Numerical Investigation of Shock Bubble Interaction using Wavelet Adaptive Multi-Resolution Method

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ABSTRACT

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When a shock interacts with a bubble having a different density than the environment or medium, the interaction causes compression and deformation of the bubble and generation of a vortex pair. Later, secondary vortices appear causing enhanced mixing. The enhanced mixing induced by the shock bubble interactions is particularly of interest in supersonic combustion and detonation. The Wavelet Adaptive Multi-resolution Representation (WAMR) method is particularly suitable for challenging continuum physics problems like shock bubble interaction which has strong multi-scale character. This method provides an efficient strategy to create a dynamically adaptive spatial grid and to obtain a verified solution. Since the wavelet amplitude provides a first-hand estimate of the local error at each point, the method is able to efficiently capture a wide spectrum of spatial scales by dynamically changing the adaptive grid. Highly resolved computations are done only in the regions where abrupt transition occurs.

In this work a detailed investigation of Shock Bubble Interaction (SBI) is carried out using shocks having Mach numbers from 1.2 to 3 for helium, nitrogen and krypton bubbles. Simulations carried out using WAMR method were used to analyze the effects of Mach number and density contrast on the shape, location and velocity of the bubble as well as vorticity and pressure in the flow field.
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Chapter 1

Introduction

When a shock interacts with a thermodynamic in-homogeneity, several processes occur that affect the geometry and propagation pattern of the shock and also the thermodynamic state of the in-homogeneous medium it propagates through. The interaction of shock with density imbalance is particularly of interest as it results in many fluid dynamic phenomena being strongly coupled. When a shock interacts with a bubble with different density, complex patterns of shock and rarefaction waves occur. The bubble undergoes compression and distortion, resulting in vorticity generation and subsequently turbulence. Meanwhile the shock experiences reflection, refraction and shock focusing effects. These interactions between a shock and misaligned density gradients create Richtmyer-Meshkov instabilities (RMI) and also cause Kelvin-Helmholtz instabilities on the bubble surface. For appropriate combination of shock strength and density contrast, secondary vortices are formed causing mass to be removed from the bubble and thus enhancing the mixing of the two fluids. This enhancement of mixing is particularly of interest in supersonic combustion and detonation.

1.1 Acceleration and Compression by shock

The passage of a shock over a medium compresses it and causes a sudden discontinuity in thermodynamic properties such as density, temperature, pressure and velocity of the medium. This jump is in accordance to the Rankine-Hugoniot conditions.

Consider flow of a shock of shock-wave velocity $W$ across a medium at rest with
density $\rho$, temperature $T$, and pressure $p$. The enthalpy of the fluid $h$ is given by $h = \gamma RT/(\gamma - 1)$. Then the post shock conditions ($\rho'$, $u'$, $p'$, $h'$, and $T'$) are given by following equations.

\begin{equation}
\rho W = \rho'(W - u'),
\end{equation}

\begin{equation}
p + \rho W^2 = p' + \rho'(W - u')^2,
\end{equation}

\begin{equation}
h + \frac{W^2}{2} = h' + \frac{(W - u')^2}{2},
\end{equation}

\begin{equation}
p' = \rho' RT
\end{equation}

\begin{equation}
h' = \frac{\gamma RT}{(\gamma - 1)}
\end{equation}

1.2 Acoustic Effects

When a shock interacts with a bubble, it undergoes reflection, refraction and diffraction. The bubble changes the shape and wave pattern of the shock through non-linear acoustic mechanisms due to acoustic impedance mismatch and curvature at the interface. Acoustic impedance ($R = \rho c$) is a thermodynamic property that quantifies the stiffness of the propagating medium. Impedance mismatch changes the shock speed as it transmits across the interface to the second fluid. If $\delta R > 0$, the shock speed is decreased after transmission. In order to maintain equilibrium at the interface the first fluid must contract i.e. the reflected wave in this case is a shock wave. If $\delta R < 0$, the shock speed increases and the reflected wave is a rarefaction wave. The curvature of the interface acts like a lens. When $\delta R > 0$, the bubble behaves like a converging
lens. The shock that is transmitted through the bubble is concave shaped and lags behind the undisturbed shock region. When $\delta \mathcal{R} < 0$, the transmitted shock moves ahead of the undisturbed shock wave. In case of convergent geometry with high density contrast, the transmitted shock wave leads to shock focusing. This generates an intense pressure jump and secondary shock waves are generated.

### 1.3 Vorticity generation and Transport

The misalignment between density and pressure gradients at the interface causes vorticity production along the interface. This misalignment is called baroclinicity. A vortex pair is formed when shock interacts with a cylindrical bubble and a vortex ring is formed for a spherical bubble. Taking curl of the compressible Navier-Stokes equation gives the vorticity transport equation

\[
\frac{D\omega}{Dt} = (\omega \cdot \nabla) U - \omega (\nabla \cdot U) + \frac{1}{\rho^2} (\nabla \rho \times \nabla p) + \nu \nabla^2 \omega. \tag{1.6}
\]

The third term, i.e. $(\nabla \rho \times \nabla p)/\rho^2$ is the baroclinic term. This corresponds to the vorticity production due to baroclinicity. In case of shock bubble interaction the vorticity is zero everywhere. So all the terms in Equation 1.6 other than the baroclinic term become zero making it an important source of vorticity production. The mechanism for vorticity generation is analogous to that in RMI. Density contrast plays a vital role in the vortex dynamics of the flow. Hence, Atwood number defined as $A = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ is commonly used in characterizing shock bubble interactions. Although $A > 0$ generally refers to the convergent case and $A < 0$ for divergent, this need not be the case every time. It is possible that $\delta \mathcal{R} > 0$ even if $A < 0$ or $\delta \mathcal{R} < 0$ when $A > 0$. 
1.4 Literature Review

The interaction of shock with an interface having density jump will result in Richtmyer Meshkov Instability (RMI). This instability occurs when an interface of two fluids is accelerated with an impulse which is generally a shock wave. It has wide applications in Astro Physics, combustion in Scramjet engine and Inertial Confinement Fusion (ICF). Several detailed experimental, numerical and theoretical works on SBI are reported in literature [1–13].

There are several numerical works on SBI. Quirk and Karni [8] studied the effect of a weak shock while Jie et al. [14] studied the interaction of planar shock wave and helium bubble for a range of Mach numbers in three dimensions; high order PPM was employed to capture the interface. With increase in Mach number, authors observed a difference in deformation of the bubble. For low Mach numbers, two vortex rings were formed. While at high Mach numbers, only one vortex ring was formed. Similar observations were reported by Bagabir and Drikakis [5]. Niederhaus et al. [4] studied the morphology of the compressible flow using three-dimensional Eulerian simulations; a range of Atwood numbers and Shock strengths were investigated. Authors observed secondary vorticity generation and acoustic effects which show the importance of turbulent effects. Hejazialhosseini et al. [15] studied the dynamics of vorticity in shock bubble interactions and its growth rates due to baroclinicity, stretching and dilation for various Mach numbers. A multi-block approach with fine grid in the bubble region was used by Diegelmann et al. [16] due to the wide range of spatial scales involved in this phenomena. Hydrogen bubble ignition using Wavelet Adaptive Multiresolution Representation (WAMR) was simulated by Paolucci et al. [17]. More comprehensive details of WAMR method can be found in the works by Paolucci et al., Wirasaet, Rastigejev and Grenga. [17,21]. A brief description of WAMR algorithm used in this work is given in the Chapter 2.
Chapter 2

Wavelet Adaptive Multiresolution Representation Method

2.1 Introduction to Wavelets

A wavelet is a mathematical function which is spatially localized wave-like oscillation. Its amplitude starts at zero, increases and then decreases back to zero. It acts as a mathematical tool for extracting information from various types of data (e.g. signals, images). A wavelet transform is analogous to Fourier transform, as it gives the information of different scales present in the data, similar to the frequency spectrum in harmonic analysis. The Fourier transform is an important tool for data analysis, but it's not good at handling sudden changes effectively. It only represents translation invariant characteristics of the data accurately and suffers with issues of non-locality. Wavelets solve this problem as they are localized in space as well. Hence, wavelets offer a compact support.

The whole domain is spatially covered by considering translations of a wavelet. This is done by shifting the wavelet by constant distances to form a collection of wavelets at the same level. The shift distance is generally an integer for simplicity.

\[
\psi_a(x) = \psi(x - a) \tag{2.1}
\]

Furthermore these wavelets are scaled to accommodate details at different resolution. Scaling refers to the process of stretching or shrinking a wavelet function.

\[
\psi_{b,a}(x) = \psi\left(\frac{x - a}{b}\right) \tag{2.2}
\]
A family of wavelets indexed by 2 integers \((j, k)\) is formed by binary dilations (i.e. dilation by \(2^j\)) and dyadic translation (i.e. translation of \(k2^{-j}\)) of the wavelet.

\[ \psi_{j,k}(x) = \psi(2^j x - k) \]  

(2.3)

### 2.2 Multi-resolution Analysis

Multi-resolution representation of a function consists of a series of approximating functional spaces \(\{V_j\}\). Consider a nested set of spaces \(\{V_j\}_{j \in \mathbb{Z}} \subset L^2(\mathbb{R})\) such that \(V_j \subset V_{j+1}\). Each space may then be decomposed as \(V_{j+1} = V_j \oplus W_j\). Here, \(W_j\) is the complement of \(V_j\) in \(V_{j+1}\). This when applied recursively generates the multi-resolution decomposition. The bases of space \(V_j\) denoted by \(\phi_{j,k}\) are called as scaling functions and that of space \(W_j\) denoted by \(\psi_{j,k}\) are called as wavelet functions. These bases are constructed by binary dilations and dyadic translations of scaling function \(\phi(x)\) and wavelet function \(\psi(x)\). These functions come in pair and are commonly referred to as father wavelet \(\phi(x)\) and mother wavelet \(\psi(x)\).

Consider the operator \(P_j\) that gives the projection or approximation of a function \(f\) over the space \(V_j\). This approximation can be given by linear combination of the bases of \(V_j\), i.e. the scaling functions at level \(j\) as shown in Equation [2.4]. Then we can get the approximation of the function \(f\) over the next level of resolution \(j+1\) by adding a member of the space \(W_j\) which is given by linear combination of wavelet functions of level \(j\) as in Equation [2.5]. Starting from the coarsest level \((j = 0)\) approximation obtained by the scaling function \(\phi_{0,k}(x)\) and adding linear combination of wavelets of finer levels up to a maximum of \(J-1\) approximation of the function on the finest level can be obtained as given by Equation [2.6]

\[ P_j f = \sum_k f_{j,k} \phi_{j,k} \]  

(2.4)
\[ P_{j+1}f = P_j f + \sum_k \langle f, \psi_{j,k} \rangle \psi_{j,k} \quad (2.5) \]

\[ P_j f = \sum_k f_{0,k} \phi_{0,k}(x) + \sum_{j=0}^{J-1} \sum_k d_{j,k} \psi_{j,k}(x) \quad (2.6) \]

Figure 2.1: Dyadic grid with base grid of 9 points and maximum resolution of level 5

### 2.3 Construction of Adaptive Grid

Consider \( N_0 \) points at the coarsest grid level. Then with the above formulation, each basis function represents a point on a regular grid with location \( \{ x_k = k(N_0 2^J)^{-1} : k \in [0,...,N_0 2^J] \} \). This creates a dyadic grid such that the space between two consecutive points of a particular refinement level is halved at every subsequent level. The amplitudes of wavelet provide an estimate of the local error at each mesh point.
So by employing a threshold for the wavelet amplitudes $\varepsilon$, the wavelet transform of a function is split depending on if the wavelet amplitudes are greater or lesser than the threshold.

$$f^J(x) = f^J_\varepsilon(x) + R^J_\varepsilon(x)$$

(2.7)

$$f^J_\varepsilon(x) = \sum_k f_{0,k}\phi_{0,k}(x) + \sum_{j=0}^{J-1} \sum_{\{k : |d_{j,k}| \geq \varepsilon\}} d_{j,k}\psi_{j,k}(x)$$

(2.8)

$$R^J_\varepsilon(x) = \sum_{j=0}^{J-1} \sum_{\{k : |d_{j,k}| < \varepsilon\}} d_{j,k}\psi_{j,k}(x)$$

(2.9)

Figure 2.2: (a) Function with an abrupt change (b) Corresponding adaptive grid

Leaving out $R^J_\varepsilon(x)$ gives an approximation to $f$ where the error at each point is less than the threshold. This approximation is given by Equation 2.8 and is called the sparse wavelet representation. This means that by leaving out the collocation point whose wavelet amplitude is less than $\varepsilon$, we get an irregular sparse grid. These points are called essential points $\nu_E$ and are given by Equation 2.10

$$\nu_E = \left\{ x_{0,k} \oplus \bigcup_{j>0} x_{j,k} : k \in \{k : |d_{j,k}| \geq \varepsilon\} \right\}$$

(2.10)

The Fast Wavelet Transform (FWT) is used to convert function values to wavelet
amplitudes. This is done by ordered application of Lagrangian interpolating polynomials. The wavelet amplitude at a particular point is evaluated as the difference between the interpolated and the actual value using the values at previous level of refinement as shown in Equation 2.11. The wavelet coefficients for level \( j \) are evaluated only at odd indices using just even indices for the interpolation. These even indices belong to the coarser level of refinement. The stencil used for the interpolation is given by Equation 2.14 [21] and shown in Figure 2.3. The inverse transform (i.e. getting function values from wavelet amplitudes) is calculated as shown in Equation 2.12. These calculations have to proceed from bottom to top (i.e. \( j = 0 \) to \( J - 1 \)) unlike in FWT which can be performed either way.

\[
d_{j,k} = f_{j,k} - \sum_{r=1}^{p} h_r f_{j,k'}
\]  

\[
f_{j,k} = d_{j,k} + \sum_{r=1}^{p} h_r f_{j,k'}
\]  

\[
h_r = \prod_{m=1 \atop m \neq r}^{p} \frac{x_{j,k} - x_{j,k_m}}{x_{j,k_r} - x_{j,k_m}}
\]  

\[
k'_r = 2 \begin{cases} 
0, \ldots, p - 1 & \quad k < p/2 \\
\left\{ \frac{1}{2} (k + 1 - p), \ldots, \frac{1}{2} (k - 1 + p) \right\} & \quad p/2 \leq k \leq N_0 2^j - p/2 \\
\left\{ N_0 2^{j-1} - p + 1, \ldots, N_0 2^{j-1} \right\} & \quad k > N_0 2^j - p/2
\end{cases}
\]  

To be able to perform adaptive fast wavelet transform and its inverse, each mesh point of the adaptive grid at a particular level needs all the points that are required for its interpolation from the previous level. This is called minimum index set. The mesh generated by sparse wavelet representation often does not satisfy this requirement. So,
additional points called the non-essential points have to be included to the irregular sparse grid. These non-essential points $\nu_N$ are given a wavelet amplitude value of zero.

The solution can advect or sharpen with time. So, neighbouring points in scale and space are also included. One level up in scale and one neighbour in space on each side is used. These neighbouring points $\nu_B$ are included in the sparse grid. The solution is advanced in time only for the active points $\nu_A$ which is the union of essential points and neighbouring points.

These methods’ extension for dimensions more than one can be done easily. Multi-dimensional basis functions are obtained by tensor products of one-dimensional wavelets and scaling functions. The index for the location becomes an n-component vector. Similarly, the base grid is also an n-component vector. The wavelet transform
in n-dimensions involves n applications of the one-dimensional transform.

2.4 Spatial Derivatives

Solving partial differential equations involves calculations of spatial derivatives of a function \( f \). Calculation of the spatial derivatives can be done by differentiation of the wavelet basis. But this approach is computationally very expensive. Hence, finite differences are used to estimate these spatial derivatives. The derivative is computed by using a non-uniform stencil of nearest \( m \) points and applying a finite difference formula on it as shown in Equation 2.15. Here \( a_n \) are finite difference coefficients obtained such that this derivative approximation is exact for Lagrange polynomials of order \( m - 1 \). The number of points used for the derivative \( m \) is chosen to be greater than \( p \). But, this does not improve the order of accuracy of the method as it is still steered by the error associated with the grid. For robustness, the ratio of highest and lowest distance between neighbouring grid points in the interpolation stencil must be under 2. This condition is usually satisfied. If it is not satisfied, the function is interpolated.

\[
f'(x_i) \approx Df(x_i) = \sum_n a_n f(x_{i-n}) \tag{2.15}\]

2.5 Time Integration

After the estimation of spatial derivatives, the partial differential equations reduce to a system of ordinary differential equations (ODE). This is subsequently integrated in time, until user specified time is reached. The time integration can be handled by an appropriate ODE integrator. The ODE integrator generally takes an initial time step as input for each integration step and refines this as needed to satisfy a specified tolerance. The initial time step is calculated according to CFL condition. This makes the time step very small. Here standard explicit Runge-Kutta-Fehlberg
(RK45) scheme is used.

2.6 Advantages of WAMR

The finite element methods have good compact support. However, it has bad continuity properties. Spectral methods exhibit a very good global support but bad spatial localization. The wavelet bases unite the advantages of both finite element and spectral methods and are able to attain good spectral and spatial resolution. This method provides a powerful method for steering spatial grid adaptation. The mesh spacing is such that it is very fine where the solution varies abruptly and coarser at locations where the solution variation is slow. Also the wavelet transform provides a first-hand estimate of the local approximation error at each mesh point. The sparse grid generated by this method provides an impressive reduction in computational complexity by significantly reducing the number of points required. In comparison to uniform grid of the same resolution, the number of mesh points used in adaptive grid generated by this method are reduced by factors of multiple orders of magnitude. Typically an adaptive mesh refinement algorithm incorporates up to 4 levels of refinement. But WAMR algorithm can easily achieve 10 levels of refinement.

2.7 Validation

The method is validated using two classic cases of Sod shock tube problem. The results of these are compared with analytical solution of Euler equation on the same problem. Although WAMR solves compressible Navier-Stokes equation the diffusivities are kept low to get near Euler solutions. Euler solutions offers accurate estimates of wave type generated in the shock tube, its speed and location. So this comparison can be used to validate the WAMR algorithm. The domain is a 1D shock tube of length 4 cm. This is split into two halves of 2 cm each and the initial conditions are set as shown in Table 2.1. The simulation is run for 15 µs. Figure 2.4 shows that
WAMR predicts the solution for both the cases accurately.

Table 2.1: Initial conditions for Sod shock tube problem

<table>
<thead>
<tr>
<th>variable</th>
<th>left</th>
<th>right case 1</th>
<th>right case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ in g/cm$^3$</td>
<td>$1 \times 10^{-3}$</td>
<td>$1.25 \times 10^{-4}$</td>
<td>$1 \times 10^{-5}$</td>
</tr>
<tr>
<td>$u$ in cm/s</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p$ in dyne/cm$^2$</td>
<td>$1.01325 \times 10^6$</td>
<td>$1.01325 \times 10^5$</td>
<td>$1.01325 \times 10^4$</td>
</tr>
</tbody>
</table>

Figure 2.4: $\rho$ vs. $x$ in Sod shock tube problem (a) case 1, (b) case 2

2.8 Error Analysis

Here we vary the error threshold and investigate its effect on the solution and on required number of points to achieve the desired accuracy.

Figure 2.5: (a) $\rho$ vs. $x$ in Sod shock tube problem for different error thresholds (b) Number of points used
Chapter 3

Physical Model

3.1 Introduction

This chapter describes the governing equations and constitutive relations used to model compressible flow problems. The details of the simulations of shock bubble interactions like the domain setting, initial conditions and boundary conditions are also comprehensively discussed. The results of these simulations are discussed in subsequent chapters.

3.2 Governing Equations

Unsteady, multi-component, compressible flows are described by the following system of partial differential equations:

\[
\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x_j} (\rho u_j), \quad (3.1)
\]

\[
\frac{\partial \rho u_i}{\partial t} = -\frac{\partial}{\partial x_j} (\rho u_i u_j + p\delta_{ij}) + \frac{\partial \tau_{ij}}{\partial x_j}, \quad (3.2)
\]

\[
\frac{\partial \rho E}{\partial t} = -\frac{\partial}{\partial x_j} [(\rho E + p)u_j] - \frac{\partial}{\partial x_j} (q_j - u_i \tau_{ij}), \quad (3.3)
\]

\[
\frac{\partial \rho Y_k}{\partial t} = -\frac{\partial}{\partial x_j} (\rho u_j Y_k) - \frac{\partial j_{k,j}}{\partial x_j}, \quad k = 1, \ldots, K - 1. \quad (3.4)
\]
This is a system of equations in strong conservative form for a K component mixture. Equations 3.1, 3.2 and 3.3 represent the conservation laws of mass, linear momentum and total energy respectively. Equation 3.4 determines the evolution of species mass fraction of K-1 species. The mass fraction of other species is evaluated from the constraint that sum of mass fractions of all the species must be unity.

\[ \sum_{k=1}^{K} Y_k = 1 \]  (3.5)

Equation 3.5 is usually applied to the species with largest mass fraction. The independent variables are the spatial coordinates \( x_j \) and time \( t \). The dependent variables in their primitive form are density \( \rho \), velocity \( u_i \), pressure \( p \), total energy \( E \), and mass fractions \( Y_k \). The total specific energy is defined as \( E = e + \frac{1}{2}u_iu_i \), where \( e \) is the specific internal energy. Closure to the remaining variables in the above equations is provided by what is called the constitutive relations.

### 3.3 Constitutive Relations

In order to obtain the variables \( p \), \( \tau_{ij} \), \( q_j \) and \( j_{k,j} \), we need additional constitutive relations to complete the system of equations. These equations ensure a complete representation of physical phenomena like conduction and diffusion of mass and momentum.

\[ p = \frac{\rho RT}{M} \]  (3.6)

\[ \tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \left( \kappa - \frac{2}{3} \mu \right) \frac{\partial u_k}{\partial x_k} \delta_{ij} \]  (3.7)

\[ q_i = -\lambda \frac{\partial T}{\partial x_i} + \sum_{k=1}^{K} \left( j_{k,i} h_k - \frac{RT}{M_k X_k} D_k T d_{k,i} \right) \]  (3.8)
\[ j_{k,i} = \frac{\rho Y_k}{M X_k} \sum_{j=1}^{K} M_j D_{kj} d_{j,i} - \frac{D^T_k}{T} \frac{\partial T}{\partial x_i} \] 

(3.9)

\[ d_{k,i} = \frac{\partial X_k}{\partial x_i} + (X_k - Y_k) \frac{1}{p} \frac{\partial p}{\partial x_i} \] 

(3.10)

3.4 Domain

The simulation of shock bubble interactions is performed on a 2D domain of length 3.0 cm and width 0.5 cm. The base grid used for this simulation consists of 60 points along the length and 8 points along the width. The maximum level of refinement allowed is 10. This is an upper limit but there should not be a point on this level of refinement to ensure that more points on higher level of refinements are not needed to achieve the required accuracy. The maximum level of refinement corresponds to a maximum grid of 61440 points along the length and 8192 points along the width.

3.5 Initial Conditions

The shock is placed at \( x_s = 0.3 \) cm and the bubble is centered at \( x_0 = 0.7 \) cm and \( y_0 = 0.0 \) cm. The bubble radius \( r_0 \) is 0.2 cm. The shock is placed at distance from the bubble so that it is allowed to evolve a bit before it impacts the bubble. The shock is initialized using the following equations:

\[ p(x_1, x_2) = \frac{1}{2} \left[ (p_{post} + p_{pre}) - (p_{post} - p_{pre}) \tanh \left( \frac{x_1 - x_s}{\delta_s} \right) \right] \] 

(3.11)

\[ T = \frac{1}{2} \left[ (T_{post} + T_{pre}) - (T_{post} - T_{pre}) \tanh \left( \frac{x_1 - x_s}{\delta_s} \right) \right] \] 

(3.12)

\[ u = \frac{1}{2} \left[ u_{post} - u_{post} \tanh \left( \frac{x_1 - x_s}{\delta_s} \right) \right] \] 

(3.13)
Here $\delta_S$ is the thickness of the shock. The post-shock conditions $p_{\text{post}}$, $T_{\text{post}}$ and $u_{\text{post}}$ are evaluated based on the Mach number of the shock using Rankine-Hugoniot relations. The pre-shock conditions $p_{\text{pre}}$ and $T_{\text{pre}}$ are 1 atm and 300 K respectively. The air and the bubble in pre-shock region are at rest (i.e. $u = 0$).

**Figure 3.1: Domain for simulations with initial location of shock and bubble**

![Figure 3.1: Domain for simulations with initial location of shock and bubble](image)

**Table 3.1: Post-shock conditions for different Mach numbers**

<table>
<thead>
<tr>
<th>$Ma$</th>
<th>$p$ (in dyne/cm$^2$)</th>
<th>$T$ (in K)</th>
<th>$u$ (in cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>$1.534 \times 10^6$</td>
<td>338.6</td>
<td>10608.1</td>
</tr>
<tr>
<td>2.1</td>
<td>$5.047 \times 10^6$</td>
<td>532.2</td>
<td>46978.7</td>
</tr>
<tr>
<td>3</td>
<td>$1.048 \times 10^7$</td>
<td>806.0</td>
<td>77149.7</td>
</tr>
</tbody>
</table>

**Figure 3.2: Initial grid for shock bubble interaction simulations**

![Figure 3.2: Initial grid for shock bubble interaction simulations](image)

$Y_m = 10^{-12}$ is the minimum mass fraction. It is used such that there are at least traces of every species everywhere. For helium/krypton bubble in air (i.e. $Y_{O_2} = 0.232, Y_{N_2} = 0.768$) the mass fractions are initialized as in Equations 3.14, 3.15 and 3.16.
\[ Y_{He/Kr} = \frac{1}{2} \left[ (1 + Y_m) - (1 - Y_m) \tanh \left( \frac{r - r_0}{\delta_b} \right) \right] \] (3.14)

\[ Y_{O_2} = \frac{1}{2} \left[ (0.232 + Y_m) + (0.232 - Y_m) \tanh \left( \frac{r - r_0}{\delta_b} \right) \right] \] (3.15)

\[ Y_{N_2} = 1 - (Y_{He/Kr} + Y_{O_2}) \] (3.16)

In case of nitrogen bubble in air, the mass fractions are initialized according to Equations 3.17 and 3.18.

\[ Y_{O_2} = \frac{1}{2} \left[ (0.232 + Y_m) + (0.232 - Y_m) \tanh \left( \frac{r - r_0}{\delta_b} \right) \right] \] (3.17)

\[ Y_{N_2} = 1 - Y_{O_2} \] (3.18)

Here \( \delta_b \) is the thickness of the bubble and \( r \) is given by Equation 3.19.

\[ r = \sqrt{(x_1 - x_0)^2 + (x_2 - y_0)^2} \] (3.19)

Density and total specific energy can be evaluated using values of pressure, temperature and mass fraction. The initial conditions are set by the variables in its conservative form as a function of spatial location \( (x_1, x_2) \).

### 3.6 Boundary Conditions

The boundary condition is such that top, bottom and right are given free slip and zero normal velocity conditions. For this, a stress boundary condition is set such that \( \tau_{ij} \) is set to zero and velocity boundary condition is set such that the normal velocity zero is specified for those 3 faces. For the left face, inflow boundary condition is
specified. The inflow conditions are determined by post shock conditions of the shock being used in the simulation.
Chapter 4

Results and Discussion

4.1 Shape of the bubble

Figure 4.1 shows the time evolution of the shape of helium bubble in air as it interacts with a shock of Mach number 1.2. Here \( t = 0 \) represents the instant when the shock touches the bubble. The subsequent times that are shown are multiples of \( D/W \). Where \( D \) is the diameter of the bubble and \( W \) is the shock speed. Figure 4.2 and 4.3 shows the time evolution of the shape of krypton bubble and nitrogen bubble in air respectively as they interact with a shock of Mach number 1.2.

Helium bubble in air represents the divergent case. The bubble is deformed such that it is concave towards the shock impact direction. Krypton bubble in air when interacting with a shock. This is a convergent case. The bubble is deformed such that it is convex towards the shock impact direction. The shape of the bubble is mushroom like, unlike in the case of helium bubble which is like a crescent. Krypton bubble also takes longer time to bend. Nitrogen bubble in air represent low density contrast between the bubble and its medium. Here, the bubble undergoes a lateral compression just like other bubbles but the bubble does not bend as in case of helium or krypton.

Figures 4.4, 4.5, 4.6 show the time evolution of shape of helium, krypton, and nitrogen bubbles respectively, as they interact with shock of Mach number 2.1. Their interaction with shock of Mach number 3 is shown in figures 4.7, 4.8, 4.9. It can be noted that with increase in Mach number of the shock although the deformation
occurs faster, the shape pattern of the bubble does not change.

Figure 4.1: Shape of $He$ bubble in air interacting with a shock (Ma=1.2) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.2: Shape of $Kr$ bubble in air interacting with a shock ($Ma=1.2$) (a) time=0, (b) time=$D/W$, (c) time=$2D/W$ (d) time=$3D/W$
Figure 4.3: Shape of $N_2$ bubble in air interacting with a shock (Ma=1.2) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.4: Shape of $He$ bubble in air interacting with a shock (Ma=2.1) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.5: Shape of Kr bubble in air interacting with a shock (Ma=2.1) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.6: Shape of $N_2$ bubble in air interacting with a shock ($Ma=2.1$) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.7: Shape of $He$ bubble in air interacting with a shock ($Ma=3$) (a) time=0, (b) time=$D/W$, (c) time=$2D/W$ (d) time=$3D/W$
Figure 4.8: Shape of Kr bubble in air interacting with a shock (Ma=3) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
4.2 Location and velocity of the bubble

Figure 4.10 shows the location of centre of mass of bubble with respect to time after being impacted by a shock of different Mach numbers. Figure 4.11 shows the velocity of the bubble with respect to time after the impact. Initially the shock accelerates the bubble to different velocities depending on the density of bubble. The lighter bubble (He) gains higher acceleration in a very short time. While the heavier bubble
(Kr) undergoes acceleration for a longer time but has a lower acceleration. The final velocity attained by the bubble is almost same for all the bubbles. The velocity attained is significantly higher for higher Mach numbers but the trends of velocity and location are similar in case of all Mach numbers. In case of helium bubble, the velocity of centre of mass rises to a high value and reduces. This reduction is due the bubble’s shape bending rearwards.

Figure 4.10: Location of bubble with respect to time. Here time = 0 is the instant when shock impacts the bubble (a) Ma = 1.2, (b) Ma = 2.1, (c) Ma = 3, (d) comparison at time = 15µs
Figure 4.11: Velocity of bubble with respect to time. Here \( time = 0 \) is the instant when shock impacts the bubble (a) \( Ma = 1.2 \), (b) \( Ma = 2.1 \), (c) \( Ma = 3 \)
4.3 Pressure

Figure 4.12 shows the time evolution of pressure field for helium bubble in air interacting with a shock of Mach number 1.2. The part of the shock that is not obstructed by the bubble experiences a rise in pressure. Also the part of the shock that passes through helium bubble moves faster. Thus, creating a diverging shock.

Figures 4.13 shows the time evolution of pressure field for Krypton bubble in air interacting with a shock of Mach number 1.2. The part of the shock that is directly obstructed by the bubble experiences a rise in pressure. Also the shock that passes through krypton bubble moves slower compared the part of the shock that is not obstructed by the bubble. Thus, creating a converging shock. At later times, this converging shock causes high pressure at points located after the position of bubble. Thus, exhibiting shock focusing effect.

Figures 4.14 shows the time evolution of pressure field for nitrogen bubble in air interacting with a shock of Mach number 1.2. It can be seen that the the pressure field is as if the bubble is absent. This is because $N_2$ offers low density contrast with air. This means it does not impact the shocks geometry or propagation pattern significantly.

The pressure fields due to interaction of these bubbles with shock of mach number 2.1 are shown in figures 4.15, 4.16 and 4.17. The pressure fields in case of a shock with Mach number 3 are shown in figures 4.18, 4.19 and 4.19.
Figure 4.12: Pressure field of $He$ bubble in air interacting with a shock ($Ma=1.2$) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.13: Pressure field of Kr bubble in air interacting with a shock (Ma=1.2) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W.
Figure 4.14: Pressure field of \( N_2 \) bubble in air interacting with a shock (Ma=1.2) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.15: Pressure field of He bubble in air interacting with a shock (Ma=2.1) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.16: Pressure field of Kr bubble in air interacting with a shock (Ma=2.1) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.17: Pressure field of $N_2$ bubble in air interacting with a shock (Ma=2.1) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.18: Pressure field of He bubble in air interacting with a shock (Ma=3) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.19: Pressure field of $Kr$ bubble in air interacting with a shock (Ma=3) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.20: Pressure field of $N_2$ bubble in air interacting with a shock (Ma=3) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
4.4 Vorticity

Figures 4.21, 4.22 and 4.23 show the time evolution of vorticity field obtained when a shock of Mach number 1.2 interacts with helium, krypton and nitrogen bubble respectively. Subsequently the vorticity field obtained by the interaction with higher Mach numbers 2.1 (4.24, 4.25 and 4.26) and 3 (4.27, 4.28 and 4.29) are shown.

The vorticity generation in case of nitrogen bubble is not significant as the density gradient is very low. The vorticity is produced along the bubble surface for the helium and krypton bubble. The direction of rotation or sign of the vorticity is opposite to each other for helium and krypton bubbles. This is due to flip in sign of density gradient. As Mach number increase the vorticity also increases due to the increase in pressure gradient.
Figure 4.21: Vorticity field for He bubble in air interacting with a shock (Ma=1.2) 
(a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.22: Vorticity field for $Kr$ in air bubble interacting with a shock ($Ma=1.2$)
(a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.23: Vorticity field for $N_2$ bubble in air interacting with a shock ($Ma=1.2$)
(a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.24: Vorticity field for He bubble in air interacting with a shock (Ma=2.1) (a) time=0, (b) time=D/W, (c) time=2D/W, (d) time=3D/W
Figure 4.25: Vorticity field for Kr bubble in air interacting with a shock (Ma=2.1)
(a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.26: Vorticity field for $N_2$ bubble in air interacting with a shock ($Ma=2.1$) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.27: Vorticity field for $He$ bubble in air interacting with a shock ($Ma=3$) (a) time=0, (b) time=$D/W$, (c) time=$2D/W$ (d) time=$3D/W$
Figure 4.28: Vorticity field for $Kr$ bubble in air interacting with a shock ($Ma=3$) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.29: Vorticity field for $N_2$ bubble in air interacting with a shock (Ma=3) (a) time=0, (b) time=D/W, (c) time=2D/W (d) time=3D/W
Figure 4.30 shows the variation of maximum absolute vorticity with Mach Number. The production of vorticity is mainly due baroclinicity. The vorticity in the flow increases due to misalignment between pressure gradient across the shock and density gradient across the bubble. The vorticity increase is higher for higher Mach numbers due to higher pressure gradients. The vorticity generation is higher for helium and krypton bubbles as compared to that of nitrogen bubble. This is due to higher density gradient in case of helium and krypton bubbles.

Figure 4.30: Maximum vorticity at $t = 3D/W$ vs. Mach number for different bubbles
4.5 Effect of Mach number and density contrast

Higher Mach number causes greater acceleration of bubble and also more vorticity production. The distortion of bubble is faster in case of higher Mach numbers. Density contrast is characterized by Atwood number. This primarily governs the morphology of the shock and the bubble. The nonlinear acoustic effects are controlled by this parameter. The vorticity production is least for near zero Atwood numbers. While the acceleration of the bubble is mainly governed by the density of the bubble. Heavier bubble attains lower velocity.

4.6 Computational load

Figure 4.31 shows the time evolution of number of essential points and the corresponding time step required for integration. When shock hits the bubble the number of points required is abruptly increased. Later the as the shock passes through the bubble the number of points is reduced. But later complicated shock propagation pattern and bubble dynamics causes the number of essential points to increase. The time step required for integration is dictated by the lowest spacing between the mesh points. This makes the time step required very low. As number of points increases the smallest grid spacing is reduced and hence time step becomes smaller.
Figure 4.31: Time step and number of essential points vs. time
Chapter 5

Concluding Remarks

Shock bubble interactions acts as a basic configuration to study Shock Accelerated Inhomogeneous Flows (SAIFs). The flow dynamics of these interactions involves multiple scales required to resolve them all. Wavelet Adaptive Multi-resolution Representation method (WAMR) provides a robust technique to generate an adaptive mesh. This mesh is very fine when the solution varies sharply and coarse when the solution varies smoothly. WAMR provides means to adapt the solution to higher number of levels of refinement as compared to conventional AMR technique. Thus making this a suitable choice for simulations of shock bubble interaction. WAMR also provides a direct estimate of the error in the solution as this error is used as an threshold for eliminating points that are not important. This method has been validated using the well-known results of Sod shock tube problems. The shock bubble interaction is modelled using unsteady, compressible, multi-component 2D Navier-Stokes equations in conservative form with detailed diffusive transport model.

The bubble is deformed by a shock forming crescent and mushroom like structures due to density contrast between the bubble and the medium. Although this deformation occurs faster in case of higher Mach number, the shape of the bubble does not depend on Mach number of the shock. The bubble is accelerated by the shock and soon it attains almost constant velocity. The acceleration and the final velocity attained is higher for lighter bubbles. Lighter bubbles like helium in air diverges the incident shock and the heavier bubbles like krypton in air converges the incident shock. Bubbles with nearly same density as that of medium do not significantly af-
fect the shock and the bubble itself is not deformed apart from being compressed. Vorticity is generated due to misalignment of pressure gradient and density gradient. Hence, the vorticity produced is higher for higher Mach numbers or greater density contrasts. The dynamics of shock bubble interactions are mainly characterized by Mach number for shock strength and Atwood number for density contrast.

**Future outlook:** Although WAMR is a very robust AMR technique it has a few limitations. As it resolves higher resolution the grid spacing gets very small. Being an explicit algorithm it suffers the requirement of a very low time step. A better reduction of computational demand can be achieved by incorporating complementary algorithms for temporal adaptivity.
REFERENCES


