Seismic Imaging and Velocity Analysis Using a Pseudo Inverse to the Extended Born Approximation

Thesis by
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In Partial Fulfillment of the Requirements
For the Degree of
Masters of Science

King Abdullah University of Science and Technology
Thuwal, Kingdom of Saudi Arabia

May, 2018
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ABSTRACT

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 Prestack depth migration requires an accurate kinematic velocity model to image the subsurface correctly. Wave equation migration velocity analysis techniques aim to update the background velocity model by minimizing image residuals to achieve the correct model. The most commonly used technique is differential semblance optimization (DSO), which depends on applying an image extension and penalizing the energy in the non-physical extension. However, studies show that the conventional DSO gradient is contaminated with artifact noise and unwanted oscillations which might lead to local minima.

To deal with this issue and improve the stability of DSO, recent studies proposed to use an inversion formula rather than migration to obtain the image. Migration is defined as the adjoint of Born modeling. Since the inversion is complicated and expensive, a pseudo inverse is used instead. A pseudo inverse formula has been developed recently for the horizontal space shift extended Born. This formula preserves the true amplitude and reduces the artifact noise even when an incorrect velocity is used. Although the theory for such an inverse is well developed, it has only been derived and tested on laterally homogeneous models. This is because the formula contains a derivative of the image with respect to a vertical extension evaluated at zero offset. Implementing the vertical extension is computationally expensive, which means this derivative needs to be computed without applying the additional extension. For laterally invariant models, the inverse is simplified and this derivative is eliminated.
I implement the full asymptotic inverse to the extended Born to account for laterally heterogeneity. I compute the derivative of the image with respect to a vertical extension without performing any additional shift. This is accomplished by applying the derivative to the imaging condition and utilizing the chain rule. The fact that this derivative is evaluated at zero offset vertical extension, makes it possible to compute the derivative without applying the extension. I also verify the newly proposed inversion formula on a laterally variant velocity model. In addition, I test the effect of the computed derivative and compare its contribution with the full formula. This additional term has overall limited influence on conventional images. Its largest impact is on vertical reflectors such as salt flanks, granted the velocity is varying laterally in the background as often is in this case. Otherwise, for most applications, we can obtain good quality extended images without this additional term.
ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my supervisors, Prof. Tariq Alkhalifah for his patience, guidance, encouragement and helpful discussions during my thesis research. I would like to sincerely thank my thesis committee, Prof. Daniel Peter and Prof. Omar Knio for the time taken to review this work. My appreciation also goes to my friends and colleagues in the Seismic Wave Analysis Group (SWAG) for all their help and support. Among the SWAG group I would like to give special thanks to Mahesh Kalita, Chao Song, Qiang Guo, Nabil Masmoudi and Bingbing Sun for sharing their knowledge and experience. I want to thank all my friends at KAUST for making this journey remarkable. I want also to express my thanks to all my family for their support and encouragement. Finally, my heartfelt gratitude is extended to the two people who support me the most, my lovely parents, my father Adnan Alali and my mother Layla Alali.
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LIST OF ABBREVIATION

CIGs    Common Image Gathers
DSO    Differential Semblance Optimization
MVA    Migration Velocity Analysis
RTM    Reverse Time Migration
SOCIG  Subsurface-Offset Common Image Gather
WEMVA  Wave Equation Migration Velocity Analysis
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Chapter 1

Introduction

1.1 Motivation

Seismic imaging plays a major role in exploring potential oil and gas traps. It is the process of creating an image of the subsurface layers by sending seismic waves into the ground and recording their reflections at the surface. These images provide crucial information about the subsurface structure and it is one of the most valuable components of the drilling decision process. The cost of drilling for oil can reach a 100s of millions of dollars [2]. Making sure we drill in the right location is crucial to the oil industry economics. An accurate seismic image helps considerably in locating and monitoring oil reservoirs.

The migration algorithm differs based on the complexity of the medium as well as the cost. In simple velocity models, Kirchhoff migration is often used to obtain the image, however, it tends to fail in complex regions [3]. In contrast, wave equation based methods are preferred in complex models. Although it has relatively high cost, reverse time migration (RTM) [4, 5] is widely used as a prestack depth migration in complex regions.

In all of the migration techniques, the kinematic velocity of the underlying structure needs to be well-estimated in order to reconstruct the image correctly. An inaccurate velocity model will lead to imaging the reflectors in incorrect positions and discrepancy in the image. Migration velocity analysis (MVA) techniques are developed to determine the accurate background velocity model needed to produce
the correct image. However, MVA methods still face some difficulties and challenges especially in complex area.

1.2 Historical Development

Traditional MVA methods split the data into subsets of common shot or common offset gathers and then migrate them separately using the same background model [6]. The image is independent of the acquisition geometry which means inconsistency between common image gathers (CIGs) resulting in moveout residuals indicate that the model is inaccurate and needs to be corrected. If the CIGs are consistent, the model is sufficient for migration. This type of MVA is called surface oriented and it is sensitive to kinematic artifacts when the model is complex [6]. Recent approaches for MVA in complex areas (depth-oriented) suggests using the full data set in migration and introducing a spatial or temporal extension in the imaging condition [6–8]. Because wave equation imaging methods are preferred in the complex areas, this type of MVA is referred to as wave equation migration velocity analysis (WEMVA).

In WEMVA the model is split into two parts [9]. The first part controls the kinematics and contains the large-scale components. The second part contains the perturbation of the model. In the Born approximation, the model perturbation linearly depends on the reflected data assuming the perturbation is small relative to the background. The adjoint of the Born operator is in fact the migration formula which is defined by a cross-correlation between source and receiver wavefields [10]. Migration maps the data residual to the model perturbation which can be considered as the reflectively. After migrating the data, the quality of the image is measured based on an objective function and the background velocity is updated if necessary. The common feature in all WEMVA techniques is that they update the velocity based on back projection of image residuals [11]. WEMVA methods differ in defining these image residuals which can be obtained by a penalty function [12]. horizontal contraction
Differential semblance optimization (DSO) introduced by Symes [6] is the most common method used in WEMVA. It is performed in the image domain by applying an image extension and penalizing the energy residing in this non-physical extension [11]. DSO measures the quality of the image in the scattering angel domain or in the subsurface offset domain [15]. In the former, deviation from flatness will indicate inaccurate velocity. In the latter, the velocity is said to be accurate when the energy is focused at the zero subsurface offset. Throughout this thesis, I will only focus on the subsurface offset domain and one can easily move to the angle domain by using the Radon transform [16].

Although DSO shows successful results in some studies [6, 15, 17, 18], it has been proven that it has some drawbacks. According to Fei et al. [19] the gradient of DSO suffers from artifacts and undesired oscillations. In addition, for RTM, the reflected waves generate unwanted artifacts in the image [20] which affect the DSO performance. A further issue can arise from the uneven illumination due to limited acquisition geometry. All these factors affect the optimization process and reduces its chances for a stable convergence [6, 11, 21].

Many studies have addressed this issue, among which I mention the following. Fei et al. [3] reduced DSO artifacts by introducing a 90-degree phase shift to the subsurface offset gathers by applying a deferential operator to the objective function; Lameloise et al. [22] improved the gradient efficiency by using quantitative migration; and most recent studies by Liu et al., Hou et al., Chauris and Cocher [1, 20, 23] suggest using an approximate inverse formula instead of migration to obtain the extended images used in DSO.

RTM is, by definition, the adjoint of Born modeling. The approximate inverse formula would be an approximate inverse to Born modeling or in other words one can call it "true amplitude" migration. In WEMVA, this kind of "true amplitude"
would include the extended subsurface offset \[23\]. Ten Kroode \[24\] inverted a space-shifted Kirchhoff operator and, inspired by this work, Hou et al.\[23\] constructed an approximate inverse for the horizontal space shift extended Born. Following that, Chauris and Cocher \[1\] derived a similar inverse formula based on the inverse of the "Generalized Radon Transform".

The inverse formula developed by Chauris and Cocher \[1\] requires a derivative of the image with respect to a vertical extension \((D_{h_z}I)\) when the velocity changes laterally. Implementing the vertical extension to take its derivative is costly, and not practical to implement in many cases. In laterally homogeneous models, the formula is simplified and the term containing this derivative is eliminated. Thus, Chauris and Cocher \[1\] assumed lateral homogeneity in their implementation to avoid computing \((D_{h_z}I)\).

In this context, this thesis will extend the work of Chauris and Cocher \[1\] to account for the laterally variant models. I implement the full asymptotic inverse formula without the need for computing the additional vertical extension. This can be achieved by applying the derivative to the image with respect to the vertical shift using the chain rule to the imaging condition. The fact that this derivative is evaluated at \(h_{h_z} = 0\) enables us to avoid performing the vertical extension.

### 1.3 Thesis Structure

The structure of the thesis consists of four main parts. The first part is imaging and Born approximation which will provide an overview of RTM and Extended RTM. It will also derive the first order Born approximation and show that its adjoint is the same as RTM. The second part is where I will introduce the the pseudo inverse to the extended Born approximation by Chauris and Cocher \[1\] and review its derivation. I will also show why the homogeneous assumption was made in \[1\]. Following this, I will extend the implementation of Chauris and Cocher \[1\] to account for the laterally
variant velocity models, which is our main contribution in this research. The third part is related to the theory behind WEMVA. I will present the DSO objective function and calculate its gradient theoretically. In the final part, two examples will be discussed. The first is the two-layer model for the lateral homogeneous case, and the second is the well-known Marmousi model as an example of a lateral heterogeneous model. Moreover, I will investigate the effects of the newly computed term \((D_{h_z}I)\) and its contribution to the full operator.
Chapter 2

Imaging and the Born Approximation

This chapter aims to review basic concepts needed throughout this research namely the concept of pre-stack depth migration, velocity analysis by extended images, Born forward modeling and its adjoint. First, we start with an overview of the pre-stack depth migration which has evolved in the recent decays from Kirchhoff to the wave equation based migration. I will focus mainly on reverse time migration (RTM) and explain its algorithm as it is the most advanced migration technique particularly in complex areas. Then, I will review the derivation of the Born approximation and show its relation to the perturbed velocity and the perturbed wavfield. At the end, I will present the adjoint operator of the Born approximation and show that it is equivalent to the RTM imaging condition.

2.1 Pre-stack Depth Migration

In recent decades, wave equation based migration became feasible and preferred to Kirchhoff migration in complex areas. This is because conventional Kirchhoff is based on the high frequency asymptotic approximation and fails to provide a correct image in an area with multi-arrival and shadow zones [3]. Hence, wave equation migration techniques are more reliable to image the complex structures. RTM is a very powerful wave equation migration technique that uses the two-way wave equation to extrapolate the wavefields in time and obtain the image. However, an extension to the image domain is needed in order to evaluate the accuracy of the velocity model
used in migration. In the following two sections we will discuss RTM and extended RTM in more details.

2.1.1 Reverse Time Migration (RTM)

RTM [4] is the most advanced technique used nowadays. Although it is relatively expensive, it has the ability to image complex medium and to handle the turning waves in addition to multiple reflections. RTM is implemented by wavefield extrapolations using the two-way wave equation and by applying an imaging condition. The Earth is in fact an elastic body, however, the acoustic approximation is sufficiently accurate to model the waves, this is a very common approximation in exploration geophysics. Therefore we used the acoustic wave-equation operator to extrapolate the wavefields giving by

\[ F = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \nabla^2, \]  

(2.1)

where \( \mathbf{x} \) represents the spatial coordinates in 2D (\( x,z \)), \( t \) is the time and \( v \) the velocity.

The algorithm of RTM can be summerized in three steps:

1. Forward extrapolation of the source wavefield

2. Backward extrapolation of the receiver wavefield

3. Applying an imaging condition

The forward extrapolation of the source wavefield \( (S) \) is obtained by solving the wave-equation

\[ FS(x, t) = f(x, t), \]  

(2.2)
with absorbing boundary conditions. Here, $f(x, t)$ is the source term. Similarly the receiver wavefield ($R$) is constructed by solving

\[
\mathcal{F}R(x, t) = 0
\]

\[R(x, z = 0, t) = d_{\text{obs}}(x, z, t),\]

in which we inject the recorded data $d_{\text{obs}}$ at the surface and propagate back in time. A very common imaging condition is the zero-lag cross-correlation between the source and receiver wavefield introduced by Claerbout [10] which is given by

\[
I(x) = \int \int S(x_s, x, t)R(x_s, x, t) \, dt \, dx_s.
\]

(2.4)

Here, $x_s$ is the shot coordinate which means that we need to sum over all the shots to obtain the image.

### 2.1.2 Extended RTM

RTM successfully images the subsurface if the correct velocity model is used. However, if the model is inaccurate, the energy will not be focused. Therefore, we need a tool to evaluate the accuracy of the model. One method is to extend the domain of the image by adding an extra dimension; extended images can be obtained either by space lag [7, 25] or time lag [8]. A horizontal space shift is most commonly used and this extension is used throughout this thesis. It can be achieved by simply applying the horizontal shift ($h$), which is defined by the half-offset between the source and receiver in the image space, to the imaging condition in equation [2.4] as follow:

\[
I(x, h) = \int \int S(x_s, x - h, t)R(x_s, x + h, t) \, dt \, dx_s.
\]

(2.5)
The cross correlation imaging condition (equation 2.4) has now become more general as it cross correlates the wavefields in different subsurface offsets. Figure 2.1 illustrates the shift applied to the wavefields where the dashed lines indicate the imaging condition without the extension and the solid lines after the extension. Provided the correct velocity is used, the energy should be focused in the zero subsurface offset ($h_x = 0$). Any Energy residing in the non-physical extension is an indication of an inaccurate background model. For simple models, using a low velocity will result in a downward curvature in the subsurface offset common image gathers (SOCIGs), whereas using a high velocity gives an upward curvature as shown in the example in chapter 5.

![Figure 2.1: Illustration of the horizontal shift in the imaging condition. Dash lines represent the zero subsurface offset which gives the physical image. Solid lines represent the non-physical extension of the image.](image)

2.2 Born Approximation

The total wavefield is not linearly proportional to velocity. To reconstruct the model, we need first to linearize the wave equation. The Born approximation is a linearization of the wave equation based on the first order perturbation theory. In the following sections we will derive the Born modeling operator in addition to its adjoint.
2.2.1 Forward Born Modeling

A point source, say a delta function $\delta(x - x_s)\delta(t)$, in the wave equation (2.1) results in the fundamental Green’s function $G(x, x_s, t)$ that satisfies

$$\left(\frac{1}{v^2(x)} \frac{\partial^2}{\partial t^2} - \nabla^2\right) G(x, x_s, t) = \delta(x - x_s) \delta(t). \quad (2.6)$$

where $x$ is the observed point and $x_s$ is the source location.

The model ($v$) can be split into two parts

$$v(x) = v_o(x) + \delta v. \quad (2.7)$$

The first part $v_o$ is smooth and controls the kinematic behavior of the model. The second part $\delta v$ is the perturbation of the model, which is assumed to be small and contains the detailed structures [9, 15]. Substituting this into equation (2.6) will give

$$\left(\frac{1}{(v_o + \delta v)^2(x)} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \left( G_o(x, x_s, t) + \delta G(x, x_s, t) \right) = \delta(x - x_s) \delta(t). \quad (2.8)$$

Here, $\delta G$ is the perturbed wavefield and $G_o$ is the solution for the smooth model that satisfies

$$\left(\frac{1}{v_o^2(x)} \frac{\partial^2}{\partial t^2} - \nabla^2\right) G_o(x, x_s, t) = \delta(x - x_s) \delta(t). \quad (2.9)$$

Subtracting the last two equations and keeping only the first order Born term we get

$$\left(\frac{1}{v_o^2(x)} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \delta G(x, x_s, t) = \frac{2\delta v(x)}{v_o^3(x)} \frac{\partial^2}{\partial t^2} G_o(x, x_s, t). \quad (2.10)$$

The solution for this equation at the receiver positions $x_r$ can be expressed in an integral form as

$$\delta G(x_r, x_s, t) = \frac{\partial^2}{\partial t^2} \int \frac{2\delta v(x)}{v_o^3(x)} G(x, x_r, t - \tau) G(x, x_s, \tau) \, d\tau \, dx. \quad (2.11)$$
The last result is the Born forward modeling operator, which relates the perturbed wavefield linearly with the perturbed velocity. One can think of it as the mapping operator from the model or the image domain to the data domain. The physical meaning of the operator can be described as follows [6]. The Green’s function $G(\tau, \mathbf{x}, \mathbf{x}_s)$ is the propagation of the wavefield from the source point $\mathbf{x}_s$ to the scattering point $\mathbf{x}$. At the scattering point, the wave interacts with the change in the medium represented by $\delta v$. The scattering is scaled by $\frac{1}{\delta v^3(x)}$ in the space domain and by $\frac{\partial^2}{\partial t^2}$ in the frequency domain. The other Green’s function $G(t-\tau, \mathbf{x}, \mathbf{x}_r)$ is the propagation of the perturbed wavefield resulting from $\delta v$ to the receiver $\mathbf{x}_r$.

Similar to extended RTM, the Born operator can be expressed in the extended domain by

$$
\delta G(\mathbf{x}_r, \mathbf{x}_s, t) = \frac{\partial^2}{\partial t^2} \int \frac{2\delta v(x)}{v^3(x)} G(\mathbf{x} + h, \mathbf{x}_r, t - \tau) G(\mathbf{x} - h, \mathbf{x}_s, \tau) d\tau d\mathbf{x} dh. \quad (2.12)
$$

### 2.2.2 Adjoint of Born Modeling

If I rewrite the Born operator in equation [2.11] as

$$
\delta G(\mathbf{x}_r, \mathbf{x}_s, t) = \mathcal{L} \delta v(x), \quad (2.13)
$$

with

$$
\mathcal{L} = \frac{\partial^2}{\partial t^2} \int \frac{2}{v^3(x)} G(\mathbf{x}, \mathbf{x}_r, t - \tau) G(\mathbf{x}, \mathbf{x}_s, \tau) d\tau d\mathbf{x}, \quad (2.14)
$$

$\mathcal{L}$ is a convolution between the Green’s functions. Therefore, the adjoint for the forward Born modeling should be a cross-correlation and can be written as

$$
\mathcal{L}^* = \frac{\partial^2}{\partial t^2} \int \frac{2}{v^3(x)} G(\mathbf{x}, \mathbf{x}_r, \tau - t) G(\mathbf{x}, \mathbf{x}_s, \tau) d\tau d\mathbf{x}_s d\mathbf{x}_r. \quad (2.15)
$$
Applying the adjoint to the data and summing over time gives

\[ I(x) = \int \frac{2}{v^3(x)} \frac{\partial^2}{\partial t^2} G(x, x_r, \tau - t) \delta d(x_s, x_r, t) G(x, x_s, \tau) \, d\tau \, dx_s \, dx_r \, dt. \quad (2.16) \]

Extending this to the subsurface offset domain reads

\[ I(x, h) = \int \frac{2}{v^3(x)} \frac{\partial^2}{\partial t^2} G(x + h, x_r, \tau - t) \delta d(x_s, x_r, t) G(x - h, x_s, \tau) \, d\tau \, dx_s \, dx_r \, dt. \quad (2.17) \]

Note the use of \( I(x) \) to denote the results of equations 2.4 and 2.16, and \( I(x, h) \) for equations 2.5 and 2.17. The reason for using the same notation is that the two expressions are actually similar. Except for the time derivatives and the factor \( \frac{2}{v^3} \), we can retrieve equation (2.4) by defining the source and receiver wavefields as

\[
S(x_s, x, t) = G(\tau, x, x_s) \\
R(x_s, x, t) = \int G(x, x_r, \tau - t) \delta d(x_s, x_r, t) \, dx_r \, d\tau.
\quad (2.18)
\]

The same definitions are applied in the extended domain by adding the subsurface offset to obtain equation 2.5. Thus, RTM is defined as the adjoint of the Born modeling.
Chapter 3

Pseudo Inverse to the Extended Born

This chapter is the core of this thesis. It will present the approximate inverse to the extended Born operator derived by Chauris and Cocher [1]. I will first review the derivation of the formula. To avoid a high computational cost and simplify the pseudo inverse formula, Chauris and Cocher [1] assumed a laterally invariant velocity model in their examples. I will show the reason behind this assumption and provide numerical examples for the case of a laterally homogeneous medium. Finally I will explain a method that enables us to implement the full asymptotic pseudo inverse of the extended Born to account for lateral heterogeneous models without going through expensive computations.

In the previous chapter, we introduced the idea of the extended domain to assess the quality of the image. Another important reason for the extension is that it is not possible to reconstruct the image when the velocity is inaccurate due to lack of information or in other word lack of dimensions. Therefore, the number of dimensions of the data space and the image space need to be equivalent. Table 3.1 illustrates the dimensions for the model and the image spaces in 1D, 2D and 3D [1]. If the model and data space have the same dimensions, then it is possible to recover the observed data from the reflectively even with incorrect model [6]. Thus the approximate inverse formula must be in the extended domain.
Dimensions | Data domain | Model domain | Extended model domain
---|---|---|---
1D | $t$ | $z$ | $z$
2D | $(x_s, x_r, t)$ | $(x, z)$ | $(x, z, h)$
3D | $(x_s, y_s, x_r, y_r, t)$ | $(x, y, z)$ | $(x, y, z, h_x, h_y)$

Table 3.1: Dimensions of data and model domains where $(x_s, y_s, x_r, y_r, t)$ are the source and receiver coordinates, $t$ is the time, $(x, y, z)$ are spatial axes and $h$ represents the subsurface offset \[1\].

### 3.1 Derivation of the Pseudo Inverse

Ten Kroode \[24\] in 2012, derived a true amplitude imaging by inverting a space shift extended Kirchhoff operator. Later in 2015 and inspired by the work of Ten Kroode \[24\] work, Hou et al. \[23\] were the first to construct an approximate inverse to the horizontal space shift extended Born. Following this development, Chauris and Cocher \[1\] derived a similar inversion formula based on the inverse of the ”Generalized Radon Transform”. Here we will review the derivation of the pseudo inverse to the extended Born operator by Chauris and Cocher \[1\]. First let’s rewrite equation \[2.12\] in the frequency domain as

$$
\beta_o(\xi)(x, h) = -(i\omega)^2 \Omega(\omega) \times \int G(x_s, x - h, \omega) \xi(x, h) G(x + h, x_r, \omega) \, dx dh.
$$

Here $\Omega(\omega)$ which is a source term usually a Ricker wavelet is introduced. The operator is denoted by $\beta_o$ as a reminder that is the forward Born. $\xi$ represents the reflectivity which is $\frac{\delta w}{\delta x}$ in equation \[2.12\].

Hou et al. \[23\] suggest a general form for the inverse operator given by

$$
\beta_{inv}(\delta d)(x, h) = k(x, h) D_p \int (i\omega)^{\nu} \tilde{\Omega}(\omega) D_s G^*(x_s, x - h, \omega) \delta d(x_s, x_r, \omega) \\
D_r G^*(x + h, x_r, \omega) \, dx_s dx_r d\omega,
$$

where $G^*$ is the Green’s complex conjugate, $D_s$ and $D_r$ are vertical derivatives with
respect to source and receiver positions respectively, $D_p$ is a derivative operator with respect to the variable $p$, and $\hat{\Omega}(\omega)$ is the inverse of the wavelet. Our goal is to determine the values of $p$ in $D_p$, $\nu$ in $(i\omega)^\nu$ and $k(\mathbf{x}, h)$ such that it compositions with the forward operator $3.1$ result in the identity, $\beta_o \circ \beta_{\text{inv}} = I$. We can formulate the product of the two operator as

$$ (\beta_{\text{inv}} \times \beta_o) \xi(y) = \int \Gamma(y, y') \xi(y') \, dy', $$

(3.3)

where $y = (\mathbf{x}, h)$. $B_{\text{inv}}$ will be an inverse in this case if $\Gamma(y, y') = \delta(y - y')$.

Before starting the derivation, certain approximations need to be made. Under the high frequency approximation, the Green’s function in the frequency domain is given by

$$ G(x_s, \mathbf{x}, \omega) \approx A(x_s, \mathbf{x}) e^{i\omega T(x_s, \mathbf{x})}, $$

(3.4)

where $T(x_s, \mathbf{x})$ is the travel time that solves the eikonal equation and $A(x_s, \mathbf{x})$ is the amplitude which is a solution to the transport equation. From one can obtain

$$ D_z G(x_s, x - h, \omega) G(x_r, x + h, \omega) \approx i\omega \left( \frac{\cos(\theta_s)}{v_-} + \frac{\cos(\theta_r)}{v_+} \right), $$

(3.7)

$$ D_h G(x_s, x - h, \omega) G(x_r, x + h, \omega) \approx i\omega \left( -\frac{\cos(\theta_s)}{v_-} + \frac{\cos(\theta_r)}{v_+} \right). $$

(3.8)

Note that $\frac{\partial T(x_s, \mathbf{x})}{\partial z} = \frac{\cos(\alpha)}{v_s}$. $D_z$ and $D_h$ in (3.7 and 3.8) are derivative operators with respect to the vertical axis and the vertical extension respectively. $\alpha_s$ and $\alpha_r$ are the take-off angles from source and receiver positions respectively, $\theta_s$ and $\theta_r$ are the incident angles associated with source and receiver positions at the image points $\mathbf{x-h}$.
and \(x+h\) respectively, \(v_s\) is the velocity at the source position and \(v_r\) is the velocity at the receiver. \(v_+\) and \(v_-\) are shifted velocities defined by \(v_+ = v(x+h)\) and similarly \(v_- = v(x-h)\).

We want to find the \(\Gamma\) operator in equation 3.3 which is obtained by the product of the forward 3.1 and the inverse 3.2 operators. using the above approximation we can obtain \(\Gamma\) by

\[
\Gamma(y,y') = k(x,h)\int (i\omega)^{2+\nu} \tilde{\Omega}(\omega) \left| \frac{\partial(x_s,x_r,\omega)}{\partial k} \right| A^2(x_s,x-h) A^2(x_r,x+h) e^{ik \cdot (y'-y)} \cos(\alpha_s) \cos(\alpha_r) \frac{v_s}{v_r} d\mathbf{k}.
\]

(3.9)

I implement a change of variables from \((x_s,x_r,w)\) to \(\mathbf{k}\) where \(\mathbf{k}\) is the wavenumber vector \(\mathbf{k} = (k_x,k_h)\).

To evaluate the determinant we use the relation between \(\mathbf{k}_s\) and \(\mathbf{k}_r\) with the incident angles \(\theta_s\) and \(\theta_r\), yeilding

\[
\left| \frac{\partial \mathbf{k}}{\partial (x_s,x_r,\omega)} \right| = \left| \frac{\partial \mathbf{k}}{\partial (\theta_s,\theta_r,\omega)} \right| \left| \frac{\partial \theta_s}{\partial x_s} \right| \left| \frac{\partial \theta_r}{\partial x_r} \right|
\]

\[
= -\frac{(i\omega)^2}{v_+ v_-} \left| \frac{\partial \theta_s}{\partial s} \right| \left| \frac{\partial \theta_r}{\partial r} \right| \times \begin{vmatrix} \cos(\theta_s) & \cos(\theta_r) & \frac{\sin(\theta_s)}{v_+} + \frac{\sin(\theta_r)}{v_-} \\ -\sin(\theta_s) & -\sin(\theta_r) & \frac{\cos(\theta_s)}{v_-} + \frac{\cos(\theta_r)}{v_+} \\ -\cos(\theta_s) & \cos(\theta_r) & -\frac{\sin(\theta_s)}{v_-} + \frac{\sin(\theta_r)}{v_+} \end{vmatrix}
\]

\[
= 2\frac{(i\omega)^2}{v_+ v_-} \left| \frac{\partial \theta_s}{\partial s} \right| \left| \frac{\partial \theta_r}{\partial r} \right| \times \left( \frac{\cos(\theta_s)}{v_+} + \frac{\cos(\theta_r)}{v_-} \right).
\]

(3.10)

Referring to equations 3.7 and 3.8 that define the the application of \(D_z\) and \(D_{hz}\), you will notice that the model \(v_-\) is associated with \(\cos(\theta_s)\) and is estimated at \(x-h\). However, in the determinant we see the term \(\frac{\cos(\theta_s)}{v_+} + \frac{\cos(\theta_r)}{v_-}\) associate \(\cos(\theta_s)\) with \(v_+\) meaning it should be estimated at \(x+h\). The same condition applies to the second term related to the receiver. The operator \(D_p\) in equatoin 3.14 is defined by
a linear combination of $D_z$ and $D_{hz}$ such that it removes the effects of $\frac{\cos(\theta_s)}{v_+} + \frac{\cos(\theta_r)}{v_-}$ that appears in the determinant. Finally the amplitude expressions are given by

$$A^2(x_s, x - h) = \frac{1}{8\pi} \frac{1}{v_s \cos(\beta_s)} \left| \frac{\partial \theta_s}{\partial s} \right|, \quad (3.11)$$

$$A^2(x_r, x + h) = \frac{1}{8\pi} \frac{1}{v_r \cos(\beta_r)} \left| \frac{\partial \theta_r}{\partial r} \right|. \quad (3.12)$$

Combining all these equations allows construction of the asymptotic inverse to the extended Born which is given by [1]

$$\beta_{inv}(x, h) = \frac{32}{v_- v_+} D_p \int \frac{\tilde{\Omega}(\omega)}{i\omega} D_{s_z} G^*(x_s, x - h, \omega)$$

$$\delta d(x_s, x_r, \omega) D_{rz} G^*(x + h, x_r, \omega) \, dx_s dx_r d\omega, \quad (3.13)$$

with

$$D_p = \frac{1}{2} \left( \frac{v_-}{v_+} + 1 \right) D_z + \frac{1}{2} \left( \frac{v_+}{v_-} - 1 \right) D_{hz}. \quad (3.14)$$

3.2 Laterally Homogeneous Models

$D_p$ in [3.14] contains two derivatives which are $D_z$ taken at the image points and $D_{hz}$ evaluated at $hz = 0$. Implementing the vertical extension to take its derivative is costly, and impractical to implement in many cases. Thus, Chauris and Cocher [1] assumed lateral homogeneity in their implementation to avoid computing $D_{hz}$.

If the velocity model does not change laterally, equation [3.13] is simplified because of the fact that $v_+ = v_- \text{ in laterally invariant media. Equation [3.13] becomes}$

$$\beta_{inv}(x, h) = \frac{32}{v^2} D_z \int \frac{\tilde{\Omega}(\omega)}{i\omega} D_{h_z} G^*(x_s, x - h, \omega)$$

$$\delta d(x_s, x_r, \omega) D_{rz} G^*(x_r + h, \omega) \, dx_s dx_r d\omega. \quad (3.15)$$

The main advantage of this simplification is that the operator $D_p$ in [3.14] simplified
to only $D_z$ and the term containing $D_{hz}$ is eliminated.

### 3.3 Lateral Heterogeneous Models

In this section, we implement the full asymptotic pseudo inverse of the extended Born (3.13) and account for lateral heterogeneity. To compute the full inverse without applying the vertical extension, we propose to compute $D_{hz}$ by using the chain rule in the extended imaging condition. The image is given by the zero lag cross correlation between the source and back-propagated receiver wavefields [10]. An extended image in both horizontal ($h_x$) and vertical ($h_z$) directions is given by

$$I(x, z, h_x, h_z) = \int S(x - h_x, z - h_z, t) R(x + h_x, z + h_z, t) dt. \quad (3.16)$$

The derivative of the image with respect to $h_z$ evaluated at $h_z = 0$ can be computed as

$$\frac{\partial I(x, z, h_x, h_z)}{\partial h_z} \bigg|_{h_z=0} = \frac{\partial I}{\partial z} \frac{\partial z}{\partial h_z} \bigg|_{h_z=0}$$

$$\Rightarrow I_{h_z}(x, z, h_x, 0) = \int \left( \frac{\partial R}{\partial z} S - \frac{\partial S}{\partial z} R \right) dt. \quad (3.17)$$

Here I denote $S(x - h_x, z, t)$ by $S$ and $R(x + h_x, z, t)$ by $R$. The vertical derivatives of the source and receiver wavefields are evaluated at the image points. The fact that $D_{hz} I$ is evaluated at $h_z = 0$ makes it possible to evaluate the derivative without applying any additional extension. For consistency, $D_z$ can be computed in a similar way by

$$I_z(x, z, h_x) = \frac{\partial I(x, z, h_x)}{\partial z} = \int \left( \frac{\partial R}{\partial z} S + \frac{\partial S}{\partial z} R \right) dt. \quad (3.18)$$

The vertical derivative of the source and receiver wavefields in the case of horizontal or nearly horizontal reflectors is almost the same. Hence $D_z$ in equation 3.18 is relatively large and $D_{hz}$ in equation 3.17 is very small. On the contrary, if there are vertical
reflectors, such as salt flanks, the image does not change with depth and in this case, $D_z$ would be extremely small and, applying a vertical shift would yield a change in the image and make $D_{hz}$ relatively large.

### 3.4 Implementation Details

The approximate inverse formula (3.13) is different to the adjoint (RTM) (2.16) in three factors [1]. The first factor is the vertical derivatives with respect to the source and receiver positions. They are implemented by using a dipole source and receiver instead of a point source. The dipole source is implemented by having two point sources. One is shifted up with a negative sign and the other is shifted down by a distance of $\Delta z$. The dipole receiver is implemented in a similar way. Figure 3.4 illustrates the layout of the dipole source and receiver wavefields. Their effect amounts to applying cosines of take-off angles at the sources and receivers positions. The second factor is the integration of the inverse wavelet $\tilde{\Omega}_{iw}$. This is implemented by constructing the source wavefield using the deconvolved version of the source wavelet and integrate it with respect to time before the cross-correlation. The final factor is a vertical derivative applied to the result of the cross-correlatio which amount to applying cosines of the half-opening angle at the image point.
Figure 3.1: Dipole source and receiver wavefields, the red line indicates the negative pole while the green line represents the positive pole.
Chapter 4

Wave Equation Migration Velocity Analysis WEMVA

4.1 Differential Semblance Optimization DSO

The target of WEMVA is to assure that the model is accurate to produce the most superior image. The most commonly used technique in WEMVA is differential semblance optimization (DSO). In the subsurface-offset domain, the DSO objective function takes the form

$$J[v] = \frac{1}{2} \left\| h I(x, h) \right\|^2 = \frac{1}{2} \int h^2 I^2(x, h) \, dx \, dh.$$  \hspace{1cm} (4.1)

The problem becomes a non-linear optimization problem in which the goal is to minimize $J$ with respect to the model velocity. The objective function is minimized by penalizing the energy residing in the nonphysical extension (i.e. away from the zero subsurface-offset $h = 0$). The image operator $I$ in the objective function can be obtained by either the adjoint operator (RTM) or by the psuedo inverse operator. Line search methods can be used to solve such a problem by updating the model iteratively using the gradient $g$

$$v_{i+1} = v_i - \alpha g_i,$$  \hspace{1cm} (4.2)

where $i$ is the iteration index and $\alpha$ is an appropriate step-length. To speed up the convergence, the conjugate gradient or the limited-memory Broyden-Fletcher-Goldfrab-Shanno (L-BFGS) algorithms could be applied.
4.2 Calculation of DSO gradient

The gradient of DSO is often computed using the adjoint-state method \[27\] which expands the domain of the model-space and introduces new variables. Here I will only give the equations needed to compute the gradient. For more details on the derivation please refer to \[27\] or \[1\]. The adjoint variables $\mu_s$, $\mu_r$ and $\lambda$ are defined as: $\mu_s$ is the adjoint-state source wavefield, $\mu_r$ is the adjoint-state receiver wavefield and $\lambda$ is the image residual. The gradient is given by cross-correlation of the source wavefield (S) with the backpropagation of the source adjoint-state wavefield ($\mu_s$) combined with the cross-correlation of the receiver wavefields (R) with its adjoint-state ($\mu_r$).

$$\nabla_v J = - \int \frac{\partial^2}{\partial t^2} S(x_s, x, t) \mu_s(x_s, x, t) \, dx_s \, dt$$
$$- \int \frac{\partial^2}{\partial t^2} R(x_s, x, t) \mu_r(x_s, x, t) \, dx_s \, dt. \quad (4.3)$$

Let $\mathcal{F}$ be the wave equation operator

$$\mathcal{F} = \frac{1}{v^2(x)} \frac{\partial^2}{\partial t^2} - \nabla^2. \quad (4.4)$$

The adjoint-state wavefields can be found by solving the following equations.

$$\lambda(x, h) = h^2 I(x, h), \quad (4.5)$$

$$\bar{\mathcal{F}} \mu_s(x, h) = \int \lambda(x + h, h) R(x_s, x + 2h, t) \, dh, \quad (4.6)$$
$$\mathcal{F} \mu_r(x, h) = \int \lambda(x - h, h) S(x_s, x - 2h, t) \, dh, \quad (4.7)$$

$\bar{\mathcal{F}}$ indicates the backward propagation operator.
Chapter 5

Numerical Example

5.1 Two-layer Model

In this section I implement the pseudo inverse operator (equation [3.15]) and compare it with the conventional RTM (equation [2.5]) on a two-layer model as an example of the lateral invariant velocity. A similar model was implemented in [1]. The velocity of the first layer is 2.5 km/s whereas the velocity for the second layer is 3.0 km/s. The reflector is located at depth of 1 km. We synthetically employ 81 shots evenly spaced at 50 m, and use a 15 Hz Ricker wavelet for the source. The data were recorded using 401 receivers placed every 10 m on the surface. To simulate the data we used a 2-6 finite difference scheme on a 401x201 grid with a 10 m spatial interval. Three cases of the background velocity were investigated which are the correct velocity \( v_c = 2.5 \) km/s, a lower velocity \( v_l = 2.35 \) km/s, and a higher velocity \( v_h = 2.65 \) km/s. Figure 5.1 shows RTM images and SOClGs taken at \( x=2 \) km for the three cases. The first row corresponds to the correct model while the second and the third represent the lower and the higher velocity cases respectively. We can see that in the incorrect velocity cases, the reflector is not in its true position and the images are not focused. In addition, low frequency artifacts appear around the reflector. Due to the acquisition geometry, migration smiles appear in the images. In a similar manner, Figure 5.2 contains the images in addition to SOClGs produced using the pseudo inverse operator (equation [3.15]). The migration smiles and the majority of the artifacts are removed. This can be easily be shown by looking at the corresponding
Figure 5.1: Imaging and SOCIG taking at x=2km using adjoint (RTM) operator. First row using correct velocity ($v_c = 2.5$), second row using low velocity ($v_l = 2.35$) and the last row using high velocity ($v_h = 2.65$).
Figure 5.2: Imaging and SOCIG taking at x=2km using inverse operator. First row using correct velocity \((v_c = 2.5)\), second row using low velocity \((v_l = 2.35)\) and the last row using high velocity \((v_h = 2.65)\).
5.2 Marmousi Model

To test the inverse formula (3.13) on laterally heterogeneous media, I apply it on the Marmousi model given by Figure 5.3. I used 61 shots spaced 150 m apart, and used a 15 Hz Ricker wavelet for the source. The data were recorded using 903 receivers spaced 10 m apart. The formula is tested in two cases: 1) Using a smoothed model of the Marmousi as a correct background velocity Figure 5.4. 2) Using a lower background velocity by scaling down the smoothed version by a factor of 0.9. For a reliable comparison with the images, I take the first derivative for the model to see the reflectors clearly and plot the result in Figure 5.5.

![Marmousi model](image1)

**Figure 5.3:** Marmousi model.

![Smooth Marmousi model](image2)

**Figure 5.4:** Smooth Marmousi model.
Figure 5.5: Marmousi’s reflectivity section.

The comparison between RTM and the inverted image for the first case is given in Figure 5.6 and for the second case in Figure 5.7. In both figures, the first row represents the RTM image, the second row shows the image produced using the inverse formula and the final row is the SOCIG taken at x=4.5 km for the RTM (left) and the inverted image (right). For both cases the inverted images have higher resolution and fewer artifacts than the RTM images. This can be clearly seen from the SOCIGs. Note that the inverted images illuminate the shallow weak reflectors better than the RTM image. This is because of the amplitude factor $\frac{32}{v_+ v_-}$ in equation (3.13) which gives a higher weight in areas with low velocity. An important point is that when the velocity used in the migration is inaccurate (Figure 5.7), the SOCIG for the inverted image clearly has less artifacts in the unfocused energy than the RTM. This will certainly improve the DSO gradient and lead to faster convergence.

The contribution of $(D_{h_z} I)$ term

In the previous sections, I account for lateral heterogeneity by computing the term containing the derivative of the image with respect to the vertical extension $(D_{h_z} I)$ in equation (3.14). At the zero subsurface-offset where the image is taken, there is no shift in the velocity and that makes the inverse behaves as the homogeneous case
(equation 3.15). Therefore the contribution of $D_{h_z} I$ term will be only in the non-physical images, namely the SOCIGs. According to the formula in section (3.3), we expect to see the largest effect and contribution of this term in the areas with dipping reflectors.

In order to investigate the role of $D_{h_z} I$ and its contribution to the full operator compared to $D_z I$, I display the SOCIGs for both operators separately, in addition to displaying the full operator, which includes both. I extract the SOCIGs for locations $x=2$ km; where the layers are gently dipping, $x=4.5$ km; where the model has steep dips, and $x=6.5$ km, as shown in Figures 5.8 and 5.9.

By examining SOCIGs for the correct velocity in Figure 5.8 we found that, unlike $D_z I$, using $D_{h_z} I$ will not focus energy to zero offset at $x=2$ km and it is almost noise which comes from the fact that the layers are nearly horizontal in that position. For the SOCIGs taken at 4.5 and 6.5 km, we see focusing at the zero offset for both $D_z I$ and $D_{h_z} I$. For the low velocity case (Figure 5.9), the frown shape of the SOCIGs of $D_{h_z} I$ is not clear and its magnitude is lower than that in $D_z I$. In general, $D_{h_z} I$ does not show a noticeable contribution to the full $D_p$ operator as demonstrated in Figures 5.8 and 5.9 which suggests it may be ignored at least in this model.
Figure 5.6: Comparison between the RTM image (first row) and the inverted image (second row). The last row is the SOCIG for RTM and inversion respectively taken at x=4.5 km for the correct velocity case.
Figure 5.7: Comparison between the RTM image (first row) and the inverted image (second row). The final row is the SOCIG for RTM and inversion respectively taken at x=4.5 km for the low velocity case.
Figure 5.8: SOCIGs for the inverted image taken at $x=2$, 4.5 and 6.5 km respectively.
Figure 5.9: SOCIGs for the inverted image taken at x=2, 4.5 and 6.5 km respectively.
Chapter 6

Concluding Remarks

6.1 Summary

In this thesis, I reviewed the asymptotic inverse to the extended Born derived by Chauris and Cocher [1] and showed its effectiveness for lateral homogeneous models. I then extend the implementation of the formula to account for lateral heterogeneous models. The obstacle for the pseudo inverse equation was that it contains a derivative for the image with respect to a vertical extension in lateral heterogeneous models. The aim was to compute this derivative without implementing the vertical shift which is very costly and impractical to compute. I succeeded in performing this derivative by applying it to the imaging condition and utilizing the dependency of the source and receiver wavefields on the vertical shift by the chain rule. I verified the approach using the Marmousi model. I compared the image obtained using classical RTM with the image obtain with the pseudo inverse formula. I validated that the inverse formula gives a better image than the classical RTM; it increases the resolution and reduces the artifacts even if inaccurate velocity is used.

Moreover, I investigated the role of the newly computed term. The final image is a linear combination of two terms; the first one takes the vertical derivative of the image. The second term has the derivative of the image with the vertical space shift. I investigated the subsurface-offset image gathers of the two terms separately and after combining them, and concluded that the second term is exceptionally small compared to the first one and therefore can be neglected.
6.2 Future Research Work

In chapter 4 of this work, I presented the theoretical part of DSO, however, I have not shown any numerical results. This research can be extended by investigating the DSO gradient of the asymptotic inverse operator in the heterogeneous case; Chauris and Cocher [1] have already proved the efficiency of the operator in the lateral invariant models. Hou et al. [28] tested the WEMVA update on the Marmosi model for the classical RTM compared to the update for an inverse Born operator which is different than the one discussed in this thesis. Their results showed a dramatic increase in the performance of the DSO update. It is expected from the pseudo inverse in equation 3.13 to give comparable results. After implementing the formula successfully on synthetic data sets, it can be applied to real data examples provided that the data is well processed and free of multiples. Obtaining the background model is also an important step in full waveform inversion (FWI). This work can be integrated with the application of FWI to assist in obtaining the large-scale of the velocity update.
REFERENCES


2015.


APPENDICES

A Papers Submitted

• Alali A, Alkhalifah T, and Sun B, “Accounting for Lateral Heterogeneity in a Pseudo Inverse Operator of the Extended Born”, Submitted to the 80th annual EAGE meeting /Journal Name, accepted.