Quantum Processing of MR Spectrum

MRS denoising using Semi-Classical Signal Analysis with Soft-Thresholding

A. Chahid[1], S. Bhaduri[2], M. Wali[1], H. Serrai[2,3], R. Achten[2] and T.M. Laleg-Kirati[1]

[1] *King Abdullah University of Science and Technology (KAUST), Thuwal, KSA.* [2] *University of Ghent, Gent, Belgium.*

[3] *University of Western Ontario,London, ON, Canada.* *Email:*`taousmeriem.laleg@kaust.edu.sa`

Abstract

A Semi-Classical Signal Analysis (SCSA) method with soft thresholding is proposed for MRSI denoising. The SCSA takes advantage of the pulseshaped MRS spectrum to decompose both real and imaginary parts, into localized basis given by squared eigenfunctions of the Schrödinger operator. An optimization-based soft-threshold is provided to find optimal semi-classical parameters, for both the real and imaginary parts of the MRS signal. The optimal SCSA parameters discard the eigenfunctions representing noise from the noisy spectrum, and conserve the eigenfunctions representing the useful information. The obtained in-vivo results show the efficiency of the SCSA with soft thresholding in removing noise and conserving metabolite signals.

Introduction

The Semi-Classical Signal Analysis method (SCSA) has been presented for the first time for signal decomposition and representation in [1] where the authors have proven that for a real positive input signal $y(t)$ and a given h, the output signal $y_h(t)$ can be approximated as follows:

$$y_h(t) = 4h \sum_{k=1}^{N_h} \sqrt{(-\mu_{k,h})}\ \psi^2_{k,h}(t), \tag{1}$$

where $h \in \mathbb{R}^*_+$, $\mu_{k,h}$ and $\psi_{k,h}(t)$, for $k = 1, \cdots, N_h < N$, refer to the negative eigenvalues and associated L_2-normalized eigenfunctions respectively of the Schrödinger operator $H(y)$ defined as follows:

$$\text{The Schrödinger operator}: \quad H(y) = -h^2 \frac{d^2}{df^2} - y(f)$$
$$\text{The Eigenvalues Problem}: \quad H(y)\psi(f) = \mu\,.\psi(f) \tag{2}$$

The SCSA can be used for signal denoising with an appropriate choice of the value of the parameter h that provides the best noise removal.

SCSA for complex MRS denoising

The noisy MRS $z(f)$ spectrum is first split into the real part $z_r(f)$ and imaginary part $z_i(f)$. Each of these to signals is considered as potential of a Schrödinger operator of the eigenvalue problem defined as follows:

$$-h_l^2 \frac{d^2}{df^2} - z_l(f)\ \psi(f) = \mu\ \psi(f) \tag{3}$$

where $h_l > 0$, $l = \{r, i\}$ defines the real part $z_r(f)$ and the imaginary part $z_i(f)$ respectively.

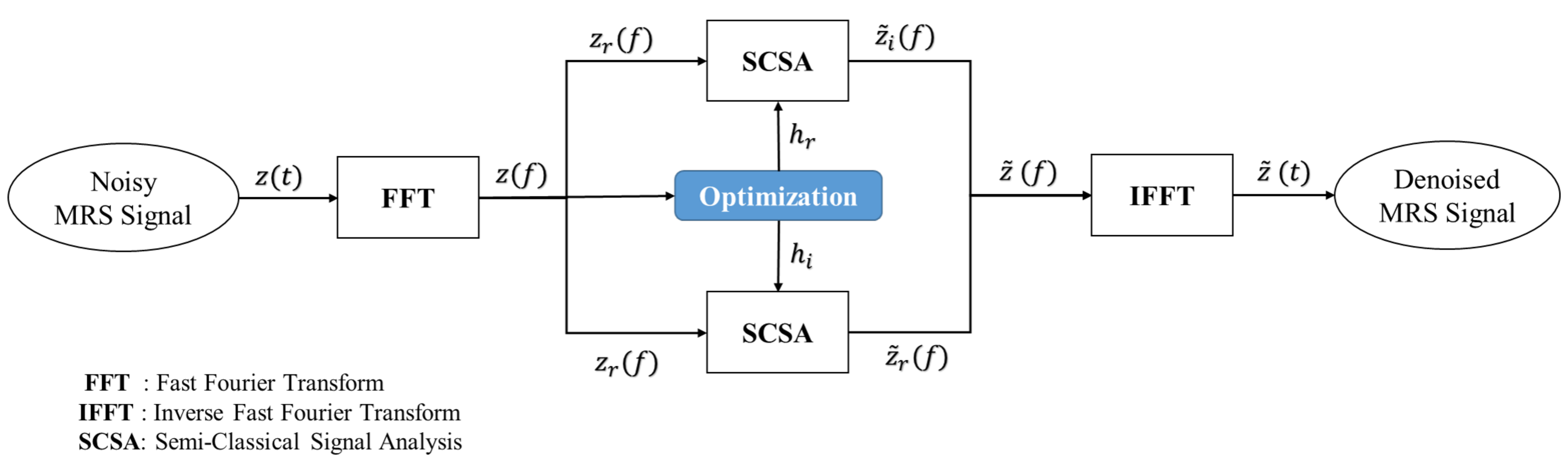

Figure 1: Complex MRS signal denoising using SCSA

The denoised MRS signal $z_h(f)$, composed of the denoised real part $z_{r,h}(f)$ and the imaginary part $z_{i,h}(f)$, is given by:

$$z_h(f) = z_{r,h}(f) + \mathbf{i}\ z_{i,h}(f) \tag{4}$$

such that :

$$z_{r,h}(f) = 4r, h \sum_{k=1}^{N_{r,h}} \sqrt{(-\mu_{r,kh})}\ \psi^2_{r,kh}(f) \quad \text{and} \quad z_{i,h}(f) = 4i, h \sum_{k=1}^{N_{i,h}} \sqrt{(-\mu_{i,kh})}\ \psi^2_{i,kh}(f) \tag{5}$$

where h_r and h_i are the optimal semi-classical parameters applied to the real part $z_r(f)$ and the imaginary part $z_i(f)$ of the MRS signal respectively. μ and ψ refer to the negative eigenvalues and associated L^2- eigenfunctions of the Schrödinger operator respectively.

Soft-Thresholding

The objective of this work is to find optimal values for the parameters h_r and h_i for efficient denoising. An optimization-based thresholding is used , defined in equation (6), based on three steps:

- Detect the location of the positive and negative peaks,
- Split the signal into a set of smaller signals such that each has one or two peaks (see Figure 2),
- Denoise each sub-signal alone by minimizing the cost function $J(h)$ (see equation (6)).

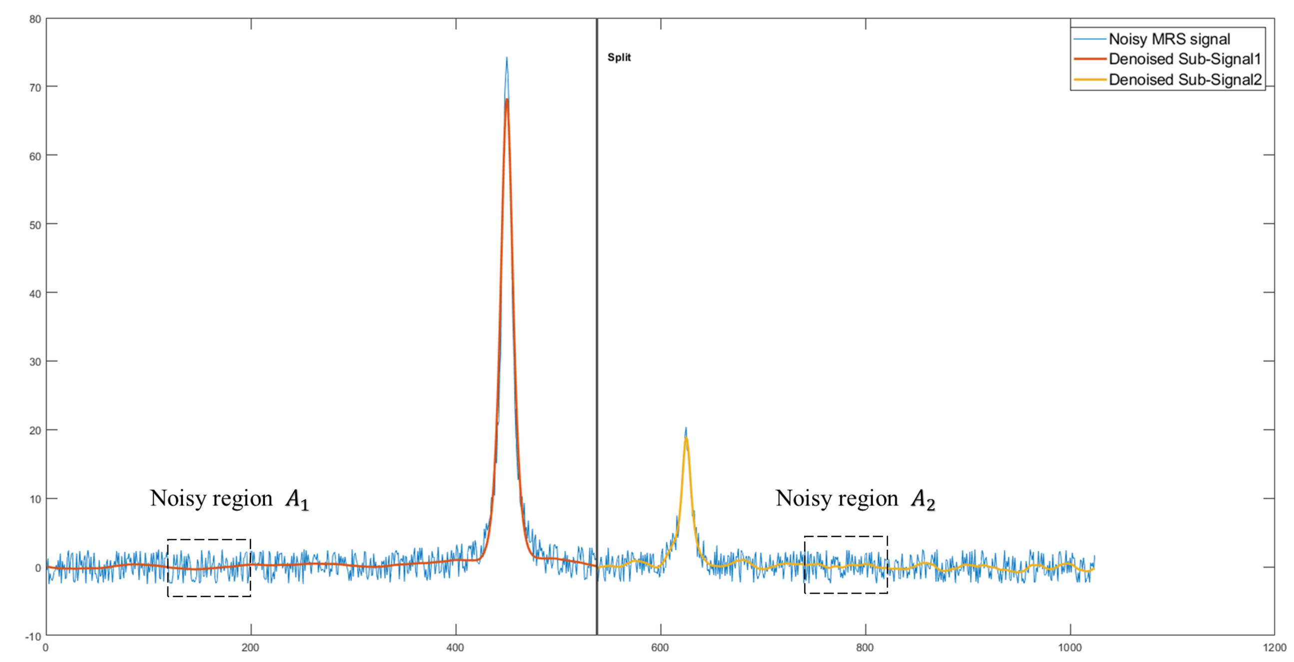

Figure 2: Example of signal splitting and denoising

$$J(h) = \begin{cases} \dfrac{STD(\frac{d^2 z_{l,h}(f_A)}{df^2})}{STD(\frac{d^2 z_l(f_A)}{df^2})} & \text{if} \quad \rho < C \\ \infty & \text{elsewhere} \end{cases} \tag{6}$$

where $l = \{r, i\}$. $f_A \in A$, where A is an interval that describe the noisy zone. The STD refers to the STandard Deviation. C is the **maximum threshold** allowed for peak attenuation defined as:

$$C = \frac{\|max(z_{l,h}) - max(z_l)\|}{\|max(z_l)\|} \tag{7}$$

Results and Discussion

The proposed SCSA efficiently reduces noise while preserving the metabolite peaks.

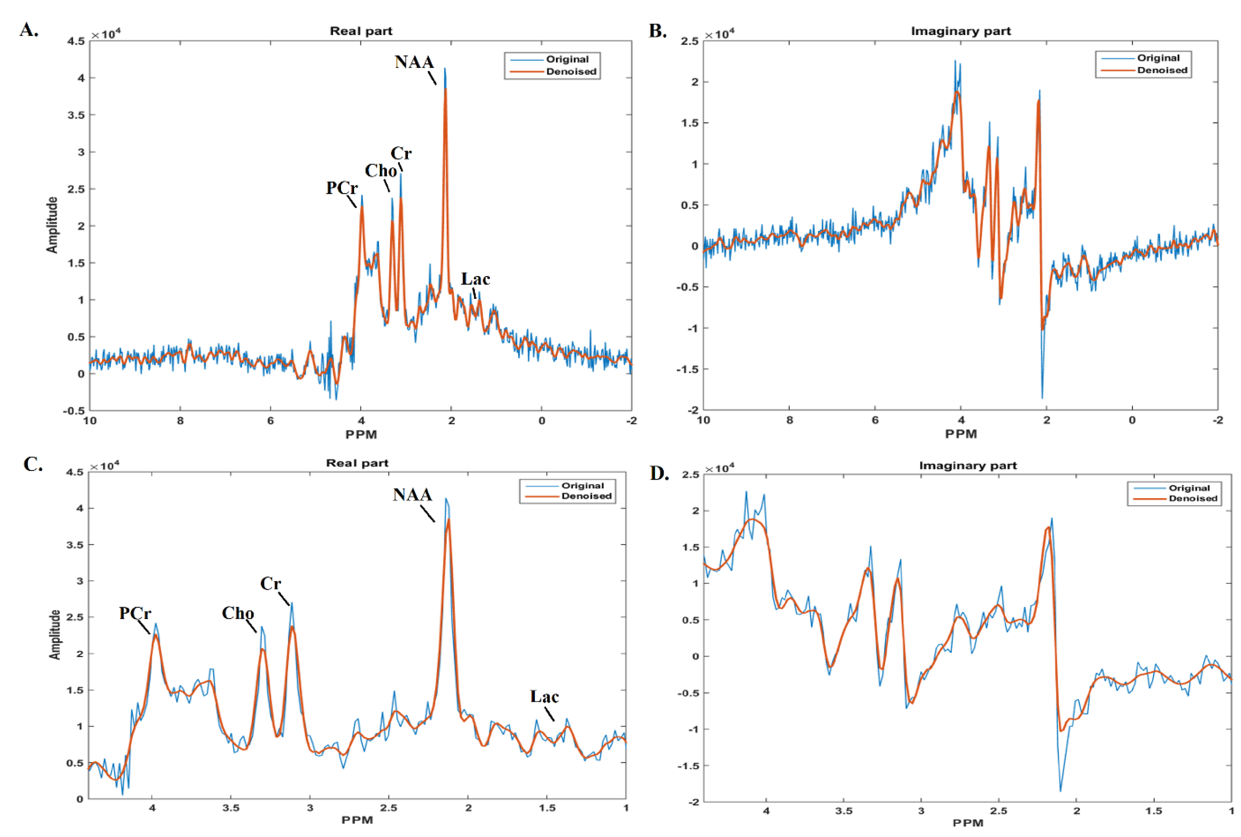

Figure 3: Result with *in vivo* data ($N_{ex} = 4$) comparing real (A. and metabolite region zoomed in C.) and imaginary parts (B. and metabolite region zoomed in D.) of original (blue) with the denoised spectra (red)

The quantification results in terms of SNR are shown in Table 1.

	SNR		NAA/Cr		Cho/Cr		Lac/Cr	
	Before	*After*	*Before*	*After*	*Before*	*After*	*Before*	*After*
Nex=2	7.1	14.9	1.55	1.78	0.92	0.89	0.21	0.18
Nex=4	11.3	24.3	1.53	1.62	0.88	0.87	0.21	0.24
Nex=8	15.5	55.8	1.61	1.61	0.86	0.85	0.22	0.22
Nex=16	20.3	65.5	1.61	1.61	0.86	0.86	0.22	0.22

Table 1: *In vivo* results showing the SNR values and metabolites ratios before and after SCSA denoising

Conclucion

The new SCSA method with adaptive thresholding achieves efficient MRS signal denoising while preserving the metabolite peaks, demonstrated by the efficient noise reduction and metabolites preservation. The obtained results are very encouraging and show the potential of the SCSA with thresholding for general pulse-shaped signal denoising.

References

[1] Taous-Meriem Laleg-Kirati, Emmanuelle Crépeau, and Michel Sorine, "Semi-classical signal analysis," *Mathematics of Control, Signals, and Systems*, vol. 25, no. 1, pp. 37–61, 2013.