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The Hydraulic Conductivity of Sediments: A Pore Size Perspective

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Abstract: This article presents an analysis of previously published hydraulic conductivity data for a wide range of sediments. All soils exhibit a prevalent power trend between the hydraulic conductivity and void ratio. Data trends span 12 orders of magnitude in hydraulic conductivity and collapse onto a single narrow trend when the hydraulic conductivity data are plotted versus the mean pore size, estimated using void ratio and specific surface area measurements. The sensitivity of hydraulic conductivity to changes in the void ratio is higher than the theoretical value due to two concurrent phenomena: 1) percolating large pores are responsible for most of the flow, and 2) the larger pores close first during compaction. The prediction of hydraulic conductivity based on macroscale index parameters in this and similar previous studies has reached an asymptote in the range of $k_{\text{meas}}/5 \leq k_{\text{predict}} \leq 5k_{\text{meas}}$. The remaining uncertainty underscores the important role of underlying sediment characteristics such as pore size distribution, shape, and connectivity that are not measured with index properties. Furthermore, the anisotropy in hydraulic conductivity cannot be recovered from scalar parameters such as index properties. Overall, results highlight the robustness of the physics inspired data scrutiny based Hagen–Poiseuille and Kozeny-Carman analyses.

Keywords: Hydraulic conductivity; Sediments; Specific surface area; Pore size; Void ratio

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1. Introduction

Theoretical and empirical equations relate the hydraulic conductivity of soils to properties such as the grain size, specific surface area, clay content, porosity and pore geometry (Taylor, 1948; Malusis et al., 2003; Zhang et al., 2005; Roque and Didier, 2006; Dolinar, 2009; Mejias et al., 2009; Chapuis, 2012; Wang et al., 2013; Ilek and Kucza, 2014; Sante et al., 2015; Ren et al., 2016; Kucza and Ilek, 2016). The analytically derived Kozeny-Carman (KC) equation considers the porous network in sediments as a bundle of tubes and assumes Poiseuille’s laminar fluid flow in the tubes. The sediment hydraulic conductivity \( k \) [m/s] can then be expressed in terms of the sediment specific surface area \( S_s \) [m\(^2\)/g] and void ratio \( e \) (Taylor, 1948):

\[
k = \frac{C_F \cdot g \cdot S_s^2 \cdot e^3}{\nu_f \cdot \rho_m \cdot (1 + e)}
\]

(1)

where \( \rho_m \) [kg/m\(^3\)] is mineral mass density, \( \nu_f \) [m/s] is the kinematic fluid viscosity, and \( C_F \approx 0.2 \) is a constant related to pore topology. In general, it is thought that the Kozeny-Carman equation more accurately predicts trends in hydraulic conductivity for coarse-grained sandy sediments than for fine-grained clayey soils.

Empirical correlations have been suggested for coarse sandy sediments and for fine-grained clayey sediments. Hazen’s equation is the most frequently cited empirical equation for coarse-grained soils and emphasizes the role of the finer fraction on a soil’s hydraulic conductivity (Hazen, 1892):

\[
k \text{ [cm/s]} \approx \left( \frac{D_{10}}{\text{mm}} \right)^2
\]

(2)

where the grain size \( D_{10} \) corresponds to the finer 10% of the soil mass (Note: the temperature correction in the original equation is not included here because prediction errors overwhelm the temperature effects). Predicted and measured values can differ in more than one order of magnitude because of grain size variability and particle shape (Lambe and Whitman, 1969; Shepherd, 1989; Carrier, 2003). Other size fractions have been considered to enhance predictability, such as \( D_5 \), \( D_{20} \) or \( D_{50} \), however the original function of \( D_{10} \) remains best
known (Sherard et al., 1984; Kenney et al., 1984; Indraratna and Nguyen, 2012). Hazen’s first-order estimate of hydraulic conductivity was based on poorly graded sands packed at medium density, and is independent of the void ratio $e$ in part due to the low compressibility of coarse grained sediments (Note: Taylor (1948) corrected the computed values for void ratio, following the Kozeny-Carman’s equation).

Empirical equations for fine-grained soils explicitly recognize the dependency of hydraulic conductivity on the void ratio. Two forms have been proposed: (a) an exponential or log-linear relation (Taylor, 1948; Nishida and Nakagawa, 1969; Lambe and Whitman, 1969; Mesri and Rokhsar, 1974; Tavenas et al., 1983),

$$\log[k/(\text{cm/s})] = a + b \cdot e$$

(3)

and (b) a power relation (Mesri and Olson, 1971; Samarasinghe et al., 1982; Carrier and Beckman, 1984; Krizek and Somogyi, 1984; Dolinar, 2009)

$$k = \alpha e^\beta$$

(4)

Model parameters in both cases have been related to either the liquid limit or the plastic limit of the soil (e.g., Carrier and Beckman, 1984; Berilgen et al., 2006; Dolinar and Škrabl, 2013). Furthermore, given the parallelism between Equation (3) and Terzaghi’s compressibility equation $e = e_0 - C_c \log(\sigma'/\sigma_0)$, model parameters $a$ & $b$ can also be associated with the sediment compressibility (Mesri and Rokhsar, 1974; Tavenas et al., 1983; Nagaraj et al., 1994). Exponential and power equations have been used for geomaterials that range from suspensions (Pane and Schiffman, 1997), to normal and overconsolidated clays (Al-Tabba and Wood, 1987; Nagaraj et al., 1994), and rocks (David et al., 1994).

This study reexamines the hydraulic conductivity of sediments. The study includes an extensive compilation of published data gathered for a wide range of sediments. The subsequent analysis seeks to identify the causal link between physics-based theoretical models and the observed empirical trends, and to identify the underlying pore-scale processes that can justify prevailing trends and anticipate potential limitations and deviations.
2. Data Compilation - The Central Role of Pore Size

The hydraulic conductivity database includes both natural and remolded sediments (coarse gravels to smectite clays, and mixtures), and of different fabrics (loose and dense packing and both flocculated and dispersed). The database is plotted in all linear-log scale combinations. We note that individual datasets plot as linear trends on the log(k)-log(e) space as shown in Figure 1. These data show two opposite trends for hydraulic conductivity as a function of void ratio: while the hydraulic conductivity increases with increasing void ratio for any single sediment, fine-grained (small pore size) soils exhibit much lower hydraulic conductivity -even at higher void ratios- than coarse-grained (large pore size) sediments. In fact, the Kozeny-Carman equation highlights the importance of “pore size” rather than “porosity” on fluid transport (anticipated by the Hagen–Poiseuille equation for a single tube).

2.1. The relevance of macroscale values e and S,

The apparent contradiction in the last statement points to the importance of “pore size” rather than “porosity” on fluid transport. The mean pore size \(d_p\) can be computed for various grain geometries and fabrics in terms of the void ratio \(e\) and the specific surface area \(S_s\) [m\(^2\)/g] (Phadnis and Santamarina, 2011). Consider the volume of voids evenly distributed around grains as a “void layer” of thickness \(t_{void}\); the inter-particle distance \(d_p=2t_{void}\) is then considered as an estimate of the mean pore size

\[
d_p = 2\frac{e}{S_s \rho_m}
\]

where \(\rho_m\) [kg/m\(^3\)] is the mineral mass density. This first order estimate of pore size is based on two macroscale parameters: void ratio \(e\) and specific surface area \(S_s\).

The specific surface area is not reported in most studies plotted in Figure 1. We estimate the specific surface area by using other published soil descriptions. In the case of fine grained soils, estimates were based on liquid limits \(w_L\) (Farrar and Coleman, 1967; see also Muhunthan, 1991 and Santamarina et al., 2002b),

\[
S_s = 1.8w_L - 34
\]
For sandy soils made of rotund grains, the specific surface area was estimated from the cumulative grain-size distribution (Santamarina et al., 2002b) - assumes linear distribution in log scale).

\[
S_s = \frac{3}{4} \frac{C_u + 7}{\rho_w G_s D_{50}}
\]  

(7)

where the coefficient of uniformity is \(C_u = D_{60}/D_{10}\), and \(D_{10}, D_{50}, D_{60}\) [mm] are the grain diameters for 10%, 50% and 60% cumulative passing fractions. The specific surface area in mixtures is estimated as a summation of the surface area contributed by the various size fractions weighted by their mass fractions.

2.2 Hydraulic conductivity vs. Pore size

We use Equation 5 to estimate the mean pore size for the data set plotted in Figure 1. Hydraulic conductivity data are then replotted as a function of the computed mean pore size \(d_p\), in Figure 2. Additional data for sands with known grain size distributions are included in this figure. We observe: (1) all experimental data gathered for soils ranging from coarse- to fine-grained sediments collapse onto a relatively narrow single trend in the \(k-d_p\) space; (2) the Hagen–Poiseuille equation for fluid flow in cylindrical tubes predicts a power-2 relationship between hydraulic conductivity and pore size \(k \propto d_p^2\). The line with slope 2 superimposed on the data in Figure 2 closely agrees with the overall trend.

These observations confirm the central role of pore size on hydraulic conductivity. Furthermore, the analysis presented above demonstrates the relevance of the two measurable macroscale parameters, the void ratio \(e\) and specific surface area \(S_s\), as captured in the Kozeny-Carman Equation 1.

3. Power Model - Parameters \(k_0\) and \(\beta\)

All data sets in Figure 1 are fitted with a straight line in log-log scale:

\[
\log \left( \frac{k}{\text{cm/s}} \right) = \log \left( \frac{k_0}{\text{cm/s}} \right) + \beta \log \left( \frac{e}{e_o} \right)
\]  

(8a)
which can be written as a power equation (refer to Equation 4):

\[
k = k_o \left( \frac{e}{e_o} \right)^\beta
\]  

(8b)

where \( k_o \) [cm/s] is the hydraulic conductivity at the reference void ratio \( e_o \) and the \( \beta \)-exponent captures the sensitivity of hydraulic conductivity to changes in the void ratio. We observe that the intercept \( \log(k_o/(\text{cm/s)}) \) at void ratio \( e_o=1 \) decreases for the finer sediments and that the slope \( \beta \) is similar for most soils and it is \( \beta>2 \).

3.1. Hydraulic conductivity \( k_o \) at the reference void ratio

The selected reference void ratio is \( e_o=1.0 \) in all cases. Therefore, the parameter \( k_o \) is the value of hydraulic conductivity at \( e_o=1.0 \). Following the previous observations, we explore the correlation between \( k_o \) and the specific surface area in Figure 3. This figure contains additional published sand and silt data, noted as open triangles; these data were published without void ratios, therefore, we estimate \( k_o \) from the reported values \( k_{rep} \) as \( k_o \approx 2.8 k_{rep} \) (based on Equation (8b) for void ratios \( e \sim 0.6-0.7 \) and \( \beta \sim 2-3 \)). Figure 3 illustrates a strong correlation between the hydraulic conductivity \( k_o \) at \( e_o=1.0 \) and the specific surface area. The best fit line is

\[
\log \left( \frac{k_o}{\text{cm/s}} \right) = -4.73 - 1.73 \log \left( \frac{S_s}{\text{m}^2/\text{g}} \right)
\]  

(9a)

The physics inspired line with slope \( \beta = -2 \) are superimposed on the plot for comparison; it has a minor increase in residual error and exhibits a better fit to the overall trend:

\[
\log \left( \frac{k_o}{\text{cm/s}} \right) = -5 - 2 \log \left( \frac{S_s}{\text{m}^2/\text{g}} \right)
\]  

(9b)

3.2. Exponent \( \beta \): Sensitivity to void ratio

Figure 4 demonstrates a weak correlation between the fitted \( \beta \)-exponent and the specific surface area (see also Dolinar (2009); Berilgen et al. (2006)). Most \( \beta \)-exponents fall within
2<β<6, with an overall increase from β=3±1 for coarse-grained soils to β=5±1 for fine-grained soils (Note: An even wider range in exponents has been observed for rocks (David et al., 1994). Interestingly, bentonite-silt mixtures exhibit a k, value in line with their high specific surface area (Figure 3), but these mixtures have relatively low sensitivity to changes in the void ratio and the exponent β=3±1 (Figure 4).

Two concurrent phenomena appear to be responsible for the high sensitivity of hydraulic conductivity to changes in the void ratio (Note: a justification based on a fractal pore structure was advanced by Costa (2006)). Firstly, flow focuses along preferential flow paths made of the larger interconnected voids in the sediment (Jang et al., 2011). Secondly, larger pores close first during sediment compaction (Delague and Lefebvre, 1984; Lipiec et al., 2012) (see also dual porosity models, Olsen (1962)). Furthermore, pores enlarge, merge and align along the principal stress direction during shear dilation. Conversely, larger pores shrink and split during shear induced volume contraction (Kang et al, 2013). Therefore, relatively small changes in macro-scale void ratios can have a pronounced effect on the hydraulic conductivity resulting in high β-exponents above the theoretical β.

3.3. Comparison with the Kozeny-Carman KC equation

Fitted parameters allow us to express the empirical power equation in a format analogous to Kozeny-Carman’s Equation 1 by combining equations (8b) and (9b) (similarly using the best fit 9a):

$$ \frac{k}{cm/s} = 10^{-5} \left( \frac{S_s}{m^2/g} \right)^{-2} e^{\beta} $$

(10)

The comparison between the physics-based KC Equation (1) and the data-bound empirical Equation (10) demonstrates that: (1) the first factor in KC is equal to \(3 \times 10^3\) cm/s, and it is in the same order as in Equation (10); (2) the hydraulic conductivity is inversely proportional to the square of the specific surface area in both cases; (3) the void ratio factor in the Kozeny-Carman equation is approximately \(e^{\beta} / (1+e) \approx e^{2.35}\), but the exponent β is higher in most measured hydraulic conductivity vs. void ratio trends.
The high sensitivity to void ratio changes prompted earlier researchers to relax the void ratio exponent in the theoretical KC equation in order to fit experimental data (Taylor, 1948; Samarasinghe et al., 1982). The sensitivity of hydraulic conductivity to changes in the void ratio needs to be properly captured in numerical simulations where hydro-mechanical coupling is anticipated, such as in the analysis of production wells or in hydraulic fracture studies (Note: inherent uncertainties can be captured within a probabilistic numerical approach).

4. Discussion - Limitations

The dataset compiled here is affected by common experimental biases (reviewed in Chapuis (2012)), and by the need to estimate the specific surface area or the void ratio from reported data when they were missing in publications. Still, trends in Figures 2 and 3 are remarkable when one considers that the data include natural and remolded specimens that range from coarse sands to very high specific surface area bentonites packed in different fabrics and tested using different devices in laboratories worldwide.

4.1. Asymptotic uncertainty

Most values predicted using Equation (10) fall within $k_{\text{meas}}/5 \leq k_{\text{predict}} \leq 5k_{\text{meas}}$ of the measured values, as shown in Figure 5. In fact, Equation (10) exhibits the same predictive power as models developed from more selective databases in previous studies (Taylor, 1948; Detmer, 1995; Mbonimpa et al., 2002; Dolinar, 2009; Chapuis, 2012; Sanzeni, 2013). Apparently, we have reached an asymptote in our ability to predict the hydraulic conductivity using basic macroscale measurements and index properties.

The uncertainty in $k$-predictions is very wide for seepage estimations given the linear dependency between the flow rate and hydraulic conductivity in Darcy’s law. Tighter predictions are attainable using the power Equation 8b when a reference value $k_o$ is measured at a known void ratio $e_o$. Finally, we recognize that the spatial variability in natural deposits adds to the uncertainty in point estimates (this study).
4.2. Specific surface and void ratio: Necessary but not sufficient

We can measure marked differences in the hydraulic conductivity for a given sediment $S_s$ at a fixed void ratio $e$. Relations $k=f(S_s,e)$ miss important information such as: bimodal-vs-monomodal pore size distribution, pore size variability or standard deviation, pore geometry and alignment, interconnectivity, tortuosity and bypassed porosity.

4.3. Anisotropy

Hydraulic conductivity models in terms of the specific surface area and void ratio $k=f(S_s,e)$ do not capture anisotropy in hydraulic conductivity. Mathematically, hydraulic conductivity is a tensor and it cannot be recovered from scalar quantities $S_s$ and $e$.

Measurements in our database are most likely to be indicative of the vertical hydraulic conductivity $k_v$ due to typical test procedures used in laboratories. The horizontal hydraulic conductivity is higher than the vertical conductivity in most natural and remolded specimens consolidated under zero-lateral strain boundary conditions, where the anisotropy ratio can reach $k_h/k_v \approx 1.5$-to-2.5 (Tavenas et al., 1983; Al-Tabba and Wood, 1987; Terzaghi et al., 1996; Mitchel and Soga, 2005; Qiu and Wang, 2015). Flow anisotropy is in agreement with frequently observed platelet alignment in the direction parallel to the bedding plane and normal to the major principal effective stress. Layering in natural sediments such as varved clays, and open micro fissures associated with sediment unloading (including sampling effects) magnify anisotropy; the horizontal hydraulic conductivity can readily exceed the vertical by an order of magnitude (Bolton et al., 2000; Kwon et al., 2004). In some cases, canaliculi left by roots, bioturbation and wormholes can result in a preferential vertical hydraulic conductivity, that is $k_h/k_v<1$. 
5. Related Observations

5.1 Specific surface area

The pressure-gradient dependent driving force equals the drag resistance along pore walls when flow reaches terminal velocity; hence the emphasis on the specific surface area in fluid conduction (Equation (9)). Furthermore, the specific surface area is inversely proportional to the grain size $S_s = (\rho \cdot d_{\text{grain}})$ (Taylor, 1948; Santamarina et al., 2001), and the finer fraction of a soil mass contributes most of the surface per unit mass of soil. Consequently, the empirical Hazen’s Equation 2 correlates hydraulic conductivity to the fines fraction $D_{10}$.

5.2. Liquid limit

The liquid limit $w_L$ of a sediment is a measure of the specific surface area (Equation (6)) and fluid-dependent fabric. Therefore, hydraulic conductivity relates to Atterberg limits (Shridharan and Nagaraj, 2005; Dolinar, 2009; Dolinar and Skrabl, 2013). It is interesting to note that the hydraulic conductivity at the liquid limit $w_L$ falls in a narrow range $k_L \approx (2.5 \pm 1) \cdot 10^{-7}$ cm/s for a wide range of clays, with $w_L = 40$-to-300 (Tavenas et al., 1983; Nagaraj et al., 1991; Sharma and Bora, 2009). This would imply that a similar pore size is dominant percolating pathways, regardless of the clay type. Figure 2 suggests that the mean pore size for $k_L \approx (2.5 \pm 1) \cdot 10^{-7}$ cm/s is less than 1 μm for all soils.

5.3. Pore or Pore throat?

Pores connect through pore throats, which are the main constrictions for fluid flow. The size of pore throats in sediments relates to the size of the two connected pores and is typically in the order of $d_{\text{throat}} \approx 0.5d_{\text{pore}}$ (Note: the following geometric relations apply to a simple cubic packing of monosized spheres: $d_{\text{pore}} \approx 0.73d_{\text{grain}}$, $d_{\text{throat}} \approx 0.41d_{\text{grain}}$ and $d_{\text{throat}} \approx 0.56d_{\text{pore}}$). So the correlation between $k$ and $d_{\text{pore}}$ (Figure 2) readily extends to $k$ and $d_{\text{throat}}$ through a correction factor.
The ratio between the pore size and the pore throat size may deviate significantly from $d_{throat} \approx 0.5d_{pore}$ in diagenetically modified and lithified formations (Saar and Manga, 1999) and special attention must be placed on the pore throat size. For example, the hydraulic conductivity of carbonates correlates best with the square of the maximum pore throat size $[\max(d_{throat})]^2$ and still preserves the quadratic relation in Hagen-Poiseuille (see Thomeer (1983) and data in Clerke et al. (2008)).

5.4. Grain breakage

A pronounced change in the hydraulic conductivity takes place during grain crushing (hypothesized in Lade et al. (1996); measured in David et al. (1994) and (2001); Al Hattamleh et al. (2013); Feia et al. (2014)). This situation may arise in various engineering systems that experience a marked increase in effective stress, from compacted fills to production wells subjected to high depressurization.

5.5 Immobile water: Bypassed pores and bound water

The specific surface area and void ratio include intra-granular surfaces and pores that do not get involved in flow, such as in clay booklets, fly ash xenospheres, diatoms, and in framboidal pyrite.

Water next to mineral surfaces has lower-mobility than bulk water; the affected layer has a thickness $t_w$ equivalent to a few monolayers of water. The mass fraction of low-mobility water $M_{lm}$ can be computed in terms of $t_w$ and specific surface area $S_s$,

$$M_{lm} = t_w S_s \rho_w \tag{11}$$

If we assume the thickness of the low-mobility water layer to be $t_w=1\text{nm}$, then the mass fraction of low-mobility water is $M_{lm} \approx 0.1\%$ for silts and sands ($S_s < 1\text{m}^2/\text{g}$), $M_{lm} \approx 2\%$ for kaolinite ($S_s = 20\text{m}^2/\text{g}$), and can reach $M_{lm} \approx 20\%$ for bentonite ($S_s = 200\text{m}^2/\text{g}$). Only the mean pore size of highly compacted bentonites approaches $d_p = 1\text{nm}$ (Figure 2). Furthermore, the largest percolating pores are responsible for most of the flow (Jang et al., 2011), and the coefficient of variation in pore size is $\approx 0.4 \pm 0.1$ for most soils (Phadnis and Santamarina,
Then, bound water has a minor effect on the hydraulic conductivity of most near-surface sediments.

5.6. Pore fluid chemistry

The diffuse double layer invades the pore space. Hydrated ions outside the shear plane can migrate to neighboring particles as long as the incoming fluid preserves electro-neutrality. Therefore, the pore-reducing effect of double layers at a constant fabric is limited to the distance to the shear plane, i.e., several monolayers.

However, changes in electrical interparticle forces cause fabric changes in fine grained sediments, and the type of particle aggregation does affect pore size distribution and interconnectivity. Soil fabrics are a function of the pore fluid chemistry at the time of sedimentation, and may change when subjected to post-depositional changes in fluid chemistry (pH, ionic concentration, the valence of prevailing ions and dielectric constant). For example, the fluid pH and ionic concentration promote: (1) either a fochculated fabric near the isoelectric point or a dispersed fabric at either high or low pH when the ionic concentration is low, and (2) face-to-face aggregation at a high ionic concentration regardless of pH (see the fabric map in Palomino and Santamarina (2005)). Consequently, pore fluid characteristics can have a pronounced effect on the hydraulic conductivity of sediments with specific surface area \( S_s \geq 1 \text{m}^2/\text{g} \) (Quirk and Schofield, 1955; Lutz and Kemper, 1959; Mesri and Olsen, 1971; Ridley et al., 1984; Dunn and Mitchell, 1984; Bowders, 1985; Evans et al., 1985; Fernandez and Quigley, 1985; Bowders and Daniel, 1987; Madsen and Mitchell, 1989; Abdul et al., 1990; Kenney et al., 1992; Rao and Mathew, 1995; Jo et al., 2001; Santamarina et al., 2002a; Kolstad et al., 2004). For example, at similar void ratios, the hydraulic conductivity of Na-bentonite can be 5-10 times lower than that of the Ca-bentonite, yet the effect diminishes as the specific surface area decreases for kaolinite (see for example Mesri and Olson (1971)).

The apparent lack of consensus on the influence of pore fluid chemistry on the hydraulic conductivity is in part due to differences in specimen preparation and permeation history. Tests must be carefully conducted and analyzed because changes in hydraulic conductivity due to changes in permeant depend on the chemistries of defending and invading fluids, the
sediment mineralogy and the specific surface area, the initial state of stress and mechanical boundary conditions during permeation. For example, brine invasion into a sediment saturated with fresh water will cause a decrease in the hydraulic conductivity in a sediment subjected to stress-controlled boundary conditions but it may cause an increase in the hydraulic conductivity if the sediment is under zero boundary strain conditions (see for example Ridley et al. (1984); see also Schmitz et al. (2004), Guimaraes et al. (2001), Powrie (1997) and Schmitz (2006)).

5.7. Concurrent processes – Potential experimental biases

In addition to common experimental mistakes (reviewed in Chapuis (2012)), there are inherent difficulties in the measurement of hydraulic conductivity. These include: sampling and edge effects (sample trimming, and preferential path along wall in rigid cells), non-linear fluid flow (high pore velocities that cause Reynolds number to be Re>10); incomplete fluid saturation; counter electromotive flow and the contribution of diffusion in high specific surface area sediments (Michaels and Lin, 1954; Mitchell and Soga, 2005), fines migration or “suffusion” (Chapuis and Aubertin, 2004), consequences related to changes in pore fluid chemistry between the fluid used to run the test and the saturating fluid at equilibrium (ensuing fabric changes - discussed above- and dissolution/precipitation), and bio-activity. Finally, the pressure gradient required to drive fluid flow causes a variation in the void ratio across the specimen (Δu/Δx→Δσ'/Δx→Δe/Δx); this implies that all hydraulic conductivity measurements are not point measurements but rather integral measurements.

6. Conclusions

Published hydraulic conductivity data were compiled for a wide range of both natural and remolded sediments, from coarse gravels to smectite clays, and of different fabrics (loose and dense packing and both flocculated and dispersed). The dataset spans 12 orders of magnitude.

The most important observations from the ensuing study follow:

(1) Trends for fine- and coarse-grained sediments exhibit prevalent power trends between the hydraulic conductivity and void ratio.
(2) All data trends collapse onto a single trend when conductivity data are plotted vs. the mean pore size estimated using the void ratio and specific surface area. Similarly, the hydraulic conductivity $k_o$ estimated at a reference void ratio $e_o=1$ falls on a single trend for all soils when plotted versus the specific surface area. These two observations highlight the robustness of physics inspired data scrutiny based Hagen–Poiseuille and Kozeny-Carman analyses.

(3) The sensitivity of hydraulic conductivity to changes in the void ratio is higher than theoretically predicted. This is a combination of two concurrent phenomena: percolating large pores are responsible for most of the flow, and the larger pores close first during compaction.

(4) The prediction of the hydraulic conductivity based on macroscale sediment properties and index parameters (such as void ratio, specific surface area, grain size distribution and Atterberg limits) has reached an asymptote in the range of $k_{\text{meas}}/5 \leq k_{\text{predict}} \leq 5k_{\text{meas}}$.

(5) The remaining uncertainty highlights the important role of underlying sediment characteristics such as pore size distribution, shape, and connectivity that are either not measured (e.g., specific surface area) or are lost in the determination of index properties (e.g., remolding for $w_l$), changes in pore topology during diagenesis and lithification. Furthermore, the anisotropy in hydraulic conductivity cannot be recovered from scalar parameters such as index properties.

(6) The prediction range $k_{\text{meas}}/5 \leq k_{\text{predict}} \leq 5k_{\text{meas}}$ is very wide for rate of seepage computations given the linear dependency between the flow rate and hydraulic conductivity in Darcy’s law. On the other hand, the sensitivity of hydraulic conductivity to changes in void ratio needs to be properly captured in numerical simulations where hydro-mechanical coupling is anticipated.

(7) Besides common experimental mistakes, there are other pending issues and inherent measurement difficulties: sampling effects, non-linear fluid flow, counter electromotive flow, fines migration, changes in pore fluid chemistry, bio-activity, the effect of shear, and the inherent changes in effective stress and void ratio associated with an imposed
pressure gradient. The latter implies that all hydraulic conductivity measurements are not point measurements but rather integral measurements.

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References


Hazen, A., 1892. Some physical properties of sand and gravel, with special reference to their use in filtration, Massachusetts State Board of Health, 24th annual report, Boston, 539-556.


Moore, R., 1991. The chemical and mineralogical controls upon the residual strength of pure and natural clays, Geotechnique, 41(1), 35–47.


Figure 1 Hydraulic conductivity versus void ratio. Data gathered for natural and remolded sediments, from coarse sands to fine-grained clays and various fabrics. Dataset: 1440 data points, 92 soils. Data sources: Mesri and Olson, 1971; Horpibulsuk et al., 2011; Michaels and Lin, 1954; Raymond, 1966; Siddique and Safiullah, 1995; Tavenas et al., 1983; Terzaghi, 1996; Deng et al., 2011; Dolinar, 2009; Sanzeni et al., 2013; Kwon et al., 2011; Kim et al., 2013; Sridharan and Nagara, 2005; Bandini and Sathiskumar, 2009; Sivapullaiah et al., 2000; Chu et al., 1954; Pandian, 2004; Kaniraj and Gayathri, 2004; Taylor, 1948; Chapuis et al., 1989; Lambe and Whitman, 1969.
Figure 2. Measured hydraulic conductivity $k$ versus estimated mean pore size $d_p$. Dataset: 1804 data points. Sources in addition to those listed in Figure 1: Indraratna et al., 2012; Louden, 1952; Bedinger, 1961; Bryant et al., 1992; Mbonimpa et al., 2002; Rowe et al., 2000; Sherard et al., 1984; Shepherd, 1989; Abichou et al., 2003; Indraratna et al., 1996; Tsai, 1990; Burmister, 1954; Keech and Rosene, 1964.
Figure 3. Hydraulic conductivity $k_o$ at reference void ratio $e_o=1$ versus specific surface $S_s$.

Best fit: $\log\left(\frac{k_o}{\text{cm/s}}\right) = -4.93 - 1.86 \log\left(\frac{S_s}{\text{m}^2/\text{g}}\right)$ shown as dotted line. The thick line has a slope -2 as anticipated by the Kozeny-Carman equation. Dataset: 718 datapoints. The open triangles correspond to hydraulic conductivity values measured at different void ratios and corrected for void ratio =1.0. Sources: refer to Figure 2.
Figure 4. Exponent $\beta$ plotted as a function of specific surface $S_s$. Dataset: 123 soils. Sources: refer to Figure 1.
Figure 5. Hydraulic conductivity predicted using Equation 10 versus the measured $k$ values (Database in figure 1).
Highlights

- Data spanning 12 orders of magnitude in hydraulic conductivity are compiled.
- Pore size rather than porosity defines a sediment hydraulic conductivity.
- Hydraulic conductivity is a power function of void ratio and specific surface.
- A simple prediction method of hydraulic conductivity for sediments is proposed.