On triangle meshes with valence 6 dominant vertices

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Abstract

We study triangulations $\mathcal{T}$ defined on a closed disc X satisfying the following condition : In the interior of X, the valence of all vertices of $\mathcal{T}$ except one of them (the irregular vertex) is 6. By using a flat singular Riemannian metric adapted to $\mathcal{T}$, we prove a uniqueness theorem when the valence of the irregular vertex is not a multiple of 6. Moreover, we exhibit non isomorphic triangulations on X with the same boundary, and with a unique irregular vertex whose valence is $6k$, for some $k > 1$.

1 Introduction

The goal of this paper is to study topological properties of triangulations on surfaces with Riemannian tools. More precisely, we study triangulations of a topological disc satisfying the following property : Every interior vertex except one has valence 6. This situation has been explored in [9], and in [2], [3] (with quad meshes instead of triangulations). Our results can be considered as their generalisation.

2 Basic definitions and notations

2.1 Weighted marked points on a surface

Let X be a topological surface X. We denote by $\overset{\circ}{X}$ its interior and by ∂X its possible boundary. A *weighted marked point* in X is a couple (v, val_v), where v is a point of X and val_v is an integer (called the *valence* of v). If

$$\mathbf{val}_{\overset{\circ}{X}} = ((v_1, \mathrm{val}_{v_1})..., (v_p, \mathrm{val}_{v_p}))$$

is a (finite) sequence of weighted points in $\overset{\circ}{X}$, and

$$\mathbf{val}_{\partial X} = ((w_1, \mathrm{val}_{w_1})..., (w_q, \mathrm{val}_{w_q}))$$

is a (finite) sequence of (ordered) weighted points in ∂X, we denote by

$$(X, \mathbf{val}_{\overset{\circ}{X}}, \mathbf{val}_{\partial X})$$

the data of X, its interior with its weighted marked points and its boundary with its weighted marked points. In this case, X is called a *weighted marked surface.* If $\overset{\circ}{X}$ admits a unique weighted point (v, val_v), we put

$$(X, (v, \mathrm{val}_v), \mathbf{val}_{\partial X}) = (X, \mathbf{val}_{\overset{\circ}{X}}, \mathbf{val}_{\partial X}).$$

2.2 Adapted triangulations

We begin with a classical definition of a triangulation, useful in our context and related to the theory of *Dessins d'Enfants*.

Definition 1 *[1] [5] [4] A* triangulation *is a couple $(X, \mathcal{T})$, where X is an (oriented) topological surface with or without boundary ∂X and $\mathcal{T}$ a finite graph on it such that $X \backslash \mathcal{T}$ is a finite union of disjoint topological discs whose boundary are triangles (that is, the union of three edges of $\mathcal{T}$).*

Let $(X, \mathbf{val}_{\overset{\circ}{X}}, \mathbf{val}_{\partial X})$ be a a weighted marked surface, with

$$\mathbf{val}_{\overset{\circ}{X}} = ((v_1, \mathrm{val}_{v_1})..., (v_p, \mathrm{val}_{v_p})) \text{ and } \mathbf{val}_{\partial X} = ((w_1, \mathrm{val}_{w_1})..., (w_q, \mathrm{val}_{w_q})).$$

Definition 2 *A triangulation $\mathcal{T}$ defined on X is* adapted *to $(\mathbf{val}_{\overset{\circ}{X}}, \mathbf{val}_{\partial X})$ if*

- *its vertices lying on the boundary of X are $\{w_1, ..., w_q\}$ with respective valences $\{\mathrm{val}_{w_1}, ..., \mathrm{val}_{w_q}\}$,*
- *$\{v_1, ..., v_p\}$ is a subset of the set of interior vertices of $\mathcal{T}$, with respective valences $\{\mathrm{val}_{v_1}, ..., \mathrm{val}_{v_p}\}$,*
- *and if other interior vertices have valence 6.*

(As usual, the *valence* of a vertex v of a triangulation is the number of edges incident to v.)

Definition 3 *Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two triangulations of X, such that $\mathcal{T}_1$ and $\mathcal{T}_2$ are adapted to $(\mathbf{val}_{\overset{\circ}{X}}, \mathbf{val}_{\partial X})$.*

- *We say that $\mathcal{T}_1$ is* equivalent *to $\mathcal{T}_2$ if there exists a homeomorphism of X inducing a isomorphism (of graph) between the graphs $\mathcal{T}_1$ and $\mathcal{T}_2$.*
- *We denote by $\mathcal{T}_{\mathbf{val}_{\overset{\circ}{X}}, \mathbf{val}_{\partial X}}$ the set of equivalence classes of such triangulations on X.*

2.3 Triangulations with a unique "irregular" vertex

In the following, we will focus on triangulations whose all interior vertices have valence 6, except one of them. This leads to the following definition.

Definition 4 *A triangulation $\mathcal{T}$ of a surface X (with or without boundary) is said* of type $(6, n)$ *if it satisfies the following property : The valence of each vertex interior to X is 6, except exactly one vertex (whose valence is $n \neq 6$).*

The goal of this paper is to prove the following result :

Theorem 1 *Let* $(X, (v, \mathrm{val}_v), \mathbf{val}_{\partial X})$ *be a weighted marked closed topological disc, and* $n \in \mathbb{N}^*$.

1. *If* n *is not a multiple of* 6, *there exists (up to an isomorphism), at most one triangulation of type* $(6, n)$ *adapted to* $(X, (v, \mathrm{val}_v), \mathbf{val}_{\partial X})$.
2. *One can build a weighted marked closed topological disc* $(X, (v, \mathrm{val}_v), \mathbf{val}_{\partial X})$ *endowed with non isomorphic triangulations of type* $(6, n)$, *where* n *is a multiple of* 6.

The proof (given in Section 5) of Theorem 1-1 consists of enlarging framework by introducing the class of Riemannian flat metrics with conical singularities on X, and use a developability argument. For Theorem 1-2, we build an explicit example, by using a covering argument.

3 Flat metric with conical singularities on a surface

3.1 On Riemannian metrics with conical singularities

We give here classical definitions and results concerning Riemannian metrics with conical singularities on a surface, [6], [7], [8].

Definition 5 *Let* X *be a surface, endowed with marked points*

$$\{v_1, \ldots, v_p\} \subset \overset{\circ}{X}, \{w_1, \ldots, w_q\} \subset \partial X.$$

Let $X' = X \backslash \{v_1, \ldots, v_p, w_1, \ldots, w_q\}$. *A metric* g with conical singularities *at* $\{v_1, \ldots, v_p, w_1, \ldots, w_q\}$ *on* X *is a Riemannian metric on* X' *such that each point* v_i *(resp.* w_i*) admits a neighborhood with polar coordinates* (r, ϕ_i), *where* r *denotes the distance to* v_i *(resp.* w_i*)and* $\phi_i \in \mathbb{R}/\theta_i$ *is the angular variable,* θ_i *being the angle of* v_i *(resp.* w_i*).*

Then, each marked point admits a neighborhood isometric to a Euclidean cone with angle θ_i. A more precise and complete definition using an analytic point of view is the following. We denote by (x, y) the coordinates of the Euclidean plane $\mathbb{E}^2$ and identify $\mathbb{E}^2$ with $\mathbb{C}$. If $z \in \mathbb{C}$ we write $z = x + iy$.

Definition 6 *Let* g *be a (singular) Riemannian metric defined on* X.

1. *A point* v *belonging to the interior of* X *is a* conical *singularity of order* $\mathrm{ord}_g(v) = \beta > -1$ *(and of angle* $\theta = 2\pi(\beta + 1)$*) for* g *if there exists a chart*
$$\psi : U \to V \subset \mathbb{E}^2,$$
where U *is an open subset of* X *containing* v, *such that*
$$\psi(v) = 0 \text{ and } g \underset{U}{=} (e^{2u}|z|^{2\beta}|dz|^2);$$
2. *A point* w *belonging to* ∂X *is a* conical *singularity for* g *of order* $\mathrm{ord}_g(w) = \gamma > -\frac{1}{2}$ *(and of angle* $\theta = 2\pi(\gamma + \frac{1}{2})$*) if there exists a chart*
$$\psi : U \to V \subset \{(x, y) \in \mathbb{E}^2, y \geq 0\},$$

where U is an open subset of X containing v, such that

$$\varphi(w) = 0 \text{ and } g \underset{U}{=} e^{2u}|z|^{4\gamma}|dz|^2.$$

*In these two items, u is a continuous function, of class C^2 on $U\backslash\{v\}$, (*resp. *$U\backslash\{w\}$). Therefore, if v (*resp. *w) is an interior (*resp. *boundary) singular point, U is isometric to a neighborhood of the vertex of a cone of angle θ in the Euclidean space $\mathbb{E}^3$ (resp . a neighborhood of the vertex of an angular sector of angle θ in the Euclidean space $\mathbb{E}^2$).*

3. *If $\{v_i, w_j\}_{i\in I, j\in J}$ is a finite set of conical singularities of g of respective order $\{\beta_i, \gamma_j\}_{i\in I, j\in J}$, the formal sum $\beta = \sum \beta_i v_i + \sum \gamma_j w_j$ is called the* divisor *representing g. The* support *of β is the set $\{v_i, w_j\}_{i\in I, j\in J}$. The* degree *of β is $|\beta| = \sum \beta_i + \sum \gamma_j$.*

The link between Definition 5 and Definition 6 is the following : The metric can be written as follows :

- Around each singular interior point,

$$g \underset{U}{=} dr^2 + r^2 d\varphi^2 \underset{U}{=} e^{2u}|z|^{2\beta}|dz|^2, \tag{1}$$

- Around each singular boundary point,

$$g \underset{U}{=} dr^2 + r^2 d\varphi^2 \underset{U}{=} e^{2u}|z|^{4\beta}|dz|^2. \tag{2}$$

Definition 7 *Let X be a (compact) Riemannian surface with divisor β, and topological Euler characteristic $\chi(X)$. The real number*

$$\chi(X, \beta) = \chi(X) + |\beta|$$

is called the Euler characteristic *of (X, β).*

Gauss-Bonnet Theorem can be stated as follows :

Theorem 2 *Let (X, g) be a (compact) Riemannian surface with conical singularities, curvature K in its interior, and whose boundary ∂X has geodesic curvature k. Then,*

$$\int_X K da + \frac{1}{2\pi}\int_{\partial X} k ds = \chi(X, \beta).$$

As a direct consequence of Theorem 2, we get :

Corollary 1 *Let (X, g) be a closed topological disc endowed with a Riemannian flat metric with conical singularities. Then,*

$$\frac{1}{2\pi}\int_{\partial X} k ds - \sum_{w_j \in \partial X} \gamma_j = 1 + \sum_{v_i \in \overset{\circ}{X}} \beta_i.$$

In particular,

- *if ∂X has a null geodesic curvature, then*

$$-\sum_{v_i\in \overset{\circ}{X}} \beta_i - \sum_{w_j\in\partial X} \gamma_j = 1.$$

- *If ∂X has a null geodesic curvature and g admits a unique conical singularity v_0 in $\overset{\circ}{X}$, then*

$$\beta_0 = -1 - \sum_{w_i\in\partial X} \gamma_j.$$

  In other words, in this special case, β_0 is completely determined by the conical singularities of the geodesic boundary.

Proof of Corollary 1 - Indeed, if X is flat with geodesic boundary, then $\chi(X,\beta) = 0$. Moreover, if X is a topological disc, then $\chi(X) = 1$. Therefore, Corollary 1 is proved.

3.2 Flat discs with a unique conical singularity in its interior

Suppose that X is a topological (compact) disc endowed with a flat Riemannian metric without any singularity in $\overset{\circ}{X}$. Then X is developable into the Euclidean plane $\mathbb{E}^2$: There exists an isometric immersion (unique, up to a rigid motion in $\mathbb{E}^2$) of X into $\mathbb{E}^2$ (in general, this immersion is not injective). We will now study the case where X admits a finite set of conical singularities, such that all of them except one belong to ∂X. One has the following uniqueness result :

Proposition 1 *Let X is a (compact) topological disc. Let g_1 (resp. g_2) be a Riemannian flat metrics defined on X with a possible finite set of conical singularities. Suppose that*

1. *all singularities of g_1 (resp. g_2) belong to ∂X, except one of them (denoted by v_1 (*resp. *v_2));*
2. *g_1 and g_2 coincide on ∂X : In particular, ${g_1}_{\partial X} = {g_2}_{\partial X}$, and g_1 and g_2 admit the same conical singularities $\{w_j\}_{j\in J} \subset \partial X$, with for all w_j, $\gamma_{g_1}(w_j) = \gamma_{g_2}(w_j)$.*

Then,

- $\mathrm{ord}_{g_1}(v_1) = \mathrm{ord}_{g_2}(v_2)$,
- $v_1 = v_2$.

Proposition 1 can be interpreted as follows : Under the assumptions of Corollary 1 the order β_0 of the unique interior conical singularity v is completely determined by the conical singularities of the geodesic boundary. Proposition 1 shows moreover that under the assumption on the angle of v, its position is also determined.

Proposition 1 is the consequence of the classical theory of developable surfaces and the following trivial lemma :

Lemma 1 *Let a and b be points in the Euclidean plane $\mathbb{E}^2$ and let θ be a real number.*

1. *If $\theta \neq 2k\pi, k \in \mathbb{Z}$ and $a \neq b$, there exist exactly two points m in the plane such that the triangle amb is isosceles. These points are on the bisector of the segment ab.*
2. *If $\theta = 2k\pi, k \in \mathbb{Z}$, for any point m in the plane, amb is a degenerate (isosceles) triangle.*

Proof of Proposition 1 -

- Since g_1 and g_2 have the same singularities on the boundary, with the same order, Corollary 1 implies that $\text{ord}g_1(v_1) = \text{ord}g_2(v_2)$.
- Let g be a Riemannian flat metric defined on X. We suppose that g admits a unique conical singularity in $\overset{\circ}{X}$, with angle $\theta \neq 2k\pi, k \geq 1$. Let γ be any (smooth) simple curve whose interior belongs to $\overset{\circ}{X}$, joining p to ∂X. Let us cut X along γ. We just built a new flat (for the metric induced by g) surface without singularity in its interior

$$\tilde{X} = X \cup \gamma \cup \gamma',$$

where we denote by γ' the curve γ with opposite orientation. Since $\tilde{X}$ is flat, simply connected and its interior has no singularity, we can develop $\tilde{X}$ into $\mathbb{E}^2$: There exists an isometry

$$i : \tilde{X} \to \mathbb{E}^2.$$

In particular, $i(\partial\tilde{X}) = i(\partial X \cup \gamma \cup \gamma')$ is a curve uniquely determined up to a rigid motion in $\mathbb{E}^2$. To simplify the notations, we put $i(v) = v$. The point p gives rise to two points p_1 and p_2 in $\mathbb{E}^2$. Moreover, because the angle of x is different to $2k\pi, k \geq 1$, $p_1 \neq p_2$. Considering the isoceles triangle $p_1 x p_2$, Lemma 1 implies that the position of x in $\mathbb{E}^2$ and then in $\tilde{X}$ and X is uniquely determined up to a symmetry with respect to the line $p_1 p_2$. We can choose p far enough to x to be sure that only one position of x is possible. Then the position of x in $\tilde{X}$ and X is uniquely determined.

Remark - The reader can be easily convinced that the assumptions of the type of singularities on the boundary can be much weakened, as far as a Gauss-Bonnet Theorem is satisfied ...

4 Euclidean structure defined on a triangulation

If $(X, \mathcal{T})$ is a topological surface endowed with a triangulation, one can define on X a *flat Riemannian metrics* (with possible conical singularities at the vertices of $\mathcal{T}$) as follows : One associates to each edge a length l in such a way that the triangular inequality is satisfied on each triangle. Then, one associates to each face t of $\mathcal{T}$ the metric of a triangle drawn in the Euclidean plane whose edges have the same lengths as the ones of t. Such a metric is called a *Euclidean structure* on $(X, \mathcal{T})$. This construction endows X with a flat metric with conical singularities.

A crucial example is obtained by associating the length 1 to each edge of $\mathcal{T}$. Such a triangulation is then called an *equilateral* triangulation. In this case, X is endowed with a flat metric g with conical singularities at some vertices of $\mathcal{T}$.

An interior vertex is singular if its valence $\mathrm{val}(v)$ is different to 6, and a vertex w belonging to the boundary of X is singular if its valence $\mathrm{val}(w)$ is different to 3. In such cases, $\mathrm{ord}_g(v) = 2\pi - \mathrm{val}(v)\frac{2\pi}{6}$ and $\mathrm{ord}_g(w) = \pi - \mathrm{val}(v)\frac{\pi}{3}$.

5 Proof of Theorem 1

We can now prove Theorem 1 by considering the canonical equilateral structure associated to $(X, \mathcal{T})$.

5.1 The case $n \neq 6k$

We endow $(X, \mathcal{T})$ with the structure of *equilateral* triangulation, by affecting the length 1 to each edge. Then, X is endowed with a structure of flat Riemannian disc with conical singularities. The result is then a direct consequence of Proposition 1.

5.2 The case $n = 6k$

We suppose now that the valence of the unique singular vertex v_0 is a multiple of 6 : $n = 6k, k \in \mathbb{N}, k \neq 1$.

5.2.1 An example

To simplify our construction, we assume that $k = 2$ (see subsection 5.2.4 for generalisations), and we show in Figure 3 an example of two triangulations $\mathcal{T}$ and $\mathcal{T}'$ of type $(6, 12)$ defined on the same (connected simply connected) domain X, adapted to the same weighted marked point $(v_0, 12)$ in the interior of X (the triangulations have a unique vertex of degree 12 in $\mathring{X}$) and the same marked points on $\mathbf{val}_{\partial X}$, that are not isomorphic. Indeed, let us introduce the classical distance $d_{\mathcal{T}}$ on a graph : If $(X, \mathcal{T})$ is a surface endowed with a triangulation, and $\mathcal{V}$ is the set of vertices of X, the distance $d_{\mathcal{T}}$ on $\mathcal{V}$ is defined as follows : If $v_1 \in \mathcal{V}$ and $v_2 \in \mathcal{V}$, $d_{\mathcal{T}}(v_1, v_2)$ is the minimum number of edges connecting v_1 and v_2. In our examples, the smallest distance $d_{\mathcal{T}}$ from m to a corner (that is, a boundary vertex with valence 2) of ∂X is different to the smallest distance $d_{\mathcal{T}'}$ from m' to a corner of $\partial X'$. We deduce that these triangulations are not isomorphic.

5.2.2 How to build such examples

Our construction is based on a ramified covering of a domain of the plane with two sheets, around the singular vertex.

5.2.3 The ramified covering of a equilateral triangulation around a conical singularity

Let us consider a regular equilateral triangulation of a domain $(X, \mathcal{T})$ of $\mathbb{E}^2$ (the valence of each interior vertex is 6), and two copies $(X_1, \mathcal{T}_1)$ and $(X_2, \mathcal{T}_2)$ of it. Let v_0 be a vertex of $\mathcal{T}$, v_1 (*resp.* v_2) the corresponding vertices in $\mathcal{T}_1$, (*resp.* $\mathcal{T}_2$) through the isometry between $\mathcal{T}$, $\mathcal{T}_1$ and $\mathcal{T}_2$. We suppose that v_0 is at distance 1 to $\partial\mathcal{T}$ (with respect to $d_{\mathcal{T}}$), and then, v_1 (*resp.* v_2) is at distance 1 to $\partial\mathcal{T}_1$

(*resp.* $\partial\mathcal{T}_2$), with respect to $d_{\mathcal{T}_1}$ (*resp.* $d_{\mathcal{T}_2}$). Let $e_1 = v_1p_1$ be an edge linking v_1 to $\partial\mathcal{T}_1$, and $e_2 = v_2p_2$ be the corresponding edge in $\mathcal{T}_2$ linking v_2 to $\partial\mathcal{T}_2$. We cut along these edges, so that the edge v_1p_1 is now replaced by two edges v_1p_1 and $v_1p'_1$ (the edge v_2p_2 is also replaced by two edges v_2p_2 and $v_2p'_2$). Now, we identify v_1 and v_2, and we glue the edge v_1p_1 with $v_2p'_2$, and $v_1p'_1$ with v_2p_2. We obtain a new domain Y whose boundary is the union of $\partial\mathcal{T}_1$ and $\partial\mathcal{T}_2$ endowed with a triangulation $\mathcal{T}$ whose all interior vertices have valence 6 except $v_1 = v_2$ whose valence is 12.

The result is a two sheets covering $\tilde{X}$ of X, branched at $\tilde{v}_0$. The domain $\tilde{X}$ is endowed with an equilateral triangulation. The valence of $\tilde{v}_0$ is 12.

To build the second example, the process is exactly the same, replacing the vertices v_1 and v_2 at distance d_{τ_1} (*resp.* d_{τ_2}) equal to 1 from their boundaries, by vertices q_1 and q_2 at distance 2 from the boundary. We obtain a new domain Z whose boundary is the union of $\partial\mathcal{T}_1$ and $\partial\mathcal{T}_2$ endowed with a triangulation $\mathcal{T}'$ whose all vertices have valence 6 except $q_1 = q_2$ whose valence is 12. Moreover, by construction, these two triangulations are not isomorphic.

Finally, we remark that these triangulations do not lie in the plane (they are immersed in $\mathbb{E}^3$ and can be embedded in $\mathbb{E}^4$). To draw them on the plane, one sends each triangle on the plane but not isometrically : By remarking that a point running on the boundary of the domain comes back to its initial position after turning twice around m, we send each triangle in the plane by dividing one of its angle by two, so that the boundary of the drawing is now a simple closed curve around the image of m.

5.2.4 Generalisations

- Analogous examples can be built by considering k-sheets ramified covering giving rise to triangulations around a singular vertex of valence $6k$, for any $k > 2$.
- Our results and constructions with triangulations can be mimicked by replacing triangles with quad meshes whose vertices except one have valence 4. We present here two figures showing two non isomorphic meshes of this type, see Figure 4.
- The reader can be easily convinced that the assumptions on the singularities of ∂X could be considerably enlightened.

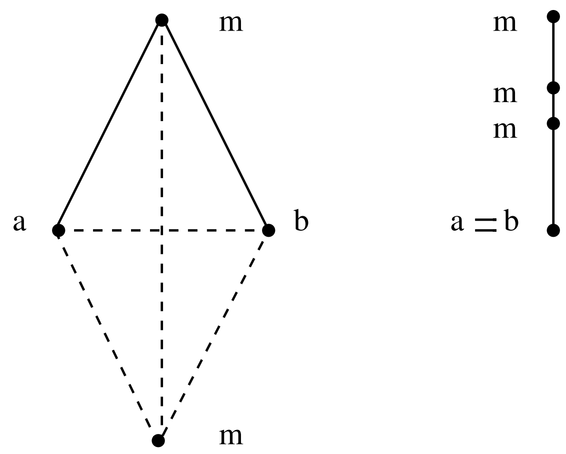

Figure 1: On the left, the angle θ at m is not a multiple of 2π. Up to a symmetry, the point m is uniquely determined by θ. It is no more true on the right, where the angle at m is a multiple of 2π.

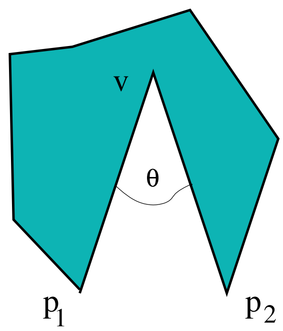

Figure 2: If one knows the position of p_1 and p_2 in the plane and the angle θ, one knows the position of v in $\tilde{X}$ and then in X.

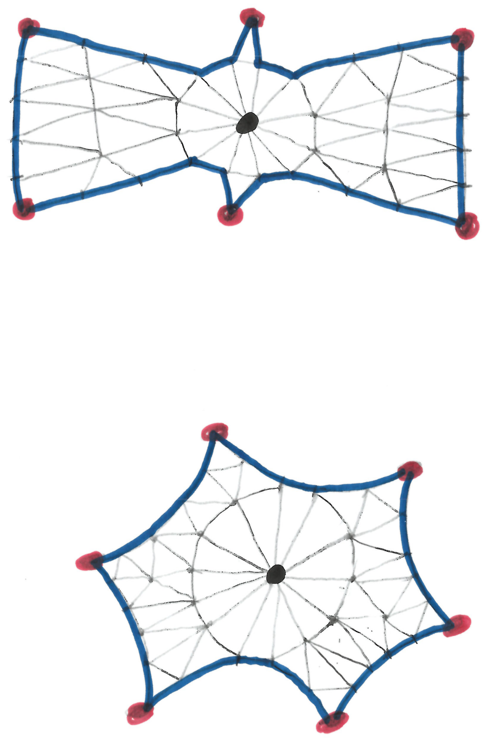

Figure 3: The smallest distance from the singular vertex to a corner of the boundary is 2 in one of the figure and 3 in the other, the red vertices are the corners on the boundary (with only two adjacent edges), the black vertex is the singular vertex in the interior

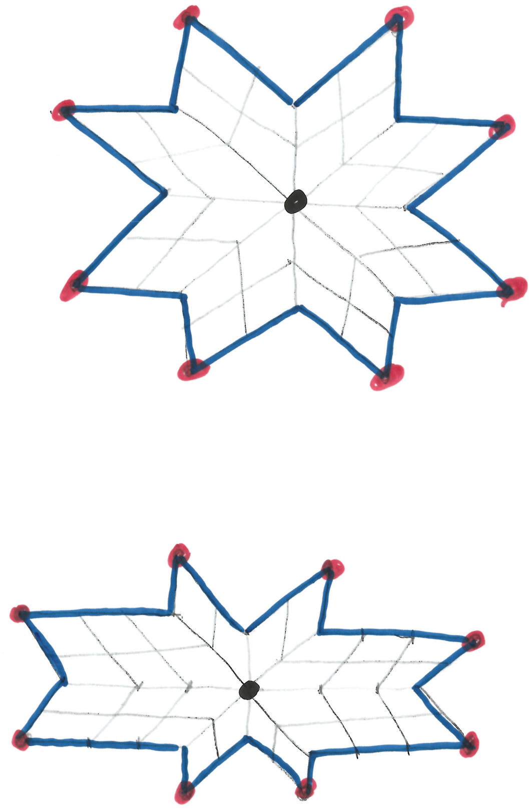

Figure 4: The distance from the boundary is 2 above and 1 below. The red vertices are the corners on the boundary (with only two adjacent edges), the black vertex is the singular vertex in the interior

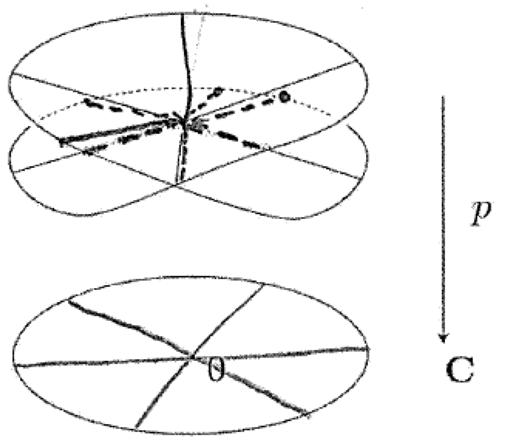

Figure 5: A ramified covering with two sheets. The map p is the projection over the disc whose boundary is the circle C

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