Optimal full-waveform inversion strategy for marine data in azimuthally rotated elastic orthorhombic media

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ABSTRACT

The orthorhombic (ORT) anisotropic description of earth layers can allow the capture of much of the earth’s anisotropic complexity. The inversion for high-resolution azimuthal variation of anisotropy is important for reservoir characterization, among other applications. A high-resolution description of the azimuth of fractures can help us to predict flow preferences. To verify the feasibility of multiparameter full-waveform inversion (FWI) for marine data assuming azimuthally rotated elastic ORT media, we have analyzed the radiation patterns and gradient directions of ORT parameters to the reflection data. First, we express the gradient direction of the ORT parameters considering the azimuthal rotation of the symmetric planes. Then, to support our observations in the gradient direction, the radiation patterns of the partial derivative wavefields from each parameter perturbation are also derived under the rotated elastic ORT assumption. To find an optimal parameterization, we compare three different parameterizations: monoclinic, velocity-based, and hierarchical parameterizations. Then, we suggest an optimal multistage update strategy by analyzing the behavior of the rotation angle as a FWI target. To analyze the trade-off among parameters in different parameterizations, we calculate the gradient direction from a hockey-puck model, in which each parameter is perturbed at the different location on a horizontal layer. The trade-off analysis supports that the hierarchical parameterization provides us with more opportunities to build up subsurface models with less trade-off between parameters and less influence of the azimuthal rotation of ORT anisotropy. The feasibility of the proposed FWI strategy is examined using synthetic marine streamer data from a simple 3D reservoir model with a fractured layer.

INTRODUCTION

Full-waveform inversion (FWI) is a seismic interpretation technique that estimates subsurface physical properties from the seismic data through mathematical optimization procedures without drilling the earth (Vireiux and Operto, 2009). The physical properties can be wave velocities, anisotropic parameters, viscosity, and so on depending on the FWI purpose. Considering such opportunities, the application of multiparameter FWI in exploration seismology has gained momentum over the past decade.

In multiparameter FWI, one important issue is finding an optimal parameterization to represent the model (Operto et al., 2013). Many geophysicists, using various physical assumptions, have carried out parameterization analysis over the years. The multiparameter FWI analysis has been presented for the isotropic, variable density, acoustic case (Operto et al., 2013; Bai and Yingst, 2014; Ma et al., 2016; Oh and Alkhalifah, 2018), and elastic earth (Tarantola, 1986; Brossier et al., 2009; Köhn et al., 2012, 2015; Prieux et al., 2013; Raknes and Weibull, 2016; Oh et al., 2018). Considering that anisotropy is prevalent inside the earth, multiparameter FWI for anisotropic media has gotten plenty of attention as an ingredient to help convergence of FWI. The vertical transverse isotropic (VTI) (Tsvankin, 2012) model is the simplest anisotropic model, which includes the intrinsic anisotropy within sedimentary layers and the apparent anisotropy caused by thin horizontal layering (Backus, 1962). The optimal parameterization for acoustic VTI media (Gholami et al., 2013; Alkhalifah and Plessix, 2014; He and Plessix, 2016; Silva et al., 2016) and elastic VTI media (Oh et al., 2015;
Kamath and Tsvankin, 2016) has been studied. Another simple anisotropic model is the horizontal transverse isotropic (HTI) model, which includes vertically aligned fracture networks (Tsvankin, 2012). Pan et al. (2016) examine the Gauss-Newton approach to invert for HTI parameters.

To take the VTI and HTI models into account, we describe the earth using an orthorhombic (ORT) anisotropic model (Tsvankin, 1997; Stovas, 2015). Although the study on ORT FWI has been rarely conducted due to the high computational cost of such an endeavor, recent developments in supercomputing allow us to work on FWI for this complex anisotropic model (Albertin et al., 2016; Masmoudi and Alkhalifah, 2016; Oh and Alkhalifah, 2016a, 2016b; Wang and Tsvankin, 2016; Xie et al., 2017).

However, in such an ORT model, we have often ignored the role that the symmetry azimuth direction plays in such an inversion. The symmetry azimuth holds crucial information on tectonic (stress) behavior of the region and more importantly fracture direction information in a potential reservoir. Conventional approaches to estimate fracture direction (Bakulin et al., 2000) rely on amplitude versus offset and azimuth inversion (Downton and Roure, 2010; Yin et al., 2013). Such approaches have a 1D preference and fail in more complex media, where lateral inhomogeneity is prevalent. With FWI, we can handle more complex media, and thus, we have an opportunity to invert for ORT azimuth in such a medium. However, most inversion approaches rely on a monoclinic (Rusmanugroho et al., 2017) or a triclinic model (Köhn et al., 2015). Because monoclinic media have more independent parameters to be inverted compared with rotated anisotropic media, a rotated ORT model is more practical if the rotation angle information is available.

In this study, we examine the possibility of inverting for the rotation angle of the symmetric axis (i.e., fracture direction) through the FWI procedure. To do that, we first formulate the azimuthally rotated elastic ORT FWI. Then, to find an optimal strategy to invert for the rotation angle, we find the most optimal parameterization and an optimal update strategy by comparing three different parameterizations: monoclinic, velocity-based, and hierarchical parameterizations: monoclinic, velocity-based, and hierarchical parameterizations conducted assuming a simple hockey-puck model. Finally, the proposed multistage strategy is examined by the marine streamer data from a synthetic 3D model with a fractured reservoir.

**INVERSE THEORY**

**Inverse problem in rotated elastic orthorhombic media**

The goal of FWI is finding the global minimum solution to minimize the data misfits between observed data and modeled data $u$.

$$E(m) = \sum_s \sum_r |u(s, r, m) - d(s, r)|^2,$$  

where $s$ and $r$ indicate the source and receiver indices, respectively. Depending on the physical model assumed in the simulation (ignoring viscosity), the model parameter vector $m$ could include from 1 (monoparameter acoustic media) to 22 parameters (triclinic elastic media) including the density. As we consider a more complex physical model, we will need more parameters to describe the physical phenomena.

The elastic wave equation for a given seismic source $f_i$ is expressed here in terms of particle displacements $u_i$, stress $\sigma_{ij}$, and strain $\varepsilon_{kl}$ as follows (Tsvankin, 2001):

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial_j \sigma_{ij} + f_i$$

and

$$\sigma_{ij} = \tilde{C}_{ijkl} \varepsilon_{kl},$$

where the parameter $\rho$ is the density. In this study, the density is ignored and is not updated during FWI (Guitton and Alkhalifah, 2016). The term $\tilde{C}_{ijkl}$ denotes the elastic constants. For the azimuthally rotated elastic ORT media, the stiffness matrix contains 13 $\tilde{C}_{ijkl}$ (in Voigt notation) nonzero parameters. In contrast to monoclinic media represented by 13 independent parameters, the rotated ORT media is represented by 10 independent parameters, including nine ORT parameters and the rotation angle $\phi$ of symmetry axis. This conversion can be expressed using the Bond transformation around the vertical $z$-axis in a counterclockwise direction (Ivanov and Stovas, 2016):

$$C^\text{ORT}_{ij} = D_\zeta(\phi) C^\text{ORT}_{ij} D_\zeta^T(\phi),$$

where

$$C^\text{ORT} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix},$$

and

$$D_\zeta(\phi) = \begin{pmatrix} \cos^2 \phi & \sin^2 \phi & 0 & 0 & 0 & -2 \sin \phi \cos \phi \\ \sin^2 \phi & \cos^2 \phi & 0 & 0 & 0 & 2 \sin \phi \cos \phi \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \phi & \sin \phi & 0 \\ 0 & 0 & 0 & -\sin \phi & \cos \phi & 0 \\ \sin \phi \cos \phi & -\sin \phi \cos \phi & 0 & 0 & 0 & \cos^2 \phi - \sin^2 \phi \end{pmatrix}.$$
\[
\begin{pmatrix}
C_{11} \\
C_{22} \\
C_{33} \\
C_{12} \\
C_{13} \\
C_{23} \\
C_{44} \\
C_{55} \\
C_{66}
\end{pmatrix} = 
\begin{pmatrix}
C_{11}\cos^2\phi + C_{22}\sin^2\phi + 2(C_{12} + 2C_{66})\sin\phi\cos\phi \\
C_{11}\sin^2\phi + C_{22}\cos^2\phi + 2(C_{12} + 2C_{66})\sin\phi\cos\phi \\
C_{11}\sin^2\phi + C_{22}\cos^2\phi + 2(C_{12} + 2C_{66})\sin\phi\cos\phi \\
C_{12}\sin^2\phi + C_{23}\cos^2\phi + 2(C_{13} + 2C_{55})\sin\phi\cos\phi \\
C_{13}\sin^2\phi + C_{23}\cos^2\phi + 2(C_{13} + 2C_{55})\sin\phi\cos\phi \\
C_{23}\sin^2\phi + C_{55}\cos^2\phi + 2(C_{13} + 2C_{55})\sin\phi\cos\phi \\
C_{44}\sin^2\phi + C_{55}\cos^2\phi + 2(C_{13} + 2C_{55})\sin\phi\cos\phi \\
C_{55}\sin^2\phi + C_{33}\cos^2\phi + 2(C_{13} + 2C_{55})\sin\phi\cos\phi \\
C_{66}\sin^2\phi + C_{33}\cos^2\phi + 2(C_{13} + 2C_{55})\sin\phi\cos\phi \\
\end{pmatrix}.
\]

### Gradient direction from monoclinic parameters

The gradient direction for each parameter, required to minimize the objective function, can be obtained by taking the partial derivative of equation 1 with respect to that model parameter. In this procedure, taking the partial derivative of the elastic wave equations, we obtain the equations for the partial derivative wavefields (PDWs) of an arbitrary parameter perturbation \( m \) at \( n \) th grid location as the same form as equation 2:

\[
\rho \frac{\partial^2 u_{\text{PDW},i}^n}{\partial t^2} = \partial_j \sigma_{\text{PDW},ij}^n + \partial_j \left( \frac{\partial \tilde{C}_{ijkl}}{\partial m_n} \epsilon_{kl} \right),
\]

where

\[
u_{\text{PDW},i}^n = \frac{\partial u_{ij}^n}{\partial m_n}
\]

and

\[
\sigma_{\text{PDW},ij}^n = \frac{\partial \epsilon_{ij}^n}{\partial m_n}
\]

The last term in equation 9 represents the virtual source for each model parameter perturbation (Pratt et al., 1998). Based on the adjoint form, the gradient direction for each \( \tilde{C}_{ijkl} \) parameter is expressed by (Kamath and Tsakanik, 2016)

\[
\nabla \tilde{C}_{ijkl} E = \int_0^T \partial_j u_j^t \partial_j u_k^t dt,
\]

where \( u^t \) and \( u^r \) are the source and back-propagated residual wavefields (Pratt et al., 1998). The gradient direction for any parameter \( m \) can be calculated using the chain rule as follows:

\[

abla_m E = \sum_{ijkl} \frac{\partial \tilde{C}_{ijkl}}{\partial m} (\nabla \tilde{C}_{ijkl} E).
\]

### Radiation-pattern analysis considering azimuthal rotation

The radiation patterns of the PDW are instructive interpretation tools in multiparameter FWI, as they provide some insights on the trade-off between the parameters (Gholami et al., 2013; Alkhalifah and Plessix, 2014). Because the gradient direction of each parameter is determined by the zero-lag crosscorrelation between PDW and residual wavefields (Operto et al., 2013), the radiation pattern of the PDW for each parameter determines which part of data contributes in recovering that parameter. Based on the definition of angle in the radiation-pattern plot, the diffraction patterns (Tarantola, 1986; Pan et al., 2016; Rusmanugroho et al., 2017), the reflection patterns (Gholami et al., 2013; Alkhalifah and Plessix, 2014; Oh and Alkhalifah, 2016a), and the transmission patterns (Alkhalifah et al., 2016; He and Plessix, 2016; Kamath and Tsakanik, 2016) are used to find an optimal parameterization that has less trade-off among parameters. Because we focus on reflections from predominately horizontal and the diving waves from velocity variations along the vertical direction, we rely on the reflection (for a horizontal reflector) patterns in this paper.

The angular dependency of PDW from the incidence P-wave in an isotropic background medium can be approximated as follows (Aki and Richards, 1980; Oh and Alkhalifah, 2016a; Pan et al., 2016):

\[
\begin{align*}
\mathbf{u}_{\text{PDW}}^P(\theta, \phi, \theta_d, \phi_d) &= \left| \mathbf{e}_{P}\mathbf{M}_{m}\mathbf{e}_{P} \right| \mathbf{e}_{P} = R_{m}^P \mathbf{e}_{P}, \\
\mathbf{u}_{\text{PDW}}^{P,SV}(\theta, \phi, \theta_d, \phi_d) &= \left| \mathbf{e}_{SV}\mathbf{M}_{m}\mathbf{e}_{SV} \right| \mathbf{e}_{SV} = R_{m}^{SV} \mathbf{e}_{SV},
\end{align*}
\]

where

\[
\mathbf{e}_{P} = (\sin \theta \cos \phi_d, \sin \theta \sin \phi_d, \cos \theta_d)
\]

and

\[
\mathbf{e}_{SV} = (\cos \theta \cos \phi_d, \cos \theta \sin \phi_d, -\sin \theta_d).
\]

The four angles, \( \theta, \phi, \theta_d, \phi_d \), denote the incidence, diffraction, and azimuth angles of the incidence and diffraction planes, respectively. The incidence and diffraction angles are measured clockwise from the vertical axis, whereas the azimuth angles are defined by a counterclockwise angle from inline direction. Matrix \( \mathbf{M} \) is the moment tensor form of the virtual source (Pratt et al., 1998) for each parameter listed in Table 1. The strains caused by the incidence P-wave can be approximated by (Pan et al., 2016)

\[
\begin{pmatrix}
\epsilon_{x,x}^P \\
\epsilon_{y,y}^P \\
\epsilon_{z,z}^P \\
\epsilon_{x,y}^P \\
\epsilon_{y,z}^P \\
\epsilon_{x,z}^P
\end{pmatrix} =
\begin{pmatrix}
\sin^2 \theta_i \cos^2 \phi_i \\
\sin^2 \theta_i \sin^2 \phi_i \\
\cos^2 \theta_i \\
\sin \theta_i \cos \theta_i \sin \phi_i \\
\sin \theta_i \cos \theta_i \cos \phi_i \\
\sin^2 \theta_i \sin \phi_i \cos \phi_i
\end{pmatrix}.
\]

The terms \( R_{m}^P \) and \( R_{m}^{SV} \) indicate the P-P and P-SV diffraction patterns caused by a perturbation of the model parameter \( m \). If we consider all 3C components or pressure data, the polarization vector \( \mathbf{e}_{P,SV} \) in equations 14 and 15 can be ignored. The radiation pattern for any parameter also can be obtained from those for monoclinic parameters using the chain rule in a similar way to equation 13. To convert diffraction patterns to the reflection patterns from a horizontal reflector in an isotropic background (Oh and Alkhalifah, 2016a), incidence and diffraction angles are converted to the opening angle \( \theta_o \), based on Snell’s law as follows:
\[
\theta^{SP}_d = \theta^p_d - \theta^S_d,
\]
where
\[
\theta^p_d = -\theta^S_d
\]
and
\[
\theta^{SV}_d = \theta^S_d - \theta^p_d,
\]
where
\[
\theta^{SV}_d = \sin^{-1}\left(\frac{v_y}{v_p} \sin(-\theta^p_d)\right).
\]

**PARAMETERIZATION STUDY**

In this section, we compare the behaviors of three possible parameterizations (Table 2), monoclinic, velocity-based, and hierarchical parameterizations based on the reflection patterns and the single-shot gradient direction. Figure 1 shows a schematic diagram illustrating a three-layer model, which has a water layer and a very weak ORT background. The strong azimuthally rotated ORT layer (L2) with rotation angle \(\phi = 45^\circ\) is interbedded. The initial model is the same as the background medium (water layer and L1), so that the residual wavefields only include reflection energy from the L2 layer. We assume single pressure source on the middle of the surface, and hydrophones are placed at all nodal points on the water surface. The maximum offset is 2 km along all azimuth directions. The gradient directions for each parameter can be calculated using equations 12 and 13. In Figure 1, the reflection patterns are calculated based on Snell’s law assuming the reflection raypath from a source (red dot) to a receiver (green dot) via subsurface reflector on the top of L2 (yellow dot). The reflection patterns (Figures 2, 3, and 4) are displayed as a 2D plot in terms of the opening angle along reflection plane (blue line) with different azimuth angles (Oh and Alkhalifah, 2016a).

**Table 1. The moment-tensor forms of the virtual source for 13 monoclinic parameters.**

<table>
<thead>
<tr>
<th>Parameterization I: Monoclinic parameterization</th>
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</thead>
<tbody>
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</tr>
<tr>
<td>Velocity-based</td>
</tr>
<tr>
<td>Hierarchical</td>
</tr>
</tbody>
</table>

**Figure 1.** The geometry of a three-layer model. The model has a very weak ORT background, so that we can ignore anisotropic propagation effects in the radiation-pattern analysis. The model also has strong azimuthally rotated ORT layer, L2 \(\phi = 45^\circ\). The red, green, and yellow dots denote the possible source, receiver, and perturbation points for reflection data, respectively. The red arrows indicate the reflection raypath from the top of anomalous layer (L2), which are assumed to derive the reflection pattern from Figures 2–4. |
coupling in the groups \([\tilde{C}_{12}, \tilde{C}_{66}]\) and \([\tilde{C}_{16}, \tilde{C}_{45}]\) is also observed. Although the radiation patterns show some overlap in the group \([\tilde{C}_{16}, \tilde{C}_{26}, \tilde{C}_{12}, \tilde{C}_{66}]\), the trade-off shown in the gradient direction is not strong thanks to the polarity change along the azimuth direction. As a result, the additional parameters \((\tilde{C}_{16}, \tilde{C}_{26}, \tilde{C}_{12}, \tilde{C}_{66})\) might be inverted well with less trade-off, as Rusmanugroho et al. (2017) also show for 2D elastic tilted transverse isotropic FWI. However, the monoclinc FWI is not necessary when we consider fracture-induced anisotropy, where vertically aligned fractures are azimuthally rotated. As the Bond transformation shows in equation 8, the number of independent parameters reduces from 13 to 10 under the assumption for the azimuthally rotated ORT medium.

### Parameterization II: Velocity-based parameterization

Under the rotated ORT assumption, we can parameterize the ORT \(C_{ij}\) parameters in several ways (Oh and Alkhalifah, 2016a). One common strategy defines a medium with P- \((v_p)\), S- \((v_s)\), and NMO velocities \((v_n)\) as follows:

\[
\begin{pmatrix}
  v_{p3} & v_{p1} & v_{p2} & v_{n1} & v_{n2} & \delta_3 & v_{s1} & v_{s2} & v_{s3} & \phi
\end{pmatrix}
\]

The parameters \(v_{p1}, v_{p2}, \text{ and } v_{p3}\) denote the P-wave velocities in the \(x-, y-, \text{ and } z\)-directions, respectively. The parameters \(v_{n1} \text{ and } v_{n2}\) are the NMO velocities for horizontal reflectors in the \(xz-\text{ and } yz\)-planes, respectively. The parameters \(v_{s1}, v_{s2}, \text{ and } v_{s3}\) indicate the vertical S-wave velocities that are polarized in the \(xz-\text{ and } yz\)-planes, respectively, and \(v_{s3}\) is the horizontal S-wave velocity in the \(xz\)-plane, but propagating in the \(x\)-direction. The parameter \(\delta_3\) is an anisotropic parameter in the \(xy\)-plane (Thomsen, 1986; Tsvankin, 1997). The ORT \(C_{ij}\) parameters are expressed using the velocity-based parameterization as follows:

\[
\begin{pmatrix}
  C_{11} \\
  C_{22} \\
  C_{33} \\
  C_{12} \\
  C_{13} \\
  C_{23} \\
  C_{44} \\
  C_{55} \\
  C_{66}
\end{pmatrix} = \begin{pmatrix}
  \rho v_p^2 \\
  \rho v_p^2 \\
  \rho v_p^2 \\
  \rho v_s^2 \\
  \rho v_s^2 \\
  \rho v_s^2 \\
  \rho v_s^2 \\
  \rho v_s^2 \\
  \rho v_s^2
\end{pmatrix}
\]

The second row shows the horizontal slices of gradient directions for each parameter on the top of anomaly (L2) acquired by a single source (red dot in Figure 1). Note that background has very weak ORT anisotropy to ignore propagation effects in anisotropic media.

**Figure 2.** The first row shows the P-P (left hemisphere) and P-SV (right hemisphere) radiation pattern for 13 monoclinic elastic constants. The angle is defined as an opening angle satisfying Snell’s law from a horizontal reflector. The Poisson’s ratio is assumed to be 0.25 to calculate P-SV radiation pattern (Oh and Alkhalifah, 2016a). Notice that \(\varphi_p\) is the azimuth angle of a source-receiver line in Figure 1, which is not related to the rotation angle of the symmetric axis \(\varphi\).
The attractive feature of the velocity-based parameterization is that each parameter has the same units (except $\delta_3$), so that we can easily scale the gradient direction. However, the velocity-based parameterization has complex behavior during FWI as we observed in the radiation patterns (Oh and Alkhalifah, 2016a). Figure 3 shows the P-P and P-SV radiation patterns of the velocity-based parameterization. At first, we observe outstanding separation of $[v_{p1}, v_{s1}, v_{p2}]$ and $[v_{p2}, v_{n2}, v_{s2}]$ along the azimuth direction, which means that the trade-off between each pair of parameters is not strong. However, this also means that the FWI updates would always be ORT if we do not apply some constraints on the parameter updates. This ORT update causes complicated wave propagation in the inversion model. For this reason, suitable constraints on the updates should be applied to stabilize the FWI procedure at an early stage.

Regarding the rotation of ORT axes, we observe that $v_{p3}$ has azimuth independent radiation pattern, whereas other parameters depend on the rotation angle of the reflection plane. The sensitivity of $v_{p3}$ is the weakest. Supporting these observations from radiation-pattern plots, the single-shot gradient direction (Figure 5) also shows azimuth-dependent characteristics of the parameter perturbations in the velocity-based parameterization. The first row is the horizontal slices of the gradient direction on the top of the anomaly. This shows azimuth-dependent characteristics of the parameter perturbations for the velocity-based parameterization. The first row is the horizontal slices of the gradient direction on the top of the anomaly. This shows azimuth-dependent characteristics of the parameter perturbations. The first row is the horizontal slices of the gradient direction on the top of the anomaly. This shows azimuth-dependent characteristics of the parameter perturbations. The first row is the horizontal slices of the gradient direction on the top of the anomaly. This shows azimuth-dependent characteristics of the parameter perturbations.

Parameterization III: Hierarchical parameterization

To avoid the aforementioned complexities in multiparameter FWI for rotated ORT media, the hierarchical parameterization has been proposed with three deviation parameters as follows (Oh and Alkhalifah, 2016a):

\[
\begin{pmatrix}
\epsilon_1 \\
\eta_1 \\
\gamma_1 \\
\delta_3 \\
\varepsilon_D \\
\eta_D \\
\gamma_D \\
\phi
\end{pmatrix},
\]

where

\[
\epsilon_D = \frac{\epsilon_2 - \epsilon_1}{1 + 2\epsilon_1},
\]

\[
\eta_D = \frac{\eta_2 - \eta_1}{1 + 2\eta_1},
\]

and

\[
\gamma_D = \frac{\gamma_1 - \gamma_2}{1 + 2\gamma_2}.
\]

The parameters $[\epsilon_1, \gamma_1]$ and $[\epsilon_2, \gamma_2]$ denote the VTI parameters $\epsilon$ and $\gamma$ in the $xz$- and $yz$-planes, respectively. Instead of VTI parameter $\delta_3$, we use $\eta_1 (v_{n1} = v_{p3}\sqrt{1 + 2\delta_3})$ and $\epsilon_1 (v_{s1} = v_{n1}\sqrt{1 + 2\eta_1})$ suggested by Alkhalifah (2003). In this VTI combination, the influence of parameter $\eta$ is negligible (Alkhalifah, 2016). The deviation parameters $[\epsilon_D, \eta_D, \gamma_D]$ determine azimuthal variations of each anisotropic parameter on the $xy$-plane (Tssvankin, 1997; Masmoudi and Alkhalifah, 2016). The ORT $C_{ij}$ parameters are expressed by the hierarchical parameterization as follows:

\[
\begin{pmatrix}
C_{11} \\
C_{12} \\
C_{13} \\
C_{21} \\
C_{22} \\
C_{23} \\
C_{31} \\
C_{32} \\
C_{33}
\end{pmatrix} = \rho \begin{pmatrix}
\rho_1^2 v_{p1}^2 & \rho_1^2 v_{s1}^2 & \rho_1^2 v_{p2}^2 \\
\rho_1^2 v_{p1}^2 & \rho_1^2 v_{s1}^2 & \rho_1^2 v_{p2}^2 \\
\rho_1^2 v_{p1}^2 & \rho_1^2 v_{s1}^2 & \rho_1^2 v_{p2}^2 \\
\rho_1^2 v_{p1}^2 & \rho_1^2 v_{s1}^2 & \rho_1^2 v_{p2}^2 \\
\rho_1^2 v_{p1}^2 & \rho_1^2 v_{s1}^2 & \rho_1^2 v_{p2}^2 \\
\rho_1^2 v_{p1}^2 & \rho_1^2 v_{s1}^2 & \rho_1^2 v_{p2}^2 \\
\rho_1^2 v_{p1}^2 & \rho_1^2 v_{s1}^2 & \rho_1^2 v_{p2}^2 \\
\rho_1^2 v_{p1}^2 & \rho_1^2 v_{s1}^2 & \rho_1^2 v_{p2}^2 \\
\rho_1^2 v_{p1}^2 & \rho_1^2 v_{s1}^2 & \rho_1^2 v_{p2}^2
\end{pmatrix}.
\]

As Oh and Alkhalifah (2016a) show from the radiation-pattern analysis, this parameterization enables us to build up subsurface models from isotropic ($v_{p3}$ and $v_{s1}$) to VTI ($\epsilon_1$, $\eta_1$, and $\gamma_1$) and finally to ORT media ($\epsilon_D$, $\eta_D$, $\gamma_D$, and $\delta_3$) by sequentially updating those parameters.

For rotated ORT media, this parameterization also offers better functionality compared with the velocity-based parameterization. In the reflection pattern (Figure 4), we observed that all five parameters ($v_{p1}$, $v_{s1}$, $\epsilon_1$, $\eta_1$, and $\gamma_1$) have azimuth-independent radiation patterns. In Figure 5, the single-shot gradient directions of these parameters are not sensitive to background rotation angle, which supports that we can recover the VTI model regardless of the rotation angle of ORT anisotropy. From this VTI model, we can further build a subsurface ORT model by updating deviation parameters ($\epsilon_D$, $\eta_D$, $\gamma_D$, and $\delta_3$).
\( \gamma_D \) and \( \delta_\theta \)). However, at this stage, the estimation of deviation parameters is affected by the initial guess of the rotation angle.

**MULTISTAGE FWI IN ROTATED ORT MEDIA**

To enhance the convergence of multiparameter FWI, an accurate rotation angle is required. First, we examine the possibility of inverting for the rotation angle as an FWI target by analyzing the gradient direction and radiation patterns, as we have done for ORT parameters. Then, we suggest an optimal FWI update strategy, in which we update the subsurface model from simple to complex anisotropy using the hierarchical parameterization.

**Rotation angle as a FWI target**

The gradient direction of the rotation angle can be derived using the chain rule in equation 13 as follows:

\[
\nabla \phi E = \partial \tilde{C}_{11,\phi} \nabla \tilde{C}_{11} E + \partial \tilde{C}_{22,\phi} \nabla \tilde{C}_{22} E + \partial \tilde{C}_{12,\phi} \nabla \tilde{C}_{12} E \\
+ \partial \tilde{C}_{13,\phi} \nabla \tilde{C}_{13} E + \partial \tilde{C}_{23,\phi} \nabla \tilde{C}_{23} E \\
+ \partial \tilde{C}_{44,\phi} \nabla \tilde{C}_{44} E + \partial \tilde{C}_{55,\phi} \nabla \tilde{C}_{55} E + \partial \tilde{C}_{66,\phi} \nabla \tilde{C}_{66} E \\
+ \partial \tilde{C}_{16,\phi} \nabla \tilde{C}_{16} E + \partial \tilde{C}_{26,\phi} \nabla \tilde{C}_{26} E + \partial \tilde{C}_{36,\phi} \nabla \tilde{C}_{36} E \\
+ \partial \tilde{C}_{45,\phi} \nabla \tilde{C}_{45} E.
\]

where

\[
\begin{pmatrix}
\partial \tilde{C}_{11,\phi} \\
\partial \tilde{C}_{22,\phi} \\
\partial \tilde{C}_{12,\phi} \\
\partial \tilde{C}_{13,\phi} \\
\partial \tilde{C}_{23,\phi} \\
\partial \tilde{C}_{44,\phi} \\
\partial \tilde{C}_{55,\phi} \\
\partial \tilde{C}_{66,\phi} \\
\partial \tilde{C}_{16,\phi} \\
\partial \tilde{C}_{26,\phi} \\
\partial \tilde{C}_{36,\phi} \\
\partial \tilde{C}_{45,\phi}
\end{pmatrix} =
\begin{pmatrix}
4 \sin \phi \cos \phi (-C_{11} + C_{12} + 2 C_{33} \cos^2 \phi - (-C_{11} + C_{12} + 2 C_{33} \sin^2 \phi)
\\
4 \sin \phi \cos \phi (-C_{11} + C_{12} + 2 C_{33} \cos^2 \phi - (-C_{11} + C_{12} + 2 C_{33} \sin^2 \phi)
\\
2 (C_{11} + C_{12} - 2 C_{33} - 4 C_{66}) \sin \phi \cos \phi \cos^2 \phi - \sin \phi \sin^2 \phi)
\\
2 (C_{11} - C_{12}) \sin \phi \cos \phi
\\
-2 (C_{11} - C_{12}) \sin \phi \cos \phi
\\
2 (C_{11} - C_{12}) \sin \phi \cos \phi
\\
2 (C_{11} - C_{12}) \sin \phi \cos \phi
\\
2 (C_{11} + C_{12} - 2 C_{33} - 4 C_{66}) \sin \phi \cos \phi \cos^2 \phi - \sin \phi \sin^2 \phi)
\\
(-C_{11} + C_{12} + 2 C_{33} \cos^2 \phi - 3 \sin^2 \phi \cos^2 \phi) + (-C_{11} + C_{12} + 2 C_{33} \sin^2 \phi - 3 \sin^2 \phi \cos^2 \phi)
\\
(C_{12} - C_{22} - 2 C_{33} \cos^2 \phi - 3 \sin^2 \phi \sin^2 \phi) + (C_{12} - C_{22} - 2 C_{33} \sin^2 \phi - 3 \sin^2 \phi \sin^2 \phi)
\\
(C_{12} - C_{12}) \cos^2 \phi - \sin^2 \phi)
\\
(C_{12} - C_{22}) \cos^2 \phi - \sin^2 \phi)
\end{pmatrix}
\]

As shown in equation 30, gradient direction of the rotation angle depends on 12 monoclinic parameters except \( C_{33} \), which causes complicated behavior of the rotation angle as an FWI target. Here, we show some interesting features in the gradient direction of the rotation angle.

At first, the gradient direction of the rotation angle depends on the model parameter deviation along slow and fast axes of ORT media such as \( C_{23} - C_{13}, \) \( C_{44} - C_{55}, \) and \( C_{11} + C_{22} - 2 C_{12} - 4 C_{66}. \) In other words, the activation of each monoclinic parameter is determined by the properties of the initial model (or the inverted model during FWI). For example, if the inversion model is isotropic and VTI, the gradient direction of the rotation angle is zero because all the coefficients in equation 31 are zero. This behavior is also physically reasonable because isotropic and VTI models are azimuthally invariant. However, once we have ORT background model, the rotation angle can be estimated through FWI procedure. Figure 6 shows the radiation pattern and single-shot gradient direction of the rotation angle. Here, four different triggers (type I, \( C_{22} > C_{11} \); type II, \( C_{23} > C_{13} \); type III, \( C_{44} > C_{55} \); and type IV, \( C_{12} > C_{66} \)), which affect the radiation patterns of the rotation angle in different ways, are displayed. We expect that, depending on the model parameter deviations in the inversion model, the sensitivity of the rotation angle can be changed abruptly during FWI.

Second, we also observe polarity change in the gradient direction of the rotation angle along the azimuth direction as is also observed from the eikonal equation in the previous literature (Masmoudi et al., 2017). This azimuthal polarity changes in the gradient direction (or sensitivity kernel) indicates that the gradient direction of the rotation angle is measured by the azimuthal variation of the wavefields. From this observation, we expect that the trade-off between the rotation angle and ORT parameters are not strong although the amplitude variations in the radiation pattern of the rotation angle resemble some ORT parameters in Figures 3 and 4.

**Multistage FWI strategy**

Based on above observations on the rotation angle as an FWI target, we suggest the optimal multistage FWI strategy using the hierarchical parameterization because the hierarchical parameterization shows better behavior than the velocity-based parameterization. As mentioned in the parameterization study, most parameters in the velocity-based parameterization are sensitive to the background rotation angle, which means that accurate rotation angle information is required. In addition, if we invert for all nine ORT parameters simultaneously using the velocity-based parameterization, inversion...
model is ORT from the beginning of FWI. Therefore, the complexity of FWI for the rotation angle increases combining all four types in Figure 6. To prevent this complexity in FWI for the rotation angle, a proper constraint needs to be applied.

On the other hand, five parameters ($v_{p1}$, $v_{s1}$, $\varepsilon_1$, $\eta_1$, and $\gamma_1$) in the hierarchical parameterization are robust to azimuthal variation of the wavefields, which means that we can build a VTI model regardless of true rotation angle. In addition, using the hierarchical parameterization, we can easily constrain the subsurface model by freezing some anisotropic parameters locally or globally. For example, if we invert for only $\varepsilon_D$ and freeze other deviation parameters ($\eta_D$, $\gamma_D$, and $\delta_3$), the sensitivity of the rotation angle is governed by only types-I and -II, as expected from equation 29, whose amplitude variations of the radiation pattern resemble those from $\varepsilon_D$ perturbation in Figure 4. This constraint on the ORT update allows us to recover the rotation angle with wide opening angle components of the data, which are also used to invert for $\varepsilon_D$.

However, because the rotation angle is estimated by measuring azimuthal variations of the wavefields as observed in Figure 6, the trade-off between $\varepsilon_D$ and the rotation angle is not strong.

As Oh and Alkhalifah (2016a) show, four parameters ($v_{s1}$, $\eta_1$, $\eta_D$, and $\gamma_D$) are strongly coupled and two parameters $\varepsilon_D$ and $\delta_3$ are also coupled. In addition, $\gamma_1$ is not sensitive to the P-P and P-SV modes. For this reason, inverting for only five parameters ($v_{p1}$, $v_{s1}$, $\varepsilon_1$, $\varepsilon_D$, and $\phi$) is a reasonable choice for a marine data inversion. Using these five parameters, we first build an isotropic background model by inverting for only $v_{p1}$ and $v_{s1}$. After the objective function reaches a certain criterion, we add $\varepsilon_1$ in the inversion process to build VTI model. Then, the deviation parameter $\varepsilon_D$ is added to recover the azimuthal variation. The parameters $\varepsilon_1$ and $\varepsilon_D$ can be added together to reduce the number stages because they are well-decoupled in the data, as supported by the radiation patterns (Figure 4). Finally, we add the rotation angle in the FWI procedure to fix the error in $\varepsilon_D$ and to find the true rotation angle.

**TRADE-OFF ANALYSIS**

To prove the trade-off patterns observed in the radiation-pattern analysis, we conduct trade-off analysis using a simple synthetic model. To design a synthetic model that reveals the trade-off between parameters, we allocate each model parameter perturbation a unique location in the model to observe imprints from other parameters (Köhn et al., 2012, 2015). To do this, we design a simple layered model with a disc-shaped anomaly, which we refer to as the hockey puck model, as shown in Figure 7.

The first layer is water, and the background ORT parameters are listed in Table 3. To verify the trade-off between parameters, each ORT parameter perturbation is located at a different part of the hockey puck from areas 1 to 9. The true rotation angle perturbation occurs at only the inner part (area 10). The detailed parameter per-
Parameterization I

Note that the perturbed values in each parameterization are determined to provide a similar influence on the data.

the background ORT medium in the center of the puck (area 10) and the end of each row, red and blue lines indicate the true fast axes of areas 1 to 10). The black arrow indicates the true location of each parameter. The imprints in other locations result from the trade-off between parameters.

In the first row of Figure 8, the velocity-based parameterization shows a nice separation of the gradient direction, as we observed in the single-shot gradient direction in Figure 5. The parameter \( v_{p3} \) is well-isolated (slice E0). Two horizontal P-wave velocities \( (v_{p1} \) and \( v_{p2} \)) are also well-decoupled, although the trade-off with \( \delta_{3} \) is observed (slices E1 and E2). Two NMO velocities \( (v_{n1} \) and \( v_{n2} \)) are barely detected due to the trade-off between the P- and S-wave velocities (slices E3 and E4). The anisotropic parameter \( \delta_{3} \) is well-isolated (slice E5). Two vertical S-wave velocities \( (v_{s1} \) and \( v_{s2} \)) suffer from the trade-off with the P-wave velocities (slices E6 and E7). The parameter \( v_{s3} \) (slice E8) is not recoverable, whereas the gradient direction of the rotation angle (slice E9) is well-detected. Because we only record P-waves using hydrophones, the S-wave velocities are generally not recovered. In addition, in these PP conversions, the S-wave velocities strongly suffer from trade-off with P-wave velocities. In the second and third rows of Figure 8, the initial guess for the rotation angle in the outer part is 45° and 90°

### Table 3. The background model parameters for the hockey-puck model in Figure 7 and perturbed values for each parameterization.

<table>
<thead>
<tr>
<th>Initial (background)</th>
<th>( v_{p1} )</th>
<th>( v_{s1} )</th>
<th>( \varepsilon_{1} )</th>
<th>( \varepsilon_{2} )</th>
<th>( \eta_{1} )</th>
<th>( \eta_{2} )</th>
<th>( \delta_{3} )</th>
<th>( \gamma_{1} )</th>
<th>( \gamma_{2} )</th>
<th>( \Phi(1 − \Phi) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2 km/s)</td>
<td>1.15 km/s</td>
<td>0</td>
<td>0.045</td>
<td>0</td>
<td>0.045</td>
<td>0</td>
<td>0.045</td>
<td>0</td>
<td>0.045</td>
<td>0</td>
</tr>
</tbody>
</table>

Parameterization I

<table>
<thead>
<tr>
<th>1.( dv_{p3} )</th>
<th>2.( dv_{p1} )</th>
<th>3.( dv_{p2} )</th>
<th>4.( dv_{p1} )</th>
<th>5.( dv_{n2} )</th>
<th>6.( dv_{n1} )</th>
<th>7.( \delta_{3} )</th>
<th>8.( dv_{s2} )</th>
<th>9.( dv_{s1} )</th>
<th>10.( dv_{\Phi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 km/s</td>
<td>0.115 km/s</td>
<td>0.2 km/s</td>
<td>0.2 km/s</td>
<td>0.2 km/s</td>
<td>0.115 km/s</td>
<td>0.115 km/s</td>
<td>0.115 km/s</td>
<td>0.115 km/s</td>
<td>0.115 km/s</td>
</tr>
</tbody>
</table>

Parameterization II

<table>
<thead>
<tr>
<th>1.( dv_{p1} )</th>
<th>2.( dv_{p3} )</th>
<th>3.( \varepsilon_{3} )</th>
<th>4.( \varepsilon_{2} )</th>
<th>5.( \delta_{1} )</th>
<th>6.( \delta_{1b} )</th>
<th>7.( \delta_{3} )</th>
<th>8.( \delta_{1b} )</th>
<th>9.( \gamma_{1} )</th>
<th>10.( dv_{\Phi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 km/s</td>
<td>0.115 km/s</td>
<td>0.105</td>
<td>0.105</td>
<td>0.105</td>
<td>0.105</td>
<td>0.105</td>
<td>0.105</td>
<td>0.105</td>
<td>0.105</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>45°</td>
</tr>
</tbody>
</table>

Note: The initial guess for the rotation angle in the outer part is 45° and 90°. The ellipsoid denotes the initial guess for the ORT axis in the whole puck (from area 1 to 10). The black arrow indicates the true location of each parameter.

Figure 8. The horizontal slices of the gradient direction on the top of the hockey puck for each parameter using the velocity-based parameterization. We assume pressure sources and hydrophone data on the water surface. Each row shows the results from three different rotation angles \( (0°, 45°, \text{and} 90°) \) in the inversion model. In the end of each row, the red and blue lines indicate the true fast axes of ORT media in the inner part (area 10) and the outer part (from area 1 to 9), respectively. The black arrow indicates the true location of each parameter. The imprints in other locations result from the trade-off between parameters.
apart from the true angle, respectively. As we expected, data fitting for each parameter except \( v_{p3} \) and \( v_{s3} \) is conducted along the wrong azimuth direction.

On the other hand, the hierarchical parameterization shows interesting features (Figure 9). In the first row, we observe that only five ORT parameters \((v_{p1}, v_{s1}, \varepsilon_1, \delta_1, \text{and } \delta_3)\) and the rotation angle are sensitive to the streamer data. Between these parameters, trade-off also exists. The P-wave velocity \((E0)\) is well-isolated, whereas \(\eta_1\) and \(\delta_3\), which depend often on diving waves, have relatively weak sensitivity. However, \(v_{p1}\) is well-decoupled from \(v_{s1}\), whereas it is slightly coupled with \(\delta_1\) (E6). All four parameters are strongly coupled with \(v_{p1}\). This observation indicates that the misfit due to \(v_{p1}\) perturbation should be resolved in the early stages; otherwise, considerable trade-off errors may be induced. The other three parameters, \((\eta_1, \eta_D, \text{and } \gamma_D)\) are strongly coupled with \(v_{s1}\), and the data are insensitive to parameter \(\gamma_1\), as we also observed in the radiation pattern of \(\gamma_1\) (Figure 4). In the second and third rows, we observe that five parameters \((v_{p1}, v_{s1}, \varepsilon_1, \eta_1, \text{and } \gamma_1)\) are independent of the background rotation angle. The convergence of the deviation parameters depends on the background rotation angle. However, at the last stage of the proposed multistage approach, the convergence of the deviation parameters can be improved by inverting for the rotation angle with those parameters simultaneously.

**NUMERICAL EXAMPLE:**
**SYNTHETIC MARINE STREAMER DATA**

To support our observations in the radiation-pattern and trade-off analyses, we first compare the behavior of the velocity-based and hierarchical parameterizations on a more realistic 3D model. Then, we show the feasibility of the proposed multistage FWI on the noise-free rotated ORT data. For simplicity, we assume a simple 3D synthetic model (Figure 10) with a high-velocity mud-filled channel structure (layer 4) and fractured rotated ORT layer (layer 6) that resembles a carbonate reservoir. The true ORT parameters are listed in Table 4. For the rotation angle in layer 6, we assume two different structures (cases-I and -II in Figure 10) to verify the trade-off of the rotation angle with other ORT parameters. The synthetic rotated elastic ORT data are acquired by fourth-order standard staggered grid finite-difference method (Graves, 1996), with the Ricker wavelet that has a peak frequency of 5 Hz. Domain decomposition (Bohlen, 2002) is applied, in which the modeling process is parallelized over shots and subdomains, simultaneously. The convolution perfectly matched layer (Roden and Gedney, 2000) is applied to absorb artificial reflected waves from the model boundaries. To avoid storing source wavefields \(u^s\) in equation 12, we choose a time-frequency hybrid-domain approach (Sirgue et al., 2008), in which forward \(u^s\) and backward \(u^r\) wavefields are transformed to the frequency domain on the fly. The frequency band used in FWI is from 3 to 10 Hz. We consider a total of 6400 pressure sources for the synthetic survey. However, we randomly chose only 100 sources at each iteration to reduce the computational cost (Warner et al., 2013). In total, 160,000 hydrophones are placed on the water surface. As an initial model (Figure 11), we use a smoothed VTI model for \(v_{p1}, v_{s1}, \text{and } \varepsilon_1\). This is because, based on the radiation pattern (Figure 4), we do not expect to obtain smoothed updates for \(v_{s1}\) and \(\varepsilon_1\) from FWI using the hierarchical parameterization. The initial \(\eta_1\) is the same as the initial \(\varepsilon_1\). The other six parameters, including the rotation angle, are set to zero.

![Hierarchical parameterization (hydrophone)](Figure 9. The same as Figure 8, but for the hierarchical parameterization.)

<table>
<thead>
<tr>
<th>Layer</th>
<th>(v_{p3})</th>
<th>(v_{s1})</th>
<th>(\varepsilon_1)</th>
<th>(\varepsilon_2)</th>
<th>(\delta_1)</th>
<th>(\delta_2)</th>
<th>(\delta_3)</th>
<th>(\gamma_1)</th>
<th>(\gamma_2)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>1.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Water</td>
</tr>
<tr>
<td>Layer 2</td>
<td>2.0</td>
<td>1.15</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Isotropic</td>
</tr>
<tr>
<td>Layer 3</td>
<td>2.2</td>
<td>1.27</td>
<td>0.12</td>
<td>0.12</td>
<td>0.04</td>
<td>0.04</td>
<td>0.08</td>
<td>0.08</td>
<td>VTI</td>
<td></td>
</tr>
<tr>
<td>Layer 4</td>
<td>3.4</td>
<td>1.63</td>
<td>0.1</td>
<td>0.1</td>
<td>0.03</td>
<td>0.03</td>
<td>0.05</td>
<td>0.05</td>
<td>VTI</td>
<td></td>
</tr>
<tr>
<td>Layer 5</td>
<td>2.6</td>
<td>1.5</td>
<td>0.06</td>
<td>0.06</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>VTI</td>
<td></td>
</tr>
<tr>
<td>Layer 6</td>
<td>2.8</td>
<td>1.7</td>
<td>0.07</td>
<td>0.15</td>
<td>0.03</td>
<td>0.08</td>
<td>0.04</td>
<td>0.08</td>
<td>rORT</td>
<td></td>
</tr>
<tr>
<td>Layer 7</td>
<td>3.5</td>
<td>2.15</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>VTI</td>
<td></td>
</tr>
<tr>
<td>Layer 8</td>
<td>4.5</td>
<td>2.9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Isotropic</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. The model parameters of each layer in Figure 10.
The true and initial densities are fixed at 1 g/cm³ in the water and at 2 g/cm³ in the layers beneath and not updated. The diagonal of the pseudo-Hessian matrix (Shin et al., 2001) is applied for ORT parameters whereas, for the rotation angle, we find the steepest descent solution to be preferable. As equation (31) shows, the virtual source as well as the gradient is controlled by the model parameter deviations along the fast and slow axes. This means that if the model deviation is too small close to VTI media, the virtual source will be very weak — almost zero, which means the pseudo-Hessian (zero-lag autocorrelation of the virtual source) will be also very small. For this reason, the gradient direction of the rotation angle can be extremely amplified by the pseudo-Hessian matrix when the model parameter deviation is very small at weak ORT areas. The FWI takes approximately 10 min per iteration with 100 computational nodes on the Cray XC40.

Figure 12 shows the FWI results from the velocity-based parameterization. All 10 parameters are inverted simultaneously, and then they are converted to those in the hierarchical parameterization for comparison. Only the five main parameters, which are sensitive to marine streamer data, are displayed. Due to the poor initial guess for the rotation angle of the ORT anisotropy, the FWI using the velocity-based parameterization was not successful. The parameters \(v_p1\) and \(v_s1\) are relatively well-recovered, whereas \(\varepsilon_1\) and \(\varepsilon_D\) are poorly inverted. Because we failed to recover background ORT parameters, the inversion for the rotation angle was also not successful.

Figure 13 shows the FWI results from the hierarchical parameterization. Following our observations in trade-off analysis, we only update five main parameters (\(v_p1\), \(v_s1\), \(\varepsilon_1\), \(\varepsilon_D\), and \(\phi\)). In this simulation, all five parameters are inverted simultaneously. The FWI results are better than those from the velocity-based parameterization. However, it is clear that \(\varepsilon_1\) and \(\varepsilon_D\) suffer from the trade-off with the P-wave velocity because this channel-like imprint is obviously caused by the P-wave velocity structure. The imprints from the P-wave velocity error decrease \(\varepsilon_1\) and increase \(\varepsilon_D\), as also supported by the trade-off analysis in Figure 9 (E2 and E6). Due to the failure in inverting for \(\varepsilon_D\), the inverted rotation angle is also not satisfactory. To reduce the trade-off, we apply the proposed multistage FWI strategy under the same FWI setting (Figure 14). Because the misfits in the data from \(v_p1\) are reduced first in an isotropic update stage, \(\varepsilon_D\) structure is well-recovered in the following stage. From this good initial ORT assumption, the rotation angle is also well-recovered. Figure 15 shows the FWI results with the multistage strategy, but for the case-II rotation angle structure. As also supported by...
radiation-pattern and trade-off analyses, VTI parameters $v_p$, $v_s$, and $\varepsilon$ are recovered regardless of the rotation angle of ORT anisotropy. From this inverted VTI model, $\varepsilon_D$ and the rotation angle are well-recovered with less trade-off each other.

**CONCLUSION**

Assuming azimuthally rotated elastic ORT media, we examine the possibility of inverting for the rotation angle of the symmetric axis as a FWI target. To do that, we formulate the gradient direction and radiation pattern of the PDW using the elastic wave equation for monoclinic media. Then, we parameterize the monoclinic elastic wave equation with the velocity-based and hierarchical parameterizations. By analyzing the radiation patterns and the gradient directions of each parameterization, we are convinced that the hierarchical parameterization, in which we use deviation parameters, provides us with the best functionality to perform FWI in rotated ORT media. In the velocity-based parameterization, most parameters depend on the azimuth direction of the ORT anisotropy and thus, the azimuth direction of the data, which means that this parameterization requires accurate information on the azimuth angle. On the other hand, thanks to the separation of the scattering potentials in the hierarchical parameterization, we build the subsurface model from simple to complex anisotropy with less trade-off with the azimuthal rotation. Trade-off analysis for the hockey-puck-shaped model clearly shows the sensitivity of each parameter, and as a result, we realized that we could mainly estimate five ORT parameters from marine streamer data using the hierarchical parameterization. The numerical examples show that a multistage FWI implementation reduces the trade-off between parameters so that, finally, the azimuth angle can be estimated.

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APPENDIX A

TRADE-OFF ANALYSIS FOR OBC ACQUISITION

In Appendix A, we show the trade-off analysis based on ocean-bottom cable (OBC) acquisition. For the same FWI setting for the hockey-puck model (Figure 7), 160,000 3C geophones (placement fields) are placed on the sea bottom. Figures A-1 and A-2 show the trade-off patterns from the OBC acquisition system. Unlike for the streamer data (Figures 8 and 9), the P and S-wave reflections are available in OBC data (Raknes and Aarnes, 2014). Thanks to S-waves, more parameters are sensitive to OBC data and resolution is improved. For the velocity-based parameterization, the recoverability of each parameter is reasonably improved when the exact azimuth angle of the ORT medium is known (the first row in Figure A-1). However, this parameterization still requires a good estimation of such an azimuth angle. The recoverability of each parameter in the hierarchical parameterization is also improved (Figure A-2). However, three parameters ($\eta_1$, $\eta_2$, and $\gamma_1$) are still weakly recovered and are strongly coupled with $v_s$. The parameter $\gamma_1$ is not sensitive even to OBC data, as supported by the radiation pattern (Figure 4) and the influence of $\delta_1$ is weaker than $\delta_2$. For this reason, five parameters ($v_p$, $v_s$, $\epsilon_1$, $\epsilon_2$, and $\phi$) in the hierarchical parameterization are still considered as the main parameters for OBC data (Oh and Alkhalifah, 2016a).

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