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**Visco-acoustic Wave-equation Traveltime Inversion and Its Sensitivity to Attenuation Errors**

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**ABSTRACT**

A visco-acoustic wave-equation traveltime inversion method is presented that inverts for the shallow subsurface velocity distribution. Similar to the classical wave equation traveltime inversion, this method finds the velocity model that minimizes the squared sum of the traveltime residuals. Even though, wave-equation traveltime inversion can partly avoid the cycle skipping problem, a good initial velocity model is required for the inversion to converge to a reasonable tomogram with different attenuation profiles. When $Q$ model is far away from the real model, the final tomogram is very sensitive to the starting velocity model. Nevertheless, a minor or moderate perturbation of the $Q$ model from the true one does not strongly affect the inversion if the low wavenumber information of the initial velocity model is mostly correct. These claims are validated with numerical tests on both the synthetic and field data sets.

**Keywords** – Visco-acoustic; Wave equation; Attenuation; Inversion

**INTRODUCTION**
Conventional full waveform inversion (FWI) finds the velocity model that minimizes the $L_2$ norm of the waveform residuals between the predicted and the observed traces (Tarantola, 2005; Virieux and Operto, 2009; Bozdağ et al, 2016). However, this misfit functional is easily trapped into a local minimum due to its strong non-linearity with respect to the velocity variations. To mitigate this non-linearity, wave-equation traveltime inversion (WT) (Luo and Schuster, 1991a,b) was proposed to robustly estimate the low-intermediate wavenumber components of the background velocity model. By inverting traveltime instead of the full waveforms, it largely mitigates the cycle skipping problem (Ma and Hale, 2013) and provides robust convergence property compared to traditional FWI (Van Leeuwen and Mulder, 2010). However, phase or traveltime measurements can also suffer from cycle skipping depending on the starting model and the frequency content of data.

WT can provide an accurate low-wavenumber starting model for FWI (Zhou et al., 1995, 1997), or jointly and gradually invert for the detailed subsurface structures with FWI (Feng and Schuster, 2016). However, WT does not take into account the subsurface attenuation, which can lead to inaccurate tomograms and the consequent mispositioning of reflectors in the migration image (Fletcher et al., 2012; Zhu et al., 2014). These problems are severe when there is strong attenuation that leads to strong phase delays in the arrivals (Tarantola, 1988; Aki and Richards, 2002; Zhu and Harris, 2015; Komatitsch et al., 2016). To solve these problems, it is necessary to take the attenuation factor into account when applying WT to near-surface data (Dutta and Schuster, 2014). With the stronger computational ability, to understand the attenuation in both theory and practice becomes more feasible to exploration seismologists. In this study, the visco-acoustic wave equation is used to invert for the shallow velocity distribution as an extension of the traditional application of WT. Numerical tests are also taken to reveal the impact of a relatively correct estimation of attenuation on the inverted tomogram.

After the introduction, the second section briefly reviews the wave equation inversion in attenuative media, where solutions to the visco-acoustic wave equation are used to compute the WT gradient. The third section presents the sensitivity analysis of the inverted velocity tomogram with respect to the attenuation factor $Q$. Then numerical tests of WT with
different \( Q \) models for both synthetic and field data sets are presented to understand the importance of accounting for \( Q \) with WT. Conclusions are drawn in the last section.

THEORY AND METHODOLOGY

Let \( J \) denote the misfit function:

\[
J = \frac{1}{2} \sum_{s,g} \left( \Delta \tau'(s,g) \right)^2 ,
\]

where \( \Delta \tau'(s, g) = \tau_{obs}(s, g) - \tau_{pred}(s, g) \) represents the traveltime residual between the observed and the synthetic data recorded at \( g \) from the source at \( s \). For convenience we assume transmission traveltimes but this method can be used for reflection traveltimes as well. A gradient optimization method can be used to iteratively update the velocity tomogram \( c(x) \) by

\[
c_{n+1}(x) = c_n(x) + \alpha_n \beta_n(x) ,
\]

where \( \alpha, \beta \) and \( n \) are respectively the step length, the search direction and the iteration index. The Fréchet derivative \( \partial \Delta \tau'/\partial c \) can be derived using the implicit function theorem (Luo and Schuster, 1991b) as

\[
\frac{\partial \Delta \tau'}{\partial c} = - \frac{\partial \hat{F}}{\partial \Delta \tau'} ,
\]

where \( \hat{F} \) is the time derivative of the crosscorrelation equation between the predicted data \( p_{pred}(g, t+\Delta \tau'|s, 0) \) and the observed data \( p_{obs}(g, t|s, 0) \):

\[
\hat{F} \left( s, g, \Delta \tau' \right) = \int_0^T p(g, t|s, 0)^{obs} \hat{p}(g, t + \Delta \tau'|s, 0) dt = 0 .
\]

The visco-acoustic wave equation characterizing the standard linear solid (SLS) mechanism (Blanch et al., 1995) for a given velocity model \( c \) and a \( Q \) model in the spatial-temporal domain, is used to compute the pressure seismogram by
\[ \frac{\partial P}{\partial t} + \kappa(\tau + 1)(\nabla \cdot \mathbf{v}) + r_p = S(x_s, t), \]  
(5)

\[ \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho} \nabla P = 0, \]  
(6)

\[ \frac{\partial r_p}{\partial t} + \frac{1}{\tau_\sigma}(r_p + \tau \kappa(\nabla \cdot \mathbf{v})) = 0, \]  
(7)

where \( \mathbf{x} = \{v_x, v_y, v_z\} \) is the particle velocity vector, \( P \) represents pressure, \( r_p \) indicates the memory variable, \( \kappa = \rho c^2 \), a product of the density \( \rho \) and the square of the velocity term \( c \), represents the bulk modulus of the medium and \( S(x_s, t) \) represents a bandlimited source wavelet for a point source at \( x = x_s \) with the listening time \( t \). The parameter \( \tau \) is related to the stress and strain relaxation parameters \( \tau_\sigma \) and \( \tau_\varepsilon \) and the quality factor \( Q \) by,

\[ \tau_\sigma = \left( \sqrt{1 + Q^2} - \frac{1}{Q} \right) / \omega, \]  
(8)

\[ \tau_\varepsilon = \left( \sqrt{1 + Q^2} + \frac{1}{Q} \right) / \omega, \]  
(9)

\[ \frac{\tau}{\tau_\sigma}. \]  
(10)

Here, \( \omega \) is the selected reference angular frequency and is usually chosen to be the centroid frequency of the source wavelet (Robertson and Walker, 1994).

Combining equations 2 and 3 yields the gradient of the misfit function in equation 1 with respect to the velocity \( c \):

\[ \beta = \frac{1}{c^3(x)} \sum_{s,g} \int_0^T dt \left[ \hat{g}_{bs}(x, t | g, 0) \ast \hat{p}(x, t | s, 0) \right] \times \delta \tau(g, t | s, 0), \]  
(11)

where the asterisk * represents temporal convolution and \( \Delta \delta \tau \) is the recorded data shifted in time and weighted by the associated traveltime residual \( E \):

\[ \delta \tau(g, t | s, 0) = -\frac{1}{E} \hat{p}(g, t - \Delta \tau' | s, 0)^{obs} \Delta \tau'(g, t | s, 0). \]  
(12)
Here the backpropagated residual wavefield $g_{bk}$ in equation 11 is calculated by solving the adjoint visco-acoustic wave equations (Blanch et al., 1995):

$$\frac{\partial q}{\partial t} + \nabla \cdot \left( \frac{1}{\rho} \mathbf{u} \right) = -\Delta d(\mathbf{x}_s, t; \mathbf{x}_c),$$

(13)

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \kappa (1 + \tau) q + \nabla \frac{\kappa \tau r_q}{\tau} = 0,$$

(14)

$$\frac{\partial r_q}{\partial t} - \frac{r_q}{\tau} - q = 0,$$

(15)

where $q$, $\mathbf{u}$ and $r_q$ are respectively the adjoint state variables of the pressure wavefield $P$, the particle velocity vector $\mathbf{v}$, and the memory variable $r_p$ in the seismogram modeled by equations 5-7. Assuming only the pressure wavefields are recorded, the residual vector $\Delta \mathbf{d}$ will have only one component as $\Delta \mathbf{d} = [\Delta d \ 0 \ 0]$, which is also known as the virtual source term. After computing the gradient in equation 11, the preconditioned conjugate gradient method is used to update the velocity tomogram (Nocedal and Wright, 2006; Luo and Schuster, 1991b).

**SENSITIVITY OF THE VELOCITY WITH RESPECT TO $Q$**

In this paper, the $Q$ does not vary with frequency over the frequency band of the source wavelet. Therefore, the constant $Q$ model (Kjartansson, 1979) is used to analyze the velocity and traveltime change with respect to different $Q$ models. For a homogeneous and lossy medium with velocity $c_0$, and a quality factor $Q$, the complex phase velocity is

$$c(\omega) = c_0 \left[ 1 + \frac{1}{\pi Q} \ln \frac{\omega}{\omega_0} \right] \left[ 1 - \frac{i}{2Q} \right],$$

(16)

where $\omega$ represents the angular frequency of the monochromatic point source, and $i$ is the imaginary unit. For $Q^{-2} \ll 1$, equation 16 (Kjartansson, 1979; Shen* et al., 2014) can be simplified as
Based on this approximation, the velocity $c$ is a function of ratio between the frequency $\omega$ and the reference frequency $\omega_0$. As a result, higher frequency components of seismic waves are attenuated more than low frequency components during the propagation (Dutta and Schuster, 2014). If $Q >> 100$, the attenuation can almost be neglected and $c \approx c_0$ holds except for very low or very high frequencies according to equation 17. On the other hand, for a small value of $Q$ (i.e. $Q \leq 20$), high-frequency components ($\omega > \omega_0$), even though attenuated, travel faster than $c_0$, so that they arrive earlier compared to the first arrivals in the non-attenuation medium with velocity $c_0$. Therefore, big $Q$ theoretically leads to large delays in the first arrivals, which can erroneously invert for slower velocities compared to the actual velocities (Groos et al., 2014). In practice, if the observed data set is recorded in the medium with attenuation, then the inverted tomogram based on the visco-acoustic model should show slower velocity distributions than the real actual cases by ignoring the attenuation or assigning a large value to $Q$. In other words, if the velocity model is fixed for forward modeling, waves travel faster in the attenuative media rather than in the non-attenuated one. We illustrate this idea in Figures 1a to 1c by setting up $Q = 20$ and 1000.

**NUMERICAL TESTS**

Visco-acoustic WT is now applied to synthetic data with the goal of assessing the importance of visco-acoustic WT in highly attenuative media. In this study, homogeneous $Q$ models are used for simplification and comparison. The first reason is a relatively local change of $Q$ may not raise enough variations in the migration image or inverted tomograms (Dutta and Schuster, 2014). Therefore, a constant $Q$ model is setup as our strategy for the numerical tests to show the impact of the overall change of $Q$ on the inversion. The other reason is, in the field data tests (Yu and Hanafy, 2014), the parameter $Q$ was already estimated and validated. This field data set is collected in a near surface area such that the $Q$ does not have strong variations due to the visible ground truth in an onsite water well. The observed data is simulated by solving equations 5-7 using the staggered grid method
(Virieux, 1986). The tested model is the Marmousi model (Figure 2(a)) with $Q = 20$. The smoothed Marmousi, shown in Figure 2(b), is used as the ground truth to compare against the WT tomograms. The command "smooth2" of SU (Seismic Unix) is used such that the original smoothed Marmousi model becomes very smooth and only three layers can be identified, as shown in Figure 3(a). The WT only reconstructs the low-intermediate wavenumber components of the velocity model.

2D Marmousi Model with A Homogeneous $Q$ Distribution

In the first test, the initial model is a highly smoothed version (Figure 3(a)) of the true Marmousi model. The model size is 1098 m in the vertical $Z$ direction and 3450 m in the horizontal $X$ direction with a grid point spacing of 6 m. The data are recorded by 190 receivers spaced at an interval of 18 m, and are triggered by 60 sources with a spacing of 54 m. The source wavelet is a Ricker wavelet with a peak frequency of 10 Hz. The input data set is from the Marmousi model with $Q = 20$. Figure 4 shows the seismograms for the initial velocity model in Figure 3(a) with different $Q$ values. The waveforms are shifted a little later with larger $Q$ values, as indicated by Figures 4(b-c), which is consistent with the sensitivity analysis in the previous section.

The velocity model is reconstructed by the proposed WT method with $Q = 20$, 50, and 1000, respectively. The tomograms after 15 iterations are shown in Figures 3(b-d). In this test, the tomogram based on the correct $Q = 20$ value is more consistent with the true velocity model while the other tomograms are not too far away from the true model. Improvements for areas at $X = 1800$ m and $Z = 720$ m can be clearly discerned in Figures 3(b) and (c) compared to Figure 3(d). In this case, the transmitted arrivals have propagated about $14 \sim 18$ wavelengths to be at the far-offset geophone of 3420 m. The data comparisons along with the model misfits are also shown in Figures 5(a-e). In Figure 5(e), all the first breaks of the four traces are almost aligned at the same moment, which suggests that a good initial velocity model can converge to a good WT inversion result regardless of the background $Q$. It deserves to point out that if there is no severe physical dispersion caused by the anelastic model as in this simulation then the above claims on the first breaks and the convergence hold. To verify if all of the inversion results with different $Q$ values approximate the true model, we then use the visco-acoustic WT tomograms (Figures 3(b-
d)) as the starting velocity models for FWI, and the results are presented in Figures 6(a-c). We conclude that a good estimate of the $Q$ model is required for an accurate FWI tomogram in both the shallow and deep parts. If one would like to use amplitudes of data, then $Q$ should be inverted simultaneously with the elastic structure. Otherwise the unknown $Q$ model is mapped into the elastic model. Here we only use the first arrival information picked from the raw or the synthetic data sets.

In the second test, a different 1D model (Figure 7(a)) is used as a starting model for the inversion based on the same data set and acquisition geometry. The inverted tomograms are presented in Figures 7(b-d) with the three different $Q$ values. With an erroneous initial model, the visco-acoustic WT method still recovers some of the low-intermediate wavenumbers from the visco-acoustic data with $Q = 20$ and $Q = 50$. However, if the attenuation is almost neglected by assigning $Q = 1000$, the inversion cannot converge, as shown in Figure 7(d). The blue line in Figure 8(e) suggests that convergence is high sensitive to an imperfect starting velocity model.

**Land Data Tests with Estimated $Q$ Profiles**

A 2D seismic survey is conducted at a wadi located to the east of KAUST (Yu and Hanafy, 2014). The survey consists of a line of 117 vertical component geophones (Figure 9), spaced every 2.0 m, and the shots are located at every receiver position so that there are $117 \times 117 = 13689$ traces. In this field experiment, a 200 lb weight drop (Figure 9) is used to generate the seismic data with 10 ~ 15 stacks at each shot location. Each CSG is recorded for 1 second with a sampling interval of 1.0 ms.

A CSG with picked first arrival traveltimes is shown in Figure 10, where the traveltimes of the first-arrival troughs are picked. We set the attenuation factor $Q = 20$ for the WT inversion, which is estimated by the frequency shift from the source to the received signals in the data (Liao and McMechan, 1997; Yu and Hanafy, 2014). For comparisons, we also set $Q = 500$ so that the attenuation can almost be neglected. We mute the data after 0.25 seconds and only use first arrivals in the WT test.

Firstly, the traveltime and WT inversions are respectively applied to invert for the shallow subsurface velocity distributions with the starting model in Figure 11(a), and the
results are shown in Figure 12. This lateral homogeneous starting model is constructed by a rough estimate of the velocities from the first arrivals. Secondly, the model in Figure 11(b) with a faster velocity distribution is used for WT inversions with $Q = 20$ and 500, respectively, and the resulting tomograms and residuals are presented in Figure 13. Both groups of numerical tests show convergence to an acceptable value of the misfit function. All the inverted tomograms are marked by black crosses which indicate the 1450 m/s contour, which is the velocity of the water table seen in a nearby well. The depth of the 1450 m/s contour in Figure 12(a2) is mostly consistent with the water table interface estimated from the zero-offset reflection section around $t = 0.03$ s in Figure 14. Every CSG is shifted in time according to the principle that all the sources start to excite at the same moment. Therefore, this aligning operation sufficiently guarantees that events in the zero offset gather can approximate the subsurface structural section. These water table depths are in the range expected from the depth of water table seen in the Figure 16. Since the root mean square velocity above 18 m is approximately 1100 m/s, the reflections from the water table are estimated to be recorded at $t = (18 \times 2)/(1100 \text{ m/s}) \approx 0.033 \text{ sec}$ (see Figure 15).

In contrast, the visco-acoustic WT tomogram in Figure 13(a2) with erroneously assumed $Q = 500$ shows unreasonable high velocities in the shallow areas compared to Figure 13(a1). Nevertheless, none of the Figures 13(a1) and (a2) with their possible water table contours are consistent with the observed water table even if their data misfits converge to acceptable ranges shown in Figures 13(d1) and (d2).

CONCLUSIONS

A visco-acoustic wave-equation traveltime inversion can be used to invert for a better background velocity model with correct attenuation profiles. In order to clarify the sensitivity of the velocity variations with respect to $Q$, we carry out numerical tests with three different homogeneous $Q$ models. If the initial background velocity model is consistent with the true model, the visco-acoustic WT method can produce a reasonable WT tomogram regardless of the $Q$ models as presented in the synthetic numerical examples. On the other hand, if the starting velocity model is not ideal, a relatively accurate estimation of the background $Q$ value is important for updating the velocity distribution because an
incorrect $Q$ model can mislead the inversion from the beginning. Both the synthetic and the field data results are consistent with these claims. Our future work will simultaneously invert for both $Q$ and the velocity.

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Figure 1: CSGs associated with (a) $Q = 20$, and (b) $Q = 1000$ generated from the Marmousi model shown in Figure 2(a). (c) Two traces at the 150th receiver location.
Figure 2: (a) The true Marmousi velocity model, and (b) its smoothed version for computing model residuals.
Figure 3: (a) The starting velocity model for WT, the WT inverted tomograms with $Q =$ (b) 20, (c) 50, and (d) 1000.
Figure 4: Calculated CSGs for the initial model in Figure 3(a) with (a) \( Q = 20 \), (b) \( Q = 50 \), and (c) \( Q = 1000 \).
Figure 5: (a) Observed CSG #7, and the CSG #7 after 15 iterations by WT with $Q =$ (b) 20, (c) 50, and (d) 1000. (e) A comparison between the observed and the predicted traces at receiver #100. (f) Model residuals of the inverted tomograms compared to Figure 2(b).
Figure 6: FWI tomograms (a-c) with $Q = 20$, 50, and 1000, using different starting models in Figures 3(b-d), respectively.
Figure 7: (a) The 1d initial velocity model for WT, the WT inverted tomograms with $Q =$ (b) 20, (c) 50, and (d) 1000.
Figure 8: CSGs #7 (a-c) calculated after 15 iterations by WT with tomograms in Figures 7(b-d) and with $Q = 20, 50, \text{ and } 1000$, respectively. (d) Comparison between the observed and the predicted traces at receiver #100, and (e) the model residuals of the inverted tomograms compared to Figure 2(a).
Figure 9: A photo looking south taken during data acquisition.
Figure 10: (a) CSG #75 with (b) the picked first arrival matrix. The red crosses delineate the picked first-arrival times in CSG #75.
Figure 11: (a) A gradient velocity model, and (b) a model with faster near surface velocity distributions as two starting models for WT with $Q = 20$ and $Q = 500$. 
Figure 12: Comparisons of (a1-d1) the traveltime inversion results ((a1) tomogram, (b1) seismogram, (c1) the first arrival matrix picked from the calculated data, and (d1) the residual first arrival matrix with Figure 10(b)), the WT inversion results with (a2-d2) Q = 20 and (a3-d3) Q = 500 using the initial model Figure 11(a). The black crosses mark the v = 1450 m/s as the sound velocity in water. The black lines in the three tomograms mark the measured water table in the well (Figure 16). The red crosses with the CSGs mark the first arrival events.
Figure 13: Comparisons of the WT inversion results ((a1) tomogram, (b1) seismogram, (c1) the first arrival matrix picked from the calculated data, and (d1) the residual first arrival matrix with Figure 9(b)) with (a1-d1) *Q* = 20 and (a2-d2) 500 using the initial model of faster velocities in Figure 11(b). The black crosses mark the v = 1450 m/s as the sound velocity in water. The black lines in the two tomograms mark the measured water table in the well (Figure 16). The red crosses with the CSGs mark the first arrival events.
Figure 14: The zero offset gather of this Wadi data set.
Figure 15: The WT tomogram in Figure 12(a2) with $Q = 20$ overlapped with the zero offset gather.
Figure 16: Two photos show the subsurface layers (a) the first layer, composed of gravel and sand with a thickness less than 5 m. (b) The second layer as shown inside a water well nearby the study area. It is composed of sand and silt with some gravel and has a thickness of 10 - 15 m.
Highlights

We propose a wave-equation traveltime inversion strategy with attenuation.

The first arrivals in traveltime are better utilized to update the background velocity.

More accurate low-wavenumber information is inverted for the success of standard FWI.