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Non-overlapped P- and S-wave Poynting vectors and its solution on Grid Method

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Right Running Head: Poynting vector

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Abstract

Poynting vector represents the local directional energy flux density of seismic waves in geophysics. It is widely used in elastic reverse time migration (RTM) to analyze source illumination, suppress low-wavenumber noise, correct for image polarity and extract angle-domain common imaging gather (ADCIG). However, the P and S waves are mixed together during wavefield propagation such that the P and S energy fluxes are not clean everywhere, especially at the overlapped points. In this paper, we use a modified elastic wave equation in which the P and S vector wavefields are naturally separated. Then, we develop an efficient method to evaluate the separable P and S poynting vectors, respectively, based on the view that the group velocity and phase velocity have the same direction in isotropic elastic media. We furthermore formulate our method using an unstructured mesh based modeling method named the grid method. Finally, we verify our method using two numerical examples.

Key words: separable, poynting vector, unmixed, unstructured mesh
1. Introduction

The energy flux density vector (Poynting vector) could be used to efficiently estimate the local propagation direction of seismic wavefield. It is convenient to implement in reverse time migration because only derivations in space and time are needed, which are easy to compute during wave extrapolation.


In elastic isotropic media, the conventional calculation for Poynting vector is obtained by multiplying the stress tensor by the particle-velocity vector (Cerveny 2001, Du et. al 2012). However, the conventional methods usually encounter certain
difficulties where P and S particle-velocity components are superimposed, the relations between particle motion and propagation direction break down; the corresponding Poynting vectors cannot indicate the propagation directions of the P- and S-waves accurately at the same time (Tang et al 2016). For this reason, Wang and McMechan (2015) introduce a new equation to accurately separate the P- and S-energies by decomposing stress tensor and particle-velocity vector. However, Tang et al (2016) find that the decomposing approach is invalid in obtaining a pure S-wave-mode stress tensor. As a result, the S-wave Poynting vector is still affected by P-wave. To solve this problem, they propose an approach to obtain the pure P- and S-wave Poynting vectors based on velocity-dilatation-rotation equations. Their results show that the two pure Poynting vector vectors can indicate the directions more accurately than the conventional methods, especially when P- and S-wave wavefields are mixed.

The paper is arranged as following: Firstly, we briefly review decoupled P- and S-wave equations. Then, we demonstrate how to efficiently calculate the non-overlapped P and S Poynting vectors. Afterwards, we try to use unstructured mesh based grid method to simulate the non-overlapped vectors numerically. Finally, we verify our schemes in the numerical example.
2. Methods

2.1. Decoupled P- and S-wave equations

The elastic wave equation in an infinite, homogeneous, isotropic media can be expressed as (Aki and Richards 1980):

\[ \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda + 2\mu) \nabla(\nabla \cdot \mathbf{u}) - \mu \nabla \times \nabla \times \mathbf{u}, \]  

(1)

where \( \mathbf{u} \) is the vector-displacement wavefield, \( t \) is time, \( \rho \) is the density, and \( \lambda \) and \( \mu \) are the Lamé parameters. Here, \( \nabla \), \( \nabla \cdot \), and \( \nabla \times \) are the gradient, divergence, and curl, respectively. According to the Helmholtz decomposition theory (Aki and Richards 1980), the vector wavefield \( \mathbf{u} \) can be decomposed into a curl-free P-wavefield and a divergence-free S-wavefield:

\[ \mathbf{u} = \mathbf{u}^p + \mathbf{u}^s. \]  

(2)

with

\[ \nabla \times \mathbf{u}^p = 0, \]  

(3)

and

\[ \nabla \cdot \mathbf{u}^s = 0. \]  

(4)

Equations (2-4) basically indicate traditional wavefield separation workflow: implementing the divergence and curl operators, respectively, to the whole elastic wavefields (Dellinger and Etgen 1990). The separated results are P-wave (potential wavefield) and S-wave (vector wavefield). In 2D case, S-wave has only one component, so we can approximately treat it as a scalar wavefield. The second approach is to obtain P- and S-waves directly, without the divergence or curl operator. Zhang and McMechan (2010) expound this method in the wavenumber domain. In
fact, it can be performed in the space domain (Du et al 2017). If we rewrite equation (1), then obtain the decoupled P- and S-wave propagation equation (Ma and Zhu 2003)

\[
\mathbf{u} = \mathbf{u}^P + \mathbf{u}^S,
\]

\[
\frac{\partial^2 \mathbf{u}^P}{\partial t^2} = b(\lambda + 2\mu)\nabla (\nabla \cdot \mathbf{u}),
\]

\[
\frac{\partial^2 \mathbf{u}^S}{\partial t^2} = -b\mu\nabla \times (\nabla \times \mathbf{u}),
\]

where \( \mathbf{u}^P = (u_{px}, u_{py}, u_{pz}) \) is the P-wave displacement vector and \( \mathbf{u}^S = (u_{sx}, u_{sy}, u_{sz}) \) is the S-wave displacement vector, \( b \) is the inverse of the density \( \rho \).

The P- and S-wave equation (5) is the second-order equation. In fact, the extra computational cost for calculating the Poynting vector is negligible because only derivatives in space and time are needed. Calculation of the Poynting vector in the first-order equation (Du et al 2012, Wang and McMechan 2015, Tang et al 2016) is more convenient and more easily because the first-order equation computes the time derivative and the space derivative of the wavefield inherently. Therefore, we use the first-order stress-particle velocity wave equations (2D) instead of equation (5):

\[
\mathbf{v} = \mathbf{v}^P + \mathbf{v}^S,
\]

\[
\frac{\partial \mathbf{v}^P}{\partial t} = \alpha^2 \nabla \theta,
\]

\[
\frac{\partial \mathbf{v}^S}{\partial t} = -\beta^2 \nabla \times \mathbf{\omega},
\]

\[
\frac{\partial \theta}{\partial t} = \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z},
\]
\[
\frac{\partial \omega}{\partial t} = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}, \quad (10)
\]

Here, \( \theta = \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \), \( \omega = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \), \( \alpha \) is the P-wave velocity \( (\sqrt{\lambda + 2\mu}/\rho) \), \( \beta \) is the S-wave velocity \( (\sqrt{\mu/\rho}) \). \( v = (v_x, v_z) \), \( v^p = (v_{px}, v_{pz}) \) and \( v^s = (v_{s}, v_{sz}) \) are the P and S particle-velocity vectors, respectively. The main advantage of these wave equations is that amplitude and phase are well preserved because we do not spatially carry out any first derivative operator to the original wavefield. No matter for the divergence or curl operator, they both could result in an additional \( \frac{i\omega}{v} \) filter, where \( i \) is imaginary part, \( \omega \) is angular frequency, \( v \) is migration velocity. The outputs of the wave equation are more physically correct compared with the traditional method.

2.2. P- and S-wave Poynting vectors

The Poynting vector is first derived for electromagnetic field. The definition of Poynting vectors is the energy flow direction; its magnitude is the amount of energy through a unit area per unit time. The components of the elastic Poynting vector can be expressed as (Cerveny 2001)

\[
s_i = -\tau_{ij} v_j, \quad (11)
\]

where \( i \) and \( j \) indicate the \( x \)- or \( z \)- component in 2D of the Poynting vector \( s \), \( \tau_{ij} \) and \( v_j \) are the stress tensor and particle velocity, respectively. For isotropic acoustic propagation, the Poynting vector \( \mathbf{S} \) can be obtained in the following form (Yoon et al 2011):

\[
\mathbf{S} = -\rho \nabla \rho, \quad (12)
\]
where \( p \) is the pressure field, \( \dot{p} \) is the time derivative of the pressure field, and \( \nabla p \) is the spatial gradient of the pressure field. Note that equation (11) indicates the group-velocity direction (the direction of energy flow), but equation (12) indicates the phase-velocity direction because it points in the direction normal to the wavefront. The directions given by equations (11) and (12) coincide in isotropic media, but they are different in anisotropic medium (Yan and Dickens 2016).

It is noticeable that equation (11) cannot distinguish between P- and S-wave Poynting vectors when these two vectors are mixed together. To overcome this limitation, Wang and McMechan (2015) propose an approach that the P- and S-wave particle-velocity and stress vector components are decomposed in the vector domain during the wavefield extrapolations. Then, they directly use the decomposed P and S stress and particle-velocity vector components to calculate the P- and S-waves Poynting vectors. As a result, P-wave Poynting vector \( s_i^p \) and S-wave Poynting vector \( s_i^s \) can be computed by the conventional way (equation (11)):

\[
\begin{align*}
\text{(13)} & \quad s_i^p = -\tau^p \nu_i^p, \\
\text{(14)} & \quad s_i^s = -\tau^s \nu_i^s,
\end{align*}
\]

where \( \tau^p = (\lambda + 2\mu) \cdot \theta \), \( \tau^s = \tau^p \delta(i-j). \) Instead of calculating a divergence-free \( \tau^i \) to obtain \( s_i^s \) in the conventional way, Tang et al (2016) study the mechanism of elastic-wave energy flux to calculate \( s_i^s \). By making some assumptions and simplifications, they firstly develop a set of velocity-dilatation rotation equations to obtain the P- and S-wave particle-velocity vectors \( \mathbf{v}_p \) and \( \mathbf{v}_s \), dilatation scalar \( \Theta \), and rotation vector \( \omega \), then, they calculate the P-wave Poynting vector by multiplying...
the P-wave particle-velocity vector by dilatation scalar, and calculate the S-wave
Poynting vector as a cross product of the S-wave particle-velocity vector and rotation
vector as follows:

\[ \mathbf{e}_P = -\theta \mathbf{v}_p, \]  \hspace{1cm} (15)

\[ \mathbf{e}_S = -\mathbf{v}_s \times \mathbf{\omega}, \]  \hspace{1cm} (16)

where \( \mathbf{v}_p = (v_{px}, v_{py}, v_{pz}) \) and \( \mathbf{v}_s = (v_{sx}, v_{sy}, v_{sz}) \) are the P-wave particle-velocity vector, and S-wave particle-velocity vector, respectively. \( \mathbf{\omega} = (\omega_x, \omega_y, \omega_z) \) in equation (16) is the rotation vector, which can also be denoted as

\[ \omega_x = \frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial z}, \omega_y = \frac{\partial u_y}{\partial z} - \frac{\partial u_z}{\partial x}, \omega_z = \frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial y}. \]

Rearranging equations (15) and (16) as 2D

\[ e_{px} = -\theta v_{px}, e_{pc} = -\theta v_{pc}, \]  \hspace{1cm} (17)

\[ e_{sx} = \omega_x v_{sx}, e_{sc} = -\omega_y v_{sx}, \]  \hspace{1cm} (18)

Note that equations (13) and (14) indicate the group-velocity direction, and equations (17) and (18) indicate the phase-velocity direction. However, these two directions differ in anisotropic media. Figure 1 demonstrates this situation. In isotropic media, group velocity and phase velocity are in the same direction as mentioned above. For the numerical examples in this paper, the differences do not need to be considered, we can directly compare equations (17) and (18) with equations (13) and (14).

2.3. Numerical solution using the grid method

Grid method is an irregular, unstructured mesh-based numerical technique proposed by Zhang and Liu (1999, 2002). The grid method is flexible in combining 2D and 3D surface and interface topographies with a natural satisfaction of the free-surface
boundary conditions (Gao and Zhang 2006). The 2D grid method has been widely used to model wave propagation in media with high velocity contrasts (Zhang 2004a), in fractured media (Zhang 2005), and in mixed elastic/poroelastic models (Zhang 2001) as well as wave propagation across fluid–solid boundaries (Zhang 2004b). Here, we try to use grid method solve the 2D isotropic elastic stress particle-velocity formulation (Virieux 1986)

\[
\rho \frac{\partial v_x}{\partial t} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z},
\]

(19)

\[
\rho \frac{\partial v_z}{\partial t} = \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z},
\]

(20)

\[
\frac{\partial \tau_{xx}}{\partial t} = (\lambda + 2\mu) \frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z},
\]

(21)

\[
\frac{\partial \tau_{zx}}{\partial t} = \lambda \frac{\partial v_x}{\partial x} + (\lambda + 2\mu) \frac{\partial v_z}{\partial z},
\]

(22)

\[
\frac{\partial \tau_{xz}}{\partial t} = \mu \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right),
\]

(23)

In equations (19-23), \(\tau_{xx}\) and \(\tau_{zz}\) are horizontal and vertical normal stresses, \(\tau_{xz}\) is the shear stress, and \(v_x\) and \(v_z\) are the horizontal and vertical particle-velocity components. The local mesh accounting for surface topography and curved interfaces is displayed in figure 2. Integrating both sides of equations (19) and (20) over the domain enclosed by the contour (dashed line) 1-2-3-4-5-6-7-8-9-10-11-12-1 leads to

\[
\int_\Omega \rho \frac{\partial v_x}{\partial t} \, dx \, dz = \oint_{1-2-3-4-5-6-7-8-9-10-11-12-1} (\alpha \tau_{xx} + \beta \tau_{xz}) \, ds,
\]

(24)

\[
\int_\Omega \rho \frac{\partial v_z}{\partial t} \, dx \, dz = \oint_{1-2-3-4-5-6-7-8-9-10-11-12-1} (\alpha \tau_{zx} + \beta \tau_{zz}) \, ds,
\]

(25)

where \(\alpha\) and \(\beta\) are direction cosines of the outward-directed normal to the contours.

By applying the lumped mass model on the left-hand side of equations (24) and (25), we finally have the expression of these two equations after some algebraic calculation.
\[ M_k \left( \frac{\partial v_x}{\partial t} \right)_k = \sum_{i=1}^{m} (\tau_{x_i})_i (b_k)_i - \sum_{i=1}^{m} (\tau_{x_i})_i (c_k)_i, \]  
\( \text{(26)} \)

\[ M_k \left( \frac{\partial v_z}{\partial t} \right)_k = \sum_{i=1}^{m} (\tau_{z_i})_i (b_k)_i - \sum_{i=1}^{m} (\tau_{z_i})_i (c_k)_i, \]  
\( \text{(27)} \)

Here, \( M_k \) is one-third of the sum of the masses of the grids around node \( k \). \( \left( \frac{\partial v_x}{\partial t} \right)_k \) and \( \left( \frac{\partial v_z}{\partial t} \right)_k \) are, respectively, time derivatives of the velocities \( v_x \) and \( v_z \) at node \( k \). Similarly, the equations (9) and (10) could be computed by

\[ M_k \left( \frac{\partial \theta}{\partial t} \right)_k = \sum_{i=1}^{m} (v_x)_i (b_k)_i - \sum_{i=1}^{m} (v_z)_i (c_k)_i, \]  
\( \text{(28)} \)

\[ M_k \left( \frac{\partial \omega}{\partial t} \right)_k = \sum_{i=1}^{m} (v_x)_i (c_k)_i - \sum_{i=1}^{m} (v_z)_i (b_k)_i, \]  
\( \text{(29)} \)

Actually, the right-hand side of equations (9) and (10) are 2D divergence and curl operators. We transform the operators to line integral using grid method, which is easily implemented during wave extrapolation. Furthermore, the Poynting vector is easy to compute via application of grid method, avoiding differential operations calculation. In the following subsection, this issue will be discussed in detail.

For a typical triangular grid \( ijk \), by utilizing the linear interpolating function pass through the nodes of the triangle, the first-order spatial derivatives have the following form (Cook 1974 page 83)

\[ \frac{\partial v_x}{\partial x} = \frac{1}{\Delta} \left[ b_i (v_x)_i + b_j (v_x)_j + b_k (v_x)_k \right], \]  
\( \text{(30)} \)

\[ \frac{\partial v_x}{\partial z} = -\frac{1}{\Delta} \left[ c_i (v_x)_i + c_j (v_x)_j + c_k (v_x)_k \right], \]  
\( \text{(31)} \)

where \( \Delta \) denotes the area of grid \( ijk \), \( b_i = (z_i - z_j)/2 \), \( c_i = (x_i - x_j)/2 \) and \( b_j \), \( c_j \) etc. can be achieved by letting \( i-j-k-i \). Here, \( (x_i, z_i) \), \( (x_j, z_j) \) and \( (x_k, z_k) \) are the coordinates of nodes \( i \), \( j \) and \( k \). Equations (30) and (31) are also valid for the velocity
component \( v_z \). It is easy to see that once equations (28) and (29) are obtained, they could be employed to compute the spatial derivatives of \( v_{px}, v_{pc}, v_{sx}, v_{sc} \) through equations (30) and (31). The geometric parameters of the triangular cells using in the grid method have been calculated and stored in advance. For a set of identical triangular cells, only a set of geometric parameters are needed to store, so it does not increase too much storage. It can be seen that the calculations of space derivatives are much simpler than conventional finite difference.

2.4. Workflow for calculating pure P- and S-wave Poynting vectors

The following steps consist of the whole procedure of P- and S-wave Poynting vectors calculation using the grid method:

1) Calculate the particle velocity \( \mathbf{v} = (v_x, v_z) \) and stress tensor \( \mathbf{\tau} = (\tau_{xx}, \tau_{zz}, \tau_{sc}) \) by using the conventional stress particle velocity wave equations (19-23).

2) Calculate the dilatation scalar wavefield \( \Theta \) and rotation vector \( \mathbf{\omega} \) by using equations (28) and (29).

3) Compute the P-wave particle-velocity \( \mathbf{v}^P = (v_{px}, v_{pc}) \) by using equation (7).

4) Compute the S-wave particle-velocity \( \mathbf{v}^S = (v_{sx}, v_{sc}) \) by using equation (8).

5) Apply the equations (17) and (18) to obtain the pure P-and S-wave Poynting vectors based on the grid method.

To illustrate the advantage of our algorithm, we do some comparisons of the efficiency and storage. In equation (11), there is one stress tensor \( \mathbf{\tau} \) and two vectors, \( \mathbf{v} \) and \( \mathbf{s} \), which need to be calculated and stored in the memory. In the approach proposed by Wang and McMechan (2015), one stress tensor \( \mathbf{\tau} \); one scalar \( \tau_p \); and
five vectors, $\mathbf{v}$, $\mathbf{v}_p$, $\mathbf{v}_s$, $\mathbf{e}_p$ and $\mathbf{e}_s$, should be calculated and stored. In the algorithm implemented by Tang et al. (2016), there is one scalar $\theta$ and the six vectors $\mathbf{v}$, $\mathbf{v}_p$, $\mathbf{v}_s$, $\mathbf{e}$, $\mathbf{e}_p$ and $\mathbf{e}_s$ need to be obtained and stored. Each vector contains three components, and the stress tensor contains six different components. Hence, memory consumption of their approach is approximately 1.7 times than conventional method. Moreover, when solving their proposed equations using the finite difference methods, the computation of solving differential operations in space is too expensive. If we use the numbers of different differential operations in space as a measurement of the computational efficiency. In the method of Wang and McMechan (2015), there are 21 different differential operations in space. There are 18 differential operations based on velocity-stress equations in the method proposed Tang et al. (2016). Obviously, the differential operator is too expensive in computation and storage. Note that the time derivation (equations (26) and (27)) can be transformed the product of particle velocity and geometric parameters ($b_i, c_i$ and $b_j, c_j$ etc.) of the triangular cells in our algorithm. In the meantime, the space derivation can also be directly obtained by similar product. For every step in time, the geometric parameters are same and do not need repeated calculation. As a result, the different differential operations can be computed by simple multiplication and addition operations. If the variables $\theta$, $\mathbf{v}_p$, $\mathbf{v}_s$, $\omega$ are not needed to output, which could be calculated by auxiliary variable in the grid method, we do not need to open extra memory space to store these variables. In summary, all calculations for Poynting vectors are linear operations and independent of neighbored points in other triangular cells compared with conventional differential
techniques, the grid method is computationally efficient and requires little additional memory space in the numerical implementation. To further improve the calculation efficiency, we perform the double-mesh grid method (Yang et al. 2017) that is very suitable to parallel arithmetic.

3. Numerical results

In this section, we demonstrate the validity of the technique presented in this paper using two models. The first model is a graben-like model and the second model is a portion of the SEG Foothill model. In these numerical examples, we solve equations using the grid method (Zhang and Liu 1999, 2002).

3.1. Garben-like model test

We first test the method by a graben-like model (the parameters of the model are the same as Tang et al 2016), displayed in figure 3. The graben-like model generated by Tang et al (2016) contains two stratum with strong velocity contrast. The interface of the two stratum forms a “valley” with two steep cliffs. The valley is just like a canyon without a flat bottom. We also choose the graben-like model for forward modeling to show the comparisons.

The grid spacing is 10 m along the $x$- and $z$- directions. The source with a peak frequency of 25 Hz is placed at the center of the model in the horizontal direction with the depth of 20 m. The grid step in time $\Delta t$ is 0.3 ms. The snapshots of the wavefield at 800 ms are shown in figure 4. In figure 4, there are no changes for amplitude and phase information when $v_p$ and $v_s$ are decomposed from $v$. These snapshots verify the advantages of the approach proposed by Tang et al (2016).
It is easy to notice that there are overlapping events of P- and S-waves in the snapshots of $v$ in figure 4(a) and (b). As we have mentioned before, the conventional method strongly depends on the assumption that there exists only one dominant direction of propagation at each grid point during wave extrapolation. Hence, the overlapping events cause inaccuracy when calculating the Poynting vector. Here, we use equation (11) to obtain the conventional Poynting vector, which is displayed as figure 5. As Wang and McMechan (2015) propose, we use equation (13) to calculate the P-wave Poynting vectors as shown in figure 6, and we demonstrate the S-wave Poynting vectors by equation (14) in figure 7. Next, we calculate the pure P- and S-wave Poynting vectors (figures 6 and 8) based on velocity-dilatation-rotation equations (Tang et al 2016).

In figure 5, the conventional Poynting vectors indicate the propagation direction of mixed P- and S-waves. In figure 5, the P-wave Poynting vectors calculated by equation (13) are affected by the S-wave. Figure 6 displays the P-wave Poynting vectors calculated by equation (17) are not interfered with by the S-wave. In figure 7, the S-wave Poynting vectors calculated as a multiplication are still affected by the P-wave when P and S particle-velocity components are superimposed. However, in figure 8, the S-wave Poynting vectors calculated as a cross product are not influenced by the P-wave. The S-wave-mode has polarity event on the opposite sides of the normal incidence point. Hence, the Poynting vector of S wave should also present the polarity change event. Compared with the conventional results (figures 6 and 7, Tang...
et al 2016), the waveform changes result from the polarity reversals of the S-wave component are well presented in this paper.

3.2. SEG Foothill model test

Next, we use a portion of the SEG Foothill model for illustration in figure 10. The ratio of the S-velocity to the P-velocity is $\frac{2}{3}$. The space step is 10 m in the $x$- and $z$- directions. A P-wave source is located at point (6.0 and 1.589 km). The time sample increment is 0.5 ms. Figure 11(a) and (b) displays the snapshots for $x$- and $z$-components of the wavefields at 1100 ms. To further verify the robustness of our procedure, a Foothill model is used in this section. The decompositions of wavefield are presented in figure 11(c) and (d). Figure 11(e-h) show the snapshots of components of $v_{px}$, $v_{pz}$, $v_{sx}$ and $v_{sz}$, respectively. Our results indicate that the quality of separation for complex model is guaranteed. We can notice that the wavefields are complex and many overlapping events appear because of wave propagation in different directions at a spatial point. Figure 12(a) and (b) demonstrates the conventional Poynting vectors calculated by equation (11). Compared with the conventional results (figure 13(c) and (d)), the overlapping events are attenuated by using proposed non-overlapped P-wave Poynting vectors as illustrated in figure 13(a) and (b). Similarly, there no exist overlap in figure 14(a) and (b) compared with figure 14(c) and (d) for S-wave Poynting vectors. Since computing angle domain common imaging gathers (ADCIG) is a very application of Poynting vectors, we illustrate the ADCIG in figure 15. The PP and PS ADCIGs are generated by migration of all common-source gathers using the separable P and S Poynting vectors and grid method.
In this paper, for the sake of simplicity of computation, we only output PP and PS single angle gathers according to Green’s Reciprocation Theorem. This numerical example illustrates that our technique based on the grid method is effective and robust even in a complex model.

4. Conclusions

In this paper, we propose a strategy that could get rid of the overlapping of P and S energy fluxes. The method is based on a P and S separable elastic wave equation. The wave equation is solved using the unstructured mesh based grid method. The conventional pointing vectors are calculated by the product of the stress tensor and particle velocity vector, in which mixed P and S energy fluxes are produced with a slightly expensive computation cost. In comparison, our separable method only need the potential wavefields multiplied by their corresponding particle velocity vectors, respectively, at a more economic computational cost, to obtain the separable Poynting vectors. We attribute this as in isotropic elastic media, the group velocity and phase velocity have the same direction. Numerical examples show that the P and S energy fluxes are separated well.

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Figure 1. Snapshots of velocity wavefields for a homogeneous anisotropic media. The parameters of the model are $V_{p0} = 3000 \text{ m/s}$, $V_{s0} = 2000 \text{ m/s}$ and $\rho = 2.4 \text{ g/cm}^3$, $\varepsilon = 0.3$, $\delta = 0.1$, tilt = 60°. (a) phase velocity direction (the red arrow), (b) group velocity direction (the blue arrow)
Figure 2. Local mesh. The triangles drawn by solid lines are basic cells in the grid method. The two velocity components are defined at the grid nodes, represented by black dot; The three stress components are defined at the cell barycenters, represented by empty dot. The dash lines link the centers of the grids and the edges.
Figure 3. Graben-like model. This model is a 2D isotropic elastic model, which contains two strataums with strong velocity contrast. The grid intervals of the $x$- and $z$-directions $\Delta x = \Delta z = 5$ m, respectively.
Figure 4. Wavefield snapshots for the graben-like model as shown figure 3, (a and b) Snapshots of components of $v_x$ and $v_z$. (c and d) Snapshots of $\theta$ and $\omega_y$. (e-h) Snapshots of components of $v_{px}$, $v_{pc}$, $v_{sx}$ and $v_{sc}$. 
Figure 5. Conventional mixed Poynting vectors calculated by equation (11). (a and b) Components of $x$- and $z$-directions of the vectors.
Figure 6. The conventional pure P-wave Poynting vectors calculated by equation (13).

(a and b) Components of $x$- and $z$-directions of the vectors.
Figure 7. The pure P-wave Poynting vectors calculated by equation (17). (a and b) Components of x- and z-directions of the vectors.
Figure 8. The conventional pure S-wave Poynting vectors calculated by equation (14).

(a and b) Components of $x$- and $z$-directions of the vector.
**Figure 9.** The pure S-wave Poynting vectors calculated by equation (18). (a and b) Components of $x$- and $z$-directions of the vectors.
Figure 10. SEG Foothill velocity model with rugged topography.
Figure 11. (a and b) Input $x$- and $z$-components of wavefields for the SEG Foothill model. (c and d) Snapshots of $\theta$ and $\omega_x$. (e-h) Snapshots of components of $v_{px}$, $v_{pc}$, $v_{sx}$ and $v_{sc}$. 
Figure 12. The mixed Poynting vectors calculated by equation (11) for partial Foothill model as shown in figure 10. (a and b) Components of x- and z-directions of the vectors.
Figure 13. The pure P-wave Poynting vectors for partial Foothill model as shown in figure 10. (a and b) Components of $x$- and $z$-directions of the vectors by equation (17). (c and d) Components of $x$- and $z$-directions of the vectors calculated by equation (13).
Figure 14. The pure S-wave Poynting vectors for partial Foothill model as shown in figure 10. (a and b) Components of $x$- and $z$-directions of the vectors calculated by equation (18). (c and d) Components of $x$- and $z$-directions of the vectors calculated by equation (14).
Figure 15. The ADCIGs for partial Foothill model. The incident angle ranges from $-60^\circ$ to $60^\circ$. (a) PP ADCIGs, (b) PS ADCIGs.