Discontinuous Galerkin Time-Domain Analysis of Power/Ground Plate Pairs with Wave Port Excitation

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***Abstract*—In this work, a discontinuous Galerkin time-domain method is developed to analyze the power/ground plate pairs taking into account arbitrarily shaped antipads. To implement proper source excitations over the antipads, the magnetic surface current expanded by the electric eigen-modes supported by the corresponding antipad is employed as the excitation. For irregularly shaped antipads, the eigen-modes are obtained by numerical approach. Accordingly, the methodology for the S-parameter extraction is derived based on the orthogonal properties of the different modes. Based on the approach, the transformation between different modes can be readily evaluated.**

I. Introduction

Similar to the on-chip interconnects, vias are also extensively employed as the off-chip interconnects in present 3-D chip designs. As the propagating of high-speed signals through the vias, unintentional radiation will be generated. As a result, strong coupling will arise among neighboring vias and simultaneously cause via-plate mode conversion and then excite propagating modes between the adjacent power-ground planes, thus leading to serious singal/power integrity issues such as the voltage fluctuation and signal distortion. Furthermore, the parasitic propagating modes would rise high order resonant modes between the plate-pairs, thereby resulting in strong edge radiation related electromagnetic interference concerns. To ensure the first-time system design success, all these problems have to be fully considered. With these aims, a large number of treatments have been proposed over the past years such as the general scattering method based on the Foldy-Lax scattering equation and mode expansion of Green's function[1], the hybrid finite-element method (FEM) [2], and the equivalent circuit models [3].

In this work, a discontinuous Galerkin time-domain (DGT-D) method [4] is developed to analyze the multilayer power-ground plate-pairs. In the DGTD analysis, the solutions across the interface of neighboring domains are allowed to be discontinuous, thus which enable the flexibility in using different basis functions in different domains. To guarantee an unique solution, the numerical flux derived from the Rankine-Hugoniot jump relation is used for information exchange. With these properties, the whole computational domain can be solved in an element-by-element scheme. Similar to finite element method (FEM), high-order hierarchical basis functions can be utilized to approximate the solution in a local manner. Consequently, DGTD integrates the advantages of finite volume method (FVM) and FEM.

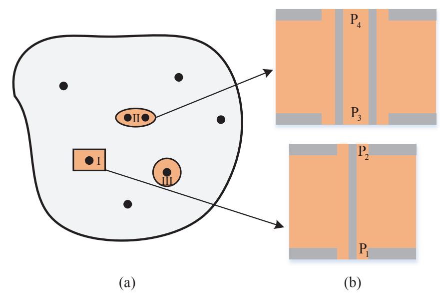

Fig. 1. Geometrical illustration of a power-ground plate-pair with various types of vias in different anti-pads. (a) The Top view of the plate-pair. (b) The side view of via I and II.

In order to enforce proper wave port excitation over the anti-pad, the magnetic current source used in this work is expanded by the eigen-modes supported by the anti-pad. In this way, only the modes of interests can be used for excitation, which makes the proposed algorithm more flexible in studying the coupling and conversion between different modes, etc. Usually, the anti-pads are in an irregular shape or have many via barrels. In this situation, the eigen-modes cannot be derived analytically but must be solved through numerical techniques. By leveraging the orthogonal property of eigen-modes, the incident, transmitted and reflected mode expansion coefficients in the time-domain can be accurately extracted. Then, the S-parameters are able to be obtained based on the pre-calculated mode coefficients.

II. Magnetic Current Based Wave Port Excitation and S-Parameter Extraction

In order to implement proper excitation over the anti-pad region, in this work, the magnetic surface current approximated by the superposition of different electric eigen-modes of the anti-pad is applied as the source. For regular concentric vias, the analytical expressions of the eigen-modes supported by the anti-pad are available, thus it is ready and convenient to apply the excitation. However, for arbitrarily shaped antipads such as rectangular or elliptic pads, the corresponding eigen-modes are not analytically available, thus they have to be numerically calculated. As shown in Fig. 1, we have vias with arbitrarily shaped anti-pads in an irregular plate-pair. It can be noted that there are three typical vias: i) traditional via with a circular anti-pad (via III); ii) normal via located in the center of a rectangular anti-pad (via I); (iii) differential via-pairs placed in an elliptic anti-pad (via II). For via III, the analytical solutions of the eigen-modes are well-known. On the other side, numerical eigen-modes are required for via I and II.

At the wave port P_i $(i = 1, \cdots, 4)$, the tangential components of the electric and magnetic fields can be approximated by the summation of the eigen-modes. Namely,

$$\mathbf{E}_t(\mathbf{r}, \omega) = a_0^{\mathrm{TEM}} \mathbf{e}_0^{\mathrm{TEM}} + \sum_{n=1}^{\infty} a_n^{\mathrm{TE}} \mathbf{e}_n^{\mathrm{TE}} + \sum_{n=1}^{\infty} a_n^{\mathrm{TM}} \mathbf{e}_n^{\mathrm{TM}} \quad (1)$$

$$\mathbf{H}_t(\mathbf{r}, \omega) = b_0^{\mathrm{TEM}} \mathbf{h}_0^{\mathrm{TEM}} + \sum_{n=1}^{\infty} b_n^{\mathrm{TE}} \mathbf{h}_n^{\mathrm{TE}} + \sum_{n=1}^{\infty} b_n^{\mathrm{TM}} \mathbf{h}_n^{\mathrm{TM}}, \quad (2)$$

where $\mathbf{e}_n$ and $\mathbf{h}_n$ denote the electric and magnetic eigen-modes, respectively; a_n and b_n are the corresponding mode expansion coefficients.

To apply the magnetic current excitation over the wave port (or the anti-pad), the electric eigen-modes should be calculated, then the required magnetic current source can be obtain as $\mathbf{M}_s = -\hat{\boldsymbol{\xi}} \times \mathbf{E}_t$ with $\hat{\boldsymbol{\xi}}$ denoting the unit normal vector pointing into the wave port (pointing from side II to I). Then we further expand the he incident electric field at the two sides of the wave port generated by the magnetic current source by the electric eigen-modes:

$$\mathbf{E}_{t,\mathrm{I}}^{\mathrm{inc}} = \tilde{a}_0^{\mathrm{TEM}} \mathbf{e}_0^{\mathrm{TEM}} + \sum_{n=1}^{\infty} \tilde{a}_n^{\mathrm{TE}} \mathbf{e}_n^{\mathrm{TE}} + \sum_{n=1}^{\infty} \tilde{a}_n^{\mathrm{TM}} \mathbf{e}_n^{\mathrm{TM}} \quad (3)$$

$$\mathbf{E}_{t,\mathrm{II}}^{\mathrm{inc}} = \tilde{b}_0^{\mathrm{TEM}} \mathbf{e}_0^{\mathrm{TEM}} + \sum_{n=1}^{\infty} \tilde{b}_n^{\mathrm{TE}} \mathbf{e}_n^{\mathrm{TE}} + \sum_{n=1}^{\infty} \tilde{b}_n^{\mathrm{TM}} \mathbf{e}_n^{\mathrm{TM}} \quad (4)$$

Due to the absence of the electric surface current, thus the tangential continuity of the incident magnetic field at the wave port is valid is kept. Based on this boundary condition, we can obtain the relations between the mode expansion coefficients at the two sides of the wave port. Namely,

$$\tilde{b}_n^{\mathrm{TEM/TE/TM}} = -\tilde{a}_n^{\mathrm{TEM/TE/TM}} \quad \text{with} \quad n = 0, \cdots, \infty \quad (5)$$

By substituting (5) into (3) and (4), we can obtain the mode coefficients of the incident electric field for a given magnetic current excitation Ms. It is expressed as:

$$\mathbf{M}_s = 2(\tilde{a}_0^{\mathrm{TEM}} \mathbf{e}_0^{\mathrm{TEM}} + \sum_{n=1}^{\infty} \tilde{a}_n^{\mathrm{TE}} \mathbf{e}_n^{\mathrm{TE}} + \sum_{n=1}^{\infty} \tilde{a}_n^{\mathrm{TM}} \mathbf{e}_n^{\mathrm{TM}}) \times \hat{\boldsymbol{\xi}} \quad (6)$$

The mode expansion coefficients for the incident electric field can be calculated by

$$\tilde{a}_n^{\mathrm{TEM/TE/TM}} = \int \hat{\boldsymbol{\xi}} \times \mathbf{M}_s / 2 \cdot \mathbf{e}_n^{\mathrm{TEM/TE/TM}} dS \quad (7)$$

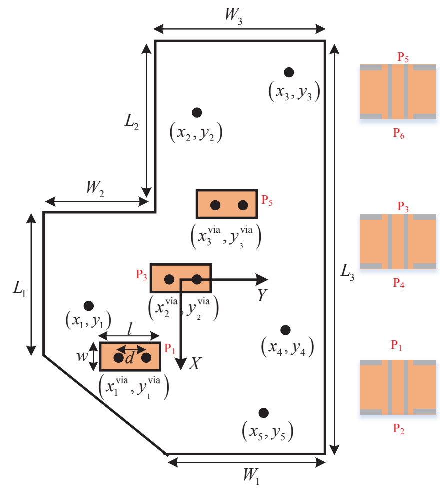

Fig. 2. Geometrical illustration of the irregular parallel plate pair with three differential via-pairs having rectangular anti-pads. The separation distance d of the differential pair is 0.508 mm, the width w and length l of the anti-pad are 0.762 mm and 1.27 mm, respectively. The radius of the PEC rods r_r is 0.127 mm, the radius of the via barrel r_b is 0.127 mm.

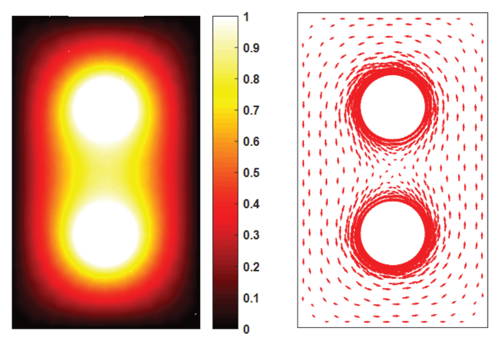

Fig. 3. Left: numerically solved common mode potential distribution $\Phi(r, \phi)$ over the circular anti-pad. Right: common mode magnetic current distribution $\mathbf{M}_s(r, \phi)$.

To extract the S-parameters, we assume that the excitation $\mathbf{M}_s$ is applied at port j and the field acquired from DGTD analysis at port i is $\mathbf{E}^i$, thus the expansion coefficient corresponding to mode n is given by

$$a_n^j = \int \mathbf{E} \cdot \mathbf{e}_n dS. \quad (8)$$

With (8) and (7), the S-parameters for mode n can be conveniently evaluated by

$$S_{ij} = \begin{cases} \frac{a_n^i - \tilde{a}_n^j}{\tilde{a}_n^j} & i = j \\ \frac{a_n^i}{\tilde{a}_n^j} & i \neq j \end{cases} \quad (9)$$

with $\tilde{a}_n^j$ denoting the incident mode expansion coefficient.

III. DGTD Formulation

To analyze the multi-layered power-ground plate-pairs by DGTD, we firstly split the computational domain of interest Ω into non-overlapping mesh cells Ω_i . In each element, the unknowns $\mathbf{E}^i$ and $\mathbf{H}^i$ are approximated by vector edge basis

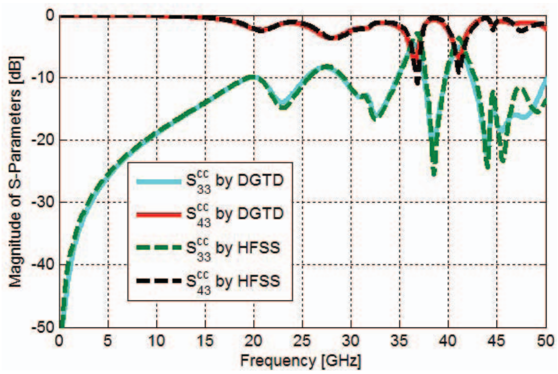

Fig. 4. The calculated common-mode S-parameters of port-pair 3 and 4 for single layer plate-pair in Fig. 2 and the results from HFSS.

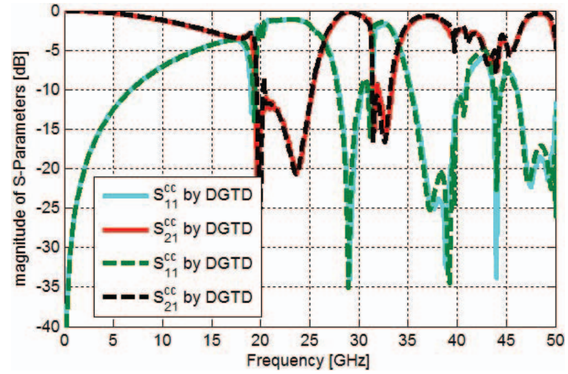

Fig. 5. The calculated common-mode S-parameters of port-pair 1 and 2 for four layer plate-pair in Fig. 2 and the results from HFSS.

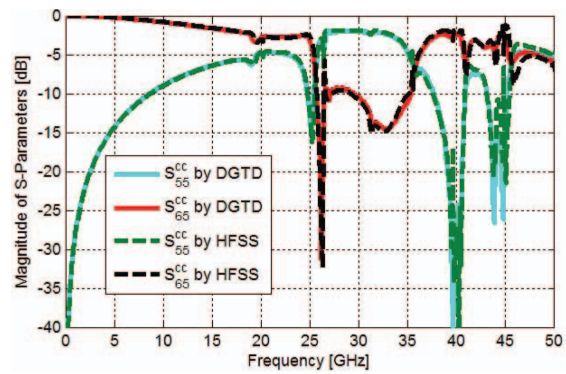

Fig. 6. The calculated common-mode S-parameters of port-pair 5 and 6 for four layer plate-pair in Fig. 2 and the results from HFSS.

functions. By conducting the Galerkin testing over the two first-order time-derivative Maxwell's equations, we can obtain the two matrix equations:

$$\bar{\mathbf{M}}_e^i \frac{\partial \mathbf{e}^i}{\partial t} = \bar{\mathbf{S}}_e^i \mathbf{h}^i + \sum_{f=1}^{n_i^f} \Big(\bar{\mathbf{F}}_{ee}^{ii,f} \mathbf{e}_f^i + \bar{\mathbf{F}}_{ee}^{ij,f} \mathbf{e}_f^j + \bar{\mathbf{F}}_{eh}^{ii,f} \mathbf{h}_f^i + \bar{\mathbf{F}}_{eh}^{ij,f} \mathbf{h}_f^j \Big) + \beta \cdot \bar{\mathbf{F}}_e^{i,\mathrm{M}_s} \quad (10)$$

$$\bar{\mathbf{M}}_h^i \frac{\partial \mathbf{h}^i}{\partial t} = -\bar{\mathbf{S}}_h^i \mathbf{e}^i + \sum_{f=1}^{n_i^f} \Big(\bar{\mathbf{F}}_{hh}^{ii,f} \mathbf{h}_f^i + \bar{\mathbf{F}}_{hh}^{ij,f} \mathbf{h}_f^j + \bar{\mathbf{F}}_{he}^{ii,f} \mathbf{e}_f^i + \bar{\mathbf{F}}_{he}^{ij,f} \mathbf{e}_f^j \Big) + \beta \cdot \bar{\mathbf{F}}_h^{i,\mathrm{M}_s} \quad (11)$$

where $\beta = 1$ if the face f is over the wave port, otherwise $\beta = 0$, $\bar{\mathbf{M}}^i_{e,h}$, $\bar{\mathbf{S}}^i_{e,h}$ and $\bar{\mathbf{F}}$ are mass, stiffness and flux matrices, respectively.

IV. Numerical Results

To validate the proposed algorithm, an irregular plate-pair including three differential via-pairs with rectangular anti-pads is benchmarked. For the common-mode analysis, the potential distribution over the rectangular anti-pad is calculated by solving the Laplace equation governed by the common-mode pertinent boundary condition. In Fig. 3 , the obtained potential distribution and the magnetic current for the common-mode are presented. As expected, the obtained results agree with the theories.

Firstly, a single-layer plate-pair is studied. The thickness of the plate-pair is $h = 0.254$ mm and the permittivity of the media is 2.2. In Fig. 4, the calculated S-parameters corresponding to the port-pair 3 and 4 are presented. For comparison, the results from HFSS are also provided. As can be seen, very good agreements are reached.

Next, the proposed algorithm is extended to investigate a four-layer power-ground plate-pair. Each layer has same thickness h=0.254 mm. In Fig. 5 and 6, the common-mode S-parameters for port-pairs 1 and 2, and port-pair 5 and 6 are presented. To validate the accuracy of the proposed algorithm, the results from HFSS simulation are given. It is noted that good agreements are achieved.

V. Conclusion

In this work, a DGTD algorithm is developed for the analysis of power-ground planes with arbitrarily-shaped anti-pads. To implement the proper excitation, the wave port is employed. Furthermore, the mode pertinent S-parameter extraction method is proposed. The accuracy and feasibility of the algorithm are verified by comparing the reference results from HFSS simulation.

References

[1] L. Tsang, H. Chen, C. C. Huang, and V. Jandhyala, "Modeling of multiple scattering among vias in planar waveguides using Foldy-Lax equations," *Microw. Opt. Technol. Lett.*, vol. 31, pp. 201-208, Nov. 2001.

[2] Y. J. Zhang, L. H. Ren, D. Z. Liu, S. De, X. X. Gu, Y. H. Kwark, C. Schuster, and J. Fan, "An efficient hybrid finite-element analysis of multiple vias sharing the same anti-pad in an arbitrarily shaped parallel-plate pair," *IEEE Trans. Microw. Theory Tech.*, vol. 63, no. 3, pp. 883-890, Mar. 2015.

[3] Y. J. Zhang and J. Fan, "An intrinsic circuit model for multiple vias in an irregular plate pair through rigorous electromagnetic analysis," *IEEE Trans. Microw. Theory Tech.*, vol. 58, no. 8, pp. 2251-2265, Aug. 2010.

[4] P. Li, L. J. Jiang and H. Bagci, "Transient analysis of dispersive power-ground plate pairs with arbitrarily shaped antipads by the DGTD method with wave port excitation," IEEE Trans. Electromagn. Compat., vol. 59, no. 1, pp. 172-183, Sept. 2016.