On the Achievable Rate of Hardware-Impaired Transceiver Systems

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***Abstract*—In this paper, we accurately model the transceiver hardware impairments (HWIs) of multiple-input multiple-output (MIMO) systems considering different HWI stages at transmitter and receiver. The proposed novel statistical model shows that transceiver HWIs transform the transmitted symmetric signal to asymmetric one. Moreover, it shows that the aggregate self-interference has asymmetric characteristics. Therefore, we propose improper Gaussian signaling (IGS) for transmission in order to improve the achievable rate performance. IGS is considered as a general signaling scheme which includes the proper Gaussian signaling (PGS) as a special case. Thus, IGS has additional design parameters which enable it to mitigate the HWI self-interference. As a case study, we analyze the achievable rate performance of single-input multiple-output systems with linear and selection combiner. Furthermore, we optimize the IGS statistical characteristics for interference alignment. This improves the achievable rate performance as compared to the PGS, which is validated through numerical results.**

I. Introduction

Hardware impairments (HWIs) such as phase noise, imperfections in analog-to-digital convertors, distortions in high power amplifier and low noise amplifier, and in-phase and quadrature (I/Q) imbalance arise in different stages in the RF front-end [1]. HWI results in phase and amplitude errors, raised noise floor pertaining to the distortion noise and inevitable mixing of image and desired signals. Therefore, developing accurate HWI models can play a major role in analyzing the system performance and proposing effective compensation techniques to meet the expected performance.

HWI statistical models adopted in the recent studies of achievable rate and outage performance analysis assumed symmetric statistical characteristics [2], [3]. However, according to the statistical signal processing studies, the I/Q imbalance (IQI) deforms the symmetrical signal characteristics [4]. Thus, we assumed asymmetric hardware distortion noise and studied the outage probability of single-input-single-output systems in [5] and single-input multiple-output systems in [6] .

Asymmetric signaling or IGS is a generalized complex Gaussian signaling scheme that relaxes the symmetric characteristics of PGS scheme allowing a correlation between the signal components and/or unequal power distribution [7]. Hence, IGS scheme offers an additional degree of freedom/design parameter pertinent to its circularly asymmetric characteristics. Interestingly, IGS scheme was proven to improve the system performance of different interference-limited configurations e.g. cognitive radio systems [8] and cooperative systems [9].

In this paper, we focus on the IQI amplitude and rotational error and the additive distortions from various RF stages. We emphasize on the precise modeling of IQI owing to its distinct characteristics. IQI is accurately modeled as a widely linear transformation which not only results in self-interfering information signals but also transforms the proper thermal noise into improper interference. This distinct behavior of IQI makes it stand out of all other forms of impairments that result in mere additive distortions. To the best of author's knowledge, this paper is the first to propose an accurate novel statistical signal model for multiple antenna systems which not only incorporates HWI but also characterizes their distinct properties. Moreover, we have carried out information theoretic achievable rate performance analysis based on the asymmetric complex interference. We further propose an optimal and adaptive asymmetric transmission scheme to mitigate the adverse effects of HWIs.

Notations: In this paper, scalars are denoted by lower-case italic letters, while vectors and matrices are denoted by boldfaced lower-and upper-case letters, respectively. $\{.\}^*$ and $|.|$ denote the conjugate and absolute of a scalar, respectively. $\|.\|_2$, $\{.\}^*$, $\{.\}^{\mathrm{T}}$, $\{.\}^{\mathrm{H}}$ and $\{.\}^{-1}$ denote the L2-norm, complex-conjugate, transpose, conjugate-transpose operator and matrix inversion respectively. The trace and determinant of matrix $\mathbf{A}$ are defined as $\mathrm{Tr}\left(\mathbf{A}\right)$ and $|\mathbf{A}|$ respectively. $\mathbb{C}^{M\times N}$ and $\mathbb{R}^{M\times N}$ describe a complex-valued and real-valued matrix with dimensions $M \times N$, respectively. The N dimensional square identity matrix is presented by I_N. In addition,$\mathrm{E}\left[.\right]$ represents the expected value operator.

II. System Description

A. Statistical Signal Characteristics

To characterize the difference between the PGS and IGS schemes, we consider a zero-mean complex Gaussian random vector $\mathbf{x} \in \mathbb{C}^N$ and introduce the following definitions:

Definition 1. *The covariance and the pseudo-covariance matrices of* $\mathbf{x}$ *are defined as* $\mathbf{C}_{\mathbf{x}} \triangleq \mathrm{E}\left[\mathbf{x}\mathbf{x}^{\mathrm{H}}\right]$ *and* $\tilde{\mathbf{C}}_{\mathbf{x}} \triangleq \mathrm{E}\left[\mathbf{x}\mathbf{x}^{\mathrm{T}}\right]$, *respectively [7].*

According to Definition 1, $\mathbf{C}_{\mathbf{x}}$ is an Hermitian positive semi-definite matrix, whereas $\tilde{\mathbf{C}}_{\mathbf{x}}$ is a symmetric matrix.

Definition 2. *A complex random vector* $\mathbf{x}$ *is called proper if its pseudo-covariance matrix* $\tilde{\mathbf{C}}_{\mathbf{x}}$ *vanishes (i.e., zero matrix), otherwise it is called improper [7].*

Definition 3. *The degree of impropriety of* $\mathbf{x}$ *is measured by the circularity coefficient* κ*, which is defined for* $N=1$*, i.e., scalar variable, as* $\kappa = \left|\tilde{\sigma}_x^2\right|/\sigma_x^2$*, where* $0 \le \kappa \le 1$*,* $\kappa = 0$ *denotes proper signal and* $\kappa = 1$ *denotes maximally improper signal [10]. On the other hand,* κ *can be defined for* $N > 1$*, i.e., vector variable, as a function of the individual circularity coefficients for each scalar in* $\mathbf{x}$*, i.e., (*$\kappa_i \;\; i = 1, \ldots, N$*) as* $\rho = \frac{1}{N}\sum_{i=1}^{r}\kappa_i^2$*, where* $r = \operatorname{rank}(\mathbf{C_x})$ *. For full rank* $\mathbf{C_x}$*, i.e.,* $r = N$*,* ρ *is also defined as a function of covariance and pseudo-covariance matrices as* $\rho = \frac{1}{N}\operatorname{Tr}\left(\mathbf{C_x}^{-1}\tilde{\mathbf{C}}_\mathbf{x}\mathbf{C_x}^{-*}\tilde{\mathbf{C}}_\mathbf{x}^{*}\right)$ *[4].*

Definition 4. *A complex-Gaussian random variable* $\mathbf{x}$ *is completely defined as* $\mathcal{CN}(\mu_\mathbf{x}, \mathbf{C_x}, \tilde{\mathbf{C}}_\mathbf{x})$ *where* $\mu_\mathbf{x}$ *is the mean of* $\mathbf{x}$*, i.e.,* $\mu_\mathbf{x} = \mathrm{E}\left[\mathbf{x}\right]$*.*

B. Multiple-Input Multiple-Output (MIMO) System Model

1) Ideal MIMO System: Consider the transmission of a complex circularly symmetric information vector $\mathbf{x} \in \mathbb{C}^{N_\mathrm{T}}$ with $\mathbf{x} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{C_x}, \mathbf{0}\right)$. Assume $\mathbf{x}$ bears the transmit power that is attained after the power amplifier. The received signal vector for the ideal MIMO system is then expressed as

$$\mathbf{y}_\mathrm{id} = \mathbf{H}\mathbf{x} + \mathbf{w}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_\mathrm{R}\mathrm{x}N_\mathrm{T}}$ is the slowly varying flat-fading Rayleigh channel and $\mathbf{w} \in \mathbb{C}^{N_\mathrm{R}}$ is the additive white complex Gaussian noise (AWGN) with $\mathbf{w} \sim \mathcal{CN}\left(0, \mathbf{C_w}, 0\right)$.

2) Non-Ideal MIMO System: Consider a MIMO RF front-end direct-conversion based architecture for spatially diverse transmitter and receiver [1] . At the transmitter side, the real and imaginary parts of information signal $\mathbf{x}$ pass through a digital-to-analog converter. Then, it is up-converted to the desired carrier frequency using homodyne architecture which introduces HWIs. These HWIs, usually referred as I/Q imbalance, include phase and amplitude errors caused by the mismatched local-oscillator and phase shifter. Transceivers with I/Q imbalance suffer from limited image rejection rendering self-interfering signals. Mismatched transmitter I/Q mixer distorts the input signal $\mathbf{x}$ yielding $\mathbf{x}_1$, defined as [1, (5.15)] and [11, (8)]

$$\mathbf{x}_1 = \mathbf{V}_1\mathbf{x} + \mathbf{V}_2\mathbf{x}^*, \tag{2}$$

where $\mathbf{V}_1$ and $\mathbf{V}_2$ are diagonal matrices which capture amplitude and rotational errors in the I/Q mixer stage with $\mathbf{V}_1 \in \mathbb{C}^{N_\mathrm{T}\times N_\mathrm{T}}$ and $\mathbf{V}_2 \in \mathbb{C}^{N_\mathrm{T}\times N_\mathrm{T}}$. They are defined, respectively, as

$$\mathbf{V}_1 = \left(\mathbf{I}_{N_\mathrm{T}} + \mathbf{A}_\mathrm{T}e^{j\theta_\mathrm{T}}\right)/2 \tag{3}$$

$$\mathbf{V}_2 = \mathbf{I}_{N_\mathrm{T}} - \mathbf{V}_1^* = \left(\mathbf{I}_{N_\mathrm{T}} - \mathbf{A}_\mathrm{T}e^{-j\theta_\mathrm{T}}\right)/2, \tag{4}$$

where $\mathbf{A}_\mathrm{T}$ and θ_T are the diagonal matrices with diagonal entries representing the amplitude and phase translation error from different transmitter branches respectively [1]. Practically these IQI parameters can be jointly estimated along with the channel estimation as discussed in [12]. It is worth to mention that the transmitters with perfect I/Q balance have $\mathbf{V}_1 = \mathrm{I}_{N_\mathrm{T}}$ and $\mathbf{V}_2 = \mathbf{0}$. Moreover, non-linear transfer function of various transceiver stages, such as analog-to-digital/ digital-to-analog converters, high power amplifier/low noise amplifier, band-pass filters and image rejection low pass filter, results in additive distortion noise in the respective transceiver branch raising the noise floor [1]. The additive distortions $\mathbf{d}_\mathrm{T} \in \mathbb{C}^{N_\mathrm{T}}$ to the transmitting stream are modeled as $\mathbf{d}_\mathrm{T} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_\mathrm{T}, \mathbf{0})$ [2]. This distorts the transmitted signal to be expressed as

$$\mathbf{x}_\mathrm{tx} = \mathbf{x}_1 + \mathbf{d}_\mathrm{T} = \mathbf{V}_1\mathbf{x} + \mathbf{V_2}\mathbf{x}^* + \mathbf{d}_\mathrm{T}. \tag{5}$$

Then, the transmitted impaired signal undergoes channel fading before reaching the destination. At the receiver side, the signal is prone to distortions from low noise amplifier, band pass filter, image rejection filters, analog-to-digital converter and thermal noise. These accumulated additive distortions at receiver $\mathbf{d}_\mathrm{R} \in \mathbb{C}^{N_\mathrm{R}}$ are assumed to be additive complex Gaussian distributed as $\mathbf{d}_\mathrm{R} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_\mathrm{R}, \mathbf{0})$ [1], [2]. Therefore, the received signal vector $\mathbf{y}_\mathrm{rx}$ is expressed as

$$\mathbf{y}_\mathrm{rx} = \mathbf{H}\mathbf{x}_\mathrm{tx} + \mathbf{d}_\mathrm{R}. \tag{6}$$

Similar to the I/Q imbalance at the transmitter, the receiver oscillator, phase shifters and mixers introduce I/Q imbalance. Therefore, the signal at the receiver output is expressed as [1]

$$\mathbf{y} = \mathbf{\Gamma}_1\mathbf{y}_\mathrm{rx} + \mathbf{\Gamma}_2\mathbf{y}_\mathrm{rx}^*, \tag{7}$$

where $\mathbf{\Gamma}_1 \in \mathbb{C}^{N_\mathrm{R}\times N_\mathrm{R}}$ and $\mathbf{\Gamma}_2 \in \mathbb{C}^{N_\mathrm{R}\times N_\mathrm{R}}$ are defined, respectively, as [1]

$$\mathbf{\Gamma}_1 = \left(\mathbf{I}_{N_\mathrm{R}} + \mathbf{A}_\mathrm{R}e^{j\theta_\mathrm{R}}\right)/2 \tag{8}$$

$$\mathbf{\Gamma}_2 = \mathbf{I}_{N_\mathrm{R}} - \mathbf{\Gamma}_1^* = \left(\mathbf{I}_{N_\mathrm{R}} - \mathbf{A}_\mathrm{R}e^{-j\theta_\mathrm{R}}\right)/2, \tag{9}$$

with $\mathbf{A}_\mathrm{R}$ and θ_R denote the diagonal matrices which capture the amplitude and rotational imbalance values from each receiver stream. It is crucial to note that $\mathbf{\Gamma}_1 = \mathrm{I}_{N_\mathrm{R}}$ and $\mathbf{\Gamma}_2 = \mathbf{0}$ represent receivers with perfect I/Q balance.

Theorem 1. *Transceiver hardware imperfections transform the transmitted symmetric signal, into asymmetric signal. Moreover, the aggregate additive noise becomes asymmetric interference, even when the additive AWGN and the nonlinear distortion are assumed to be symmetric signals.*

Proof. By substituting (5) and (6) in (7), we obtain

$$\mathbf{y} = \bar{\mathbf{H}}_1\mathbf{x} + \bar{\mathbf{H}}_2\mathbf{x}^* + \mathbf{z}, \tag{10}$$

where $\mathbf{z} = \mathbf{\Gamma_1}\left(\mathbf{H}\mathbf{d}_\mathrm{T} + \mathbf{d}_\mathrm{R}\right) + \mathbf{\Gamma_2}\left(\mathbf{H}^*\mathbf{d}_\mathrm{T}^* + \mathbf{d}_\mathrm{R}^*\right)$, $\bar{\mathbf{H}}_1 = \mathbf{\Gamma}_1\mathbf{H}\mathbf{V}_1 + \mathbf{\Gamma}_2\mathbf{H}^*\mathbf{V}_2^*$ and $\bar{\mathbf{H}}_2 = \mathbf{\Gamma}_1\mathbf{H}\mathbf{V}_2 + \mathbf{\Gamma}_2\mathbf{H}^*\mathbf{V}_1^*$ with $\bar{\mathbf{H}}_1, \bar{\mathbf{H}}_2 \in \mathbb{C}^{N_\mathrm{R}\times N_\mathrm{T}}$. Since $\mathbf{z}$ is a sum of two widely linear transformed variables, thus $\mathbf{z}$ is assumed to transform into improper interference. To confirm the IGS characteristics of $\mathbf{z}$, we find the pseudo-covariance as follows

$$\tilde{\mathbf{C}}_\mathbf{z} = \mathbf{\Gamma}_1\left(\mathbf{H}\mathbf{C}_\mathrm{T}\mathbf{H}^\mathrm{H} + \mathbf{C}_\mathrm{R}\right)\mathbf{\Gamma}_2^\mathrm{T} + \mathbf{\Gamma}_2\left(\mathbf{H}^*\mathbf{C}_\mathrm{T}\mathbf{H}^\mathrm{T} + \mathbf{C}_\mathrm{R}\right)\mathbf{\Gamma}_1^\mathrm{T}, \tag{11}$$

which is not zero matrix, proving the asymmetric characteristics of $\mathbf{z}$. As for covariance matrix, one can find it from

$$\mathbf{C}_\mathbf{z} = \mathbf{\Gamma}_1\left(\mathbf{H}\mathbf{C}_\mathrm{T}\mathbf{H}^\mathrm{H} + \mathbf{C}_\mathrm{R}\right)\mathbf{\Gamma}_1^\mathrm{H} + \mathbf{\Gamma}_2\left(\mathbf{H}^*\mathbf{C}_\mathrm{T}\mathbf{H}^\mathrm{T} + \mathbf{C}_\mathrm{R}\right)\mathbf{\Gamma}_2^\mathrm{H}. \tag{12}$$

Similar to the aggregate noise, the transmitted signal is also widely linearly transformed as $\bar{\mathbf{H}}_1\mathbf{x} + \bar{\mathbf{H}}_2\mathbf{x}^*$ due to transceiver HWIs. Thus, considering the transmitted information signal vector $\mathbf{x} \in \mathbb{C}^{N_\mathrm{T}}$ with $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C_x}, \mathbf{0})$ and the aggregated

HWI self-interference $\mathbf{z}\sim\mathcal{CN}(\mathbf{0},\mathbf{C}_{\mathbf{z}},\tilde{\mathbf{C}}_{\mathbf{z}})$, the received signal vector has a covariance $\mathbf{C}_{\mathbf{y}_{\mathrm{P}}}$ and a pseudo-covariance $\tilde{\mathbf{C}}_{\mathbf{y}_{\mathrm{P}}}$ expressed, respectively, as

$$\mathbf{C}_{\mathbf{y}_{\mathrm{P}}}=\bar{\mathbf{H}}_1\mathbf{C}_{\mathbf{x}}\bar{\mathbf{H}}_1^{\mathrm{H}}+\bar{\mathbf{H}}_2\mathbf{C}_{\mathbf{x}}\bar{\mathbf{H}}_2^{\mathrm{H}}+\mathbf{C}_{\mathbf{z}} \tag{13}$$

$$\tilde{\mathbf{C}}_{\mathbf{y}_{\mathrm{P}}}=\bar{\mathbf{H}}_1\mathbf{C}_{\mathbf{x}}\bar{\mathbf{H}}_2^{\mathrm{T}}+\bar{\mathbf{H}}_2\mathbf{C}_{\mathbf{x}}\bar{\mathbf{H}}_1^{\mathrm{T}}+\tilde{\mathbf{C}}_{\mathbf{z}}, \tag{14}$$

assuming perfect CSI and HWIs coefficients. From (14), it is obvious that the received signal has a non-zero pseudo-covariance matrix of $\tilde{\mathbf{C}}_{\mathbf{y}_{\mathrm{P}}}$, which proves the asymmetric characteristics of the received useful signal. □

It is crucial to note that the hardware-impaired received signal mathematical model in (10) reduces to the ideal model in (1) when $\mathbf{\Gamma}_1=\mathbf{V}_1=\mathbf{I}$, $\mathbf{\Gamma}_2=\mathbf{V}_2=\mathbf{0}$, $\mathbf{d}_{\mathrm{T}}=\mathbf{0}$ and $\mathbf{d}_{\mathrm{R}}=\mathbf{w}$. Thus, $\bar{\mathbf{H}}_1=\mathbf{H}$ and $\bar{\mathbf{H}}_2=0$ which reduces the pseudo-covariance matrices to zero, i.e., $\tilde{\mathbf{C}}_{\mathbf{z}}=\mathbf{0}$ and $\tilde{\mathbf{C}}_{\mathbf{y}_{\mathrm{P}}}=\mathbf{0}$.

IGS represents the general signaling scenario, which motivates us to adopt it for transmission. As such, the transmitted signal is assumed to be $\mathbf{x}\sim\mathcal{CN}(\mathbf{0},\mathbf{C}_{\mathbf{x}},\tilde{\mathbf{C}}_{\mathbf{x}})$. Therefore, the covariance $\mathbf{C}_{\mathbf{y}_{\mathrm{I}}}$ and the pseudo-covariance $\tilde{\mathbf{C}}_{\mathbf{y}_{\mathrm{I}}}$ matrices of the received signal are given, respectively, by

$$\mathbf{C}_{\mathbf{y}_{\mathrm{I}}}=\mathbf{C}_{\mathbf{y}_{\mathrm{P}}}+\mathbf{H}_1\tilde{\mathbf{C}}_{\mathbf{x}}\mathbf{H}_2^{\mathrm{H}}+\mathbf{H}_2\tilde{\mathbf{C}}_{\mathbf{x}}^{\mathrm{H}}\mathbf{H}_1^{\mathrm{H}} \tag{15}$$

$$\tilde{\mathbf{C}}_{\mathbf{y}_{\mathrm{I}}}=\tilde{\mathbf{C}}_{\mathbf{y}_{\mathrm{P}}}+\mathbf{H}_1\tilde{\mathbf{C}}_{\mathbf{x}}\mathbf{H}_1^{\mathrm{T}}+\mathbf{H}_2\tilde{\mathbf{C}}_{\mathbf{x}}^{\mathrm{H}}\mathbf{H}_2^{\mathrm{T}}. \tag{16}$$

Therefore, the received signal at MIMO receiver under both PGS and IGS transmission schemes exhibits asymmetric characteristics as deduced from the non-zero pseudo-covariance matrices in (14) and (16), respectively. As such, using IGS is preferable since it introduces another design flexibility which can be tuned to reduce the impact of HWIs and improve the overall system performance.

C. Single-Input Multiple-Output System with Linear Combiner

The SIMO system introduces receive diversity benefits through N_{R} receiving streams, which can be jointly used to detect the transmitted signal. Linear combination is one of the traditional techniques which is adopted to exploit the array and diversity gain. The transmitted impaired signal is given by

$$x_{\mathrm{SIMO}}=\nu_1x+\nu_2x^*+d_{\mathrm{T}}, \tag{17}$$

where the impact of I/Q amplitude and phase errors at the transmitter are captured in ν_1 and ν_2, with $\nu_1=\left(1+a_{\mathrm{T}}e^{j\theta_{\mathrm{T}}}\right)/2$ and $\nu_2=\left(1-a_{\mathrm{T}}e^{-j\theta_{\mathrm{T}}}\right)/2$. Moreover, the aggregated distortion noise from various blocks is assumed to be $d_{\mathrm{T}}\sim\mathcal{CN}(0,\sigma_{\mathrm{T}}^2,0)$ [1]. All the received observations $\mathbf{y}$ are combined using the linear combiner (LC) $\varphi\in\mathbb{C}^{N_{\mathrm{R}}}$ and are expressed as

$$y_{\mathrm{SIMO}}=\varphi^H\mathbf{y}=\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1x+\varphi^{\mathrm{H}}\bar{\mathbf{h}}_2x^*+\varphi^{\mathrm{H}}\mathbf{z}_1, \tag{18}$$

where, $\bar{\mathbf{h}}_1=\mathbf{\Gamma}_1\mathbf{h}\nu_1+\mathbf{\Gamma}_2\mathbf{h}^*\nu_2^*$, $\bar{\mathbf{h}}_2=\mathbf{\Gamma}_1\mathbf{h}\nu_2+\mathbf{\Gamma}_2\mathbf{h}^*\nu_1^*$ and $\mathbf{z}_1=\mathbf{\Gamma}_1\left(\mathbf{h}d_{\mathrm{T}}+\mathbf{d}_{\mathrm{R}}\right)+\mathbf{\Gamma}_2\left(\mathbf{h}^*d_{\mathrm{T}}^*+\mathbf{d}_{\mathrm{R}}^*\right)$. $\mathbf{h}\in\mathbb{C}^{N_{\mathrm{R}}}$ is the flat-fading channel, $\mathbf{d}_{\mathrm{R}}\in\mathbb{C}^{\mathbf{N}_{\mathrm{R}}}$ is the additive distortion at the receiver with $\mathbf{d}_{\mathrm{R}}\sim\mathcal{CN}(\mathbf{0},\mathbf{C}_{\mathrm{R}},\mathbf{0})$ and $d_{\mathrm{T}}\in\mathbb{C}$ is the additive distortion at the transmitter with $d_{\mathrm{T}}\sim\mathcal{CN}(0,\sigma_{\mathrm{T}}^2,0)$. Moreover, the aggregated distortion noise at the receiver is distributed as zero-mean improper complex Gaussian vector $\mathbf{z}\sim\mathcal{CN}(\mathbf{0},\mathbf{C}_{\mathbf{z}},\tilde{\mathbf{C}}_{\mathbf{z}})$, which can be easily derived using (11) and (12) by imposing $N_{\mathrm{T}}=1$.

III. Achievable Rate Performance

In this section, we analyze the achievable rate of hardware-impaired MIMO and SIMO systems when PGS or IGS is used for transmission. First for the purpose of comparison, we consider the ideal hardware scenario, where the additive noise is PGS. Thus, we choose the transmitted signal from PGS to maximize the achievable rate [4]. Then, we consider the scenario of non-ideal hardware where both the received signal and the aggregate noise component are IGS, irrespective of the PGS or IGS transmitted signal.

A. MIMO Systems

1) Ideal Hardware MIMO Scenario: In the absence of any HWI, the only undesired signal $\mathbf{z}$ is the AWGN thermal noise $\mathbf{w}$ at the receiver branches, which motivates us to adopt PGS scheme for the transmission [4]. Therefore, the entropy of the received signal and the AWGN noise are expressed, respectively, as

$$h\left(\mathbf{y}\right)=\log_2\left(\left(\pi e\right)^{N_{\mathrm{R}}}\left|\mathbf{C}_{\mathbf{y}}\right|\right), \tag{19}$$

$$h\left(\mathbf{w}\right)=\log_2\left(\left(\pi e\right)^{N_{\mathrm{R}}}\left|\mathbf{C}_{\mathbf{w}}\right|\right), \tag{20}$$

where $\mathbf{C}_{\mathbf{y}}$ and $\mathbf{C}_{\mathbf{w}}$ are found to be

$$\mathbf{C}_{\mathbf{y}}=\mathbf{H}\mathbf{C}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}+\mathbf{C}_{\mathbf{w}}, \tag{21}$$

$$\mathbf{C}_{\mathbf{w}}=\sigma_w^2\mathbf{I}_{\mathrm{N_R}}. \tag{22}$$

Given channel state information (CSI) at receiver (CSIR), the achievable rate of ideal hardware MIMO system is found as

$$R_{\text{MIMO-Ideal}}=\log_2\left|\mathbf{I}_{N_{\mathrm{R}}}+\frac{1}{\sigma_w^2}\mathbf{H}\mathbf{C}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}\right|. \tag{23}$$

2) Hardware Impaired MIMO Scenario: To analyze the achievable rate of MIMO systems with HWIs, we introduce the following definitions and lemmas that deal with asymmetric signals, i.e., IGS.

Definition 5. *The complex augmented random vector* $\underline{\mathbf{x}}$ *is defined as* $\underline{\mathbf{x}}=\left[\begin{array}{cc}\mathbf{x}^{\mathrm{T}} & \mathbf{x}^{\mathrm{H}}\end{array}\right]^{\mathrm{T}}$, $\mathbf{x}\in\mathbb{C}^N$ *[4].*

Lemma 1. *The entropy of an IGS random variable* $\mathbf{x}\in\mathbf{C}^N$, *with augmented covariance matrix* $\underline{\mathbf{C}}_{\mathbf{x}}$, *is in general a function of both covariance and pseudo-covariance matrices and is given as [4]*

$$h\left(\mathbf{x}\right)=\frac{1}{2}\log_2\left(\left(\pi e\right)^{2\mathrm{N}}\left|\underline{\mathbf{C}}_{\mathbf{x}}\right|\right), \tag{24}$$

where $\underline{\mathbf{C}}_{\mathbf{x}}$ *is defined as*

$$\underline{\mathbf{C}}_{\mathbf{x}}\triangleq\mathrm{E}\left(\underline{\mathbf{x}}\underline{\mathbf{x}}^{\mathrm{H}}\right)=\left[\begin{array}{cc}\mathbf{C}_{\mathbf{x}} & \tilde{\mathbf{C}}_{\mathbf{x}}\\ \tilde{\mathbf{C}}_{\mathbf{x}}^* & \mathbf{C}_{\mathbf{x}}^*\end{array}\right]. \tag{25}$$

For a PGS random variable $\mathbf{x}$ *with* $\tilde{\mathbf{C}}_{\mathbf{x}}=0$, *the entropy reduces to the well-known expression.*

$$h\left(\mathbf{x}\right)=\log_2\left(\left(\pi e\right)^{N}\left|\mathbf{C}_{\mathbf{x}}\right|\right) \tag{26}$$

Lemma 2. *Employing Hermitian positive semidefinite property of covariance matrix $\mathbf{C_x}$, symmetry property of pseudo-covariance matrix $\tilde{\mathbf{C}}_\mathbf{x}$ and Schur complement, the determinant of augmented covariance matrix $\underline{\mathbf{C}}_\mathbf{x}$ is given as [13]*

$$|\underline{\mathbf{C}}_\mathbf{x}| = |\mathbf{C_x}|^2 \left| \mathrm{I} - \mathbf{C_x}^{-1}\tilde{\mathbf{C}}_\mathbf{x}\mathbf{C_x}^{-\mathrm{T}}\tilde{\mathbf{C}}_\mathbf{x}^{\mathrm{H}} \right|. \tag{27}$$

First, we assume PGS transmission with $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C_x}, \mathbf{0})$. In addition, (12) and (13) induce non-singular $\mathbf{C_z}$ and $\mathbf{C_y}$. Therefore, based on Lemma 1 and Lemma 2, the differential entropy of the received signal $\mathbf{y}$ and the improper undesired signal $\mathbf{z}$ is given as

$$h(\mathbf{y}) = \log_2\left((\pi e)^N |\mathbf{C}_{\mathbf{y}_\mathrm{P}}|\right) + \frac{1}{2}\log_2\left|\mathrm{I}_{\mathrm{N_R}} - \mathbf{C}_{\mathbf{y}_\mathrm{P}}^{-1}\tilde{\mathbf{C}}_{\mathbf{y}_\mathrm{P}}\mathbf{C}_{\mathbf{y}_\mathrm{P}}^{-\mathrm{T}}\tilde{\mathbf{C}}_{\mathbf{y}_\mathrm{P}}^{\mathrm{H}}\right|$$
$$h(\mathbf{z}) = \log_2\left((\pi e)^N |\mathbf{C_z}|\right) + \frac{1}{2}\log_2\left|\mathrm{I}_{\mathrm{N_R}} - \mathbf{C_z}^{-1}\tilde{\mathbf{C}}_\mathbf{z}\mathbf{C_z}^{-\mathrm{T}}\tilde{\mathbf{C}}_\mathbf{z}^{\mathrm{H}}\right|. \tag{28}$$

Given CSIR, the achievable rate for PGS transmission is given by

$$R_{\text{MIMO-PGS}} = \log_2\frac{|\mathbf{C}_{\mathbf{y}_\mathrm{P}}|}{|\mathbf{C_z}|} + \frac{1}{2}\log_2\frac{\left|\mathbf{I}_{N_\mathrm{R}} - \mathbf{C}_{\mathbf{y}_\mathrm{P}}^{-1}\tilde{\mathbf{C}}_{\mathbf{y}_\mathrm{P}}\mathbf{C}_{\mathbf{y}_\mathrm{P}}^{-\mathrm{T}}\tilde{\mathbf{C}}_{\mathbf{y}_\mathrm{P}}^{\mathrm{H}}\right|}{\left|\mathbf{I}_{N_\mathrm{R}} - \mathbf{C_z}^{-1}\tilde{\mathbf{C}}_\mathbf{z}\mathbf{C_z}^{-\mathrm{T}}\tilde{\mathbf{C}}_\mathbf{z}^{\mathrm{H}}\right|} \tag{29}$$

Similarly, the achievable rate expression can be obtained using the mutual information between $\mathbf{y}$ and $\mathbf{z}$ for HWI MIMO systems that employ IGS transmission

$$R_{\text{MIMO-IGS}} = \log_2\frac{|\mathbf{C}_{\mathbf{y}_\mathrm{I}}|}{|\mathbf{C_z}|} + \frac{1}{2}\log_2\frac{\left|\mathbf{I}_{N_\mathrm{R}} - \mathbf{C}_{\mathbf{y}_\mathrm{I}}^{-1}\tilde{\mathbf{C}}_{\mathbf{y}_\mathrm{I}}\mathbf{C}_{\mathbf{y}_\mathrm{I}}^{-\mathrm{T}}\tilde{\mathbf{C}}_{\mathbf{y}_\mathrm{I}}^{\mathrm{H}}\right|}{\left|\mathbf{I}_{N_\mathrm{R}} - \mathbf{C_z}^{-1}\tilde{\mathbf{C}}_\mathbf{z}\mathbf{C_z}^{-\mathrm{T}}\tilde{\mathbf{C}}_\mathbf{z}^{\mathrm{H}}\right|}. \tag{30}$$

In the presence of channel state information at transmitter (CSIT), IGS provides additional degrees of freedom by appropriately designing $\tilde{\mathbf{C}}_\mathbf{x}$ besides the transmit covariance matrix $\mathbf{C_x}$, to improve the achievable rate of MIMO systems with HWIs.

B. SIMO Systems

1) Ideal Hardware Scenario: In the absence of hardware impairments, the received signal for ideal SIMO system is derived from (18) and is found to be $y_{\text{SIMO}} = \varphi^H \mathbf{y} = \varphi^{\mathrm{H}} h x + \varphi^{\mathrm{H}} w$ with $x \sim \mathcal{CN}(0, \sigma_x^2, 0)$. Hence, the undesired signal z is only comprised of PGS thermal noise with variance $\sigma_z^2 = \varphi^{\mathrm{H}}\mathbf{C_w}\varphi$ and the superposed received signal y_{SIMO} has variance $\sigma_y^2 = (\varphi^{\mathrm{H}}\mathbf{h}\mathbf{h}^H\varphi)\sigma_x^2 + \sigma_z^2$. Therefore, the achievable rate is expressed as

$$R_{\text{SIMO - Ideal}} = \log_2\left(1 + \sigma_x^2\frac{\varphi^{\mathrm{H}}\mathbf{h}\mathbf{h}^{\mathrm{H}}\varphi}{\varphi^{\mathrm{H}}\mathbf{C_w}\varphi}\right). \tag{31}$$

Given CSIR, the optimal combiner that maximizes $R_{\text{SIMO - Ideal}}$ is the same one that maximizes the signal-to-noise ratio and is derived using Cauchy-Schwarz inequality to be

$$\varphi = \frac{\mathbf{C_w^{-1}h}}{\left\|\mathbf{C_w^{-1}h}\right\|_2}. \tag{32}$$

2) Non-Ideal Hardware with PGS: The achievable rate of LC-SIMO systems with PGS transmission $x \sim \mathcal{CN}(0, \sigma_x^2, 0)$ is obtained using Lemma 1

$$R_{\text{SIMO - PGS}} = \frac{1}{2}\log\left(\frac{\sigma_{y_\mathrm{P}}^4 - \left|\tilde{\sigma}_{y_\mathrm{P}}^2\right|^2}{\sigma_z^4 - \left|\tilde{\sigma}_z^2\right|^2}\right), \tag{33}$$

where $\sigma_{y_\mathrm{P}}^2$ and $\tilde{\sigma}_{y_\mathrm{P}}^2$ are the variance and pseudo-variance of the superposed signal y_{SIMO} with PGS transmission and are expressed as

$$\sigma_{y_\mathrm{P}}^2 = \sigma_x^2\varphi^{\mathrm{H}}\left(\bar{\mathbf{h}}_1\bar{\mathbf{h}}_1^{\mathrm{H}} + \bar{\mathbf{h}}_2\bar{\mathbf{h}}_2^{\mathrm{H}}\right)\varphi + \varphi^{\mathrm{H}}\mathbf{C_z}\varphi \tag{34}$$
$$\tilde{\sigma}_{y_\mathrm{P}}^2 = \sigma_x^2\varphi^{\mathrm{H}}\left(\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{T}} + \bar{\mathbf{h}}_2\bar{\mathbf{h}}_1^{\mathrm{T}}\right)\varphi^* + \varphi^{\mathrm{H}}\tilde{\mathbf{C}}_\mathbf{z}\varphi^*. \tag{35}$$

Moreover, the variance and pseudo-variance of superposed distortion signal $\varphi^{\mathrm{H}} z_1$ are given by $\sigma_z^2 = \varphi^{\mathrm{H}}\mathbf{C_z}\varphi$ and $\sigma_z^2 = \varphi^{\mathrm{H}}\tilde{\mathbf{C}}_\mathbf{z}\varphi^*$, respectively, where $\mathbf{C_z}$ and $\tilde{\mathbf{C}}_\mathbf{z}$ are given as

$$\mathbf{C_z} = \mathbf{\Gamma}_1\left(\sigma_\mathrm{T}^2\mathbf{h}\mathbf{h}^{\mathrm{H}} + \mathbf{C}_\mathrm{R}\right)\mathbf{\Gamma}_1^{\mathrm{H}} + \mathbf{\Gamma}_2\left(\sigma_\mathrm{T}^2\mathbf{h}^*\mathbf{h}^{\mathrm{T}} + \mathbf{C}_\mathrm{R}\right)\mathbf{\Gamma}_2^{\mathrm{H}} \tag{36}$$
$$\tilde{\mathbf{C}}_\mathbf{z} = \mathbf{\Gamma}_1\left(\sigma_\mathrm{T}^2\mathbf{h}\mathbf{h}^{\mathrm{H}} + \mathbf{C}_\mathrm{R}\right)\mathbf{\Gamma}_2^{\mathrm{T}} + \mathbf{\Gamma}_2\left(\sigma_\mathrm{T}^2\mathbf{h}^*\mathbf{h}^{\mathrm{T}} + \mathbf{C}_\mathrm{R}\right)\mathbf{\Gamma}_1^{\mathrm{T}}. \tag{37}$$

3) Non-Ideal Hardware with IGS: Assume IGS transmission with $x \sim \mathcal{CN}(0, \sigma_x^2, \tilde{\sigma}_x^2)$ and follow the same analysis as in non-ideal hardware SIMO under PGS, the achievable rate expression is expressed as

$$R_{\text{SIMO - IGS}} = \frac{1}{2}\log\left(\frac{\sigma_{y_\mathrm{I}}^4 - \left|\tilde{\sigma}_{y_\mathrm{I}}^2\right|^2}{\sigma_z^4 - \left|\tilde{\sigma}_z^2\right|^2}\right), \tag{38}$$

where $\sigma_{y_\mathrm{I}}^2$ and $\tilde{\sigma}_{y_\mathrm{I}}^2$ are expressed, respectively, as

$$\sigma_{y_\mathrm{I}}^2 = \sigma_{y_\mathrm{P}}^2 + \varphi^{\mathrm{H}}\left(\tilde{\sigma}_x^2\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{H}} + (\tilde{\sigma}_x^*)^2\bar{\mathbf{h}}_2\bar{\mathbf{h}}_1^{\mathrm{H}}\right)\varphi \tag{39}$$
$$\tilde{\sigma}_{y_\mathrm{I}}^2 = \tilde{\sigma}_{y_\mathrm{P}}^2 + \varphi^{\mathrm{H}}\left(\tilde{\sigma}_x^2\bar{\mathbf{h}}_1\bar{\mathbf{h}}_\mathbf{1}^{\mathrm{T}} + (\tilde{\sigma}_x^*)^2\bar{\mathbf{h}}_\mathbf{2}\bar{\mathbf{h}}_\mathbf{2}^{\mathrm{T}}\right)\varphi^*. \tag{40}$$

The achievable rate performance of the IGS-based SIMO system can be improved by optimizing the variance, pseudo-variance and the receiver combiner.

IV. Signal Design of Linear Combining SIMO Systems with HWI

In this section, we consider SIMO system with linear combiner (LC) and design the statistical signal characteristics represented by σ_x^2 and $\tilde{\sigma}_x^2$ to maximize the achievable rate under the assumption of perfect CSIT. First, based on the strictly increasing nature of the logarithmic function of $\mathrm{R_{SIMO-IGS}}$, maximizing $\mathrm{R_{SIMO-IGS}}$ is is equivalent to maximizing $\Xi_{\text{SIMO}-\text{IGS}}$, which is defined as

$$\Xi_{\text{SIMO}-\text{IGS}}\left(\sigma_\mathrm{x}^2, \tilde{\sigma}_\mathrm{x}^2\right) = \sigma_{y_\mathrm{I}}^4 - \left|\tilde{\sigma}_{y_\mathrm{I}}^2\right|^2 \tag{41}$$

As such, the optimization problem is written as,

$$\begin{aligned} \mathbf{P1}: \quad & \max_{\sigma_x^2, \tilde{\sigma}_x^2} \Xi_{\text{SIMO - IGS}}\left(\sigma_\mathrm{x}^2, \tilde{\sigma}_\mathrm{x}^2\right) \\ & \text{s. t.} \quad 0 \leqslant \left|\tilde{\sigma}_x^2\right| \leqslant \sigma_x^2 \leqslant \mathrm{P_T}. \end{aligned}$$

To solve **P1**, we present all complex quantities in form of their real and imaginary parts such that the objective function is in real form as

$$\begin{aligned} \Xi_{\text{SIMO}-\text{IGS}} = \; & q_{11}\Re\{\tilde{\sigma}_x^2\}^2 + q_{22}\Im\{\tilde{\sigma}_x^2\}^2 + q_{33}\sigma_x^4 \\ & + 2q_{12}\Re\{\tilde{\sigma}_x^2\}\Im\{\tilde{\sigma}_x^2\} + 2q_{13}\sigma_x^2\Re\{\tilde{\sigma}_x^2\} + 2q_{23}\sigma_x^2\Im\{\tilde{\sigma}_x^2\} \\ & + p_1\Re\{\tilde{\sigma}_x^2\} + p_2\Im\{\tilde{\sigma}_x^2\} + p_3\sigma_x^2 + r, \end{aligned} \tag{42}$$

where q_{11}, q_{22}, q_{33}, q_{12}, q_{13} and q_{23} are defined as

$$q_{11}=4\Re\{\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{H}}\varphi\}^2-\left|\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_1^{\mathrm{T}}\varphi^*\right|^2-\left|\varphi^{\mathrm{H}}\bar{\mathbf{h}}_2\bar{\mathbf{h}}_2^{\mathrm{T}}\varphi^*\right|^2-2\Re\{m_4\}$$
$$q_{22}=4\Im\{\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{H}}\varphi\}^2-\left|\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_1^{\mathrm{T}}\varphi^*\right|^2-\left|\varphi^{\mathrm{H}}\bar{\mathbf{h}}_2\bar{\mathbf{h}}_2^{\mathrm{T}}\varphi^*\right|^2+2\Re\{m_4\}$$
$$q_{33}=n_1^2-|n_2|^2$$
$$q_{12}=-4\Re\{\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{H}}\varphi\}\Im\{\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{H}}\varphi\}+2\Im\{m_4\}$$
$$q_{13}=2n_1\Re\{\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{H}}\varphi\}-\Re\{m_2\}$$
$$q_{23}=-2n_1\Im\{\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{H}}\varphi\}+\Im\{m_2\}, \tag{43}$$

p_1, p_2 and p_3 are defined, respectively, as

$$p_1=4\Re\{\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{H}}\varphi\}\left(\varphi^{\mathrm{H}}\mathbf{C}_{\mathbf{z}}\varphi\right)-2\Re\{m_3\}$$
$$p_2=-4\Im\{\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{H}}\varphi\}\left(\varphi^{\mathrm{H}}\mathbf{C}_{\mathbf{z}}\varphi\right)+2\Im\{m_3\}$$
$$p_3=2n_1\left(\varphi^{\mathrm{H}}\mathbf{C}_{\mathbf{z}}\varphi\right)-m_1 \tag{44}$$

and the coefficients r, n_1, are defined, respectively, as

$$r=\left(\varphi^{\mathrm{H}}\mathbf{C}_{\mathbf{z}}\varphi\right)^2-\left|\varphi^{\mathrm{H}}\tilde{\mathbf{C}}_{\mathbf{z}}\varphi^*\right|^2,$$
$$n_1=\varphi^{\mathrm{H}}\left(\bar{\mathbf{h}}_1\bar{\mathbf{h}}_1^{\mathrm{H}}+\bar{\mathbf{h}}_2\bar{\mathbf{h}}_2^{\mathrm{H}}\right)\varphi,\qquad n_2=\varphi^{\mathrm{H}}\left(\bar{\mathbf{h}}_1\bar{\mathbf{h}}_2^{\mathrm{T}}+\bar{\mathbf{h}}_2\bar{\mathbf{h}}_1^{\mathrm{T}}\right)\varphi^*$$
$$m_1=n_2\left(\varphi\tilde{\mathbf{C}}_z^*\varphi^T\right)+n_2^*\left(\varphi^{\mathrm{H}}\tilde{\mathbf{C}}_{\mathbf{z}}\varphi^*\right)$$
$$m_2=n_2\left(\varphi\bar{\mathbf{h}}_1^{\mathrm{H}}\bar{\mathbf{h}}_1^*\varphi^{\mathrm{T}}\right)+n_2^*\left(\varphi^{\mathrm{H}}\bar{\mathbf{h}}_2\bar{\mathbf{h}}_2^{\mathrm{T}}\varphi^*\right)$$
$$m_3=\left(\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_1^{\mathrm{T}}\varphi^*\right)\left(\varphi\tilde{\mathbf{C}}_z^*\varphi^{\mathrm{T}}\right)+\left(\varphi\bar{\mathbf{h}}_2^{\mathrm{H}}\bar{\mathbf{h}}_2^*\varphi^{\mathrm{T}}\right)\left(\varphi^{\mathrm{H}}\tilde{\mathbf{C}}_{\mathbf{z}}\varphi^*\right)$$
$$m_4=\left(\varphi^{\mathrm{H}}\bar{\mathbf{h}}_1\bar{\mathbf{h}}_1^{\mathrm{T}}\varphi^*\right)\left(\varphi\bar{\mathbf{h}}_2^{\mathrm{H}}\bar{\mathbf{h}}_2^*\varphi^{\mathrm{T}}\right). \tag{45}$$

Moreover, **P1** can be equivalently written in a vector form as

$$\mathbf{P2}:\max_{\mathbf{s}}\quad \mathbf{s}^{\mathrm{T}}\mathbf{Q}\mathbf{s}+\mathbf{s}^{\mathrm{T}}\mathbf{p}+r$$
$$\text{subject to}\quad \mathbf{A}_1\mathbf{s}\leqslant\mathbf{b},\qquad \mathbf{s}^{\mathrm{T}}\mathbf{A}_2\mathbf{s}\leqslant\mathbf{0},$$

where the vector $\mathbf{s}$ captures the transmit variance, and real and imaginary parts of pseudo-variance as $\mathbf{s}=\left[\Re\{\tilde{\sigma}_x^2\}\ \Im\{\tilde{\sigma}_x^2\}\ \sigma_x^2\ \right]^{\mathrm{T}}$, the elements of symmetric matrix $\mathbf{Q}\in\mathbb{C}^{3\times3}$ and vector $\mathbf{p}\in\mathbb{C}^3$, are defined as q_{ij} and p_i with $i=1,2,3$, $j=1,2,3$ in (43) and (44), $\mathbf{A}_1=\mathrm{diag}\,[0\ 0\ 1]^{\mathrm{T}}$, $\mathbf{b}=[0\ 0\ \mathrm{P_T}]^{\mathrm{T}}$ and $\mathbf{A}_2=\mathrm{diag}\,[1\ 1\ -1]^{\mathrm{T}}$. The equivalent problem $\mathbf{P2}$ is a concave quadratic-constraint quadratic-programming and can be optimally solved using the interior-point method owing to the concave quadratic nature of the objective function and convex set constraints. SIMO systems with selection combining (SC) scheme follows the similar analysis which has not been included here for the sake of brevity.

V. Numerical Results

In this section, we numerically investigate the degradation in the achievable rate performance due to HWI in SIMO-LC as well as SIMO-SC in order to reduce system complexity. Moreover, we study the impact of changing the signal characteristics on the achievable rate performance. As for the system parameters, we assume, unless otherwise specified, N_{R} receiver branches and a unit transmitter branch with power budget of $P_{\mathrm{T}}=1$W along with transmitter and receiver distortion levels $\sigma_{\mathrm{T}}^2=1$ and $\sigma_{\mathrm{R}}^2=1$ respectively. As for

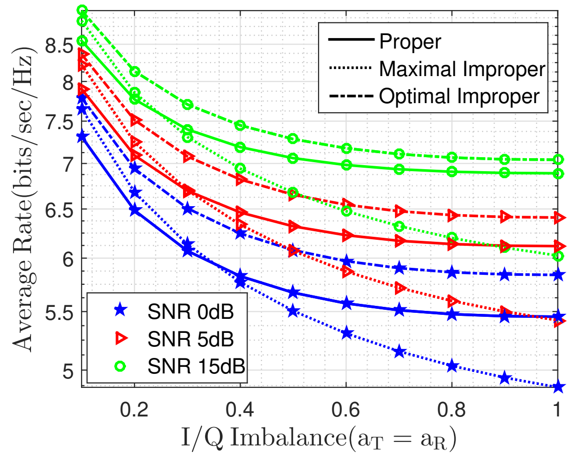

Fig. 1: SIMO-LC average achievable rate vs. $a_{\mathrm{T}}=a_{\mathrm{R}}$ for various SNR

IQI estimates, we assume identical amplitude translation error $a_{\mathrm{T}}=a_{\mathrm{R}}=0.9$ and phase translation error $\theta_{\mathrm{T}}=\theta_{\mathrm{R}}=3^o$ for transmitter and receiver branches respectively [1]. Using common oscillator for demodulation to the desired carrier frequency at all receivers justifies identical I/Q imbalance in adopted SIMO system.

A. Effect of IQI Amplitude Errors

The first numerical example compares the average achievable rate of three different signaling schemes named as, PGS, maximally IGS and optimized IGS versus the I/Q imbalance amplitude errors assuming $a_{\mathrm{T}}=a_{\mathrm{R}}$. We consider 3 antenna SIMO-LC system that is subjected to HWI with $\sigma_{\mathrm{T}}^2=\sigma_{\mathrm{R}}^2=0.6$ and $\theta_{\mathrm{T}}=\theta_{\mathrm{R}}=1^o$ based on the experimental investigations in [1]. We have employed equal gain linear combiner for simplicity. The achievable rate is studied for different levels of signal-to-noise ratio as shown in Fig 1. First, we observe that the achievable rate system performance decreases drastically with increasing I/Q amplitude error. However, this degradation can be dampened by increasing SNR. Moreover, optimal IGS scheme outperforms the maximal IGS and PGS schemes by efficiently mitigating HWI in the presence of CSIT. It is important to highlight that a low-complexity adaptive scheme can be developed by switching between maximal IGS and PGS depending on the estimated I/Q amplitude error and SNR. Generally, maximal IGS outperforms PGS for low amplitude I/Q errors, whereas PGS is the preferred choice for medium to high amplitude errors in the absence of CSIT.

B. Effect of SNR on Achievable Rates

Next, the averaged achievable rate analysis has been carried out with signal-to-noise ratio ranging from 0dB to 30dB for the hardware impaired SISO and SIMO-LC system with $N_{\mathrm{R}}=3$ equal gain combiner. We assume $\sigma_{\mathrm{T}}^2=0.5$, $\sigma_{\mathrm{R}}^2=1$, $a_{\mathrm{T}}=a_{\mathrm{R}}=0.2$ and $\theta_{\mathrm{T}}=\theta_{\mathrm{R}}=5^o$ to simulate severely degraded system. Fig 2 represents that increasing N_{R} adds to system diversity and offers improved data-rates. However, no significant improvement is observed owing to the presence of

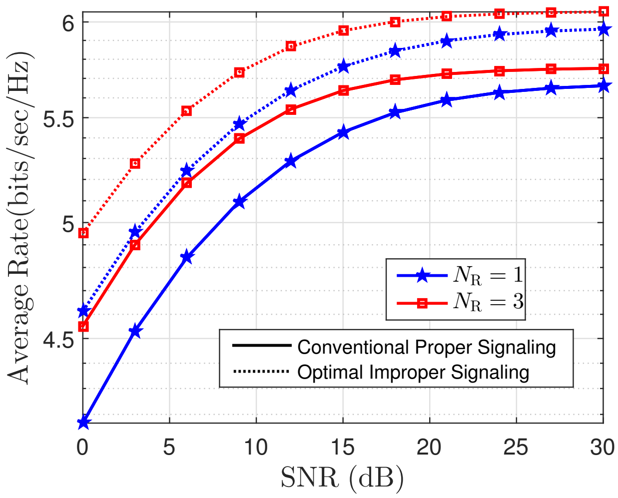

Fig. 2: SISO and SIMO-LC average achievable rate vs. SNR

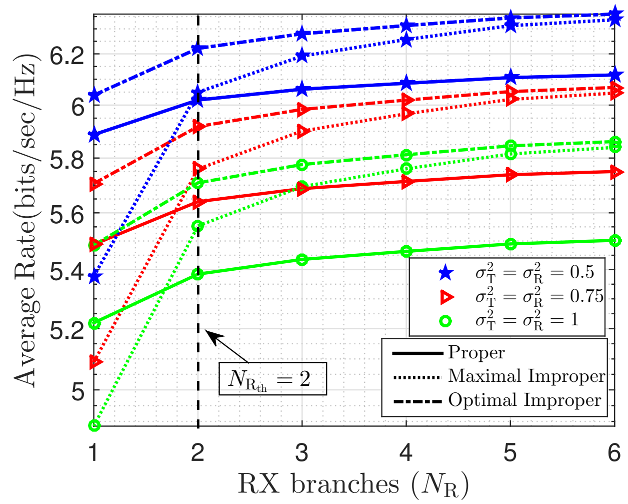

Fig. 3: SIMO-SC average achievable rate vs. N_{R}

inevitable and drastic HWI. In addition, increasing SNR also boosts average achievable rate. However, the gain reaped by boosting SNR is more prominent for fewer receive branches but undergoes saturation for higher order receive branches. Therefore, presence of HWI imposes a huge challenge on multi-antenna systems in quest of attaining higher order data-rates. In addition, careful analysis depict that the deteriorating effect of HWIs is considerably mitigated by using IGS as compared to it's counterpart PGS for given N_{R} and SNR value.

C. Effect of Receiver Branches and Distortion Noise Level

Eventually, we investigate the effect of increasing receiver branches $\mathrm{N_R}$ from 1 to 6 for varying distortion levels due to the non-linear transfer function of various aforementioned transceiver blocks. For simulation, we assume identical distortion levels at transmitter and receiver branches $\sigma_{\mathrm{T}}^2 = \sigma_{\mathrm{R}}^2 = 0.5, 0.75, 1$. In addition, we assume identical amplitude $a_{\mathrm{T}} = a_{\mathrm{R}} = 1$ and phase error $\theta_{\mathrm{T}} = \theta_{\mathrm{R}} = 3^o$ at all transmitter and receiver branches. Fig 3 depicts, the degradation caused by the varying level of additive TX and RX distortions given 25 dB SNR. Evidently, increasing additive distortions significantly deteriorate system performance. Whereas, increasing number of receiver branches counters HWI and struggles to improve the overall achievable rate. Verily, optimal IGS outperforms PGS and maximally IGS in all presented scenarios. Intriguingly, maximally IGS can be adopted to improve system performance relative to the PGS transmission when receive branches exceed $N_{\mathrm{R_{th}}} = 2$ for the adopted system. Interestingly, for higher N_{R}, maximal IGS has equal performance gain relative to the optimal scheme. Thus it eliminates the optimization overhead. The same trend has been observed even for the lower SNR levels such as 0dB.

VI. Conclusion

In this paper, we proposed an accurate novel statistical model for HWI systems which captures the aggregated transceiver hardware imperfections. An efficient signaling is inevitably required to mitigate the effect of HWIs. Therefore, we adopted the IGS scheme transmission in order to reduce the self-interference induced by the HWI. Closed form expressions are developed for the achievable rate for MIMO and SIMO with linear combining systems assuming PGS or IGS. The proposed approach has a performance advantage over more traditional signal processing techniques to correct magnitude and phase errors in the transceivers as it reduces the transmission overhead. The IQI estimates once obtained by sending pilots can be used later for fine-tuning in future transmissions.

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