Localization of Energy Harvesting Empowered Underwater Optical Wireless Sensor Networks

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Abstract

In this paper, a received signal strength (RSS) based localization technique is developed for energy harvesting underwater optical wireless sensor networks (EH-UOWSNs), where the optical noise sources and channel impairments of seawater pose significant challenges for range estimation. Energy limitation is another major problem due to the limited battery power and difficulty in replacing or recharging the battery of an underwater sensor node. In the proposed framework, sensor nodes with insufficient battery, harvest the energy and starts communicating once it has sufficient energy storage. Network localization is carried out by measuring the RSSs of active nodes, which are modeled based on the underwater optical communication channel characteristics. Thereafter, block kernel matrices are computed for the RSS based range measurements. Unlike the traditional shortest-path approach, the proposed technique reduces the shortest path estimation for each block kernel matrix. Once the complete block kernel matrices are available, a closed form localization technique is developed to find the location of every optical sensor node in the network. Furthermore, an analytical expression for Cramer Rao lower bound (CRLB) is derived as a benchmark to compare the localization performance of the proposed technique. Finally, extensive simulations show that the proposed technique outperforms the well-known network localization techniques.

Index Terms

Energy Harvesting, Underwater Optical Sensor Networks, Localization, Optical Communication

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I. INTRODUCTION

A. Motivations and Background

Underwater wireless sensor networks (UWSNs) are enablers of many commercial, scientific and military underwater applications including instrument monitoring, climate recording, prediction of natural disasters, exploration for the oil industry, search & rescue missions, control of the autonomous underwater vehicles (AUVs), and marine life study [2], [3]. The need for high quality of service communication necessitates high data rate, low latency, and long-range networking solutions poses a daunting challenge because of the highly attenuating medium of seawater for most electromagnetic frequencies.

Today’s UWSNs are mostly based on acoustic communication systems which suffer from unique aquatic conditions and display severe attenuation characteristics, frequency dispersion, multipath fading, and limited bandwidth. In addition, the signal propagation delays and variable speed of sound create a set of unique challenges. Therefore, underwater acoustic communication has low achievable rates (10-100 kbps) due to the limited bandwidth and high latency because of the low propagation speed of acoustic waves (1500 m/s) [4]. On the other hand, underwater optical wireless communication (UOWC) has the advantages of higher bandwidth, lower latency, and enhanced security [5]. Nevertheless, UOWC has a very limited range attainability (10-100 m) due to absorption, scattering, and turbulence impairments of seawater. It is also susceptible to many noise sources such as sunlight, background, thermal, and dark current noises [6].

Furthermore, underwater sensor nodes have limited energy resources, which has a substantial impact on the network lifetime. Taking the engineering hardship and monetary cost of battery replacement into account, an energy self-sufficient UWSN is essential to maximize the network lifetime. In this regard, energy harvesting is a promising solution to collect energy from the ambient sources in the aquatic environment and storing it in an energy buffer. As surveyed in [7], ongoing research efforts on terrestrial communications have shown that energy harvesting plays a significant role in enhancing the performance. However, most of the energy harvesting techniques are designed for outdoor environments and not applicable in the aquatic environment. Moreover, albeit the notable research body on designing different protocols for underwater communication networks, no significant research is carried out on the energy harvesting methods for UOWSNs.

As the gathered data is useful only if it refers to a particular position of the sensor node, localization of nodes in UOWSN is of utmost importance. Network localization is especially useful
for a number of applications such as target detection, intruder detection, routing protocols, and data tagging. Hence, a number of acoustic underwater sensor networks localization techniques have been proposed in the past [8]–[14]. The performance of every localization technique mainly relies on the initial reference position, number of sensor nodes, ranging technique, number of anchors and the position of the anchors in the network [15]. However, aforementioned optical communication challenges and energy constraints do not allow the use of existing acoustic localization techniques for underwater sensor nodes. The only UOWSN localization technique is addressed in [16] where RSS and time of arrival methods are investigated for an optical code-division multiple access networks. Accordingly, in this paper, we investigate an RSS based localization for energy harvesting underwater optical sensor networks (EH-UOWSNs). To the best of our knowledge, the problem of UOWSN localization with energy harvesting is not addressed in the literature yet.

B. Main contributions

The contributions of this paper are two-folded: First, we propose an EH-UOWSN where sensor nodes are quipped with multiple energy harvesters to collect renewable energy in the aquatic environment. Therefore, we consider an energy harvester module as a combination of microbial fuel cells (MFCs, acoustic harvester (AH), and an ultra-capacitor with a high storing efficiency. Based on the harvested energy availability, the sensor nodes communicate with its neighbor nodes and computes the RSS ranges. Secondly, a closed form localization technique is developed to find the location of every optical sensor node in EH-UOWSN. The proposed localization technique accurately minimizes the error function by partitioning the kernel matrix into smaller block matrices. Furthermore, a novel matrix completion strategy is used to complete the missing elements in block matrices, which results in better approximation. An analytical expression for Cramer Rao lower bound (CRLB) is also derived from the localization performance of the proposed technique. Simulation results show that the root means square positioning error (RMSPE) of the proposed technique is more robust and accurate compared to well-known network localization techniques such as Isomap [17] and multidimensional scaling (MDS) [18], [19]. We further show that the network connectivity increases as more nodes switch to active mode by harvesting energy, which improves the accuracy of the proposed network localization scheme in return.
C. Paper Organization

The rest of the paper is organized as follows: Section II introduces the system model and proposed localization technique for an EH-UOWSN. Section III-C presents the performance analysis in terms of CRLB for the proposed technique. In section IV, simulations are conducted for the performance evaluation of the proposed technique and section V concludes the proposed work.

II. System Model

Consider an EH-UOWSN consisting of $N$ energy harvesting nodes and $M$ anchors in a $\rho$-dimensional Euclidean space. While the location of the anchors is known apriori, our goal is to determine the positions of $N$ sensor nodes by means of anchor locations and RSS measurements. Fig. 1 shows a simple EH-UOWSN scenario which consists of energy harvesting optical sensor nodes, anchor nodes, surface buoys and a surface station which behaves as a sink, collects all the data from nodes and forwards it to the onshore stations.

A. Energy Harvesting for UOWSNs

One of the major concern of UOWSNs is the power management because it’s difficult to replace or recharge the battery of a sensor node, especially in deep oceanic water. Therefore,
designing an efficient and reliable energy harvester for a self-sustainable operation of UOWSNs is essential. We assume that each sensor nodes buffer the harvested energy in super-capacitors, which are known to achieve a storing efficiency of 99% [20]. Uninterrupted operations of the nodes can be achieved if the following condition is satisfied

\[ P_b(t) \geq P_c(t), \quad (1) \]

where \( t \) is the time, \( P_b(t) \) is the available output power of energy buffer at time \( t \), and \( P_c(t) \) is the energy consumption of a sensor node at time \( t \). Accordingly, the amount of stored energy for duration \( T \) is given by

\[ \int_0^T P_b(t) \, dt + \nu \geq \int_0^T P_c(t) \, dt, \quad (2) \]

where \( \nu \) is the initially stored energy in the buffer. In the remainder, we consider \( P_c(t) \) as a binary function such that the sensor node is either active at a certain power or passive/idle when there is no available power.

For underwater energy harvesting, different techniques are proposed such as microbial fuel cells, acoustic resonators, piezoelectric cantilevers, ionic polymer metal composites, etc.

1) Microbial Fuel Cells (MFCs): The MFCs produce the electric current from the metabolic interactions of bacteria. The bacteria colony grows on the positive surface, degrades the substance, and charges the positive terminal [21]. The charges then move to the negative chamber and dissolve with oxygen to produce water. The flow of charges from the positive terminal is fed to the sensor node by using an external resistor. The ideal power and current densities of the MFC are computed as

\[ P_m = \frac{v_m}{R_{ex} \kappa}, \quad (3) \]

and

\[ I_m = \frac{v_m}{R_{ex} \kappa}, \quad (4) \]

respectively, where \( v_m \) is the MFC output voltage, \( R_{ex} \) is the external resistance, and \( \kappa \) represents the volume of the MFC. Indeed, the practical output voltage from the MFC is less than the theoretical voltage due to different losses such as activation potential \( \mu_a \), concentration over potential \( \mu_c \) and ohmic over potential \( \mu_o \). The actual output voltage of MFC is expressed as

\[ v_{mfc} = v_m - (\mu_a + \mu_c + \mu_o). \quad (5) \]
Using Monod [22] and Erdey-Gruz-Volmer equations [23], the losses $\mu_a$, $\mu_c$ and $\mu_o$ are modeled as

$$\mu_a = \Omega \tau (K_a + B_a) \ln \left( \frac{r_A}{k_1} \right),$$

(6)

$$\mu_c = \Omega \tau (K_a + B_a) \ln \left( \frac{I_c}{\gamma \varphi_{o_2}} \right),$$

(7)

and

$$\mu_o = I_c R_{in},$$

(8)

where $\Omega$ is universal gas constant, $\tau$ is the temperature, $K_a$ is acetate half velocity constant, $B_a$ is concentration of acetate for MFC, $\Phi$ is positive terminal charge rate coefficient, $\psi$ is Faraday’s constant, $r_A$ is the substrate utilization rate, $k_1$ represents the reaction rate at positive terminal, $\zeta$ represents Tafel slope, $\gamma$ is the current density, $I_c$ is the cell current, $\varphi_{o_2}$ is the concentration of oxygen and $R_{in}$ is the internal resistance of the cell. Substituting (6), (7) and (8) in (5), the voltage generated by MFC is

$$v_{mfc} = \frac{\Delta T (K_a + B_a)}{\Phi \psi B_a} \ln \left( \frac{r_A}{k_1} \right) - I_c R_{in} - \frac{\Delta T (K_a + B_a)}{\zeta \psi} \ln \left( \frac{I_c}{\gamma \varphi_{o_2}} \right).$$

(9)

Finally, the expression for net output power becomes

$$P_m = \frac{v_{mfc}}{R_{ex} \kappa}.$$ 

(10)

2) **Acoustic Harvester (AH):** Acoustics resonators are used for numerous applications such as sound amplification and noise attenuation. An incident wave excites an acoustic resonator to produce acoustic energy at their resonating frequencies [24]. By placing a piezoelectric beam in acoustic resonator creates a pressure difference which generates the electric current. The natural frequency of the resonator is given by

$$\omega = c \sqrt{\frac{Q}{LV}},$$

(11)

where $c$ is the speed of sound, $Q$ is the cross sectional area of the resonator, $L$ is the resonator effective length, and $V$ is the volume of the resonator cavity. The voltage produced by the
piezoelectric energy harvester at resonating frequency is then derived as [25]

\[
    v_{pz} = \frac{\omega R_{ext} E_p t_p}{\sqrt{R_{ext} E_p \omega^2 (4j^2 + u^2) + 4j^2 u^2 \omega R_{ext} E_p}}
\]  

(12)

where \( E_p \) is the piezoelectric capacitance, \( \iota \) is piezoelectric constant, \( t_p \) is the thickness of the material, \( j \) is the damping ratio and \( u_p \) is the coupling coefficient. Then accordingly the power generated by AH is given by

\[
    P_{pz} = \frac{v_{pz}}{R_{ext}}.
\]

(13)

3) Multi-Source Energy Harvesting: In this paper, we consider a multi-source energy harvesting system which is able to get the energy from different sources and stores it into a buffer. The output power of a multi-source energy harvesting system is formulated as by

\[
    P_b = P_m + P_{pz},
\]

(14)

where \( P_m \) and \( P_{pz} \) are the powers collected from microbial fuel cells and acoustic harvester, respectively. Please note that multi-source case is generic enough to accommodate any type of additional power source, which will appear as an extra summation term in (14).

III. PROPOSED NETWORK LOCALIZATION METHOD

A. Ranging in UOWSNs

Optical light passing through the aquatic medium suffers from widening and attenuation in angular, temporal and spatial domains [16]. The widening and attenuation of the underwater optical signals are dependent on the wavelength. The extinction coefficient is modeled as a combination of the absorption coefficient \( a(\lambda) \) and scattering coefficient \( s(\lambda) \) as follows [26]

\[
    e(\lambda) = a(\lambda) + s(\lambda).
\]

(15)

Accordingly, the propagation loss as a function of distance \( d_{ij} \) and wavelength \( \lambda \) between any two sensor nodes is given by

\[
    L_{ij} = \exp^{-e(\lambda)d_{ij}}
\]

(16)
In this paper, we consider line of sight communication, where the sensor node $i$ directs the optical light to sensor node $j$. Then, the received power at sensor node $j$ is given as [27],

$$P_{r_j} = P_t \eta_i \eta_j L_{ij} \frac{A_j \cos \theta}{2\pi d_{ij}^2 (1 - \cos \theta_0)},$$

(17)

where $P_t$ is the optical power transmitted by node $i$, $\eta_i$ and $\eta_j$ are the optical efficiencies of node $i$ and $j$, respectively, $A_j$ is the aperture area of node $j$, $\theta$ is the angle between the transmitter trajectory node $i$ and node $j$ receiver, and $\theta_0$ is the divergence angle of the transmitted signal. One of the most common modulation schemes for optical wireless communications presented is the intensity modulation with direct detection (IM/DD). The bit error rate (BER) expression for IM/DD with on-off keying is developed as a function of the number of photons arriving at the photon counter using Poisson model. The number of photons arriving at node $j$ in time duration $T$ is

$$r_j = P_{r_j} \eta_j \lambda \frac{T R_d h c}{T R_d h c},$$

(18)

where $R_d$ is the data rate, $h$ is the Planck’s constant and $c$ is the free space speed of light. The BER for the photons arriving at node $j$ is expressed as

$$b_j = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{T}{2}} \left( \sqrt{r_1} - \sqrt{r_0} \right) \right),$$

(19)

where $r_1 = r_d + r_j + r_b$ and $r_0 = r_d + r_b$ represent the photons required for the transmission of binary 1 and 0 respectively, while $r_d$ is the dark count noise and $r_b$ is the noise produced due to background illumination. Replacing the values of $r_1$ and $r_0$ in (19), $r_j$ is given as

$$r_j = \left( \sqrt{r_d + r_b} + \sqrt{\frac{2}{T} \text{erfc}^{-1}(2b_j)} \right)^2 - r_d - r_b.$$

(20)

Substituting (17) and (18) in (20), then estimated distance $\tilde{d}_{ij}$ between node $i$ and $j$ is obtained as

$$\tilde{d}_{ij} = \frac{2 \cos \theta}{e(\lambda)} W_0 \left( \frac{e(\lambda)}{2 \cos \theta \sqrt{2\pi \hbar c R_d r_j (1 - \cos \theta_0) \eta_j A_j \lambda P_t \eta_i \eta_j A_j \cos \theta}} \right),$$

(21)

where $W_0(.)$ is the real part of Lambart $W$ function [28].
For the sensor coverage area, we consider a circular disk model, that is, \(i\)th sensor will communicate directly with any other sensor in its transmission capability \(r_i\). Note that \(i\)th sensor can also communicate with any sensor located outside of its transmission capability in a multi-hop transmission fashion. The initial number of active and passive sensor nodes are denoted by \(N_a\) and \(N_p\) (i.e., \(N = N_a + N_p\)), respectively.

Based on this assumption the network can be represented as a weighted graph \(G_{\rho,K}(Y, \Lambda, D)\) where \(Y = \{y_1, \ldots, y_M, y_{M+1}, \ldots, y_K\}\) are the coordinates of vertices/sensors, \(K = M + N_a\), \(\Lambda = \{\Lambda_{i,j}\}_{i,j=1,i\neq j}^K\) are edge weights characterized by the link quality, and \(D = \{\tilde{d}_{i,j}\}_{i,j=1,i\neq j}^K\) are the associated distance estimations. The edges represent the connecting links and the associated weights are the estimated ranges between the sensor nodes derived in (21). The estimated ranges \(\tilde{d}_{ij}\) in (21) can be re-written as

\[
\tilde{d}_{ij} = \sqrt{(y_i - y_j)(y_i - y_j)^T} + \xi_{ij},
\]

where the first term is the actual distance and \(\xi_{ij} \sim \mathcal{N}(0, \sigma_j^2)\), \(\forall i,\) is the additive range estimation noise. The error function for the noisy distances is given as

\[
S(Y) = \sum_{i < j < K} \Lambda_{ij} (\tilde{d}_{ij} - d_{ij}(Y))^2,
\]

where \(d_{ij}\) is the actual Euclidean distance and \(\Lambda_{ij}\) represents the quality of the link between node \(i\) and \(j\) and defined as \(\Lambda_{ij} = 1/\sigma_j^2, \forall i,\). The error function in (23) can expanded as

\[
S(Y) = \sum_{i < j} \Lambda_{ij} \tilde{d}_{ij}^2 + \sum_{i < j} \Lambda_{ij} d_{ij}(Y) - 2 \sum_{i < j} \Lambda_{ij} \tilde{d}_{ij} d_{ij}(Y).
\]

Notice in (24) that the first term is constant and is only dependent on \(\Lambda_{ij}\) and \(\tilde{d}_{ij}\). Second term is quadratic in \(Y\) and is weighted sum of the \(d_{ij}(Y)\). The last term is bounded by Cauchy-Schwarz inequality, i.e.,

\[
d_{ij}(Y) = \| y_i - y_j \| = \| y_i - y_j \| \frac{\| v_i - v_j \|}{\| v_i - v_j \|} \geq \frac{(y_i - y_j)^T (\Lambda_i - \Lambda_j)}{\| v_i - v_j \|},
\]
where \( \Lambda = \{ \Lambda_1, ..., \Lambda_K \}^T \in \mathbb{R}^{K \times \rho} \) and \( v_i \) is the column vectors of \( V = \Lambda \circ A \) where \( \circ \) is the dot product and \( A \in \{0, 1\}^{K \times K} \) is the adjacency matrix representing the connectivity. Therefore, the last term is bounded by

\[
\sum_{i<j} \Lambda_{ij} \tilde{d}_{ij}(Y) \geq \sum_{i<j} \Lambda_{ij} \tilde{d}_{ij} \frac{(y_i - y_j)^T (\Lambda_i - \Lambda_j)}{\| v_i - v_j \|}.
\]  

(26)

Following from (24)-(26), the error function in (23) is bounded by \( G(Y, \Lambda) \) which is given by

\[
S(Y) \leq G(Y, \Lambda) = \sum_{i<j} \Lambda_{ij} \tilde{d}_{ij}^2 + \sum_{i<j} \Lambda_{ij} \tilde{d}_{ij}^2(Y)
- 2 \sum_{i<j} \Lambda_{ij} \tilde{d}_{ij} \frac{(y_i - y_j)^T (\Lambda_i - \Lambda_j)}{\| v_i - v_j \|}.
\]  

(27)

The function \( G(Y, \Lambda) \) can be put into matrix form as

\[
G(Y, \Lambda) = B + \text{Tr}(Y^T Z Y) - 2 \text{Tr}(Y^T C(\Lambda) A).
\]  

(28)

where elements of \( Z \) and \( C \) are defined as

\[
z_{ij} = \begin{cases} 
\sum_{i=1, i \neq j}^K \Lambda_{ij}, & \text{if } i \neq j, \\
0, & \text{if } i = j,
\end{cases}
\]  

(29)

and

\[
c_{ij} = \begin{cases} 
\sum_{i=1, i \neq j}^K \Lambda_{ij} \tilde{d}_{ij}(Y), & \text{if } i \neq j, \\
0, & \text{if } i = j.
\end{cases}
\]  

(30)

respectively. While \( z_{ij} \) is the cumulative link qualities from all other nodes to node \( j \), \( c_{ij} \) is obtained by weighting \( z_{ij} \) by the distance estimate normalized by the actual distance. Finally, estimated locations can be obtained by minimizing \( G(Y, \Lambda) \) with respect to the actual locations \( Y \) as follows

\[
\hat{Y} = \min_Y G(Y, \Lambda) = Z^{-1} C(\Lambda) A
\]  

(31)

which can be calculated by partitioning \( Y, \Lambda, C(\Lambda) \) and \( Z \), as follows

\[
Y_{Na} = [y_1, ..., y_{Na}] \in \mathbb{R}^{Na \times \rho},
\]  

(32)
\[ Y_M = [y_1, \ldots, y_{N_a}] \in \mathbb{R}^{N_a \times \rho}, \quad (33) \]
\[ \Lambda_{N_a} = [\Lambda_1, \ldots, \Lambda_{N_a}] \in \mathbb{R}^{N_a \times \rho}, \quad (34) \]
\[ \Lambda_M = [\Lambda_1, \ldots, \Lambda_{N_a}] \in \mathbb{R}^{N_a \times \rho}, \quad (35) \]
\[ C(\Lambda) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, \quad (36) \]

and
\[ Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}, \quad (37) \]

where the block matrices \( C_{11}/z_{11} \) is of length \( N_a \times N_a \) and related to node-to-node elements, \( C_{12}/z_{12} \) is \( M \times N_a \) long and corresponds to anchor-to-node elements, and \( C_{22}/z_{22} \) is of length \( M \times M \) and related to anchor-to-anchor elements. The missing elements for the block matrices\(^1\) \( C_{11} \) and \( C_{12} \) are approximated by
\[ \hat{c}_{ij} = \frac{\hat{R}_{\text{min}} + \hat{R}_{\text{max}}}{2}, \quad (38) \]

where \( \hat{R}_{\text{max}} \) is the longest edge in the network and \( \hat{R}_{\text{min}} \) is the shortest edge. Note that the approach in (38) is different than the shortest path distance estimation. In [17]–[19] the authors considered shortest path estimation for the missing elements of the kernel matrix. Nevertheless, the shortest path distance estimation leads to a large error due to the accumulated error over the multi-hop path. The shortest path estimation also depends on the distribution of sensor nodes in the network, if nodes are densely and uniformly deployed, then the shortest path can be a good approximation. However, in case of sparsely deployed nodes, it is not an optimal solution. Therefore, in this paper, we proposed a novel technique to approximate the missing elements

\(^1\)Element \( c_{ij} \) is missed if there is no connectivity thus no signal reception from node \( i \) to node \( j \)
for the block kernel matrix. Evaluating (28) in terms of the partitioned matrices, we obtain

$$G(\mathbf{Y}, \Lambda) = \mathbf{B} + \sum_{k=1}^{K} \left( \mathbf{Y}_{N_a(k)}^T \mathbf{z}_{11} \mathbf{Y}_{N_a(k)} \right)$$

$$+ 2 \mathbf{Y}_{N_a(k)}^T \mathbf{z}_{12} \mathbf{Y}_{M(k)} + \mathbf{Y}_{M(k)}^T \mathbf{z}_{22} \mathbf{Y}_{M(k)} \right)$$

$$- 2 \sum_{k=1}^{K} \left( \mathbf{Y}_{N_a(k)}^T \mathbf{C}_{11} \Lambda_{N_a(k)} \right)$$

$$+ 2 \mathbf{Y}_{N_a(k)}^T \mathbf{C}_{12} \Lambda_{M(k)} + \mathbf{Y}_{M(k)}^T \mathbf{C}_{22} \Lambda_{M(k)} \right).$$

(39)

Differentiating (39) with respect $\mathbf{Y}_{N_a(k)}$

$$\frac{\partial G(\mathbf{Y}, \Lambda)}{\mathbf{Y}_{N_a(k)}} = 2(\mathbf{z}_{11} \mathbf{Y}_{N_a(k)} + \mathbf{z}_{12} \mathbf{Y}_{M(k)}$$

$$- \mathbf{C}_{11} \Lambda_{N_a(k)} - \mathbf{C}_{12} \Lambda_{M(k)}).$$

(40)

By setting (40) equal to zero, the location estimation $\hat{\mathbf{Y}}_{N_a(k)}$ of the $N_a$ sensor nodes are given as

$$\hat{\mathbf{Y}}_{N_a(k)} = \mathbf{z}_{11}^{-1}(\mathbf{C}_{11} \Lambda_{N_a(k)} + \mathbf{C}_{12} \mathbf{Y}_{M(k)} + \mathbf{z}_{12} \mathbf{Y}_{M(k)})$$

(41)

which can also be put into matrix form as

$$\hat{\mathbf{Y}}_{N_a} = \mathbf{z}_{11}^{-1}(\mathbf{C}_{11} \Lambda_{N_a} + \mathbf{C}_{12} \mathbf{Y}_{M} + \mathbf{z}_{12} \mathbf{Y}_{M})$$

(42)

Notice that the solution for additional sensor nodes which are activated by the energy harvesting source is straightforward from (42) by adding $N_p$ number of nodes to $N_a$.

C. Performance Comparison to CRLB Benchmark

Since the localization error function is generally characterized by Cramer Rao lower bound (CRLB) [29], we take it as our benchmark for the performance comparison of the proposed localization technique. The Fisher information matrix (FIM) is required to be obtained first to derive the CRLB. Consider that the sensor node positions $\mathbf{y}$ are stochastic processes with density $f(\mathbf{y})$, which means that the RSS based ranging measurements are also random variables. Accordingly, we derive the expression for FIM [30],

$$I(\mathbf{y}) = -E \left[ \frac{\partial^2 \ln(f(\mathbf{d}/\mathbf{y}))}{\partial \mathbf{y}} \right].$$

(43)
It is assumed that the noise added to the ranging measurements is zero mean Gaussian process with variance $\sigma_j$, $1 \leq j \leq N_a$. Therefore, the RSS based noise co-variance matrix become $\Gamma_d = \text{diag}(\sigma_1, \ldots, \sigma_j, \ldots, \sigma_{N_a})$. Then, the likelihood ratio is computed as

$$
\ln(f(\tilde{d}/y)) = \ln\left(\frac{1}{(2\pi)^{N_a/2}}\right) - \frac{1}{2} \left(\tilde{d} - d(y)\right)^T \Gamma_d^{-1} \left(\tilde{d} - d(y)\right).$$

The FIM is constructed from the likelihood ratios, given by

$$
\chi_d = \Theta_d^T \Gamma_d^{-1} \Theta_d,
$$
where
\[
\Theta_d = -\beta \begin{bmatrix}
\frac{\Delta x}{d^2} \\
\frac{\Delta y}{d^2}
\end{bmatrix},
\]  
and \(\beta\) is the path loss exponent. The elements of \(\chi_d\) are derived as
\[
\{\chi_d\}_{1,1} = \frac{\nabla \left( \tilde{d} - d(y) \right)^T}{\nabla x} \Gamma_d^{-1} \frac{\nabla \left( \tilde{d} - d(y) \right)}{\nabla x},
\]  
\[
\{\chi_d\}_{1,2} = \frac{\nabla \left( \tilde{d} - d(y) \right)^T}{\nabla x \nabla y} \Gamma_d^{-1} \frac{\nabla \left( \tilde{d} - d(y) \right)}{\nabla x \nabla y},
\]  
and
\[
\{\chi_d\}_{2,2} = \frac{\nabla \left( \tilde{d} - d(y) \right)^T}{\nabla y} \Gamma_d^{-1} \frac{\nabla \left( \tilde{d} - d(y) \right)}{\nabla y}.
\]
Simplifying (47), (48) and (49) we get
\[
\{\chi_d\}_{1,1} = \frac{\beta^2 \Delta x^T \Gamma_d^{-1} \Delta x}{d^4},
\]  
\[
\{\chi_d\}_{1,2} = \frac{\beta^2 \Delta x^T \Gamma_d^{-1} \Delta y}{d^4},
\]  
and
\[
\{\chi_d\}_{2,2} = \frac{\beta^2 \Delta y^T \Gamma_d^{-1} \Delta y}{d^4}.
\]
The CRLB is computed as the inverse of the diagonal elements of the FIM, i.e.,
\[
\text{CRLB} = \{\chi_d\}_{1,1}^{-1} + \{\chi_d\}_{2,2}^{-1}.
\]  
Finally the root mean square positioning error (RMSPE) should satisfy the following condition
\[
\sqrt{\frac{\sum_{i=1}^{N_a} (y_i - \hat{y}_i)^2}{N_a}} \geq \{\chi_d\}_{1,1}^{-1} + \{\chi_d\}_{2,2}^{-1}.
\]

IV. SIMULATION RESULTS

A number of Monte Carlo simulations are conducted in MATLAB to analyze the performance of the proposed approach. Initially, we consider 100 nodes which are randomly distributed in 10 × 10 m² area, where active nodes \(N_a\) are 10, passive nodes \(N_p\) are 80, and anchor nodes
$M = 10$ as shown in Fig. 2. The transmission capability of each sensor node is kept constant to 3 m. It is clear from the figure that due to the limited energy availability in the network, most of the nodes are in the passive mode and thus disconnected from the network.

As a result of the limited network connectivity, proposed network localization approach is not able to determine the node positions as it requires more range measurements to estimate the entire network. In practice, energy arrivals to the harvesters have an intermittent nature and the passive nodes can harvest energy from the ambient energy sources and gets connected to the network. Once the network is connected it is possible to localize all the nodes in the network as shown in Fig. 3 - Fig. 5 where 10-80 of the passive nodes harvest energy from the ambient energy sources and gets connected to the network, respectively. Fig. 4 - Fig. 5 also show that when the active nodes are increased from 10 to 90, i.e., all the nodes in the network are active, the localization performance is significantly improved and RMSPE is reduced by 50%. In Fig. 6 - Fig. 8, proposed network localization technique is compared and shown to provide better results due to energy harvesting, block kernel matrices, and better shortest path estimation. Fig. 6 shows the relationship between energy harvesting and RMSPE. Energy harvesting increases the number of active nodes in the network, thus reduces the multi-hop error in the network. The RMSPE is also compared to well-known network localization techniques such as Isomap [17] and multidimensional scaling (MDS) [18], [19]. In Fig. 7, the RMSPE performance of the proposed
Figure 7 shows the RMSPE vs. transmission capability, where the RMSPE improves with the increase in the transmission range up to a certain level, i.e., 6m for the given scenario and after that, it increases due to the problem of flipping ambiguity.

Figure 8 shows the impact of increasing the number of anchors in the network where there exists 90 sensor nodes deployed in a $10 \times 10$ square area with the EH-UOWSNs localization compared with Isomap and MDS in terms of transmission range capability of the sensor nodes. The RMSPE improves with the increase in the transmission range up to a certain level, i.e., 6m for the given scenario and after that, it increases due to the problem of flipping ambiguity. Fig 8 shows the impact of increasing the number of anchors in the network where there exists 90 sensor nodes deployed in $10 \times 10$ square area with the
constant communication range of 3 m. Furthermore, it is clear from Fig. 8 that due to the energy harvesting capabilities, block kernel matrices and better shortest path estimation the proposed technique outperforms Isomap and MDS.

V. CONCLUSIONS

In this paper, an energy harvesting based localization technique is developed for underwater optical sensor networks using the RSS measurements. In aquatic environment its difficult to replace or recharge the battery of a sensor node. Therefore designing an efficient and reliable energy harvester for continual operation of UOWSN is required. In this paper, a mathematical model is developed which can harvest energy from multiple sources to the sensor nodes. The RSS measurements for underwater optical communication are inaccurate and introduces large localization error. The proposed technique takes into account the energy from the energy harvesting sources, thus making it more robust compared to other network localization techniques. Also the proposed method reduces the shortest path estimation error in block kernel matrices by introducing novel matrix completion technique. Furthermore, the CRLB is derived for the proposed EH-UOWSN localization technique. Simulations show that the proposed technique for underwater optical sensor networks localization is a good strategy to get robust and accurate results.

REFERENCES