Supplementary Material for:

Modeling spatial processes with unknown extremal dependence class

1 Supporting information for Section 3

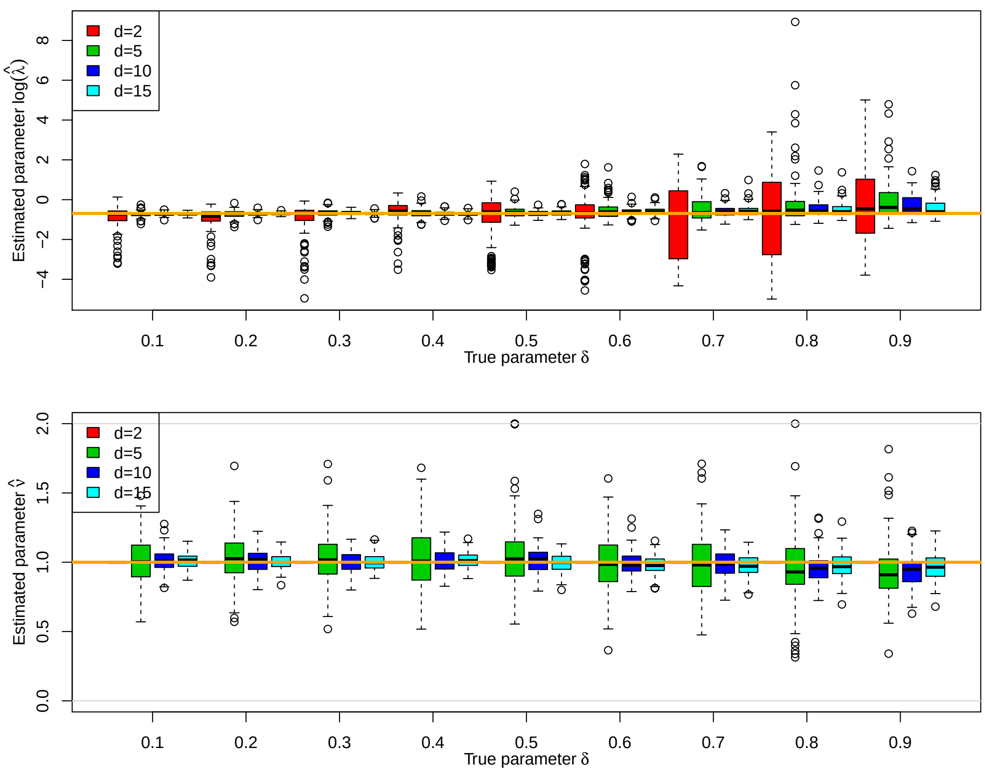

Figure 1: Boxplots for the MLEs $\log(\hat{\lambda})$ and $\hat{\nu}$, estimated concurrently with $\hat{\delta}$ as in Figure 3 of Section 3.2.

2 Supporting information for Section 4

2.1 Bootstrap procedure

To demonstrate that the stationary bootstrap procedure described in §4.1 adequately reproduces the temporal dependence in the extremes, we consider a spatial extension of the *extremal index* for univariate time series. For a stationary time series $\{X_t\}$, the extremal index, $\theta \in [0, 1]$, can be defined as

$$\theta = \lim_{n\to\infty} \mathrm{P}(X_2 \leq u_n, \ldots, X_{p_n} \leq u_n | X_1 > u_n),$$

where $p_n = o(n)$ and u_n is a series such that $n\{1-F(u_n)\} \to \tau \in (0, \infty)$. The extremal index describes the degree of temporal clustering of extremes, with $1/\theta$ the limiting mean cluster size. A popular estimator for θ is the so-called Runs Estimator (Smith and Weissman, 1994). The estimate is formed by taking the reciprocal of the mean cluster size, whereby threshold exceedances are determined to be part of different clusters (the same cluster) if they are separated by a run of at least m (fewer than m) consecutive non-exceedances.

In our application we have a time series of spatial processes $\{X_t(s)\}$, which, as we consider winter months only, may reasonably be deemed stationary. In analogy to the univariate case, we define clusters of spatial threshold exceedances as follows. A realization of the process is deemed to be a "threshold exceedance" if the observation at any site exceeds a given threshold. Clusters are then defined as sequences of threshold exceedances separated by a run of at least m non-exceedances, and θ as the reciprocal mean cluster size. Figure 2 displays a histogram of estimated θs, using a value of $m = 1$, from 200 bootstrap samples, along with that from the original dataset of 50 sites temporally thinned to one observation per day. The threshold value used was the 95%-quantile, as in the model fit. The agreement between the original and bootstrap samples indicates that the temporal structure of the extremes is adequately reproduced.

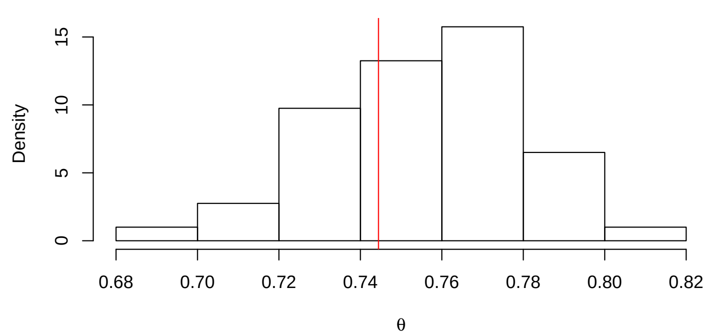

Figure 2: Estimates of the extremal index from the time series of spatial processes using the stationary bootstrap sampling procedure described in §4.1. The vertical line is the value from the original sample.

2.2 Additional model fit diagnostics

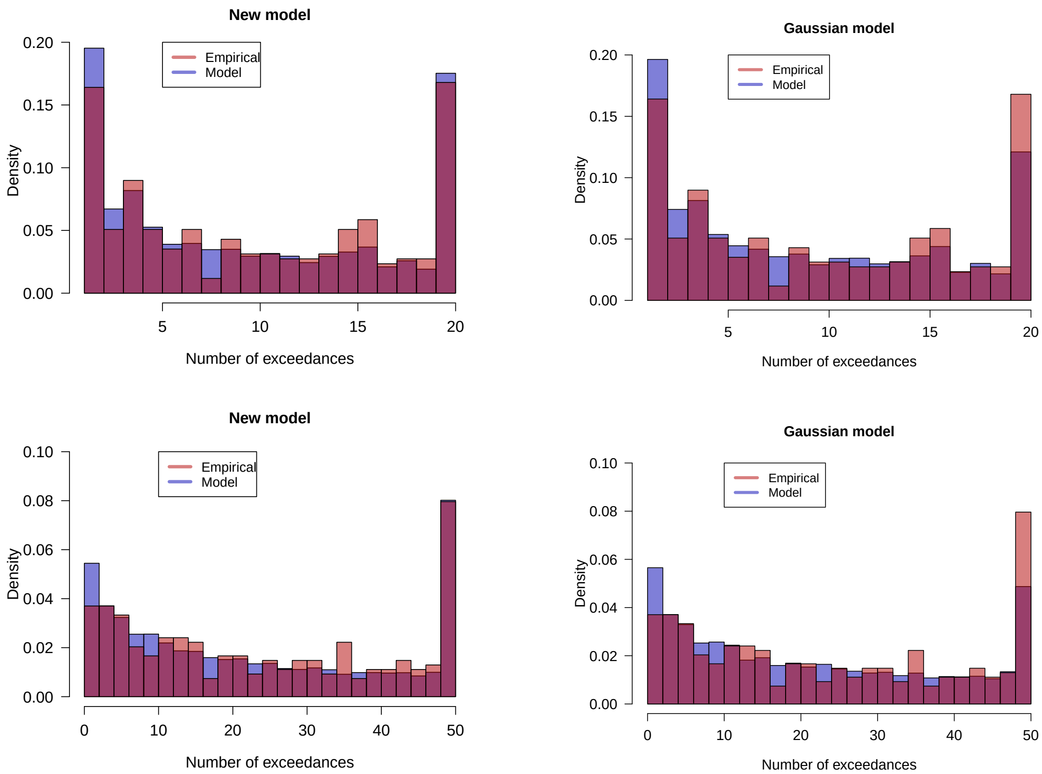

Figure 3: Distribution of the number of exceedances, given at least one exceedance of the 95%-quantile threshold. These histograms are based on the data at the 20 sites used for fitting the model (top) or all 50 sites (bottom). Model-based quantities are calculated for our new model (left) and the Gaussian model (right) by simulating 10^5 values from the fitted dependence models.

References

Smith, R. L. and Weissman, I. (1994). Estimating the extremal index. *J. Roy. Statist. Soc. B*, pages 515–528.