

# On Physical Layer Security of Multiple-Relay Assisted NOMA Systems

Hongjiang Lei<sup>†</sup>, Zixuan Yang<sup>†</sup>, Ki-Hong Park<sup>§</sup>, Imran Shafique Ansari<sup>¶</sup>, Gaofeng Pan<sup>‡</sup>, and Mohamed-Slim Alouini<sup>§</sup>

<sup>†</sup>School of Communication and Information Engineering,

Chongqing University of Posts and Telecommunications, Chongqing 400065, China

<sup>§</sup>CEMSE Division, King Abdullah University of Science and Technology, Thuwal 23955-6900, Saudi Arabia

<sup>¶</sup>School of Engineering, University of Glasgow, Glasgow G12 8QQ, United Kingdom

<sup>‡</sup>Chongqing Key Laboratory of Nonlinear Circuits and Intelligent Information Processing,  
Southwest University, Chongqing 400715, China

**Abstract**—This paper considers the secrecy outage performance of a multiple-relay assisted non-orthogonal multiple access (NOMA) network over Rayleigh fading channels. Two slots are utilized to transmit signals from the base station to destination. In the first slot, the base station broadcasts the superposition signal of two users to all decode-and-forward relays by message mapping strategy. Then the selected relay transmits superposition signal to the two users via power-domain NOMA. Two relay selection (RS) schemes, i.e., optimal single relay selection (OSRS) scheme and two-step single relay selection (TSRS) scheme, are proposed. As a benchmark, we also examine the secrecy outage performance of the traditional multiple relays combining (TMRC) scheme. The closed-form expressions for the security outage probability (SOP) of the proposed schemes are derived and validated via simulations. To get more insights, we also derive the closed-form expressions for the asymptotic SOP for all the schemes with fixed and dynamic power allocations. Monte-Carlo simulations are performed to verify our analytical results.

## I. INTRODUCTION

Non-orthogonal multiple access (NOMA) has been considered as one of the most promising technologies to deal with the shortage of bandwidth resources and enhance user fairness in the fifth generation (5G) mobile network [1]- [3]. Compared to traditional orthogonal multiple access schemes, NOMA systems can obtain superior merits, such as higher bandwidth efficiency, better user fairness, ultra-connectivity, and higher flexibility.

Cooperative communication is one of the most attractive technique that not only extends the network's coverage but also enhances the system performance when diversity technology is utilized at the destinations. Ding *et al.* analyzed the outage probability (OP), diversity order of cooperative NOMA systems, and proposed an approach based on user pairing to reduce system complexity in [4]. A new cooperative simultaneous wireless information and power transfer NOMA protocol was proposed and the closed-form expressions for the OP and system throughput were derived in [5]. Furthermore, a dedicated amplify-and-forward (AF) relay with multiple antennas was utilized in the cooperative NOMA systems and the closed-form expressions for the exact and lower bound of OP were derived in [6]–[8].

Relay selection (RS) technique is an effective scheme in making full use of space diversity with low implementation

complexity and it can straightforwardly improve the spectral efficiency of cooperative systems [9], [10]. Considering the quality of service (QoS) requirements for the two users might be different, a two-stage single-relay-selection strategy and dual-relay-selection were studied in [11] and [12], respectively.

To deal with the security issues followed by the explosive increase in cellular data, physical layer security (PLS) is emerging as one of the most promising ways to ensure secure communication. In [13], Liu *et al.* investigated the security performance of a NOMA-based large scale network, where both the NOMA users and eavesdroppers were modeled by homogeneous PPPs. In [14], we investigated the secrecy outage performance of a multiple-input single-output (MISO) NOMA system with transmit antenna selection (TAS) schemes and the closed-form expressions for secrecy outage probability (SOP) were derived. The secrecy outage performance of multiple-input multiple-output (MIMO) NOMA systems with multiple legitimate and illegitimate receivers was analyzed in [15] and the closed-form expressions for SOP were subsequently obtained. Secrecy beamforming schemes were proposed for MISO-NOMA systems, cognitive MISO-NOMA systems, and MIMO-NOMA systems in [16], [17], [18], and [19], respectively. The secrecy performance of a NOMA system with multiple eavesdroppers was investigated while zero-forcing and minimum mean-square error decoding schemes were utilized on the legitimate destinations in [20]. A new joint subcarrier (SC) assignment and power allocation scheme was proposed to improve the security of the two-way relay NOMA systems in [21]. The secrecy performance of a cooperative NOMA system with a dedicated decode-and-forward (DF) relay and an amplify-and-forward (AF) relay was investigated and the closed-form expressions for the secrecy outage probability (SOP) were obtained in [22].

In this work, we analyze a cooperative NOMA system with multiple relays and an eavesdropper employing RS schemes to enhance the secrecy performance. Optimal single relay selection (OSRS) scheme and two-step single relay selection (TSRS) scheme are proposed. For the purpose of comparison, we also investigate the secrecy performance of the NOMA systems with traditional multiple relays combining (TMRC) scheme in which all the relays that successfully decode signals from the

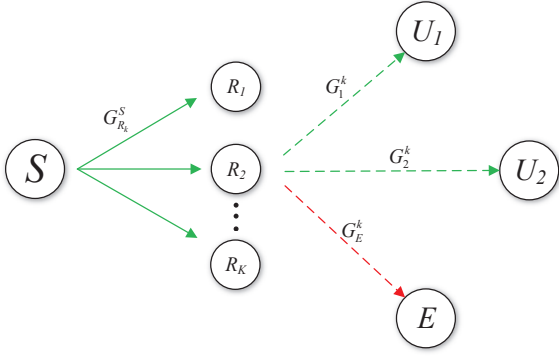


Fig. 1. System model demonstrating a cooperative NOMA system with one source ( $S$ ), multiple relays ( $R_k, k = 1, \dots, K$ ), two legitimate users ( $U_1$  and  $U_2$ ), and an illegitimate eavesdropper ( $E$ ).

source forward signals to the NOMA users with equal power. Considering the correlation between the secrecy capacity of two users, the closed-form expressions for the SOP under different RS schemes are derived and validated via simulations. To obtain more insights, we also derive the closed-form expressions for the asymptotic SOP under different RS schemes with fixed power allocation (FPA) and dynamic power allocation (DPA) schemes. The results demonstrate that all the RS schemes with FPA obtain zero secrecy diversity order (SDO), which can be improved thereby achieving non-zero SDO with DPA scheme.

This paper is organized as follows, in Section II, the system model is introduced. In Section III, we analyze the security outage performance of the proposed system. The asymptotic SOP for cooperative NOMA systems with FPA and DPA are derived in Sections IV and V, respectively. Numerical and simulation results are presented in Section VI to demonstrate the our analysis results. Finally, Section VII concludes the paper.

## II. SYSTEM MODEL

As shown in Fig. 1, we consider a cooperative downlink NOMA system that consists of a base station ( $S$ ),  $K$  ( $K \geq 1$ ) DF half duplex relays ( $R_k, k = 1, \dots, K$ ), and two users ( $U_1$  and  $U_2$ ). An eavesdropper ( $E$ ) wants to wiretap the information through decoding the received signals. It is assumed that all nodes are equipped with a single antenna and the direct link between  $S$  and both users are unavailable due to deep fading. The communication links between  $S$  and all the receivers ( $U_1, U_2$ , and  $E$ ) are relayed by  $R_k$ . All the channels are assumed to undergo independent and slow-fading Rayleigh model and remain unchanged during each fading block but independently vary from one block to another. Then the probability density function (PDF) and the cumulative distribution function (CDF) of the channel power gains of link between the node  $v$  and the node  $\ell$  can be expressed by

$$f_{G_\ell^v}(x) = \frac{1}{\lambda_\ell} e^{-\frac{x}{\lambda_\ell}}, \quad (1)$$

$$F_{G_\ell^v}(x) = 1 - e^{-\frac{x}{\lambda_\ell}}, \quad (2)$$

where  $v \in \{S, R_k\}$  signifies the transmitter,  $\ell \in \{R_k, U_1, U_2, E\}$  denotes the receiver, and  $\lambda_\ell^v$  are average channel power gains. To make analysis simple, it is assumed that the

links of the two hops are independent and identically distributed (i.i.d.), which means  $\lambda_{R_k}^S = \lambda_R$ ,  $\lambda_{U_1}^{R_k} = \lambda_1$ ,  $\lambda_{U_2}^{R_k} = \lambda_2$ , and  $\lambda_E^{R_k} = \lambda_E$ .

Two time slots are assumed to utilize to the communication between  $S$  and the NOMA users. In the first time slot,  $S$  broadcasts the superposition signal to all relays by message mapping strategy, which is proved to be optimal to achieve the minimal common OP in [23]. During the second time slot, a relay is selected using the RS scheme proposed below to send superposition signal to the two NOMA users. We assume all the relays are close enough such that the relative distance between the two users and the relay is determined.

To make the representation clear and simple, we use  $G_{U_i}^{R_k} = G_i^k$ . It is also assumed that  $U_1$  has a better channel condition than  $U_2$  ( $G_1^k > G_2^k$ ), which is adopted in many NOMA studies, e.g., [14] and [23]. Then the signal-to-interference-noise ratio (SINR) of the NOMA users can be written as

$$\gamma_1^k = \alpha_1 \rho G_1^k, \quad (3)$$

$$\gamma_2^k = \frac{\alpha_2 \rho G_2^k}{\alpha_1 \rho G_2^k + 1}, \quad (4)$$

where  $\alpha_i$  ( $i = 1, 2$ ) represents the power allocation coefficients at the  $k$ th relay,  $\alpha_1 + \alpha_2 = 1$ ,  $\alpha_1 > \alpha_2$ , and  $\rho = \frac{P_R}{\sigma^2}$  signifies the transmit signal to noise ratio (SNR).

## III. SECURITY OUTAGE PROBABILITY ANALYSIS

The set of relays that can correctly decode mixed signals from  $S$  can be expressed as

$$\Phi \triangleq \left\{ k : 1 \leq k \leq K, \frac{1}{2} \ln(1 + \rho_S G_{R_k}^S) \geq R_1^{\text{th}} + R_2^{\text{th}} \right\}, \quad (5)$$

where the factor  $\frac{1}{2}$  arises from the fact that two slots are required to complete the data transmission,  $\rho_S$  denotes the transmit SNR, and  $R_i^{\text{th}}$  ( $i = 1, 2$ ) is the data rate threshold for  $U_i$ . Thus the SOP of cooperative NOMA systems can be written as

$$P_{\text{out}} = \sum_{n=0}^K \Pr(|\Phi| = n) P_{\Phi_n}, \quad (6)$$

where  $P_{\Phi_n}$  denotes the SOP under the condition that there are  $n$  relays that correctly decode the mixed signals. One can easily have  $P_{\Phi_0} = 1$ .

It is assumed that all the links between the source and the relays are i.i.d., so we have

$$\begin{aligned} \Pr\{|\Phi| = n\} &= C_K^n (\Pr\{G_{R_k}^S \geq \eta\})^n (\Pr\{G_{R_k}^S < \eta\})^{K-n} \\ &= C_K^n e^{-\frac{n\eta}{\lambda_R}} \left(1 - e^{-\frac{\eta}{\lambda_R}}\right)^{K-n}, \end{aligned} \quad (7)$$

where  $\eta = \frac{e^{2R_1^{\text{th}} + 2R_2^{\text{th}}} - 1}{\rho_S}$ .

### A. Traditional Multiple Relays Combining Scheme

As a benchmark, the traditional multiple relays combining (TMRC) scheme is presented in this subsection, where all the  $n$  relays can successfully decode and forward the signal to both the users with equal power. Both the legitimate and illegitimate receivers combine their received signals with maximal ratio combining (MRC) scheme to maximize their SINR. To make

fair comparison, it is assumed that the total transmit power at these relays is given by  $P_R$ . Then the  $P_{\Phi_n}$  with this scheme can be expressed as

$$P_{\Phi_n} = \Pr \{ C_{s,1}^{\text{TMRC}} < R_1^s \text{ or } C_{s,2}^{\text{TMRC}} < R_2^s \mid |\Phi| = n \}, \quad (8)$$

where  $C_{s,i}^{\text{TMRC}} = \ln \left( \frac{1+\gamma_i^{\text{TMRC}}}{1+\gamma_{E,i}^{\text{TMRC}}} \right)$  means the secrecy capacity of  $U_i$ ,  $i = 1, 2$ ,  $\gamma_1^{\text{TMRC}} = \rho_1 \alpha_1 G_1^{\text{TMRC}}$ ,  $\gamma_2^{\text{TMRC}} = \frac{\rho_1 \alpha_2 G_2^{\text{TMRC}}}{1+\rho_1 \alpha_1 G_2^{\text{TMRC}}}$ ,  $G_i^{\text{TMRC}} = \sum_{k=1}^n G_i^k$ ,  $\gamma_{E,i}^{\text{TMRC}}$  signifies the SNR at  $E$  when  $U_i$  is wiretapped,  $R_i^s$  is the secrecy rate threshold for  $U_i$ ,  $\rho_1 = \frac{P_R}{n\sigma^2}$ , and  $\sigma^2$  is the noise power.

The CDF of  $G_i^{\text{TMRC}}$  can be expressed as

$$F_{G_i^{\text{TMRC}}}(x) = 1 - e^{-\frac{x}{\lambda_i}} \sum_{k=0}^{n-1} \frac{x^k}{\lambda_i^k k!}. \quad (9)$$

where  $i = 1, 2$ .

As similar to [13], [15], and [22], we assume that  $E$  has enough capabilities to detect multiuser data. Then the SNR at  $E$  can be expressed as

$$\gamma_{E,i}^{\text{TMRC}} = \rho_1 \alpha_i G_E^{\text{TMRC}}, \quad (10)$$

where  $i = 1, 2$  and  $G_E^{\text{TMRC}} = \sum_{k=1}^n G_E^k$ . Similarly, the PDF of  $G_E^{\text{TMRC}}$  can be expressed as

$$f_{G_E^{\text{TMRC}}}(x) = \frac{x^{n-1} e^{-\frac{x}{\lambda_E}}}{\lambda_E^n (n-1)!}. \quad (11)$$

Based on (8),  $P_{\Phi_n}$  in this case can be rewritten as

$$\begin{aligned} P_{\Phi_n}^{\text{TMRC}} &= \Pr \{ (C_{s,1}^{\text{TMRC}} < R_1^s \text{ or } C_{s,2}^{\text{TMRC}} < R_2^s) \mid |\Phi| = n \} \\ &= 1 - \int_0^{a_1} \Pr \{ G_1^{\text{TMRC}} > \delta_1(x) \} \\ &\quad \times \Pr \{ G_2^{\text{TMRC}} > \delta_2(x) \} f_{G_E^{\text{TMRC}}}(x) dx \\ &= 1 - \int_0^{a_1} \left( 1 - F_{G_1^{\text{TMRC}}}(\delta_1(x)) \right) \times \\ &\quad \left( 1 - F_{G_2^{\text{TMRC}}}(\delta_2(x)) \right) f_{G_E^{\text{TMRC}}}(x) dx, \end{aligned} \quad (12)$$

where  $a_1 = \frac{1-\theta_2\alpha_1}{\rho_1\alpha_1\alpha_2\theta_2}$ ,  $\delta_1(x) = b_1 + \theta_1 x$ ,  $b_1 = \frac{\theta_1-1}{\alpha_1\rho_1}$ ,  $\theta_1 = e^{2R_1^s}$ ,  $\delta_2(x) = c_1 + \frac{\alpha_2}{d_1-e_1x}$ ,  $c_1 = -\frac{1}{\alpha_1\rho_1}$ ,  $d_1 = \alpha_1\rho_1(1-\alpha_1\theta_2)$ ,  $e_1 = \rho_1^2\alpha_1^2\alpha_2\theta_2$ , and  $\theta_2 = e^{2R_2^s}$ .

To facilitate the following analysis, we define

$$\begin{aligned} g(a, b, c, r, q, f, h, k, j) &= \int_0^a x^{b-1} e^{-fx - \frac{h}{1-qx}} \\ &\quad \times (1+cx)^k \left( 1 + \frac{r}{1-qx} \right)^j dx. \end{aligned} \quad (13)$$

To the authors' best knowledge, it is very difficult to obtain the closed-form expression of  $g(a, b, c, r, q, f, h, k, j)$ . Here by making use of Gaussian-Chebyshev quadrature, we obtain

$$\begin{aligned} g(a, b, c, r, q, f, h, k, j) &= \left( \frac{a}{2} \right)^b \sum_{i=1}^N w_i s_i^{b-1} e^{-\frac{a f s_i}{2} - \frac{2h}{2-aq s_i}} \\ &\quad \times \left( 1 + \frac{acs_i}{2} \right)^k \left( 1 + \frac{2r}{2-aq s_i} \right)^j, \end{aligned} \quad (14)$$

where  $N$  is the number of terms,  $s_i = t_i + 1$ ,  $t_i$  is the  $i$ th zero of Legendre polynomials, and  $w_i$  is the Gaussian weight.

Substituting (9) and (11) into (12) and with some simple algebraic manipulations, we have

$$P_{\Phi_n}^{\text{TMRC}} = 1 - \frac{e^{-p_1}}{\lambda_E^n (n-1)!} \sum_{k=0}^{n-1} \sum_{j=0}^{n-1} \frac{b_1^k c_1^j \Xi_1}{\lambda_1^k \lambda_2^j k! j!}, \quad (15)$$

where  $\Xi_1 = g \left( a_1, n, \frac{\theta_1}{b_1}, \frac{\alpha_2}{c_1 d_1}, \frac{e_1}{d_1}, p_0, \frac{\alpha_2}{\lambda_2 d_1}, k, j \right)$ ,  $p_0 = \frac{\theta_1}{\lambda_1} + \frac{1}{\lambda_E}$ , and  $p_1 = \frac{b_1}{\lambda_1} + \frac{c_1}{\lambda_2}$ .

### B. Optimal Single Relay Selection Scheme

In this subsection, we propose OSRS scheme to minimize the overall SOP of the proposed cooperative NOMA system. One can easily obtain the SOP for  $U_1$  with  $m$ th relay as

$$P_{out,1} = 1 - \Pr \{ G_1^m > \delta_1(G_E^m) \} = \Pr \left\{ \frac{G_1^m}{\delta_3(G_E^m)} < 1 \right\}, \quad (16)$$

where  $\delta_3(x) = b_2 + \theta_1 x$ ,  $b_2 = \frac{\theta_1-1}{\alpha_1\rho_2}$ , and  $\rho_2 = \frac{P_R}{\sigma^2}$ .

Similarly, we can express SOP for  $U_2$  as

$$\begin{aligned} P_{out,2} &= \Pr \left\{ \frac{G_2^m}{\delta_4(G_E^m)} < 1 \mid G_E^m < a_2 \right\} \Pr \{ G_E^m < a_2 \} \\ &\quad + \Pr \{ G_E^m > a_2 \}, \end{aligned} \quad (17)$$

where  $a_2 = \frac{1-\theta_2\alpha_1}{\theta_2\alpha_1\alpha_2\rho_2}$ ,  $\delta_4(x) = c_2 + \frac{\alpha_2}{d_2-e_2x}$ ,  $c_2 = -\frac{1}{\alpha_1\rho_2}$ ,  $d_2 = \alpha_1\rho_2 - \theta_2\alpha_1^2\rho_2$ , and  $e_2 = \theta_2\alpha_1^2\alpha_2\rho_2^2$ . One can observe that when  $G_E^m > a_2$  the secrecy outage would always occur at  $U_2$ , which means the cooperative NOMA system is insecure.

For the  $m$ th relay, we define

$$X_m = \begin{cases} \min \left\{ \frac{G_1^m}{\delta_3(G_E^m)}, \frac{G_2^m}{\delta_4(G_E^m)} \right\}, & G_E^m < a_2 \\ 0, & G_E^m \geq a_2 \end{cases}. \quad (18)$$

Then, to maximize the secrecy performance, the relay is selected with the following criterion

$$m^* = \arg \max_{m \in \Phi} (X_m). \quad (19)$$

The SOP of the cooperative NOMA system conditioned on  $|\Phi| = n$  can be written as

$$\begin{aligned} P_{\Phi_n}^{\text{OSRS}} &= (\Pr \{ X_m < 1 \})^n \\ &= \left( 1 - \underbrace{\Pr \{ G_1^m \geq \delta_3(G_E^m), G_2^m \geq \delta_4(G_E^m), G_E^m < a_2 \}}_{\triangleq \Delta_1} \right)^n. \end{aligned} \quad (20)$$

Based on (1) and (2), we obtain

$$\begin{aligned} \Delta_1 &= \Pr \{ G_1^m \geq \delta_3(G_E^m), G_1^m \geq \delta_4(G_E^m), G_E^m < a_2 \} \\ &= \int_0^{a_2} (1 - F_{G_1^m}(\delta_3(x))) (1 - F_{G_2^m}(\delta_4(x))) f_{G_E^m}(x) dx \\ &= \frac{e^{-p_2} \Xi_2}{\lambda_E}, \end{aligned} \quad (21)$$

where  $\Xi_2 = g \left( a_2, 1, 0, 0, \frac{e_2}{d_2}, p_0, \frac{\alpha_2}{d_2\lambda_2}, 0, 0 \right)$  and  $p_2 = \frac{b_2}{\lambda_1} + \frac{c_2}{\lambda_2}$ .

### C. Two-Step Single Relay Selection Scheme

In some scenarios stated in [11], [12], the QoS requirements for the two users are different. We can easily obtain the similar conclusion: the secrecy QoS for one user is higher than that of the other user. In this subsection, a new two-step RS (TSRS) scheme is proposed.

The TSRS scheme is presented as follows. In the first step, the following subset is built in the relays by focusing on  $U_1$ 's target secrecy rate

$$\Psi = \{i : i \in \Phi, C_{s,1}^i > R_1^s\}, \quad (22)$$

where  $C_{s,1}^i = \frac{1}{2} \ln \left( \frac{1+\alpha_1 \rho_2 G_1^i}{1+\alpha_1 \rho_2 G_E^i} \right)$  signifies the secrecy capacity for  $U_1$  from  $R_i$ . At the second step, the relay to maximize the secrecy capacity of  $U_2$  is selected, i.e.,

$$j^* = \arg \max_{j \in \Psi} (C_{s,2}^j), \quad (23)$$

where  $C_{s,2}^j = \frac{1}{2} \ln \left( \frac{1+\alpha_2 \rho_2 G_2^j}{1+\alpha_2 \rho_2 G_E^j} \right)$  signifies the secrecy capacity for  $U_2$  from  $R_j$  ( $j \in \Psi$ ).

The SOP with this scheme can be achieved as

$$P_{\Phi_n}^{\text{TSRS}} = \sum_{j=1}^n C_n^j \Pr \left\{ \min_{1 \leq i \leq j} \{C_{s,1}^i\} > R_1^s, \right. \\ \left. \max_{1 \leq k \leq n-j} \{C_{s,1}^k\} < R_1^s, \max_{1 \leq p \leq j} \{C_{s,2}^p\} < R_2^s \right\} \quad (24) \\ = (\Pr \{C_{s,1}^i < R_1^s \text{ or } C_{s,1}^i < R_1^s\})^n \\ = (1 - \Pr \{C_{s,1}^i > R_1^s, C_{s,2}^i > R_2^s\})^n.$$

**Remark 1:** An interesting result can be observed from (24) that the expression for the SOP under TSRS scheme is the same as that under OSRS.

## IV. ASYMPTOTIC SECRECY OUTAGE PROBABILITY ANALYSIS

To obtain more insights, in this section we analyze the asymptotic SOP in the higher SINR regime. Similar to [14] and [15], we assume that  $\lambda_2 \rightarrow \infty$ ,  $\lambda_1 = \varepsilon_1 \lambda_2$  ( $\varepsilon_1 > 1$ ), and  $\lambda_R = \varepsilon_2 \lambda_2$ , where  $\varepsilon_1$  and  $\varepsilon_2$  are constant. The asymptotic CDF of  $G_v^k$  ( $v \in \{1, 2\}$ ) can be expressed as [24]

$$F_{G_v^k}^\infty(x) = \varphi_v x^n + \mathcal{O}(x^n), \quad (25)$$

where  $\varphi_1 = \frac{1}{(\varepsilon_1 \lambda_2)^n n!}$ ,  $\varphi_2 = \frac{1}{\lambda_2^n n!}$ , and  $\mathcal{O}(\cdot)$  denotes higher order terms. Then the asymptotic SOP of cooperative NOMA systems is obtained as

$$P_{\text{out}}^\infty = \sum_{n=0}^K C_K^n \varphi_R^{K-n} \eta^{(K-n)} P_{\Phi_n}^\infty, \quad (26)$$

where  $\varphi_R = \frac{1}{\varepsilon_2 \lambda_2}$ . The  $P_{\Phi_n}^\infty$  for all the RS schemes is given as follows.

### A. Traditional Multiple Relays Combining Scheme

Based on (12), we have the closed-form for SOP with TMRC scheme as

$$P_{\Phi_n}^{\text{TMRC}, \infty} \\ = 1 - F_{G_E}(a_1) \\ + \frac{\varphi_1}{\lambda_E^n (n-1)!} \sum_{i=0}^n C_n^i b_1^{n-i} \theta_1^i \lambda_E^{n+i} \Upsilon \left( n+i, \frac{a_1}{\lambda_E} \right) \\ + \frac{\varphi_2 c_1^n}{\lambda_E^n (n-1)!} g \left( a_1, n, 0, \frac{\alpha_2}{c_1 d_1}, \frac{e_1}{d_1}, \frac{1}{\lambda_E}, 0, 0, n \right) \\ - \frac{\varphi_1 \varphi_2 b_1^n c_1^n}{\lambda_E^n (n-1)!} g \left( a_1, n, \frac{\theta_1}{b_1}, \frac{\alpha_2}{c_1 d_1}, \frac{e_1}{d_1}, \frac{1}{\lambda_E}, 0, n, n \right), \quad (27)$$

where  $\Upsilon(\cdot, \cdot)$  is the lower incomplete Gamma function.

### B. Optimal Single Relay Selection Scheme

Based on (20), we obtain

$$P_{\Phi_n}^{\text{OSRS}, \infty} = (1 - \Delta_1^\infty)^n, \quad (28)$$

where  $\Delta_1^\infty$  is given as

$$\Delta_1^\infty \\ = \int_0^{a_2} \left( 1 - F_{G_1^\infty}^\infty(\delta_3(x)) \right) \left( 1 - F_{G_2^\infty}^\infty(\delta_4(x)) \right) f_{G_E^m}(x) dx \\ = 1 - e^{-\frac{a_2}{\lambda_E}} - \varphi_3 b_2 + \varphi_3 b_2 e^{a_2} - \theta_1 \lambda_E \varphi_3 \Upsilon \left( 2, \frac{a_2}{\lambda_E} \right) \\ - \frac{\varphi_4 c_2}{\lambda_E} g \left( a_2, 1, 0, \frac{\alpha_2}{c_2 d_2}, \frac{e_2}{d_2}, \frac{1}{\lambda_E}, 0, 0, 1 \right) \\ + \frac{\varphi_3 \varphi_4 b_2 c_2}{\lambda_E} g \left( a_2, 1, \frac{\theta_1}{b_2}, \frac{\alpha_2}{c_2 d_2}, \frac{e_2}{d_2}, \frac{1}{\lambda_E}, 0, 1, 1 \right), \quad (29)$$

where  $\varphi_3 = \frac{1}{\varepsilon_1 \lambda_2}$  and  $\varphi_4 = \frac{1}{\lambda_2}$ .

## V. ASYMPTOTIC SECRECY OUTAGE PROBABILITY ANALYSIS WITH DYNAMIC POWER ALLOCATION SCHEME

Since there is a ceiling for the SINR of  $U_2$ , which equals to  $\frac{\alpha_2}{\alpha_1}$ , there is a floor for the SOP of NOMA systems when the transmit SNR increases. A new DPA scheme was proposed in [14] and the PA coefficients are given as

$$\begin{cases} \alpha_1^{\text{DPA}} = \frac{1}{1+\mu\lambda_2^\varpi} \\ \alpha_2^{\text{DPA}} = \frac{\mu\lambda_2^\varpi}{1+\mu\lambda_2^\varpi}, \end{cases} \quad (30)$$

where  $0 < \varpi < 1$  and  $\mu > 1$ . It was testified that non-zero diversity order can be obtained due to  $\frac{\alpha_2^{\text{DPA}}}{\alpha_1^{\text{DPA}}} = \mu\lambda_2^\varpi \rightarrow 0$  with  $\Omega_2 \rightarrow \infty$  in [14] and [15]. In this section, the asymptotic SOP is analyzed while this DPA scheme is utilized. Based on (30), we have  $a_i^{\text{DPA}} = \frac{(1+\mu\lambda_2^\varpi)^2 - \theta_2 \mu \lambda_2^\varpi - \theta_2}{\theta_2 \mu \lambda_2^\varpi \rho_i}$ ,  $b_i^{\text{DPA}} = \frac{\theta_1 - 1}{\rho_i} (1 + \mu \lambda_2^\varpi)$ ,  $c_i^{\text{DPA}} = -\frac{1+\mu\lambda_2^\varpi}{\rho_i}$ ,  $d_i^{\text{DPA}} = \frac{(1+\mu\lambda_2^\varpi - \theta_2)\rho_i}{(1+\mu\lambda_2^\varpi)^2}$ ,  $e_i^{\text{DPA}} = \frac{\theta_2 \mu \lambda_2^\varpi \rho_i^2}{(1+\mu\lambda_2^\varpi)^3}$ , and  $i = 1, 2$ .

### A. Traditional Multiple Relays Combining Scheme with Dynamic Power Allocation Scheme

Substituting (30) into (27), we achieve the closed-form expression for asymptotic SOP under TMRC-DPA scheme. The expression of  $P_{\Phi_n}^{\text{TMRC},\infty,\text{DPA}}$  can be given by

$$\begin{aligned} P_{\Phi_n}^{\text{TMRC},\infty,\text{DPA}} &= 1 - F_{G_E}(a_1^{\text{DPA}}) \\ &+ \frac{\varphi_1}{\lambda_E^n (n-1)!} \sum_{i=0}^n C_n^i (b_1^{\text{DPA}})^{n-i} \theta_1^i \lambda_E^{n+i} \Upsilon\left(n+i, \frac{a_1^{\text{DPA}}}{\lambda_E}\right) \\ &+ \tau_1 g\left(a_1^{\text{DPA}}, n, 0, \frac{\alpha_2}{c_1^{\text{DPA}} d_1^{\text{DPA}}}, \frac{e_1^{\text{DPA}}}{d_1^{\text{DPA}}}, \frac{1}{\lambda_E}, 0, 0, n\right) \\ &- \tau_2 g\left(a_1^{\text{DPA}}, n, \frac{\theta_1}{b_1^{\text{DPA}}}, \frac{\alpha_2}{c_1^{\text{DPA}} d_1^{\text{DPA}}}, \frac{e_1^{\text{DPA}}}{d_1^{\text{DPA}}}, \frac{1}{\lambda_E}, 0, n, n\right), \end{aligned} \quad (31)$$

where  $\tau_1 = \frac{\varphi_2 (c_1^{\text{DPA}})^n}{\lambda_E^n (n-1)!}$  and  $\tau_2 = \frac{\varphi_1 \varphi_2 (b_1^{\text{DPA}})^n (c_1^{\text{DPA}})^n}{\lambda_E^n (n-1)!}$ .

### B. Optimal Single Relay Selection Scheme with Dynamic Power Allocation Scheme

Similar to (28), we have

$$P_{\Phi_n}^{\text{OSRS},\infty,\text{DPA}} = \left(1 - \Delta_1^{\infty,\text{DPA}}\right)^n. \quad (32)$$

By substituting (30) into (29),  $\Delta_1^{\infty,\text{DPA}}$  can be given as

$$\begin{aligned} \Delta_1^{\infty,\text{DPA}} &= 1 - e^{-\frac{a_2^{\text{DPA}}}{\lambda_E}} \\ &- \varphi_3 b_2^{\text{DPA}} + \varphi_3 b_2^{\text{DPA}} e^{a_2^{\text{DPA}}} - \theta_1 \lambda_E \varphi_3 \Upsilon\left(2, \frac{a_2^{\text{DPA}}}{\lambda_E}\right) \\ &- \tau_3 g\left(a_2^{\text{DPA}}, 1, 0, \frac{\alpha_2}{c_2^{\text{DPA}} d_2^{\text{DPA}}}, \frac{e_2^{\text{DPA}}}{d_2^{\text{DPA}}}, \frac{1}{\lambda_E}, 0, 0, 1\right) \\ &+ \tau_4 g\left(a_2^{\text{DPA}}, 1, \frac{\theta_1}{b_2^{\text{DPA}}}, \frac{\alpha_2}{c_2^{\text{DPA}} d_2^{\text{DPA}}}, \frac{e_2^{\text{DPA}}}{d_2^{\text{DPA}}}, \frac{1}{\lambda_E}, 0, 1, 1\right), \end{aligned} \quad (33)$$

where  $\tau_3 = \frac{\varphi_4 c_2^{\text{DPA}}}{\lambda_E}$  and  $\tau_4 = \frac{\varphi_3 \varphi_4 b_2^{\text{DPA}} c_2^{\text{DPA}}}{\lambda_E}$ .

## VI. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present numerical results and Monte-Carlo simulations to testify our analysis. The main adopted parameters are set to  $P_S = P_R = P$ ,  $R_1^{\text{th}} = 0.2$  nat per channel use,  $R_2^{\text{th}} = 0.1$  nat per channel use, and  $\sigma^2 = 1$ . In all the figures, ‘Sim’ denotes the simulation results<sup>1</sup>. One can observe that the analysis results match perfectly with simulation results and the secrecy performance with OSRS always outperforms that of TMRC.

One can easily observe from Figs. 2 - 3 that the secrecy performance of cooperative NOMA systems is enhanced and then becomes worse by improving the transmit SNR, which is similar to the conclusions in [14] and [15]. This is because the SINR of  $U_2$  has a ceiling in the higher- $\rho$  region. It is demonstrated in Fig. 2 that the SOP is enhanced by increasing  $K$  due to the improved diversity of the cooperative NOMA systems. From Fig. 3 we can observe that  $R_2^s$  has significant

<sup>1</sup>We omit the results for TSRS scheme since it is exactly the same as that for OSRS.

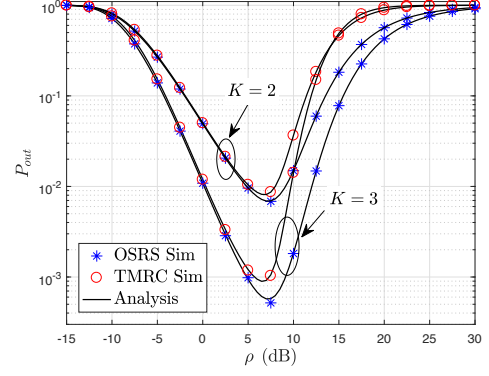


Fig. 2. SOP for various  $K$  with  $R_1 = 0.1$ ,  $R_2 = 0.2$ ,  $\lambda_1 = 12$  dB,  $\lambda_R = \lambda_2 = 10$  dB, and  $\lambda_E = -10$  dB.

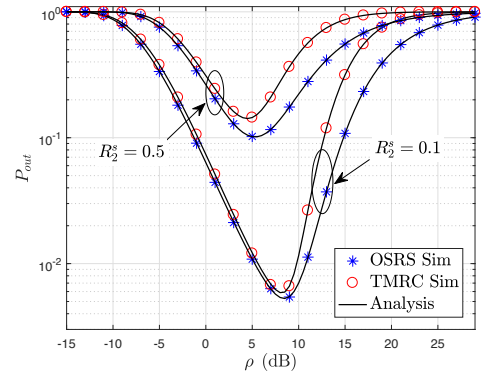


Fig. 3. SOP for various  $R_2^s$  with  $R_1^s = 0.2$ ,  $K = 2$ ,  $\alpha_1 = 0.2$ ,  $\lambda_1 = 12$  dB,  $\lambda_R = \lambda_2 = 10$  dB, and  $\lambda_E = -10$  dB.

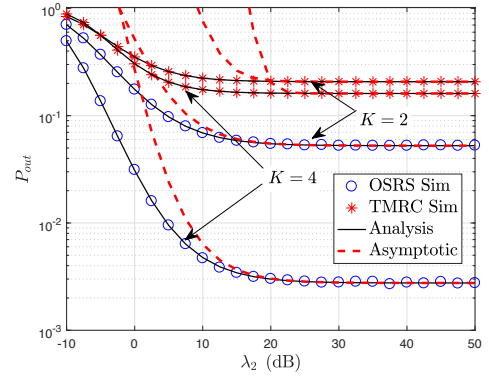


Fig. 4. SOP for various  $K$  with  $R_1 = 0.1$ ,  $R_2 = 0.2$ ,  $P = 15$  dB,  $\alpha_1 = 0.2$ ,  $\varepsilon_1 = 1.5$ ,  $\varepsilon_2 = 2$ , and  $\lambda_E = -12$  dB.

effect on the SOP of NOMA systems in high- $\rho$  region. This proves that in high- $\rho$  region, weaker user must be given more attention.

The SOP for various  $\lambda_2$  with FPA is presented in Figs. 4 - 5. The results prove that increasing  $K$  and  $l$  or decreasing  $\alpha_1$  enhances the SOP of cooperative NOMA systems. One can also observe that the asymptotic SOP approaches the exact ones at high- $\lambda_2$  region.

The impact of PA parameter on the SOP is investigated in Fig. 6. One can observe that DPA scheme achieves a non-zero secrecy diversity order, which depends on the number of relays,

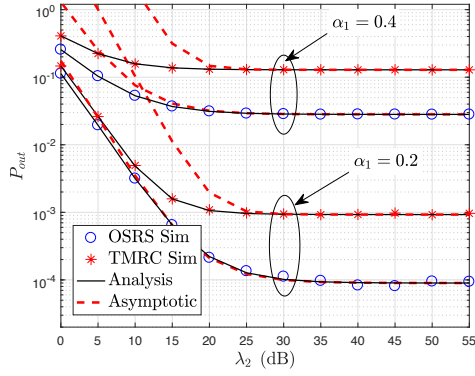


Fig. 5. SOP for various  $\alpha_1$  with  $R_1 = 0.1$ ,  $R_2 = 0.2$ ,  $K = 2$ ,  $P = 10$  dB,  $\varepsilon_1 = 1.5$ ,  $\varepsilon_2 = 2$ , and  $\lambda_E = -12$  dB.

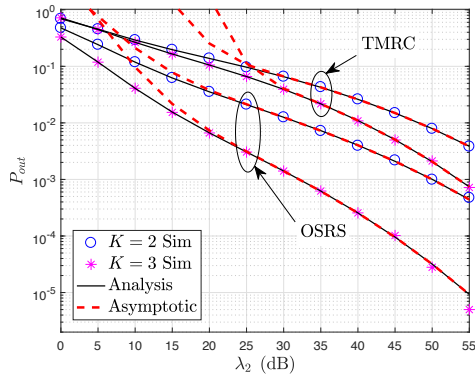


Fig. 6. SOP with DPA scheme for various  $K$  with  $R_1 = 0.1$ ,  $R_2 = 0.2$ ,  $P = 10$  dB,  $\varepsilon_1 = 1.5$ ,  $\varepsilon_2 = 2$ ,  $\varpi = 0.1$ ,  $\mu = 5$ , and  $\lambda_E = -5$  dB.

the fading parameters of  $S - R_k$  and  $S_k - U_i$ . Thus in the high-SNR regime, the cooperative NOMA system with DPA can achieve better security performance compared to FPA scheme.

## VII. CONCLUSION

In this work, two RS schemes were proposed to enhance the secrecy performance of cooperative NOMA systems. We investigated the secrecy outage performance of these schemes considering the presence of dependence between the two NOMA users with different secrecy rate thresholds. The closed-form expressions for the exact and asymptotic SOP were obtained and verified by Monte Carlo simulation results. The results demonstrate that relative to TMRC scheme, OSRS and TSRS schemes enhance the security of the cooperative NOMA systems. In the high-SNR regime, a more secure system can be realized via DPA scheme, which achieves non-zero SDO.

## ACKNOWLEDGEMENTS

This work was supported by the Project of Fundamental Science and Frontier Technology Research Plan of Chongqing under Grant cstc2017jcyjAX0204 and cstc2015jcyjBX0085.

## REFERENCES

[1] L. Dai, B. Wang, Y. Yuan, S. Han, C.-L. I, and Z. Wang, "Non-orthogonal multiple access for 5G: solutions, challenges, opportunities, and future research trends," *IEEE Commun. Mag.*, vol. 53, no. 9, pp. 74-81, Sept. 2015.

[2] Y. Wang, B. Ren, S. Sun, S. Kang and X. Yue, "Analysis of non-orthogonal multiple access for 5G," *China Commun.*, vol. 13, no. Supplement2, pp. 52-66, N/A 2016.

[3] Z. Ding, Y. Liu, J. Choi, Q. Sun, M. Elkashlan, I. C. L, and H. V. Poor, "Application of non-orthogonal multiple access in LTE and 5G networks," *IEEE Commun. Mag.*, vol. 55, no. 2, pp. 185-191, Feb. 2017.

[4] Z. Ding, M. Peng, and H. V. Poor, "Cooperative non-orthogonal multiple access in 5G systems," *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1462-1465, Aug. 2015.

[5] Y. Liu, Z. Ding, M. Elkashlan, and H. V. Poor, "Cooperative non-orthogonal multiple access with simultaneous wireless information and power transfer," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 4, pp. 938-953, Apr. 2016.

[6] J. Men and J. Ge, "Non-orthogonal multiple access for multiple-antenna relaying networks," *IEEE Commun. Lett.*, vol. 19, no. 10, pp. 1686-1689, Oct. 2015.

[7] J. Men, J. Ge, and C. Zhang, "Performance analysis of non-orthogonal multiple access for relaying networks over Nakagami- $m$  fading channels," *IEEE Trans. Veh. Technol.*, vol. 66, no. 2, pp. 1200-1208, Feb. 2017.

[8] Y. Xiao, L. Hao, Z. Ma, Z. Ding, Z. Zhang, and P. Fan, "Forwarding strategy selection in dual-hop NOMA relaying systems," *IEEE Commun. Lett.*, vol. 22, no. 8, pp. 1644-1647, Aug. 2018.

[9] Y. Zou, X. Wang, and W. Shen, "Optimal relay selection for physical-layer security in cooperative wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 10, pp. 2099-2111, Oct. 2013.

[10] H. Lei, H. Zhang, I. S. Ansari, Z. Ren, G. Pan, K. A. Qaraqe, and M.-S. Alouini, "On secrecy outage of relay selection in underlay cognitive radio networks over Nakagami- $m$  fading channel," *IEEE Trans. Cogn. Commun. Netw.*, vol. 3, no. 4, pp. 614 - 627, Dec. 2017.

[11] Z. Ding, H. Dai, and H. V. Poor, "Relay selection for cooperative NOMA," *IEEE Wireless Commun. Lett.*, vol. 5, no. 4, pp. 416-419, Aug. 2016.

[12] J. Zhao, Z. Ding, P. Fan, Z. Yang, and G. K. Karagiannidis, "Dual relay selection for cooperative NOMA with distributed space time coding," *IEEE Access*, vol. 6, pp. 20440-20450, Apr. 2018.

[13] Y. Liu, Z. Qin, M. Elkashlan, Y. Gao, and L. Hanzo, "Enhancing the physical layer security of non-orthogonal multiple access in large-scale networks," *IEEE Trans. Wireless Commun.*, vol. 16, no. 3, pp. 1656-1672, Mar. 2017.

[14] H. Lei, J. Zhang, K. Park, P. Xu, I. S. Ansari, G. Pan, B. Alomair, and M.-S. Alouini, "On secure NOMA systems with transmit antenna selection schemes," *IEEE Access*, vol. 5, pp. 17450-17464, Sep. 2017.

[15] H. Lei, J. Zhang, K.-H. Park, P. Xu, Z. Zhang, G. Pan, and M.-S. Alouini, "Secrecy outage of max-min TAS scheme in MIMO-NOMA systems," *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 6981-6990, Aug. 2018.

[16] L. Lv, Z. Ding, Q. Ni, and J. Chen, "Secure MISO-NOMA transmission with artificial noise," *IEEE Trans. Veh. Technol.*, vol. 67, no. 7, pp. 6700-6705, Jul. 2018.

[17] F. Zhou, Z. Chu, H. Sun, R. Q. Hu, and L. Hanzo, "Artificial noise aided secure cognitive beamforming for cooperative MISO-NOMA using SWIPT," *IEEE J. Sel. Areas Commun.*, vol. 36, no. 4, pp. 918-931, Apr. 2018.

[18] N. Zhao, W. Wang, J. Wang, Y. Chen, Y. Lin, Z. Ding, and N. C. Beaulieu, "Joint beamforming and jamming optimization for secure transmission in MISO-NOMA networks," *IEEE Trans. Commun.*, doi: 10.1109/TCOMM.2018.2883079, 2018.

[19] Z. Zhu, Z. Chu, F. Zhou, H. Niu, Z. Wang, and I. Lee, "Secure beamforming designs for secrecy MIMO SWIPT systems," *IEEE Wireless Commun. Lett.*, vol. 7, no. 3, pp. 424-427, Jun. 2018.

[20] K. Jiang, T. Jing, Y. Huo, F. Zhang, and Z. Li, "SIC-based secrecy performance in uplink NOMA multi-eavesdropper wiretap channels," *IEEE Access*, vol. 6, pp. 19664-19680, Apr. 2018.

[21] H. Zhang, N. Yang, K. Long, M. Pan, G. K. Karagiannidis, and V. C. M. Leung, "Secure communications in NOMA system: Subcarrier assignment and power allocation," *IEEE J. Sel. Areas Commun.*, vol. 36, no. 7, pp. 1441-1452, Jul. 2018.

[22] J. Chen, L. Yang, and M. Alouini, "Physical layer security for cooperative NOMA systems," *IEEE Trans. Veh. Technol.*, vol. 67, no. 5, pp. 4645-4649, May. 2018.

[23] P. Xu, Z. Yang, Z. Ding, and Z. Zhang, "Optimal relay selection schemes for cooperative NOMA," *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7851-7855, Aug. 2018.

[24] H. Lei, C. Gao, I. S. Ansari, Y. Guo, Y. Zou, G. Pan, and K. Qaraqe, "Secrecy outage performance of transmit antenna selection for MIMO underlay cognitive radio systems over Nakagami- $m$  channels," *IEEE Trans. Veh. Technol.*, vol. 66, no. 3, pp. 2237-2250, Mar. 2017.