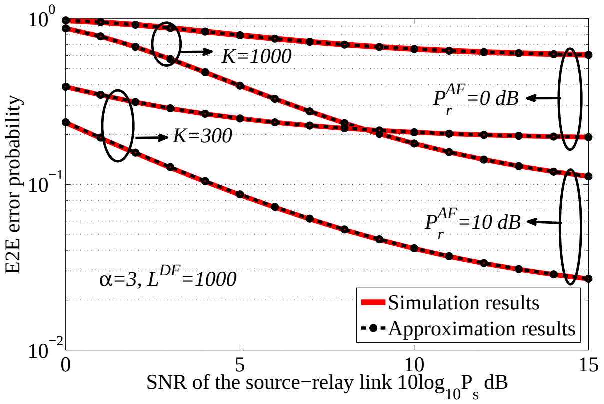

Figure 1. On the accuracy of the theoretical results, $L^{\mathrm{DF}} = 1000$, $\alpha = 3$. For different values of K and $P_{\mathrm{r}}^{\mathrm{AF}}$, the values of L^{AF} and $P_{\mathrm{r}}^{\mathrm{DF}}$ are determined by parts II and III of Theorem 1, respectively, such that the same E2E error probability and packet transmission delay are obtained in both AF- and DF-based schemes.

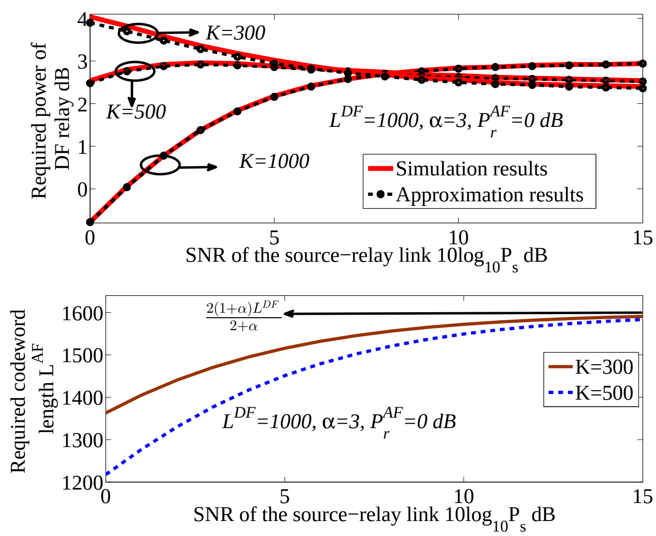

Figure 2. (a): On the accuracy of the approximation (19). The required power of the DF-relay node is determined such that the same E2E error probability is obtained in the DF- and AF-based schemes, $\alpha = 3, L^{\mathrm{DF}} = 1000, P_{\mathrm{r}}^{\mathrm{DF}} = 0$ dB. The length of the codeword in the AF-based approach is determined by (15) to guarantee the same E2E packet transmission delay for both AF- and DF-based schemes. (b): The required length of the codewords in the AF-based approach to guarantee the same E2E packet transmission delay in the AF- and DF-based schemes, $L^{\mathrm{DF}} = 1000, \alpha = 3, P_{\mathrm{r}}^{\mathrm{AF}} = 0$ dB.

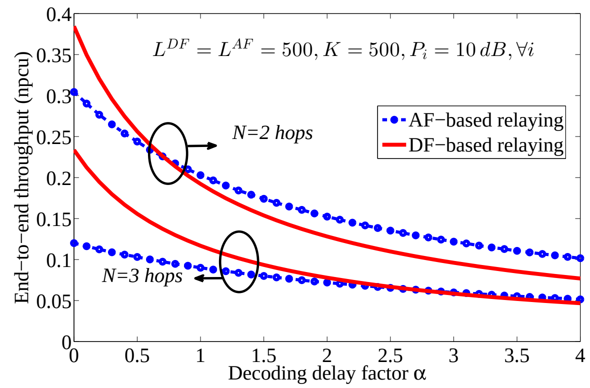

Figure 3. Throughput for different relaying methods and numbers of hops, $L^{\mathrm{DF}} = L^{\mathrm{AF}} = 500, K = 500$ nats, $P_i = 10$ dB, $\forall i$.

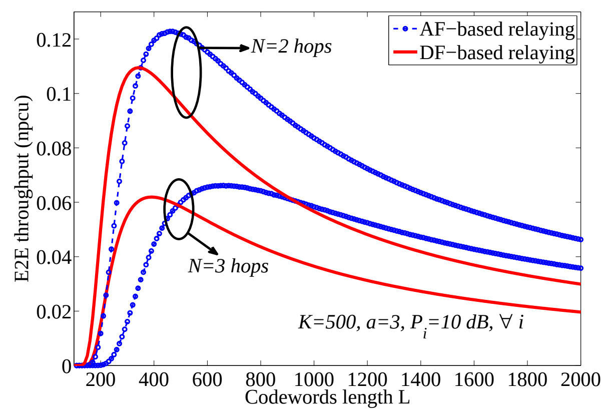

Figure 4. Throughput vs the codeword length L, $\alpha = 3, K = 500$ nats, $P_i = 10$ dB, $\forall i$.

is considerably affected by the codewords length as well as the message decoding delay. Finally, depending on the codewords length, the message decoding delay of the nodes and the number of hops, each of the AF- or DF-based methods can lead to better E2E system performance.

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