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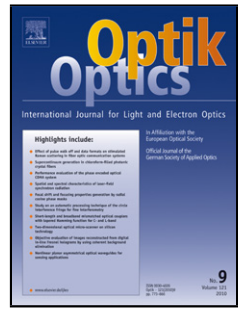

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Performance Analysis of Multihop Heterodyne Free-Space Optical Communication Over General Malaga Turbulence Channels with Pointing Error

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Abstract

This work investigates the end-to-end performance of a free space optical amplify-and-forward (AF) channel-state-information (CSI)-assisted relaying system using heterodyne detection over Malaga turbulence channels at the presence of pointing error employing rectangular quadrature amplitude modulation (R-QAM). More specifically, we present exact closed-form expressions for average bit-error rate for adaptive/non-adaptive modulation, achievable spectral efficiency, and ergodic capacity by utilizing generalized power series of Meijer's G-function. Moreover, asymptotic closed form expressions are provided to validate our work at high power regime. In addition, all the presented analytical results are illustrated using a selected set of numerical results. Moreover, we applied the bisection method to find the optimum beam width for the proposed FSO system.

Keywords: Free-space optical (FSO), multihop relaying, atmospheric turbulence, CSI-assisted relaying, Malaga channel, bit error rate (BER), adaptive transmission, achievable spectral efficiency (SE)

1. Introduction

Recently, free-space optical (FSO) communication systems have attracted attention thanks to their high capacity and low cost over unlicensed optical spectrum compared to radio frequency (RF) and fiber optic communication [1].

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Specifically, traditional RF spectrum band has been scarce and it requires high cost to reserve a licensed spectrum band. Moreover, it is not cost-and time-effective to deploy fiber optic technology due to the difficulty of wiring fiber optic cables especially in rugged environments. These features of FSO communication systems potentially enable solving the issues that the radio frequency (RF) communication systems face due to the expensive and scarce spectrum [1]. However, atmospheric turbulence and pointing errors may lead to a significant degradation in the performance of the FSO communication systems. Specifically, atmospheric turbulence is highly variable and unpredictable due to weather effect [2]. Furthermore, pointing error is usually caused by thermal expansion, dynamic wind loads, weak earthquakes and misalignment. These factors impose exaggerated fading and power loss on FSO links [3, 4].

The degradation effects of atmospheric turbulence and pointing errors on the design and performance of FSO links have been investigated theoretically. For instance, [5] investigated an FSO link over K distributed turbulence fading channels subjected to pointing error. Additionally, [6] analyzed the performance of FSO links over lognormal distribution. Actually, most of the contributions were restricted to specific turbulence models such as Lognormal distribution for weak turbulence and Gamma-Gamma distribution for weak-to-strong turbulence [7]. As a generalization, Malaga turbulence model has been developed as a unified distribution where most of the fading models are considered as special cases such as K-distribution, HK-distribution, Rice-Nagakami distribution, etc., and characterizes a wide range of (weak to strong) turbulence conditions [8]. Performance analysis of optical wireless communications with pointing errors over Malaga fading model was presented in [9, 10]. [11] investigated the performance of FSO systems utilizing intensity modulation and direct detection (IM/DD) and heterodyne detection over Malaga channels with pointing error model. They proposed closed-form of bit error rate (BER), ergodic capacity, achievable spectral efficiency (SE), and outage probability (OP) using Meijer's G-function. However, the utilization of Meijer's G-function in the closed form expression does not give an explicit insights on the dependence of performance metric on the channel parameters.

Recently, multihop relaying has gained an increasing attention due to its efficient technique to expand the coverage of wireless networks with low power requirements while offering high data-rate at the end-to-end communication [12]. Moreover, multihop relaying has been proposed to reduce the turbulence fading and increase the reliability of the FSO channel. In [12], the OP of a multihop relaying FSO system with amplify-and-forward (AF) or decode-and-forward (DF) relays over Gamma-Gamma turbulence is discussed. In addition, [13] investigated the performance of end-to-end multihop FSO system using variable and fixed-gain relays over strong turbulence fading channels. More recently, pointing error was introduced to analyze the multihop FSO systems with heterodyne detection over Gamma-Gamma fading [14].

Furthermore, to enhance the spectral efficiency, adaptive modulation is considered as a promising solution. It varies modulation orders according to the channel condition in turbulent fading channels. For each instantaneous fading state, this technique enables to reach the maximum achievable data rate under the predefined channel quality requirement. Such system provides higher spectral efficiency than non-adaptive modulation scheme while maintaining fixed allowable transmit power and target BER requirement. In [15], the performance behaviour of subcarrier intensity modulation (SIM) systems over the Gamma-Gamma turbulence channels has been emphasized. They derived closed-form solutions of non-adaptive and adaptive modulation employing M-ary phase shift keying (M-PSK) and rectangular quadrature amplitude modulation (R-QAM), but they focused only on strong turbulence. In [16], we generalized the performance analysis of adaptive SIM systems over general Malaga channel without pointing error.

In this paper, we analyze multihop FSO relaying with heterodyne detection over Malaga atmospheric turbulence channel with pointing error. Here we summarize the contribution of paper:

- We first derive the multihop probability density function (PDF) of AF channel-state information (CSI)-assisted multihop relaying system using heterodyne detection over Malaga turbulence channels with pointing error. Unlike [14, 17], the generalized power series of Meijer's G-function are employed to present the performance metrics in closed form. Therefore, the mathematical expressions of PDF, cumulative density function (CDF) and moment generating function (MGF) are tractable to be applied to calculate the performance metrics.

- We derive BER performance employing R-QAM of multihop relaying based on non-adaptive and adaptive modulation. Moreover, we obtained the closed form results of SE, OP and ergodic capacity.
- Moreover, asymptotic results at high signal-to-noise ratio (SNR) can be easily deductible from the closed form results.
- The closed form expressions and their asymptotic results will be validated with numerical results.
- Numerical optimization using bisection method has been proposed to find out the optimum beam width of end-to-end multihop FSO system to minimize the BER performance.

The rest of this paper is structured as follows: Section 2 describes the system and channel models. Section 3 investigates and validates the BER performance of non-adaptive and adaptive systems utilizing R-QAM. Section 4 derive the ergodic capacity results. Optimum beam width of the proposed system of N-hop relays has been obtained in section 5. Numerical results and discussions are presented in Section 6. Finally, Section 7 concludes the presented work.

2. System and Channel Models

2.1. System Model

In this work, we present an FSO system using coherent modulation with heterodyne detections. In this system model, the transmitter converts modulated electric signal into optical intensity signal. Then, the modulated beam will

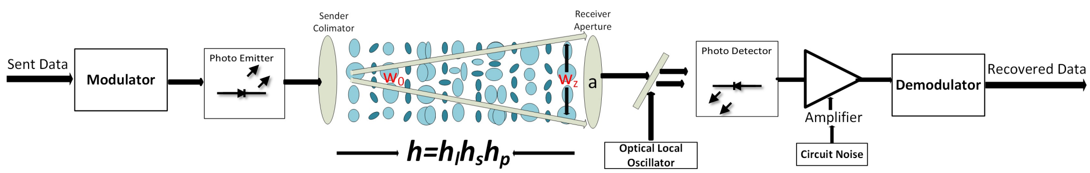

Figure 1: Block diagram of a coherent detection optical receiver.

be sent over the atmospheric turbulence channel with pointing error. As the channel coherence time is on the order of msec and the data rate is assumed to be on the order of Gbps, we can therefore assume in our analysis a slow fading channel model [18]. At the receiver, the received optical signal is mixed with a local oscillator (LO). Then, it converted into an electrical signal using photodetector and demodulated to recover the transmitted data as shown in Fig. 1, which is elaborated in [19]. For heterodyne detection [20], the instantaneous electrical SNR is computed as

$$\gamma = \frac{\eta_e h}{N_0}, \tag{1}$$

where h represents the channel fading coefficients . η_e and N_0 are the effective photo current conversion ratio and the additive white Gaussian noise (AWGN) variance, respectively. In line-of-sight (LOS) FSO links channel state can be expressed as $h = h_l h_p h_a$, with h_l being the path loss, h_p denoting the pointing error, and h_a representing the turbulence fading. Here h_l is a deterministic component as no randomness in its behaviour (it could be realized on order of hours), where h_p and h_a are both time-variant factors (considered as random variables (RVs)). In our work, we consider multihop system proposed in [14, 17]. In this model, the source S sends the signal to the receiver D through $R-1$ serially intermediate hops $R_1, R_2, \cdots, R_N$ which act as AF relays as shown in Fig. 2. For computational tractability, we used the upped bound approximation. Then, the end-to-end SNR of CSI-assisted relaying system can be derived for

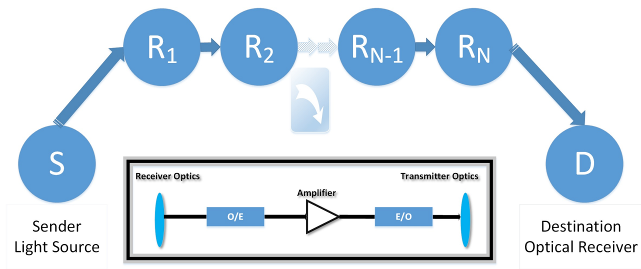

Figure 2: Multihop FSO communication system.

upper bound as [21]

$$\gamma_{ub} = \frac{1}{N}\prod_{i=1}^{N}\gamma_i^{(1/N)}, \tag{2}$$

where γ_i is the instantaneous SNR for the i-th hop.

2.2. Atmospheric Turbulence Models

The Malaga turbulence model [8] is a general turbulence model representing several turbulence models as special case [8, Table 1]. Malaga distribution providing an excellent agreement with published simulation data over a wide range of turbulence conditions (weak to strong) [8]. The unified PDF of the irradiance h with integer and non-integer β is given by [8, Eq. (24)] [1]

$$f_a(h_a) = A\sum_{m=1}^{X} a_m h_a^{\frac{\alpha+m}{2}-1} K_{\alpha-m}\left(2\sqrt{\xi h_a}\right), \quad h_a > 0 \tag{3}$$

where

$$(X, A, a_m, \xi) = \begin{cases} \beta, A^{(1)}, a_m^{(1)}, \frac{\alpha\beta}{g\beta+\overline{\Omega}}, & \text{integer } \beta \\ \infty, A^{(2)}, a_m^{(2)}, \frac{\alpha}{g}, & \text{non-integer } \beta \end{cases},$$

$$A^{(1)} \triangleq \frac{2\alpha^{\frac{\alpha}{2}}}{g^{1+\frac{\alpha}{2}}\Gamma(\alpha)}\left(\frac{g\beta}{g\beta+\overline{\Omega}}\right)^{\beta+\frac{\alpha}{2}}$$

[1] we will omit the subscript for hop index in subsections 2.2, 2.3 and 2.4 for easy of understanding the fading model.

$$a_m^{(1)} \triangleq \begin{pmatrix} \beta - 1 \\ m - 1 \end{pmatrix} \frac{\left(g\beta + \overline{\Omega}\right)^{1-\frac{m}{2}}}{(m-1)!} \left(\frac{\overline{\Omega}}{g}\right)^{m-1} \left(\frac{\alpha}{\beta}\right)^{\frac{m}{2}},$$

$$A^{(2)} \triangleq \left(\frac{2\alpha^{\frac{\alpha}{2}}}{g^{1+\frac{\alpha}{2}}\Gamma\left(\alpha\right)}\right) \left(\frac{g\beta}{g\beta + \overline{\Omega}}\right)^{\beta},$$

$$a_m^{(2)} \triangleq \frac{(\beta)_{m-1}(\alpha g)^{\frac{m}{2}}\overline{\Omega}^{m-1}}{[(m-1)!]^2 g^{m-1}\left(g\beta + \overline{\Omega}\right)^{m-1}}.$$

In (3), α is a positive parameter related to the effective number of large-scale cells of the scattering process, β is the amount of fading parameter and is a natural number, $g = 2b_0(1-\rho)$ denotes the average power of the scattering component received by off-axis eddies, $2b_0$ is the average power of the total scatter components, the parameter $0 \leq \rho \leq 1$ represents the amount of scattering power coupled to the LOS component, $\overline{\Omega} = \Omega + 2b_0\rho + 2\sqrt{2b_0\rho\Omega}\cos(\theta_A - \theta_B)$ symbolizes the average power through the coherent advantages, Ω is the regular power of the LOS aspect, θ_A as well as θ_B are the deterministic levels of the LOS and also the coupled-to-LOS spread terms, respectively, $\Gamma(.)$ is the Gamma function as presented in [20, Eq. (8.310)], and $K_\nu(.)$ is the νth-order modified Bessel function of the second kind [20, Sec. (8.432)]. Moreover, the superscript 1 and 2 has been used to indicate that the definitions of A and a_m parameters are specified to integer or non-integer β.

2.3. Pointing Error Model

We consider the existence of pointing error impairments for which the PDF of the irradiance h_p is given by [4]

$$f_p\left(h_p\right) = \frac{\zeta^2}{A_0^{\zeta^2}} h_p^{\zeta^2-1}, 0 \leq h_p \leq A_0, \tag{4}$$

where ζ is the ratio between the equivalent beam radius at the receiver and the standard deviation of pointing error displacement (jitter) at the receiver, A_0 is a constant term that defines the pointing loss given by $A_0 = [\mathrm{erf}(\nu)]^2$, $\mathrm{erf}(\cdot)$ is the error function defined as $\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-t^2)dt$, $\nu = \sqrt{\pi}a/(\sqrt{2}w_z)$, a is the radius of the detection aperture, and w_z is the beam waist.

2.4. Composite Atmospheric Turbulence-Pointing Error Model

The PDF of random variables $h = h_l h_a h_p$ can be computed by utilizing

$$f_h(h) = \int_t^{\infty} f_a(h_a)\frac{1}{h_a h_l} f_p(h/(h_a h_l))dh_a, \tag{5}$$

where $t = h/(h_l A_0)$. By applying random variable transformation on (4) and using (5), we obtain the PDF of received composite channel gain h which experiences Malaga channel at the presence of pointing error impairments, which is

$$\begin{aligned} f_h(h) &= \frac{\zeta^2}{(h_l A_0)^{\zeta^2}} h^{\zeta^2-1} \int_t^{\infty} \sum_{m=1}^{X} a_m h_a^{\frac{\alpha+m}{2}-\zeta^2-1} \\ &\times K_{\alpha-m}\left(2\sqrt{\xi I_a}\right) dh_a. \end{aligned} \tag{6}$$

Hence, by using [22, Eq. (07.34.03.0605.01))] we can express the modified Bessel function of the second kind, $\mathrm{K}_v(\cdot)$ as a special case of the Meijer's G-function [22, Eq. (07.34.02.0001.01)], defined as

$$G_{0\,2}^{2\,0}\left(a, b|x\right) = 2x^{(a+b)/2} k_{a-b}\left(2\sqrt{x}\right). \tag{7}$$

Finally, by applying (7) on (6) and using [22, Eq. (07.34.21.0085.01)] we can solve (6), given as

$$f_h(h) = \frac{\zeta^2 A}{2h} \sum_{m=1}^{X} y_m G_{1\,3}^{3\,0}\left({}^{\zeta^2+1}_{\zeta^2,\alpha,m} \middle| \xi \frac{h}{A_0 h_l} \right), \tag{8}$$

where $y_m = a_m \xi^{\frac{-(\alpha+m)}{2}}$. The average SNR for heterodyne detection technique could be derived based on (1) and [9, Eq. (24)] as $\mu = \mathrm{E}_\gamma[\gamma] = \overline{\gamma} = \eta_e \mathrm{E}_h[h]/N_0 = h_l \eta_e \zeta^2 A_0(g+\overline{\Omega})/[(1+\zeta^2)N_0]$. Therefore, by utilizing PDF transformation rule on (8) we can get a unified PDF of instantaneous SNR over Malaga channel as

$$f_\gamma(\gamma) = \frac{\zeta^2 A}{2\gamma} \sum_{m=1}^{X} y_m G_{1\,3}^{3\,0}\left({}^{\zeta^2+1}_{\zeta^2,\alpha,m} \middle| \gamma \frac{D}{\mu} \right), \tag{9}$$

where $D = \frac{\zeta^2(g+\overline{\Omega})}{\zeta^2+1}\xi$.

2.5. Statistics of End-to-End Received SNR

In this section, we will calculate the PDF of end-to-end SNR of CSI-assisted relaying in [14]. So, to determine this expression we have to derive the closed form expression of MGF of CSI-assisted relaying, respectively, as explained in the next sub-sections.

2.5.1. MGF of End-to-End SNR

MGF of end-to-end SNR γ_{ub} in [14] can be expressed as

$$\begin{aligned} M_{\gamma_{ub}}(s) &= \frac{\sqrt{N}\prod_{i=1}^{N}(\zeta_i^2 A_i)}{(2\pi)^{\frac{(N-1)}{2}}\,2^N}\prod_{j=1}^{N}\sum_{m_j=1}^{X} y_{m_j} \\ &\times\ G_{3N,\ 3N}^{3N,\ N}\left(\begin{smallmatrix} k_1 \\ k_2 \end{smallmatrix}\middle|\left(\frac{N}{s}\right)^N\prod_{x=1}^{N}\frac{D_x}{\mu_x}\right), \end{aligned} \tag{10}$$

where $k_1 = \Delta(N,0), \Delta(1, 1+\zeta_1^2), \ldots, \Delta(1, 1+\zeta_N^2)$, $k_2 = \Delta(1,\zeta_1^2), \Delta(1,\alpha_1), \Delta(1,m_1), \ldots, \Delta(1,\zeta_N^2), \Delta(1,\alpha_N), \Delta(1,m_N)$, and we define $\Delta(x,y) = y/x, (y+1)/x, \ldots, (y+x-1)/x$. Proof: See Appendix A.

2.5.2. PDF of End-to-End SNR

By utilizing $f_{\gamma_{ub}}(\gamma) = \mathcal{L}^{-1}\{M_{\gamma_{ub}}(s)\}$ [23, Eq. (3.40.1.1)] on (10) the PDFs of γ_{ub}, can be expressed as closed-from as

$$\begin{aligned} f_{\gamma_{ub}}(\gamma) &= \frac{N\gamma^{-1}\prod_{i=1}^{N}\zeta_i^2 A_i}{2^N}\prod_{j=1}^{N}\sum_{m_j=1}^{X} y_{m_j} \\ &\times\ G_{3N,\ 3N}^{3N,\ 0}\left(\begin{smallmatrix} k_5 \\ k_6 \end{smallmatrix}\middle| N^N\gamma^N\prod_{x=1}^{N}\frac{D_x}{\mu_x}\right), \end{aligned} \tag{11}$$

where $k_5 = 1+\zeta_N^2, \cdots, 1+\zeta_1^2$, $k_6 = \zeta_1^2, \alpha_1, m_1, \cdots, \zeta_N^2, \alpha_N, m_N$. To obtain an explicit insights into the effects of the channel parameters on (11), Meijer's G-function can be expressed in terms of the generalized power series given by,

$$\begin{aligned} G_{p\ q}^{m\ n}\left(\begin{smallmatrix} a_1,\ldots,a_n,\ldots,a_p \\ b_1,\ldots,b_m,\ldots,b_q \end{smallmatrix}\middle| z\right) &= \sum_{t=1}^{m}\frac{\prod_{j=1,j\neq t}^{m}\Gamma(b_j-b_t)}{\prod_{j=n+1}^{p}\Gamma(a_j-b_t)} z^{b_t} \\ &\times\ \sum_{i=0}^{\infty}\frac{\prod_{j=n+1}^{p}\Gamma(1-a_j-b_t)_i\prod_{j=1}^{n}\Gamma(1-a_j+b_t+i)}{\prod_{j=1}^{m}\Gamma(1-b_j-b_t)_i\prod_{j=m+1}^{q}\Gamma(1-b_j+b_t+i)} \\ &\times\ ((-1)^{p-m-n}z)^i. \end{aligned} \tag{12}$$

where $\Gamma(\cdot)$ is gamma function defined as $\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}dx, \quad t > 0$. The PDFs of upper bound for end-to-end SNR for CSI-assisted relaying can be derived

$$f_{\gamma_{ub}}(\gamma) \ = N\prod_{i=1}^{N}\frac{\zeta_i^2 A_i}{2}\prod_{j=1}^{N}\sum_{m_j=1}^{X} y_{m_j}\sum_{y=0}^{\infty}\psi(y)(N^N\gamma^N\prod_{x=1}^{N}\frac{D_x}{\mu_x})^{b_t+y} \times \ (-1)^{-N}N^{N(b_t+y)}\gamma^{N(b_t+y)-1}, \tag{13}$$

where $\psi(i) = \sum_{t=1}^{3N}\frac{\prod_{j=1,j\neq t}^{3N}\Gamma(b_j-b_t)\prod_{j=1}^{N}(b_t-\zeta_j^2)_i}{\prod_{j=1}^{N}\Gamma(\zeta_j^2+1-b_t)\prod_{j=1}^{3N}(1-b_j+b_t)_i}$, b_t is the tth element in order of $\{\zeta_1^2, \alpha_1, m_1, \cdots \zeta_N^2, \alpha_N, m_N\}$ and $(a)_n$ is the Pochhammer number defined as $(a)_n = a(a+1)\ldots(a+n_1)$, with $(a)_0 = 1$.

2.6. Moments of End-to-End SNR

In this study, we investigate the moments metric to be used for computing asymptotic results of ergodic capacity at high SNR, as it will be shown later in the ergodic capacity section. The moments are defined as $\mathrm{E}[\gamma^n]$. Hence, based (2), the moments for CSI-assisted relaying expressed as

$$\mathrm{E}[\gamma_{ub}] = \frac{1}{N^n}\mathrm{E}\left[\prod_{i=1}^{N}\gamma_i^{n/N}\right] = \frac{1}{N^n}\prod_{i=1}^{N}\mathrm{E}\left[\gamma_i^{n/N}\right]. \tag{14}$$

The moments $\mathrm{E}[\gamma_i^n]$ in [11, Eq. (20)] given by

$$\mathrm{E}[\gamma_i^n] = \frac{\zeta_i^2 A_i\Gamma(n+\alpha_i)}{2(n+\zeta_i^2)D_i^n}\sum_{m=1}^{X} y_m\Gamma(n+m)\mu_i^n. \tag{15}$$

Using (14) on (15), we obtain the moments of γ_{ub} as

$$\mathrm{E}[\gamma_{ub}^n] = \frac{1}{N^n}\prod_{i=1}^{N}\frac{\zeta_i^2 A_i\Gamma(n+\alpha_i)}{2(n+\zeta^2)D_i^{n/N}}\sum_{m=1}^{X} y_m\Gamma(n+m)\mu_i^{n/N}. \tag{16}$$

3. Bit Error rate analysis

In this section, we present an average BER based on adaptive and non-adaptive modulation systems employing R-QAM. A closed form expression over Malaga turbulence channels have been derived. In addition, we validated our results asymptotically for high SNR. For a Gray coded M-ary QAM with modulation order $M = (I \times J)$, BER over the AWGN channel is given by [24, Eq. (22)] as

$$P_b(I,J,\gamma) = \frac{1}{\log_2(I\cdot J)}\left(\frac{1}{I}\sum_{k=1}^{\log_2 I}\sum_{i=0}^{(1-2^{-k})I-1} C_i(k,I)\,\mathrm{erfc}\left(A_i\sqrt{\gamma}\right) + \frac{1}{J}\sum_{l=1}^{\log_2 J}\sum_{j=0}^{(1-2^{-l})J-1} C_j(l,J)\,\mathrm{erfc}\left(A_j\sqrt{\gamma}\right)\right), \quad (17)$$

where erfc$(\cdot)$ is the complementary error function defined as $\mathrm{erfc}(z) = \frac{2}{\pi}\int_z^\infty \exp(-t^2)dt$, and $C_i(k,I)$, and A_i, and are given, respectively, as

$$C_i(k,I) = (-1)^{\frac{i2^{k-1}}{I}}\left(2^{k-1} - \left\lfloor \frac{i2^{k-1}}{I} + \frac{1}{2}\right\rfloor\right), \quad (18)$$

and

$$A_i = (2i+1)\sqrt{\frac{3}{I^2+J^2-2}}. \quad (19)$$

For in-phase components, $C_j(l,J)$, and A_j can be obtained similarly from (18) and (19) by using change of variables. Moreover, we note that, throughout the paper, we adopted the change of variables to compute the performance results in the in-phase components (J) of QAM but only show the analytic results for the quadrature components (I) for simplicity. Also, in (18), $\lfloor x \rfloor$ operator is the nearest integer less than or equal to x.

3.1. Non-Adaptive Modulation System

3.1.1. Average BER of R-QAM Based on Non-adaptive Modulation

Average BER of a non-adaptive modulation system in a turbulence channel is calculated by

$$\overline{P_b}(I,J) = \int_0^\infty P_b(I,J,\gamma) f_{\gamma_{ub}}(\gamma)d\gamma, \quad (20)$$

By applying (13) and (17) on (20) and utilizing the integration identity of erfc [22, Eq. (06.27.21.0132.01)], the average BER for CSI-assisted relaying can be computed as

$$\overline{P_b}(I,J) = \frac{\prod_{i=1}^{N} \zeta_i^2 A_i}{2^N \log_2 (I \cdot J)} \left(P_I + P_J\right), \tag{21}$$

where the definition of P_I is .

$$P_I = \frac{1}{I} \sum_{k=1}^{\log_2 I} \sum_{i=0}^{\left(1-2^{-k}\right)I-1} C_i(I,J) \prod_{j=1}^{N} \sum_{m_j=1}^{X} y_{m_j} \left(\sum_{y=0}^{\infty} \psi(y) N^{N(b_t+y)} \prod_{x=1}^{N} (D_x/\mu_x)^{b_t+y} \times \frac{A_i^{-(2N(b_t+y))}}{\sqrt{\pi} N\left(b_t+y\right)} \Gamma\left(\frac{1}{2} + N(b_t+y)\right)\right). \tag{22}$$

3.1.2. Asymptotic BER

At high average SNR, the leading term for CSI-assisted in (22) with $m_j = 1$, $y = 0$ and $\underline{b}_t = \min\{\zeta_1^2, \alpha_1, m_1, \cdots, \zeta_N^2, \alpha_N\}$ becomes dominant. Therefore, asymptotic values in high SNR regimes can be directly obtained as

$$P_I^{asym} = \frac{1}{I} \sum_{k=1}^{\log_2 I} \sum_{i=0}^{\left(1-2^{-k}\right)I-1} C_i(I,J) \prod_{j=1}^{N} y_1 \left(\psi(0) N^{N\underline{b}_t} \prod_{x=1}^{N} (D_x/\mu_x)^{b_t} \frac{A_i^{-2N\underline{b}_t}}{\sqrt{\pi} N \underline{b}_t} \times \Gamma\left(\frac{1}{2} + N\underline{b}_t\right)\right). \tag{23}$$

By applying (23) on (21), asymptotic BER for CSI-assisted relaying derived as

$$\overline{P}_b^{asym}(I,J) = \frac{\prod_{i}^{N} \zeta_i^2 A_i}{2^N \log_2 (I \cdot J)} \left(P_I^{asym} + P_J^{asym}\right). \tag{24}$$

At high SNR, average BER in fading channels can be expressed as $p_e^{\infty} = (G_c{\cdot}\mu_r)^{-G_d}$, where G_d is the diversity order and G_c is the coding gain [11]. Based on (23), the diversity order for N-hop CSI-assisted system is G_d=min$(\zeta_1^2, \alpha_1, m_1, \cdots, \zeta_N^2, \alpha_N, m_N)$.

3.2. Adaptive Modulation

The main goal of adaptive transmission technique is to send the transmit signal with the largest possible modulation order while satisfying the desired BER P_0. Specifically, the transmitter selects one of Z available modulation orders $\{M_1, M_2, ..., M_Z\}$, according to the receiver estimated SNR γ_r and the desired BER P_0.

$$M_n = I_n J_n = 2^n, \text{ if } \gamma_n < \gamma_r < \gamma_{n+1} \quad n = 1, 2, \ldots, Z. \tag{25}$$

If n is even, $I_n J_n = 2^n$, and if n is odd, $I_n = 2^{(n+1)/2}$ and $J_n = 2^{(n-1)/2}$. The region boundaries γ_n is set to the required values of SNR in order to achieve the target BER P_0 over the AWGN channels using (17) for R-QAM. If the received SNR is below the SNR threshold of the minimum modulation order, i.e., $\gamma_r < \gamma_1$ (for R-QAM), the transmission will be terminated and the adaptive system will suffer an outage.

3.2.1. Achievable Spectral Efficiency

SE is the data rate transmitted per unit bandwidth and it is defined as [25].

$$S = \frac{R}{W} = \sum_{n=1}^{Z} a_n \log_2 M_n, \tag{26}$$

where R and W represent the transmitted data rate and bandwidth measured in bits/s and in Hz, respectively. In (26) a_n is the probability that the received SNR falls in the nth region, and it is defined as

$$a_n = \Pr(\gamma_n \le \gamma_r \le \gamma_{n+1}) = F(\gamma_{n+1}) - F(\gamma_n), \tag{27}$$

where $F_\gamma(\cdot)$ is the CDF of SNR. From (13), CDF of SNR γ for CSI-assisted relay can be obtained as

$$\begin{aligned} F(\gamma_n) &= \frac{\prod_{i=1}^{N} \zeta_i^2 A_i}{2^N} \prod_{j=1}^{N} \sum_{m_j=1}^{X} y_{m_j} \\ &\times \left(\sum_{i=0}^{\infty} \psi(i) (N^N \prod_{x=1}^{N} (D_x/\mu_x))^{b_t+i+1} \frac{\gamma_n^{N(b_t+i)+1}}{N(b_t+i)+1} \right). \end{aligned} \tag{28}$$

Hence, with (27) and $M_n = 2^n$, (26) can be written as

$$S = \sum_{n=1}^{Z} n \left(F(\gamma_{n+1}) - F(\gamma_n) \right) = Z - \sum_{n=1}^{Z} F(\gamma_n), \tag{29}$$

where we have used the fact $F\left(\gamma_{n+1}\right) = 1$. It is obvious from (29), for high SNR, i.e., when $\mu \to \infty$, the SE becomes Z, the largest modulation order.

3.2.2. Average BER of R-QAM Based on Adaptive Modulation

The BER of a constant-power/discrete-rate system can be computed as the ratio of the number of bits in error to the total number of transmitted bits which is given by [25]

$$\overline{P}_b = \frac{\sum_{n=1}^{Z} \left\langle P_b \right\rangle_n \log_2 M_n}{S}, \tag{30}$$

where $\left\langle P_b \right\rangle_n$ is the BER of an M_n-order R-QAM transmitted in the SNR range $[\gamma_n, \gamma_{n+1})$, and it is defined as

$$\left\langle P_b \right\rangle_n = \int_{\gamma_n}^{\gamma_{n+1}} P_n(I_n, J_n, \gamma) f_{\gamma_{ub}}(\gamma)\, d\gamma. \tag{31}$$

Applying (13) to (31) and using an integral identity [22, Eq. (06.27.21.0005.01)], we can evaluate $\left\langle P_b \right\rangle_n$ as

$$\left\langle P_b \right\rangle_n = \frac{\prod_{i=1}^{N} \zeta_i^2 A_i}{2^N \log_2 \left(I_n \cdot J_n\right)} \left(P_{I_n} + P_{J_n}\right), \tag{32}$$

where $\Gamma(a, b)$ is incomplete gamma function [22, Eq. (06.06.02.0001.01)], and

$$\begin{aligned} P_{I_n} &= \frac{1}{I} \sum_{k=1}^{\log_2 I} \sum_{i=0}^{\left(1-2^{-k}\right)I-1} C_i(I,J) \prod_{j=1}^{N} \sum_{m_j=1}^{X} y_{m_j} \left(\sum_{y=0}^{\infty} \psi(y) \left(N^N \prod_{x=1}^{N} \frac{D_x}{\mu_x} \right)^{b_t+y} \right. \\ &\times \left(\frac{{\gamma_{n+1}}^{N(b_t-y)}}{N(b_t+y)} \left(1 - \mathrm{erf}\left(A_i \sqrt{\gamma_{n+1}}\right) - A_i \sqrt{\gamma_{n+1}} \left(\gamma_{n+1} A_i^2\right)^{-N(b_t+y)+\frac{1}{2}} \right. \right. \\ &\times \left. \frac{\Gamma(N(b_t+y)+\frac{1}{2}, A_i^2 \gamma_{n+1})}{\sqrt{\pi}} \right) - \frac{{\gamma_n}^{N(b_t-y)}}{N\left(b_t+y\right)} \\ &\times \left(1 - \mathrm{erf}\left(A_i \sqrt{\gamma_n}\right)\right. \\ &- \left. \left. \left. \frac{A_i \sqrt{\gamma_n} \left(\gamma_n A_i^2\right)^{-N(b_t+y)+\frac{1}{2}} \Gamma(N(b_t+y)+\frac{1}{2}, A_i^2 \gamma_n)}{\sqrt{\pi}} \right) \right) \right). \end{aligned} \tag{33}$$

3.2.3. Asymptotic BER

It is essential to derive the asymptotic results of both numerator and denominator of (30) when the average SNR γ_i approaches infinity. Using a similar steps in deriving (24), asymptotic value of $\langle P_b \rangle_n$ for R-QAM based adaptive modulation in high SNR regimes can be obtained as

$$\langle P_b \rangle_n^{asym} = \frac{\prod_{i=1}^{N} (\zeta_i^2 A_i)}{2^N \log_2 (I_n \cdot J_n)} \left(P_{I_n}^{asym} + P_{J_n}^{asym} \right), \tag{34}$$

where

$$\begin{aligned} P_I^{asym} \underset{\mu \gg 1}{\approx} & \frac{1}{I} \sum_{k=1}^{\log_2 I} \sum_{i=0}^{\left(1-2^{-k}\right)I-1} C_i(I,J) \prod_{j=1}^{N} y_1 \Bigg(\psi(0)(N^N \prod_{x=1}^{N} D_x/\mu_x)^{\underline{b}_t} \\ \times & \left(\frac{\gamma_{n+1}}{N\underline{b}_t}^{N\underline{b}_t} \Big(1 - \text{erf} \left(A_i \sqrt{\gamma_{n+1}} \right) - A_i \sqrt{\gamma_{n+1}} (\gamma_{n+1} A_i^2)^{-N(\underline{b}_t) + \frac{1}{2}} \right. \\ \times & \left. \frac{\Gamma(N\underline{b}_t + \frac{1}{2}, A_i^2 \gamma_{n+1})}{\sqrt{\pi}} \Big) - \frac{\gamma_n}{N\underline{b}_t}^{N\underline{b}_t} \right. \\ \times & \left(1 - \text{erf} \left(A_i \sqrt{\gamma_n} \right) \right. \\ - & \left. \frac{A_i \sqrt{\gamma_n} (\gamma_n A_i^2)^{-N\underline{b}_t + \frac{1}{2}} \Gamma(N\underline{b}_t + \frac{1}{2}, A_i^2 \gamma_n)}{\sqrt{\pi}} \right) \Bigg). \end{aligned} \tag{35}$$

For the denominator, when $\mu \to \infty$ the SE will converge to largest modulation order. Therefore, we calculate the asymptotic BER of adaptive modulation in high SNR as

$$\overline{P}_b^{asym} = \frac{\sum_{n=1}^{Z} \langle P_b \rangle_n^{asym} \log_2 M_n}{Z}. \tag{36}$$

4. Ergodic Capacity

In this paper, we consider the effect of pointing error impairments as well as the Malaga turbulence fading over FSO channel. Ergodic capacity represents the best average achievable data rate over the wireless optical link. In this section the exact and asymptotic ergodic capacity for N-hops FSO system utilizing CSI-assisted relaying is investigated.

4.1. Exact Ergodic Capacity

According to Shannon ergodic capacity, the ergodic capacity for heterodyne detection is formulated as $C = \mathrm{E}[\log_2(1+\gamma)]$. Hence, the upper bounds for the ergodic capacity of both CSI-assisted relay by utilising $C = \mathrm{E}[\log_2(1 + \gamma_{ub})]$ can be derived by representing $\ln(1+\gamma)$ in terms of Meijers G function as $G_{2,\,2}^{1,\,2}\left({}^{1,1}_{1,0}\middle|\gamma\right)$. Integrating the ergodic capacity over (11) and utilizing [23, Eq. (2.24.1.1)], the ergodic capacity for CSI-assisted is computed as

$$\begin{aligned}\overline{C}_{\gamma_{ub}}(\gamma) &= \frac{\prod_{i=1}^{N}\zeta_i^2 A_i}{\ln(2)(2\pi)^{N-1}2^N}\prod_{j=1}^{N}\sum_{m_j=1}^{X} y_{m_j} \\ &\times\ G_{2N+1,\ 4N+1}^{4N+1,\ \ N}\left(\begin{matrix}k_9\\k_{10}\end{matrix}\middle| N^N\prod_{i=1}^{N}\frac{D_i}{\mu_i}\right), \end{aligned} \tag{37}$$

where $k_9 = \Delta(N,0), 1, k_5\ and, k_{10} = k_6, \Delta(N,0), 0$.

By utilising (12) in (37), we obtain an insight overview of the ergodic capacity equations. Hence, these equations could be written as shown in (38)

$$\begin{aligned}\overline{C}_{\gamma_{ub}}(\gamma) &= \frac{\prod_{i=1}^{N}\zeta_i^2 A_i}{\ln(2)(2\pi)^{N-1}2^N}\prod_{j=1}^{N}\sum_{m_j=1}^{X} y_{m_j}\sum_{y=0}^{\infty}\psi(y)(N^N\prod_{x=1}^{N}\frac{D_x}{\mu_x})^{b_t+y} \\ &\times\ (-1)^{-3N}N^{N(b_t+y)}, \end{aligned} \tag{38}$$

where $\psi(i) = \sum_{t=1}^{4N+1}\frac{\prod_{j=1,j\neq t}^{4N+1}\Gamma(b_j-b_t)\prod_{j=N+1}^{2N+1}(1-a_j+b_t)_y\prod_{j=1}^{N}\Gamma(1-a_j+b_t+y)}{\prod_{j=1}^{2N+1}\Gamma(a_j-b_t)\prod_{j=1}^{4N+1}(1-b_j+b_t)_i}$, b_t is the tth element in order of $\{k_6, \Delta(N,0), 0\}$, a_j is the jth element in order of $\Delta(N,0), 1, k_5$.

4.2. Asymptotic Ergodic Capacity

At high SNR, the dominated terms in ergodic capacity in (38) at $m_j = 1$ and $y = 0$. Hence, the the asymptotic formula for N-hops utilizing CSI- assisted relaying can be approximated as

$$\begin{aligned}\overline{C}_{\gamma_{ub}}(\gamma) &\underset{\mu\gg 1}{\approx} \frac{\prod_{i=1}^{N}\zeta_i^2 A_i}{\ln(2)(\pi)^{N-1}2^{2N-1}}\prod_{j=1}^{N} y_1\psi(0)(N^N\prod_{x=1}^{N}\frac{D_x}{\mu_x})^{\underline{b}_t} \\ &\times\ \ (-1)^{-3N}N^{N\underline{b}_t}, \end{aligned} \tag{39}$$

Alternatively, simple asymptotic expressions for the ergodic capacity in (37) at the high SNR may also be obtained from the first derivative of the nth-order moment of γ_{ub} [11, Eq. (34)] as where $\boldsymbol{\psi}(\cdot)$ is the psi (digamma) function.

5. System Beam Width Design

The transmitted beam width at the transmitter side w_0 is selected to minimize end-to-end BER over Malaga turbulence channel based on CSI-assisted

$$\overline{C}_{\gamma_{ub}}(\gamma) \approx \frac{\partial}{\partial n}\mathrm{E}[\gamma_{ub}^{n}]\,|_{n=0} \underset{\mu\gg 1}{\approx} -\log(N) + +\log(\prod_{i}^{N}\mu_i^{1/N}) + \frac{1}{N}\sum_{i}^{N}\frac{A_i\Gamma(A_i\alpha_i)}{2} \times \sum_{m_i=1}^{\beta_i} y_{m_i}\Gamma(m_i)\{-1/\zeta_i^2 - \log(B_i) + \boldsymbol{\psi}(\alpha_i) + \boldsymbol{\psi}(m_i)\} \quad (40)$$

relay. Consequently, the beam width optimization balances the impact of pointing error and atmospheric turbulence factors on the BER performance. Increasing the beam width mitigates pointing errors at the expense of received power while narrowing the beam limits the geometric loss but increases the impact of misalignment. Therefore, there exist an optimum beam width to show the minimum BER. To obtain the optimum beam width w_{op}, we applied numerical finding-roots method using bisection optimization method. Initially, we specify the maximum and minimum value of w_z, initial beam width value w_0 and the step size d_w, then we compute the BER value at w_0 and $w_0 + d_w$. After that, we check the sign of its derivative, and based on the sign we update wz_{max}, wz_{min} and w_0 as explained in the Algorithm 1 to approach optimal beam width iteratively.

6. Numerical Results

In this section, we validated our performance analysis of multihop CSI-assisted relaying FSO heterodyne system over Malaga turbulence under pointing error. We consider that channel conditions for the all hops are equal. Moreover, we consider Malaga turbulent channel with integer β as strong ($\beta = 1, \alpha = 2.04$), moderate ($\beta = 2, \alpha = 2.296$) and weak ($\beta = 3, \alpha = 2.4$), which are considered in [16], Also, these values are subject to the standards in [8, 26]. For non-integer β, the results will not be presented, while the results are approximately close to integer β case. We investigated the BER performance under non-adaptive and adaptive transmission technique. The pointing error parameter ζ for the whole analysis has been defined for strong and weak pointing error, respectively,

Algorithm 1: Finding Roots Using Bisection Method

Result: Optimum Beam Width with the N hop FSO system

initialize $wz_{\max}$ and $wz_{\min}$

$w_0 = (z_{\max} + z_{\min})/2$

$d_w = 0.01$

$i = 0$

$i_{\max}$ = maximum number of iterations

while $i < i_{\max}$ **do**

  Compute $\zeta(w_0)$ *and* $A_0(w_0)$

  Compute $P_1 = \overline{P_b}(\zeta, A_0)$ *using* Eq.(23)

  Compute $\zeta(w_0 + d_w)$ *and* $A_0(w_0 + d_w)$

  Compute $P_2 = \overline{P_b}(\zeta, A_0)$ *using* Eq.(23)

  $d = \frac{p_1 - p_2}{dw}$

  if $d > 0$ **then**

    $wz_{\min} = w_0$

  else

    $wz_{\max} = w_0$

  end

  $w_0 = (wz_{\max} + wz_{\min})/2;$

  $i = i + 1$

end

as $\zeta = 0.5$ and $\zeta = 6.5$. From the simulation, we emphasized the number of iteration of generalized power series that we used to converge. For low SNR, especially at $0, 10, 20$ dB, $j = 25$ where for the rest of the SNR $j = 10$. Also, we introduced the the asymptotic results for the whole analysis at high SNR. The obtained analysis has been validated numerically by comparing with direct integration. On the other hand, to evaluate the adaptive BER performance, we assigned the pre-defined target BER to $P_o = 10^{-5}$. Throughout the numerical results, we observe excellent match between our derived results and numerical integration.

In Fig. 3, we demonstrate the 2-hop BER performance of 4-QAM under non-adaptive transmission for integer β utilizing multihop CSI-assisted relaying. The figures show that the diversity order of CSI-assisted relaying at $\zeta = 6.7$ is depending on the fading parameter $G_d = \min(\alpha_i, 1)$. In contrast, at $\zeta = 0.5$ is depending on the fading parameter $G_d = \min(\zeta_i^2)$. It is shown that the asymptotic BER performance agree with the exact derived SNR at high SNR regime.

Fig. 4 shows the 2-hop BER performance of R-QAM based on adaptive FSO heterodyne system for integer under CSI-assisted relaying. We noticed that the obtained BER performance of adaptive transmission satisfies the predefined requirements. The diversity order is $G_d = \min(\alpha_i, 1)$ where $\zeta = 6.7$. While, at $\zeta = 0.5$ the diversity order is depending on $G_d = \min(\zeta_i^2)$.

In Fig. 5, we present the ergodic capacity of CSI-assisted relaying heterodyne FSO system. The figure shows the ergodic capacity metric with $\zeta = 6.7$ and $\zeta = 0.5$. The analytical results for the upper bound on the ergodic capacity obtained in (38) have been verified by (37) numerically, and a perfect agreement is observed.

Fig. 6 presents the relation between the BER of 2-hop CSI-assisted relaying and the beam width under the system design parameters used in [3, Table III] . It shows that the performance of 2-hop BER under CSI-assisted relay was decreasing with the increase of the beam width by compensating for misalignment. Then, it start to increase because of sever received power loss due to the

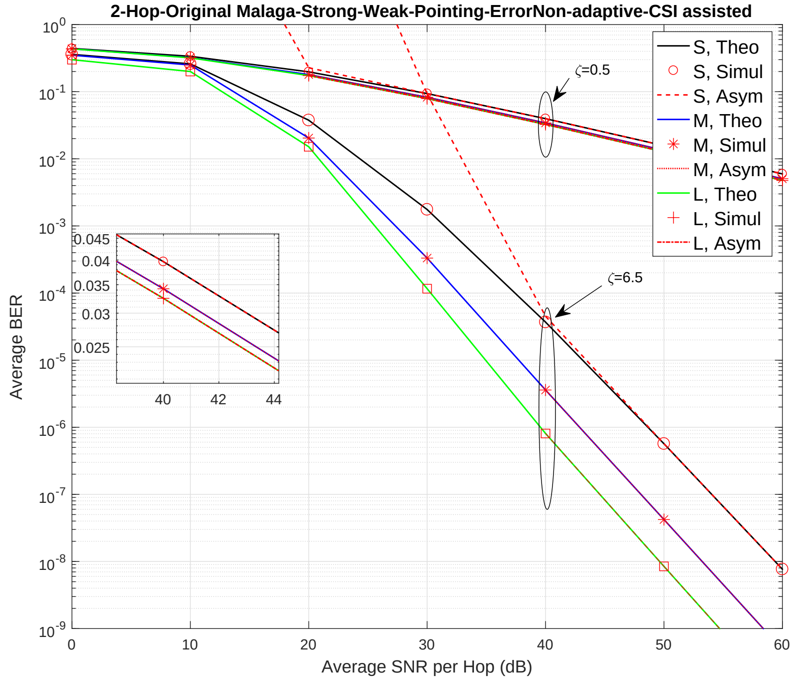

Figure 3: Average BER of 4-QAM utilizing non-adaptive modulation in 2-hop relaying with variable gain over the Malaga turbulence channels.(S: strong, M: moderate and W: weak)

diverse beam width. That is because the aperture of the FSO system start to loss some fraction of the beam power gradually with wider beam width, while reducing pointing error. To select the optimum beam width w_{op} we applied the bisection method as seen in Fig 7. The figure shows the optimum beam width w_{op} under fixed turbulence parameters ($\alpha_i = 2.296, \beta_i = 2$). The proposed algorithm find the optimum beam width for each SNR value. The average BER under optimal beam width provide the best performance compared with that under fixed beam width.

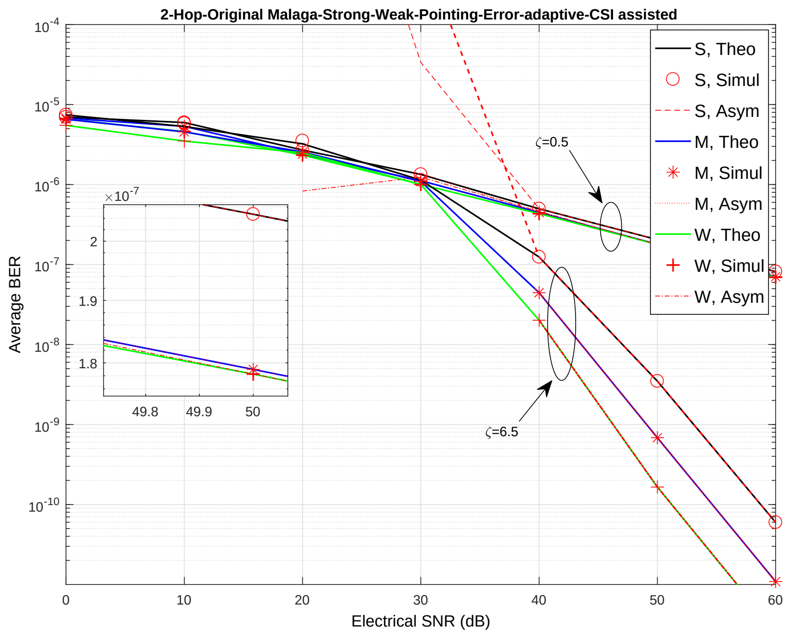

Figure 4: Average BER of R-QAM utilizing adaptive modulation in 2-hop relaying with variable gain over the Malaga turbulence channels. (S: strong, M: moderate and W: weak).

7. Conclusion

We proposed highly accurate closed-form of average BER performance for utilizing non-adaptive and adaptive modulation, SE, and ergodic capacity of N-hop heterodyne AF relaying using CSI-assisted relays over the general Malaga turbulence channels with pointing errors. The presented power series expressions are more traceable and observable than using Meijer's G-function, especially, for adaptive scheme. Furthermore, the performance analysis at high SNR regions has been verified asymptotically. Our analysis emphasized that the performance results based on generalized power series perfectly match the numerical results. Moreover, we show from our asymptotic BER that the diversity order of the proposed system for CSI-assisted relay is $\min_i \min\{\zeta_i^2, \alpha_i, m_i\}$. Also, by applying bisection finding-root method we can find optimal beam width

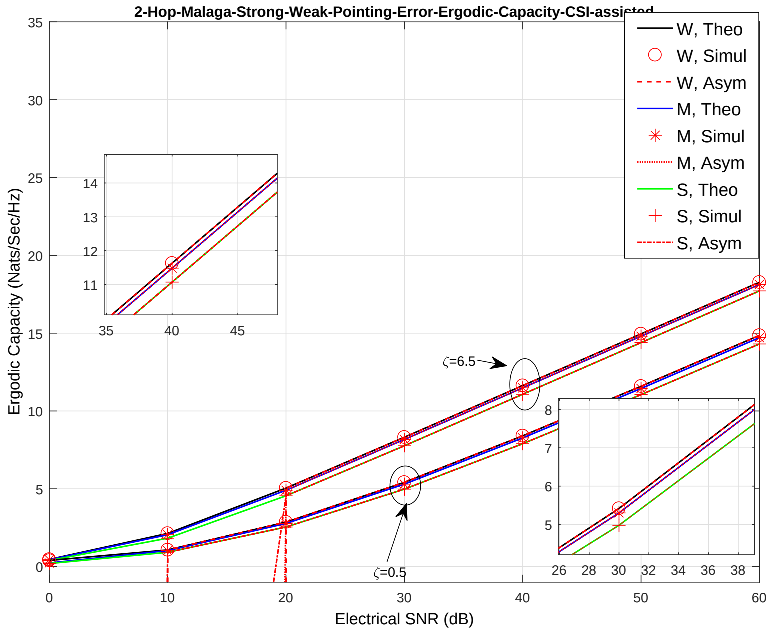

Figure 5: Ergodic capacity of 4-QAM in 2-hop relaying with variable gain over the Malaga turbulence channels. (S: strong, M: moderate and W: weak).

to minimize average BER for end-to-end CSI-relaying FSO system.

Appendix A. MGF of multihop relays

Firstly, for simplicity we defined a new random variable as $Y = N^{-1} \prod_{i=1}^{N} \gamma_i^{1/N}$. Then, the MGF of Y based on upper bound definition in (2) can be defined as

$$M_Y(s) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-s \prod_{i=1}^{N} \gamma_i^{1/N}} \prod_{i=1}^{N} f_{\gamma_i}(\gamma_i) d\gamma_i. \tag{A.1}$$

The first integration in (A.1) can be derived as

$$\begin{aligned} I_1 &= \frac{\zeta_i^2 A_i}{2} \sum_{m_1=1}^{X_1} y_{m_1} \int_0^\infty \gamma_1^{-1} \\ &\times G_{1\,3}^{3\,0}\left(\left. {\zeta_1^2+1 \atop \zeta_1^2, \alpha_1, m_1} \right| \gamma \frac{D_1}{\mu_1} \right) G_{0,\,1}^{1,\,0}\left(\left. {- \atop 0} \right| s P_2 \gamma_1^{1/N} \right) d\gamma_1, \end{aligned} \tag{A.2}$$

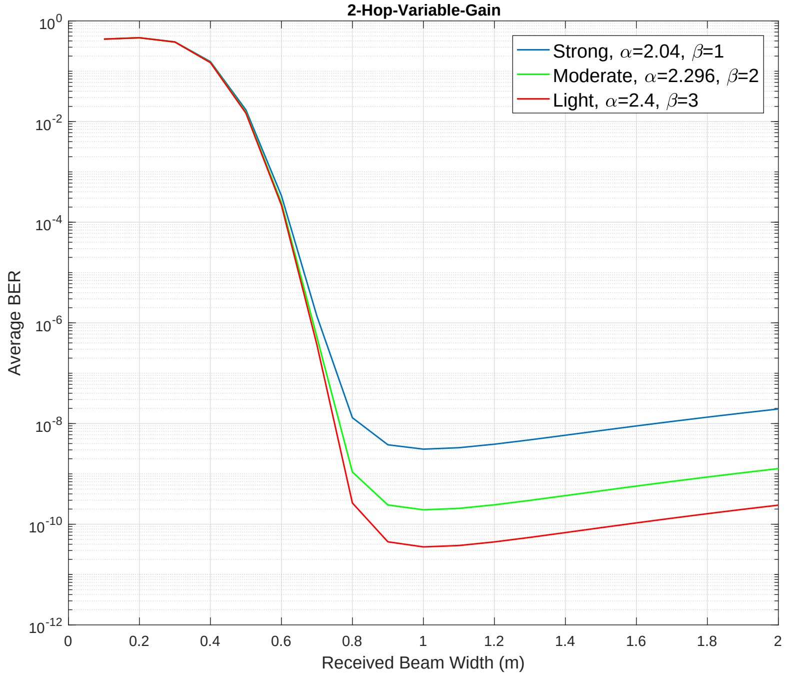

Figure 6: Relation between beam width and average BER of 4-QAM in 2-hop relaying with variable gain over the Malaga turbulence channels. (S: strong, M: moderate and W: weak)

where $P_i = \prod_{j=i}^{N} \gamma_j^{1/N}$. Utilizing [22, Eq. (01.03.26.0005.01)] to represent $e^{-sP_2\gamma_1^{1/N}}$ as $G_{0,\,1}^{1,\,0}\left(\begin{smallmatrix}-\\0\end{smallmatrix}\middle| sP_2\gamma_1^{1/N}\right)$ then applying the integral identity [27, Eq. (2.24.1.1)], we get after some manipulations I_1 as

$$I_1 = \frac{\sqrt{N}\zeta_1^2}{2(2\pi)^{\frac{N-1}{2}}} \sum_{m_1=1}^{X_1} y_{m_1} G_{3,N+1}^{N,3}\left(\begin{matrix}\delta_1\\ \delta_2\end{matrix}\middle| \left(\frac{sP_2}{N}\right)^N \frac{\mu_1}{D_1}\right), \qquad \text{(A.3)}$$

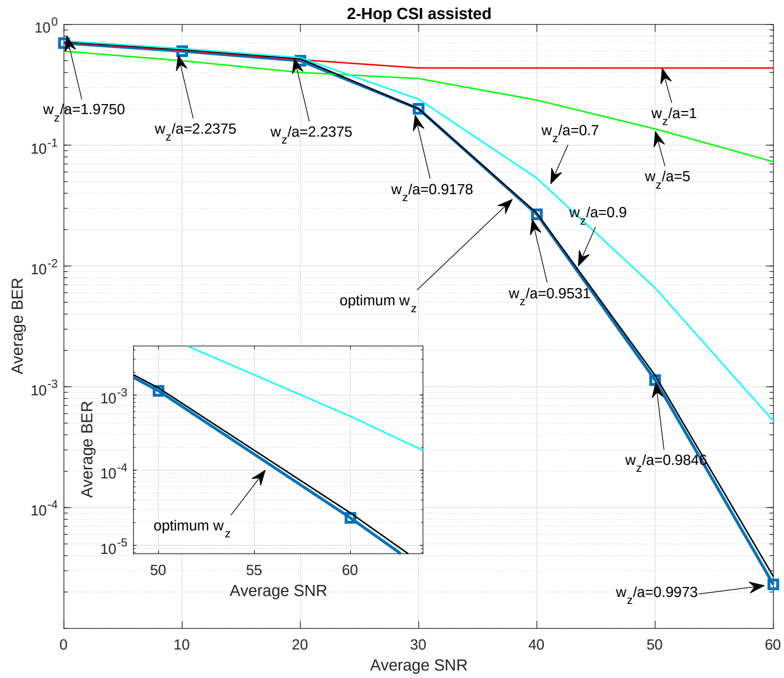

Figure 7: Optimum beam width based on average BER of 4-QAM in 2-hop relaying with variable gain over the Malaga turbulence channels.

where $\delta_1 = \Delta(1, 1-\zeta_1^2), \Delta(1, 1-\alpha_1), \Delta(1, 1-m_1)$, $\delta_2 = \Delta(N, 0), \Delta(1, 1-\zeta_1^2)$. The second integration in (A.5) can be expressed as

$$
\begin{aligned}
I_2 &= \frac{\sqrt{N}\zeta_1^2\zeta_2^2}{4\,(2\pi)^{\frac{N-1}{2}}} \sum_{m_1=1}^{X_1} y_{m_1} \sum_{m_2=1}^{M_2} y_{m_2} \qquad \text{(A.4)} \\
&\times \int_0^\infty \gamma_2^{-1} G_{3,N+1}^{N,3}\left(\begin{matrix} \delta_1 \\ \delta_2 \end{matrix} \middle| \left(\frac{sP_3}{N}\right)^N \frac{\gamma_2}{D_1} \right) \\
&\times \; G_{1,3}^{3,0}\left(\begin{matrix} 1+\zeta_2^2 \\ \zeta_2^2, \alpha_2, m_2 \end{matrix} \middle| \gamma_2 \frac{D_2}{\mu_2} \right) d\gamma_2,
\end{aligned}
$$

Similarly, by applying the integral identity [27, Eq. (2.24.1.1)], I_2 can be expressed as

$$I_2 = \frac{\sqrt{N}\zeta_1^2\zeta_2^2}{4(2\pi)^{\frac{N-1}{2}}} \sum_{m_1=1}^{X_1} y_{m_1} \sum_{m_2=1}^{X_2} y_{m_2}$$

$$\times G_{6,\ N+3}^{N,\ 6}\left(\begin{matrix} \delta_3 \\ \delta_4 \end{matrix} \middle| \left(\frac{sP_3}{N}\right)^N \frac{\mu_1\mu_2}{D_1 D_2} \right), \tag{A.5}$$

where $\delta_3 = \Delta(1, 1-\zeta_1^2), \Delta(1, 1-\alpha_1), \Delta(1, 1-m_1), \Delta(1, 1-\zeta_2^2), \Delta(1, 1-\alpha_2), \Delta(1, 1-m_2)$, $\delta_4 = \Delta(1, 0), \Delta(1, -\zeta_1^2), \Delta(1, -\zeta_2^2)$. Following the N-fold integral and applying a PDF transformation based on $\gamma_{ub} = N^{-1}Y$ we will reach (10).

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