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A parameterization study for elastic VTI Full-Waveform Inversion of hydrophone components: synthetic and North Sea field data examples

Antoine Guitton\(^1\) & Tariq Alkhalifah\(^2\)

(August 7, 2017)

Running head: Parameterization

ABSTRACT

Choosing the right parameterization to describe a transversely isotropic medium with a vertical symmetry axis (VTI) allows us to match the scattering potential of these parameters to the available data in a way that avoids potential tradeoff and focus on the parameters to which the data are sensitive. For 2-D elastic full-waveform inversion in VTI media of pressure components and for data with a reasonable range of offsets (as with those found in conventional streamer data acquisition systems), assuming that we have a kinematically accurate NMO velocity \((v_{nmo})\) and anellipticity parameter \(\eta\) (or horizontal velocity, \(v_h\)) obtained from tomographic methods, a parameterization in terms of horizontal velocity \(v_h\), \(\eta\) and \(\epsilon\) is preferred to the more conventional parameterization in terms of \(v_v\), \(\delta\) and \(\epsilon\). In the \(v_h, \eta, \epsilon\) parameterization and for reasonable scattering angles (< 60°), \(\epsilon\) acts as a “garbage collector” and absorbs most of the amplitude discrepancies between modeled and observed data, more so when density \(\rho\) and shear-wave velocity \(v_s\) are not inverted for (a standard practice with streamer data). On the contrary, in the \(v_v, \delta, \epsilon\) parameterization, \(\epsilon\) is mostly sensitive to large scattering angles, leaving \(v_v\) exposed to strong leakages from \(\rho\) mainly. These assertions will be demonstrated on the synthetic Marmousi II as well as a North Sea OBC dataset, where inverting for the horizontal velocity rather than the vertical velocity yields more accurate models.
and migrated images.
INTRODUCTION

The scattering potentials of perturbations in the anisotropic parameters reveal the data dependency on the parameters used to describe the anisotropic model (Gholami et al., 2013; Alkhalifah and Plessix, 2014). They also expose our ability to invert for these parameters given the seismic acquisition set-up used in the experiment (Jin et al., 1992; Forgues and Lambaré, 1997). The scattering potentials of anisotropic model parameter perturbations are based on the linearized approximation of the wave equation with respect to these parameters given by the first term of the Born series (Tarantola, 1986). Since this term constitutes the gradient for full-waveform inversion (FWI), it also reveals important information on the parameter tradeoff and their resolvability.

Since the FWI process is highly nonlinear, the story can only be complete when such scattering potential inferences are supported by an FWI implementation, the main goal of this paper. Alkhalifah (2016) studied the short and long wavelength influences of perturbations in the parameters for parameterizations promoted by Alkhalifah and Plessix (2014) for acoustic VTI media. He concluded that a parameterization given by the horizontal velocity $v_h$, the anellipticity parameter $\eta$, and the parameter that relates the horizontal-to-the vertical velocity, $\epsilon$, was optimal for FWI using conventional surface seismic PP-waves data. In this case, the long wavelength information of $v_{nmo}$ and $\eta$ (or $v_h$) are assumed to be included in the initial model and we, thus, need to invert only for $v_h$ and $\epsilon$. The role of $\epsilon$ in this case is to provide the perturbations necessary to fit the amplitudes of reflections at short offsets to accommodate the limitations of the acoustic model in properly fitting elastic amplitudes.

Here we study the parameterization effects in the elastic case assuming pressure recordings and a reasonable range of offsets ($< 10$ km) or scattering angles (most likely, $< 60^\circ$), requirements often met with marine streamer or ocean-bottom acquisition systems. Using an elastic propagator
with hydrophone data might seem to be an overkill, but there is an undeniable trend in the FWI community to model data more accurately taking into account all known parameters affecting phase and amplitude, including elastic ones. Therefore, our study embraces this trend and attempts to expand conclusions drawn from the acoustic world into the more realistic elastic one. Our vision is that using accurate physics to describe wave propagation will allow practitioners to rely more on seismic amplitudes to derive accurate models (as opposed to mostly phase information in the acoustic approximation). Now, focusing on pressure data only simplifies the analysis because S-waves are coming from mode conversions and become second-order events. Also, the inversion of these data is usually simpler than the inversion of often-noisy land data (due to the complex near-surface effects), thus giving us a chance to test our findings on field datasets more easily. Therefore, our goal is to bring more insights into the influence of some important parameters in the Elastic Full-Waveform Inversion (EFWI) of field data, where the Earth is assumed to be closer to an elastic than acoustic medium.

In this paper, we first start by comparing the radiation patterns of an optimal \( v_h, \eta \) and \( \epsilon \) parameterization with those of the more conventional \( v_v, \delta \) and \( \epsilon \) parameterization for an elastic medium. We show that the conclusions drawn by Alkhalifah (2016) in an acoustic medium hold in an elastic one. Then, using a modified Marmousi II synthetic dataset, we analyze both parameterizations when a kinematically accurate NMO velocity and \( \eta \) (or horizontal velocity, \( v_h \)) obtained, for example, from tomographic methods are available. Having an inaccurate \( \delta \) (equal to zero in these first tests) caused the inversion parameterized by the \( v_v, \delta, \) and \( \epsilon \) to yield worse results than the inversion parameterized by \( v_h, \eta \) and \( \epsilon \). Nevertheless, both inversion results are slightly effected by (1) missing long wavelength depth information and (2) the elastic nature of the modeled data. The degradation, however, is far more severe using the conventional \( v_v, \delta, \epsilon \) parameterization.

Finally, we conduct our parameterization study on a 2-D OBC line from the North Sea. We show
that our VTI elastic FWI models using $v_h$, $\eta$ and $\epsilon$ yield better elastic RTM images than the starting models and the models obtained using the standard $v_v$, $\delta$ and $\epsilon$ parameterization, thus confirming our mathematical analysis with the radiation patterns and our conclusions from the synthetic data case.

**COMPARING $v_v$, $\delta$, $\epsilon$ AND $v_h$, $\eta$, $\epsilon$ PARAMETERIZATIONS**

In this section, we compare two parameterizations of VTI elastic FWI by first presenting and analyzing radiation patterns and then illustrating our findings on a modified Marmousi II model. Radiation patterns yield useful information regarding the sensitivity of all parameters as a function of scattering angles. However, they ignore other effects such as band-limited data, travel-time sensitivity (da Silva et al., 2016) or model complexities that might influence the parameterization as well. Nonetheless, we think that their analysis is one of the most practical way to make sense of the complex interactions between different parameters and their resolvability. This analysis is for hydrophone data and focuses on the inversion of PP-waves only. Again, we target essentially marine acquisition systems where pressure sources are used.

**Scattering potentials in VTI elastic media**

For acoustic VTI media, Alkhalifah and Plessix (2014) derived such patterns for different anisotropic parameter combinations that they deem to be the most practical. Later, Alkhalifah (2016) made the argument for one of these combinations, $v_h$, $\eta$, and $\epsilon$, for FWI of conventionally acquired surface seismic PP-wave data. Considering the asymptotic Green’s function $G(x, k, \omega)$, expressed in the frequency domain $\omega$, and a plane wave described by the wavenumber vector $k$ for either the source $(k_s)$ or receiver $(k_r)$ wavefields approaching location $x$, then we can write the single-scattered
wavefield (Alkhalifah and Plessix, 2014)

$$u_s(k_s, k_r, \omega) = -\omega^2 s(\omega) \int dx \frac{G(k_s, x, \omega) G(k_r, x, \omega)}{v_0^2(x) \rho(x)}.$$  \hspace{1cm} (1)

with $s(\omega)$ is the source function, $\rho$ is the density, $v_0$ is the background isotropic velocity. The vector $r(x)$ includes the perturbations of the individual parameters. For a medium parameterized with $v_h, \epsilon, \eta, \rho$ and $v_s$, the perturbation vector $r_v(x)$ becomes

$$r_v(x) = \begin{pmatrix} r_{v_h}(x) \\ r_{v_s}(x) \\ r_\eta(x) \\ r_\epsilon(x) \\ r_\rho(x) \end{pmatrix}$$  \hspace{1cm} (2)

while for a medium parameterized with $v_h, \epsilon, \delta, \rho$ and $v_s$, the perturbation vector $r_h(x)$ becomes

$$r_h(x) = \begin{pmatrix} r_{v_v}(x) \\ r_{v_s}(x) \\ r_\delta(x) \\ r_\epsilon(x) \\ r_\rho(x) \end{pmatrix}$$  \hspace{1cm} (3)

The coefficients of $a(x)$ define the radiation patterns of each parameter for the given parameterization (Aki and Richards, 1980). For a medium parameterized with $v_h, \epsilon, \eta, \rho$ and $v_s$, the radiation
patterns are given by $a(x) = a_{PP}^{v_h}(x)$ where

$$a_{PP}^{v_h}(x) = \begin{pmatrix} 2 \\ \sin(2\theta_i)\sin(2\theta_r)+2\cos(2\theta_i)\cos(2\theta_r)-2 \\ -\cos^2(\theta_i)\sin^2(\theta_i)-\cos^2(\theta_r)\sin^2(\theta_r) \\ -\cos^2(\theta_i)-\cos^2(\theta_r) \\ 2 - \frac{(1-2\sigma)\sin^2(\theta_r-\theta_i)}{1-\sigma} + 2\cos(\theta_i-\theta_r) \end{pmatrix}, \quad (4)$$

where $\sigma$ is the Poisson’s ratio, $\theta_i$ and $\theta_r$ are the incident and reflection angles, respectively. For a medium parameterized with $v_h$, $\epsilon$, $\delta$, $\rho$ and $v_s$, the radiation patterns are given by $a(x) = a_{PP}^{v_s}(x)$ where

$$a_{PP}^{v_s}(x) = \begin{pmatrix} 2 \\ \sin(2\theta_i)\sin(2\theta_r)+2\cos(2\theta_i)\cos(2\theta_r)-2 \\ \cos^2(\theta_i)\sin^2(\theta_i)+\cos^2(\theta_r)\sin^2(\theta_r) \\ 2\sin^2(\theta_i)\sin^2(\theta_r) \\ 2 - \frac{(1-2\sigma)\sin^2(\theta_r-\theta_i)}{1-\sigma} + 2\cos(\theta_i-\theta_r) \end{pmatrix}, \quad (5)$$

In Figures 1a and 1b, we show the reflection PP-wave radiation patterns for perturbations in the elastic VTI parameters for the two different parameterizations. The radiation pattern for $v_s$ holds regardless of the parameterization, and the radiation patterns for the rest of the VTI parameters are the same as in the acoustic case. The radiation pattern for a perturbation in $v_s$ has a behavior similar to that of $\delta$ or $\eta$. There is, thus, an unfortunate tradeoff between perturbations in the shear wave.
velocity and that in either $\eta$ or $\delta$ in each of the parameterization. However, for conventional offset surface seismic data, the scattering influence of $v_s$, $\eta$ or $\delta$ on surface PP-wave data is small, and thus, can be neglected (Alkhalifah, 1998). The amplitude disparity will be absorbed by another parameter, specifically $\epsilon$ in the suggested parameterization, and $v_v$ in the standard one.

Figures 1a and 1b help us better understand our choice of parameterization. In the $v_v$, $\delta$, $\epsilon$ parameterization, $\epsilon$ can only be recovered from the long offsets, large scattering angles, which might be missing from conventional datasets. In addition, density effects will be absorbed by the velocity due to crosstalk between these two parameters at small angles. In contrast, with the $v_h$, $\eta$, $\epsilon$ parameterization, $\epsilon$ can absorb amplitude effects keeping $v_h$ relatively unaffected (as we will show in our examples). Indeed, the radiation pattern for $\epsilon$ shows that small angle scatterings will influence its recovery the most. These small angles are where the amplitude information, from $\rho$ perturbations and other effects, prevails. In other words if we don’t invert for $\rho$ (and/or $v_s$), then $\epsilon$ can be used to absorb the reflectivity, playing the role of a so-called “garbage collector”.

**Illustration on an elastic VTI Marmousi II model**

The elastic Marmousi II model was developed to provide a challenging dataset to the advocates of the use of multi-wave modes. The true VTI parameters used in the elastic modeling are shown in Figures 2a, 2c and 2d for $v_v$, $\delta$ and $\epsilon$, respectively. The corresponding true VTI parameters for $v_h$ and $\eta$ are shown in Figures 2b and 2e, respectively. The true shear wave and density models (not shown here) follow the same structure as the $v_v$ velocity model. In all the examples, the modeling engine is the same as the one used in the inversion (the so-called inversion crime), as the purpose of this work is to focus on the parameter tradeoff only. For the same reason, we invert eight frequency bands: 0-1 Hz, 0-2 Hz, 0-3 Hz, 0-4 Hz, 0-5 Hz, 0-7 Hz, 0-9 Hz, and 0-11 Hz. Starting
at such low frequencies is unrealistic but, given our starting models (detailed below), is needed to obtain meaningful results. Our FWI implementation minimizes the sum of the differences (in a least-squares sense) between observed and modeled data for all traces and time samples, thus incorporating both amplitude and phase information in the misfit function. An example of the derivations of the elastic FWI gradients in VTI media is provided in Kamath and Tsvankin (2016) and will not be repeated in this manuscript.

Our synthetic dataset mimics a marine acquisition survey with 67 shots spaced at 225 m and a maximum offset of 5 km. The shots were modeled with a finite-difference code using an explosive source and a Ricker wavelet for the shot waveform (maximum frequency of 15 Hz). This short offset spread fits our assumptions regarding the scattering angles present in the data. Since we can usually obtain smooth $v_{nmo}$ and $\eta$ from surface seismic PP-waves data using, for example, tomographic methods, the starting models are constructed by smoothing the exact $v_{nmo}$ and $v_h$ models with a window length of 1.5 km. The $\delta$ model is set to zero (Figure 3c), corresponding to the usual practice in the absence of well information. In this configuration, the corresponding starting models for $v_v$ (equal to the smoothed version of $v_{nmo}$) and $v_h$ are shown in Figures 3a and 3b, respectively. Such smoothing actually results in even a smoother starting model for $\epsilon$ and $\eta$ (Figure 3d), which we tend to expect from tomographic inversion methods (compared to velocity). In this case, we expect a considerable depth error in the inverted parameters. Our objective here is to test the tradeoff and convergence for the various parameterizations, thus we use the true $\delta$ (Figure 2c) to map the inverted results to their expected depth. The mapping process given by $z' = z\sqrt{1 + 2\delta}$ is an approximate correction as lateral variation in $\delta$ influences data recorded on the surface (Alkhalifah et al., 2001). Finally, as it is commonly done with streamer data, $v_s$ and $\rho$ are not updated in the inversion and are equal to a constant value (average value of exact models) in the whole sedimentary section below the water bottom.
This parameterization is widely used in the industry (Vigh et al., 2014; Baumstein, 2014) and Figure 1a shows the corresponding radiation patterns. For conventional offset-to-depth ratios (< 2), the scattering wavelengths of $\delta$ have little influence on the data. Despite that $\delta$ has a small imprint for this 5 km offset data, we will invert for it as well. After 15 iterations of EFWI per frequency scale using an L-BGFS approximation of the Hessian (Nocedal, 1980) and inverting the parameters simultaneously, we end up with the inverted models shown in Figures 4a, 4b, 4c for $v_h$, $\delta$ and $\epsilon$, respectively (after applying the aforementioned depth correction to the inverted models). The inverted vertical velocity model shows generally some features of the true model structure, but with higher velocities in some places, as illustrated in the vertical velocity profiles of Figures 6a and 6b at x=8 km and x=12 km, respectively. In a sense, in addition to the leakage of other elastic parameters into $v_h$, the inversion yields an average of the horizontal and vertical velocity models, thus explaining the higher-than-expected $v_h$. The $\delta$ model, as anticipated, looks erroneous, with limited information added to the initial $\delta$ model. Finally, the $\epsilon$ model also, because of limited data sensitivity to it with such parameterization and geometry, looks erroneous, especially at shallow depths.

Here, the radiation patterns (Figure 1b) resemble that of the previous parameterization with a change in the role that $\epsilon$ plays. Now $\epsilon$ helps in fitting the reflectivity whereas before in Figure 1a, $\epsilon$ would get mostly updated from the diving waves/long offset data. The parameter $\eta$, like $\delta$, has a minor role to play in FWI, but similar to what we have done with $\delta$ before, we are inverting for it anyway. Following the same frequency continuation strategy with the same number of iterations, we end up with the models shown in Figures 5a, 5b, 5c for $v_h$, $\eta$ and $\epsilon$, respectively. The horizontal
velocity looks similar to the inverted $v_v$ in Figure 4a, but with more accurate values, as illustrated in the vertical profiles of Figures 6c and 6d. Clearly, there is a closer match between the exact and inverted velocity profiles. It is interesting to notice that $v_v$ in Figure 4a seems to have slightly higher wavenumbers in the shallow parts (< 1.5 km) compared to $v_h$ in Figure 5a, a difference that could be explained by the different resolution vertically- or horizontally-traveling waves bring to the model. More importantly, $\epsilon$ now captures some reflectivity, as illustrated by the layering present in Figure 5c. As anticipated, $\epsilon$ seems to absorb most of the amplitude mismatches of the elastic assumption, which caused over estimation of velocity in the case of the vertical velocity parameterization. Again, as already seen with $\delta$ in Figure 4b, the update $\eta$ in Figure 5b doesn’t show any useful information due to its sensitivity to large scattering angles only (see radiation pattern of $\eta$ in Figure 1b). In addition, some leakage of density and $v_s$ is also occurring as discussed in Guitton and Alkhalifah (2016).

**Obtaining $v_h$ from $v_v$ and $\epsilon$**

We advocate an EFWI parameterization based on $v_h$ mostly. A question then arises: how does the inverted $v_h$ in our optimal parameterization compares to a $v_h$ computed from $v_v$ and $\epsilon$ in the standard parameterization? In other words, should we invert for $v_h$ or should we compute $v_h$ from inverted $v_v$ and $\epsilon$? In Figure 7a, we see the result of deriving $v_h$ from $v_v$ (Figure 4a) and $\epsilon$ (Figure 4c).

In Figure 7b we see the result of our optimal parameterization (similar to Figure 5a). We obtain a higher resolution in the top part of Figure 7a due to $v_v$. As we go deeper ($z > 2.5$ km.), however, $v_h$ in Figure 7a is underestimated because we don’t recover $\epsilon$ very well in the standard parameterization.

The direct inversion of $v_h$ in Figure 7b is relatively immune to this defect and yields more accurate results. Therefore, computing $v_h$ from $v_v$ and $\epsilon$ doesn’t provide the same accuracy that a direct inversion of $v_h$ brings. Therefore, we should always invert for $v_h$, and not derive it from the results
of other parameterizations.

We now study the effects of the standard and optimal parameterizations on an OBC 2-D line from the North Sea. These field data results corroborate our findings and support the assertion that inverting for $v_h$ yields better models and images than inverting for $v_v$.

APPLICATION TO A NORTH SEA OBC DATASET

Now we present field data results using a 2-D OBC line shot over the Volve field. We start by a presentation of the data and of the FWI strategies. Then we demonstrate that our proposed parameterization yields improved inversion results and RTM images compared to the traditional one.

Data and parameterization of the inversion

We apply our VTI parameterization to a 2-D receiver line of the OBC Volve dataset (Szydlik et al., 2007). One of the main imaging goal is to delineate the chalk layers for a proper identification of the reservoirs below the base Cretaceous unconformity (BCU). The data given to us are separated PP and PS wavemodes and thus do not exactly correspond to what our modeling operator produces (pressure components with all mode conversions). This discrepancy will have a limited impact on the inversion wherever S-velocity contrasts are small. Around the chalk layer where S-velocity contrasts are likely to be strong, the impact might be more pronounced (but not quantified in this study). The data were processed to remove all surface-related multiples. In addition, all post-critical events were muted out and all traces with offsets greater than 5 km discarded. While sufficient for imaging, the data are therefore not ideal for many FWI implementations, especially elastic ones, where longer offsets (and therefore, wider angles) are preferred.

For our inversion, we select 122 PP receiver gathers, 100 m apart and treat them as pressure...
data. Each receiver gather has a maximum of 401 shots at the surface, 25 m apart. The phase
and amplitude of the receiver gathers were modified to take into account the 2-D geometry of our
inversion (Pica et al., 1990). We assume that the PP receiver gathers are equivalent to hydrophone
records at the water-bottom ($z \approx 92$ m) from an acoustic source at the surface ($z \approx 6$ m). Our wavelet
is estimated by a matching procedure where synthetic traces computed in the starting models are
compared to the observed data. The reflections coming from the chalk layers are used for this step.
The estimated wavelet is kept constant for all FWI iterations. For the modeled shots, we simulate
the source ghost only by adding a mirrored source across the water-surface with opposite polarity.
For OBC data, this allows us to model the ghost effect without adding a free-surface, a very useful
feature when multiples are not present in the observed data (due to processing) but when the source
ghost is still present.

We use eight frequency scales for the inversion, starting at 2-3 Hz, finishing at 2-11 Hz. To
limit the computational cost, the inversions stop when we either reach twenty iterations or thirty
function-gradient evaluations per frequency scale. We use an L-BFGS solver preconditioned with
the source illumination estimated at each iteration. In multi-parameter inversion, it is especially
important to impose bounds on the model values. We impose constraints on the model parameters
(mostly clipping) at each new function-gradient evaluation to make sure that the models don’t create
numerical problems and are within a reasonable range of values.

Because the density is not inverted for, we add a masking function in our objective function to
attenuate its influence at near offsets. With the limited range of offsets present (only 5 km at best),
this simple procedure will not completely prevent the mapping of density into velocity (Guitton,
2014). However, our proposed parameterization with $v_h$, $\epsilon$, and $\eta$ helps to mitigate this leakage by
treating $\epsilon$ as a so-called “garbage collector”, where most amplitude discrepancies between modeled
and observed data will map into. For the traditional parameterization with $v_r$, $\epsilon$, and $\delta$, the crosstalk
is particularly strong between $v_v$ and $\rho$ at short offsets (see Figure 1a), and our masking will not remove it entirely.

The starting models are shown in Figures 8a, 8b and 8c for $v_h$, $v_v$ and $\rho$, respectively. The density model was obtained from $v_v$ using Gardner’s relation (Gardner et al., 1974). The starting anisotropic models are shown in Figures 9a, 9b and 9c for $\epsilon$, $\eta$ and $\delta$, respectively. These models are smoothed versions of the original ones provided to us by Statoil. Their constructions involved layer-stripping using a layer-based tomography method, making sure that the main geological features in migrated images would tie with well markers (Szydlik et al., 2007). The shear wave velocity is kept constant in the sediments and set to zero in the water layer. Remember that both $\rho$ and $v_s$ are kept stationary during the inversions.

**FWI and RTM results**

Now we present our inversion results. Figures 10a and 10b show a constant offset section ($h=1$ km) and a receiver gather with the offset mask applied ($x=6$ km) for the observed data at the last frequency band (2-11 Hz), respectively. Note that at this location, the offset range is limited to 3 km only. The top of the chalk layer corresponds to the strong event at $t=2.5$ s. Figures 10c and 10d are the corresponding residual panels for the $v_v$, $\epsilon$, $\delta$ parameterization while Figures 10e and 10f are the corresponding residual panels for the proposed $v_h$, $\epsilon$, $\eta$ parameterization. The residuals are essentially the same: the parameterization affects the models, i.e. where the information is going, not so much the data fit.

The main differences between the two parameterizations are clearly visible in the model space, however. Figures 11a and 11c show the estimated models for the $v_v$, $\epsilon$, $\delta$ parameterization and Figures 11b and 11d show the estimated models for the $v_h$, $\epsilon$, $\eta$ parameterization. The top of the
chalk layer is visible in both velocity images at z=2.8 km. We also notice a slow velocity zone at z=3.2 km and x=6 km, below the chalk, in both Figures 11a and 11b, which might indicate the presence of a reservoir. Overall, the estimated $v_h$ model seems to display more lateral continuity than the estimated $v_v$ model: for instance, a strong reflector (marked as “1”) at z=0.9 Km appears to have more consistent velocity values across the model in Figure 11b than in Figure 11a. Also, inverted events between z=1.5 km to z=2.2 km above the chalk layer (marked as “2”) appear less continuous in the $v_v$ model than in the $v_h$ model.

Looking now at the estimated $\epsilon$ models in Figures 11c and 11d, we see that our proposed parameterization is behaving as expected: a layering due to the mapping of amplitude discrepancies between observed and modeled data appears (mostly density effects). With the $v_v$, $\epsilon$, $\delta$ parameterization, because of the very small offset range and angle coverage for PP reflections, the estimated $\epsilon$ model doesn’t change much. In this more traditional parameterization, $v_v$ might absorb more of these amplitude discrepancies, which would explain why Figure 11a looks more noisy than Figures 11b.

We now look into the migration results of the pressure data with an elastic VTI RTM engine. Figure 12a shows the migration result with the smooth starting models. Figures 12b and 12c show the migration results with the inverted models for the $v_v$ and $v_h$ parameterizations, respectively. Because $\epsilon$ in Figure 11d has many fake horizontal events due to its role as a “garbage collector” (i.e., crosstalks), the migration shown in Figure 12c utilizes the smooth starting model of $\epsilon$ instead (Figure 3d), not the inverted one. This decision re-emphasizes our inversion strategy that sacrifices $\epsilon$ to better estimate $v_h$. Other datasets with more offsets and inversions with other parameters to invert for might require a different approach. Compared to Figure 12a, Figures 12b and 12c have stronger top chalk reflections at z=2.8 km., suggesting an improvement of the velocity model after inversion. The top chalk layer is even stronger and more continuous in Figure 12c than in Figure
12b, proving the ability of the $v_v$, $\epsilon$, $\eta$ parameterization to deliver reliable velocities. One geological element that seems to be attenuated after inversion is the anticline structure at the base of chalk at $z=3$ km and $x=5.25$ km. We offer three possible explanations to this. First, our 2-D line is very close to the edge of this anticline (as the 3-D models given to us indicate), making our 2-D geometry far from ideal (side reflections are mapped wrongly). Second, we didn’t use any well information to constrain our results: the inversion is global and didn’t make use of local information, contrary to what was done to build the starting models. We think that a better control of the well ties with our inverted models would probably yield even better results. Finally, we don’t update either $\delta$ and $\eta$, which control the depth of the reflectors. Longer offsets would help in this matter.

Because differences outside the chalk layer are hard to see between the different migration results, Figures 13a and 13b show migrated surface-offset panels for the initial and inverted model ($v_h$, $\epsilon$, $\eta$ parameterization only). Gathers are flatter for the top of the chalk layer ($z=2.8$ km) after inversion as well as for shallow sediments ($z=0.6$ km). To better qualify the differences between the two parameterizations, we show in Figures 14a an 14b the surface-offset panels above the chalk layer only. The gathers are generally flatter with the proposed parameterization, as exemplified by the layer at $z=0.6$ km. Overall, compared to the initial smooth model, our VTI EFWI of the PP reflections improves the images. Our proposed parameterization yields flatter gathers than the standard one, especially shallow. More offsets (and therefore, more scattering angles), 3-D data, as well as more control on the inversion (in the form of regularization terms) might improve our results further and help matching known geological features (such as anticlines) better.
Discussion

For 2-D pressure data, we numerically tested VTI EFWI parameterized by \( v_h, \eta, \) and \( \epsilon \) and by \( v_v, \delta, \) and \( \epsilon \). As the radiation patterns for these parameterizations suggest, the shear wave velocity \( v_s \), \( \delta \) and \( \eta \) have minor influence on the inversion of seismic PP-waves data with a reasonable offset range. In the \( v_h, \eta, \epsilon \) parameterization, the density effect is absorbed by \( \epsilon \) as they share almost the same scattering behavior. In the contrary, in the \( v_v, \delta, \epsilon \) parameterization, \( \epsilon \) is only sensitive to large scattering angles and the density effect is absorbed by \( v_v \), as \( \rho \) and \( v_v \) radiation patterns are similar at small angles. Thus, using an initial velocity model given by an accurate background NMO velocity and \( \eta \) (\( \delta \) is set to zero), the \( v_h \) parameterization, despite the inaccurate \( \delta \) model, provides a reasonable velocity, better than that given by the conventional parameterization.

Two noticeable absentees from our analysis are \( v_s \) and \( \rho \) as well as PS waves. In the elastic case, the influence of \( v_s \) and \( \rho \) on the inversion depends largely on their contrasts (Barnes and Charara, 2009). For PS waves, Guitton and Alkhalifah (2016) show that they mostly help recovering \( \eta \) in the \( v_h, \eta, \epsilon \) parameterization. More work needs to be done to better understand how the wave modes and parameters, given the data (especially when multi-components are present), are influencing the inversion.

Finally, our work focused on the mitigation of crosstalke by careful parameterization of the inversion only. It is now well-known that better multi-parameter inversion methods taking into account the inverse Hessian can help resolving some of these issues as well (Operto et al., 2013; Pan et al., 2016). One promising technique worth mentioning is the truncated-Newton method which might help alleviating leakage effects between parameter spaces (Métivier et al., 2013, 2014).
CONCLUSION

An improved FWI parameterization in terms of $v_h$, $\eta$ and $\epsilon$ for VTI media proposed in the acoustic approximation holds in the elastic case for pressure data: it mitigates leakage effects and assigns $\epsilon$ as a so-called “garbage collector” to handle amplitude discrepancies between modeled and observed data. Inversion examples on synthetic data, assuming accurate background NMO velocity and $\eta$ fields, seem to confirm that the $v_h$ parameterization yields improved results compared to the $v_v$ one. Inversion results on an OBC 2-D line from the North Sea corroborate these findings in the model as well as in the image space: gathers are flattened and reflectors’ strength are improved.

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LIST OF FIGURES

1 The reflection radiation patterns from a horizontal reflector for the two sets of parameters ((a) $v_v$, $\delta$, $\epsilon$ and (b) $v_h$, $\eta$, $\epsilon$) describing an elastic VTI model and PP-waves scattering. The polar component describes the opening angle $\theta = \theta_i + \theta_r$ between the incidence and reflected wave path (for opening angles between $0^\circ$ to $180^\circ$), while the radial component expresses the relative amplitude of scattering.

2 True models for (a) $v_v$, (b) $v_h$, (c) $\delta$, (d) $\epsilon$ and (e) $\eta$.

3 Starting models for (a) $v_v$, (b) $v_h$, (c) $\delta$ and (d) $\epsilon$ and $\eta$.

4 Inverted models for (a) $v_v$, (b) $\delta$, and (c) $\epsilon$ for standard parameterization after depth shifting, thus resulting in a blue band at the bottom of the models.

5 Inverted models for (a) $v_h$, (b) $\eta$, and (c) $\epsilon$ for optimal parameterization after depth shifting, thus resulting in a blue band at the bottom of the models.

6 Vertical profiles at 8km (a and c) and 12 km (b and d) laterally of the $v_v$ model (a and b) from Figure 4a and the $v_h$ model (c and d) from Figure 5a. Overall, $v_v$ is overestimated (a and b) while $v_h$ is closer to the true values (c and d).

7 Estimated $v_h$ from (a) $v_v$ and $\epsilon$ of standard parameterization (i.e. Figures4a and 4c) and (b) optimal parameterization (i.e. Figure 5a). Color scale is the same as the one used in Figure 5a (left). Note how $v_h$ in (a) is too slow in the deepest parts of the model compared to $v_h$ in (b).

8 Starting models for (a) $v_h$, (b) $v_p$ and (c) $\rho$.

9 Starting models for (a) $\epsilon$, (b) $\eta$ and (c) $\delta$.

10 Data space comparisons (2-11 Hz) showing in (a), (c) and (e) constant offset panels (h=1 km) for the observed data, residual for the $v_v$, $\epsilon$, $\delta$ parameterization, and residual for the $v_h$, $\epsilon$, $\eta$ parameterization, respectively. Receiver gathers (x=6 km) in (b), (d) and (f) display the observed data, residual for the $v_v$, $\epsilon$, $\delta$ parameterization, and residual for the $v_h$, $\epsilon$, $\eta$ parameterization, respectively.
No obvious differences exist between the parameterizations and a similar data fit is obtained.

Inverted models for (a) $v_v$ and (c) $\epsilon$ in the $v_v$, $\epsilon$, $\delta$ parameterization. Inverted models for (b) $v_h$ and (d) $\epsilon$ in the $v_h$, $\epsilon$, $\eta$ parameterization. Arrows “1” and “2” show areas where the $v_h$ model seems to have more lateral continuity and more consistent velocities horizontally than the $v_v$ model.

Elastic VTI RTM images for (a) the starting models, (b) the inverted models with $v_v$, $\epsilon$, $\delta$ parameterization, and (c) the inverted models with $v_h$, $\epsilon$, $\eta$ parameterization. For (c), we use the smooth starting model of $\epsilon$ in Figure 9a instead of the inverted one in Figure 11d.

Surface-offset gathers (a) before and (b) after inversion using the $v_h$, $\epsilon$, $\eta$ parameterization only. Notice how the gathers flatten after inversion for the chalk layer and shallow sediments. Arrows point to locations where gathers are generally flatter after inversion.

Surface-offset gathers above the chalk layer after (a) inversion using the $v_v$, $\epsilon$, $\delta$ parameterization and (b) inversion using the $v_h$, $\epsilon$, $\eta$ parameterization. Arrows point to locations where gather are flatter using the $v_h$, $\epsilon$, $\eta$ parameterization.
1a. The reflection radiation patterns from a horizontal reflector for the two sets of parameters ((a) \( v_v, \delta, \epsilon \) and (b) \( v_h, \eta, \epsilon \)) describing an elastic VTI model and PP-waves scattering. The polar component describes the opening angle \( \theta = \theta_i + \theta_r \) between the incidence and reflected wave path (for opening angles between 0° to 180°), while the radial component expresses the relative amplitude of scattering.

190x158mm (300 x 300 DPI)
1b. The reflection radiation patterns from a horizontal reflector for the two sets of parameters 
((a) $v_v$, $\delta$, $\epsilon$ and (b) $v_h$, $\eta$, $\epsilon$) describing an elastic VTI model and PP-waves scattering. The polar component describes the opening angle $\theta = \theta_i + \theta_r$ between the incidence and reflected wave path (for opening angles between $0^\circ$ to $180^\circ$), while the radial component expresses the relative amplitude of scattering.

190x157mm (300 x 300 DPI)
2a. True models for (a) $v_v$, (b) $v_h$, (c) $\delta$, (d) $\varepsilon$ and (e) $\eta$
2b. True models for (a) $v_v$, (b) $v_h$, (c) $\delta$, (d) $\varepsilon$ and (e) $\eta$

96x31mm (300 x 300 DPI)
2c. True models for (a) $v_v$, (b) $v_h$, (c) $\delta$, (d) $\epsilon$ and (e) $\eta$.
2d. True models for (a) $v_v$, (b) $v_h$, (c) $\delta$, (d) $\epsilon$ and (e) $\eta$
2e. True models for (a) $v_v$, (b) $v_h$, (c) $\delta$, (d) $\epsilon$ and (e) $\eta$

$98\times32\text{mm} \ (300 \times 300 \text{ DPI})$
3a. Starting models for (a) $v_v$, (b) $v_h$, (c) $\delta$ and (d) $\varepsilon$ and $\eta$. 

97x32mm (300 x 300 DPI)
3b. Starting models for (a) \( v_v \), (b) \( v_h \), (c) \( \delta \) and (d) \( \varepsilon \) and \( \eta \).
3c. Starting models for (a) $v_v$, (b) $v_h$, (c) $\delta$ and (d) $\epsilon$ and $\eta$. 

98x32mm (300 x 300 DPI)
3d. Starting models for (a) $v_v$, (b) $v_h$, (c) $\delta$ and (d) $\epsilon$ and $\eta$. 

98x32mm (300 x 300 DPI)
4a. Inverted models for (a) $v_v$, (b) $\delta$, and (c) $\varepsilon$ for standard parameterization after depth shifting, thus resulting in a blue band at the bottom of the models.
4b. Inverted models for (a) $v_v$, (b) $\delta$, and (c) $\epsilon$ for standard parameterization after depth shifting, thus resulting in a blue band at the bottom of the models.
4c. Inverted models for (a) ν, (b) δ, and (c) ε for standard parameterization after depth shifting, thus resulting in a blue band at the bottom of the models.
5a. Inverted models for (a) \( v_h \), (b) \( \eta \), and (c) \( \epsilon \) for optimal parameterization after depth shifting, thus resulting in a blue band at the bottom of the models.
5b. Inverted models for (a) $v_h$, (b) $\eta$, and (c) $\varepsilon$ for optimal parameterization after depth shifting, thus resulting in a blue band at the bottom of the models.
5c. Inverted models for (a) $v_h$, (b) $\eta$, and (c) $\varepsilon$ for optimal parameterization after depth shifting, thus resulting in a blue band at the bottom of the models.

98x32mm (300 x 300 DPI)
6a. Vertical profiles at 8km (a and c) and 12 km (b and d) laterally of the $v_v$ model (a and b) from Figure 4a and the $v_h$ model (c and d) from Figure 5a. Overall, $v_v$ is overestimated (a and b) while $v_h$ is closer to the true values (c and d).

169x101mm (300 x 300 DPI)
6b. Vertical profiles at 8km (a and c) and 12 km (b and d) laterally of the $v_v$ model (a and b) from Figure 4a and the $v_h$ model (c and d) from Figure 5a. Overall, $v_v$ is overestimated (a and b) while $v_h$ is closer to the true values (c and d).
6c. Vertical profiles at 8 km (a and c) and 12 km (b and d) laterally of the $v_v$ model (a and b) from Figure 4a and the $v_h$ model (c and d) from Figure 5a. Overall, $v_v$ is overestimated (a and b) while $v_h$ is closer to the true values (c and d).

169x101mm (300 x 300 DPI)
6d. Vertical profiles at 8km (a and c) and 12 km (b and d) laterally of the $v_v$ model (a and b) from Figure 4a and the $v_h$ model (c and d) from Figure 5a. Overall, $v_v$ is overestimated (a and b) while $v_h$ is closer to the true values (c and d).

169x101mm (300 x 300 DPI)
7a. Estimated $v_h$ from (a) $v_r$ and $s$ of standard parameterization (i.e. Figures 4a and 4c) and (b) optimal parameterization (i.e. Figure 5a). Color scale is the same as the one used in Figure 5a (left). Note how $v_h$ in (a) is too slow in the deepest parts of the model compared to $v_h$ in (b).
7b. Estimated $v_h$ from (a) $v$, and $\varepsilon$ of standard parameterization (i.e. Figures 4a and 4c) and (b) optimal parameterization (i.e. Figure 5a). Color scale is the same as the one used in Figure 5a (left). Note how $v_h$ in (a) is too slow in the deepest parts of the model compared to $v_h$ in (b).
8a. Starting models for (a) $v_h$, (b) $v_p$ and (c) $\rho$

153x80mm (300 x 300 DPI)
8b. Starting models for (a) $v_h$, (b) $v_p$ and (c) $\rho$

153x80mm (300 x 300 DPI)
8c. Starting models for (a) $v_h$, (b) $v_p$ and (c) $\rho$

154x81mm (300 x 300 DPI)
9a. Starting models for (a) $\epsilon$, (b) $\eta$ and (c) $\delta$.

153x80mm (300 x 300 DPI)
9b. Starting models for (a) $\varepsilon$, (b) $\eta$ and (c) $\delta$.

153x80mm (300 x 300 DPI)
9c. Starting models for (a) \( \varepsilon \), (b) \( \eta \) and (c) \( \delta \).

153x80mm \((300 \times 300 \text{ DPI})\)
10a. Data space comparisons (2-11 Hz) showing in (a), (c) and (e) constant offset panels (h=1 km) for the observed data, residual for the $v_v$, $\epsilon$, $\delta$ parameterization, and residual for the $v_h$, $\epsilon$, $\eta$ parameterization, respectively. Receiver gathers ($x=6$ km) in (b), (d) and (f) display the observed data, residual for the $v_v$, $\epsilon$, $\delta$ parameterization, and residual for the $v_h$, $\epsilon$, $\eta$ parameterization, respectively. No obvious differences exist between the parameterizations and a similar data fit is obtained.
10b. Data space comparisons (2-11 Hz) showing in (a), (c) and (e) constant offset panels \( h=1 \) km) for the observed data, residual for the \( v_r, \varepsilon, \delta \) parameterization, and residual for the \( v_h, \varepsilon, \eta \) parameterization, respectively. Receiver gathers \( x=6 \) km in (b), (d) and (f) display the observed data, residual for the \( v_r, \varepsilon, \delta \) parameterization, and residual for the \( v_h, \varepsilon, \eta \) parameterization, respectively. No obvious differences exist between the parameterizations and a similar data fit is obtained.

219x169mm (300 x 300 DPI)
10c. Data space comparisons (2-11 Hz) showing in (a), (c) and (e) constant offset panels (h=1 km) for the observed data, residual for the $v_v$, $\varepsilon$, $\delta$ parameterization, and residual for the $v_h$, $\varepsilon$, $\eta$ parameterization, respectively. Receiver gathers (x=6 km) in (b), (d) and (f) display the observed data, residual for the $v_v$, $\varepsilon$, $\delta$ parameterization, and residual for the $v_h$, $\varepsilon$, $\eta$ parameterization, respectively. No obvious differences exist between the parameterizations and a similar data fit is obtained.
10d. Data space comparisons (2-11 Hz) showing in (a), (c) and (e) constant offset panels ($h=1$ km) for the observed data, residual for the $v_v$, $\epsilon$, $\delta$ parameterization, and residual for the $v_h$, $\epsilon$, $\eta$ parameterization, respectively. Receiver gathers ($x=6$ km) in (b), (d) and (f) display the observed data, residual for the $v_v$, $\epsilon$, $\delta$ parameterization, and residual for the $v_h$, $\epsilon$, $\eta$ parameterization, respectively. No obvious differences exist between the parameterizations and a similar data fit is obtained.

219x169mm (300 x 300 DPI)
10e. Data space comparisons (2-11 Hz) showing in (a), (c) and (e) constant offset panels (h=1 km) for the observed data, residual for the $v_h$, $\varepsilon$, $\delta$ parameterization, and residual for the $v_h$, $\varepsilon$, $\eta$ parameterization, respectively. Receiver gathers (x=6 km) in (b), (d) and (f) display the observed data, residual for the $v_h$, $\varepsilon$, $\delta$ parameterization, and residual for the $v_h$, $\varepsilon$, $\eta$ parameterization, respectively. No obvious differences exist between the parameterizations and a similar data fit is obtained.

219x169mm (300 x 300 DPI)
10f. Data space comparisons (2-11 Hz) showing in (a), (c) and (e) constant offset panels (h=1 km) for the observed data, residual for the $v_v$, $\varepsilon$, $\delta$ parameterization, and residual for the $v_h$, $\varepsilon$, $\eta$ parameterization, respectively. Receiver gathers (x=6 km) in (b), (d) and (f) display the observed data, residual for the $v_v$, $\varepsilon$, $\delta$ parameterization, and residual for the $v_h$, $\varepsilon$, $\eta$ parameterization, respectively. No obvious differences exist between the parameterizations and a similar data fit is obtained.
11a. Inverted models for (a) \( v \) and (c) \( \varepsilon \) in the \( v, \varepsilon, \delta \) parameterization. Inverted models for (b) \( v_h \) and (d) \( \varepsilon \) in the \( v_h, \varepsilon, \eta \) parameterization. Arrows "1" and "2" show areas where the \( v_h \) model seems to have more lateral continuity and more consistent velocities horizontally than the \( v \) model.
11b. Inverted models for (a) $v_v$ and (c) $\varepsilon$ in the $v_v$, $\varepsilon$, $\delta$ parameterization. Inverted models for (b) $v_h$ and (d) $\varepsilon$ in the $v_h$, $\varepsilon$, $\eta$ parameterization. Arrows “1” and “2” show areas where the $v_h$ model seems to have more lateral continuity and more consistent velocities horizontally than the $v_v$ model.
11c. Inverted models for (a) $v_v$ and (c) $\varepsilon$ in the $v_v$, $\varepsilon$, $\delta$ parameterization. Inverted models for (b) $v_h$ and (d) $\varepsilon$ in the $v_h$, $\varepsilon$, $\eta$ parameterization. Arrows "1" and "2" show areas where the $v_h$ model seems to have more lateral continuity and more consistent velocities horizontally than the $v_v$ model.
11d. Inverted models for (a) $v_v$ and (c) $\epsilon$ in the $v_v$, $\epsilon$, $\delta$ parameterization. Inverted models for (b) $v_h$ and (d) $\epsilon$ in the $v_h$, $\epsilon$, $\eta$ parameterization. Arrows "1" and "2" show areas where the $v_h$ model seems to have more lateral continuity and more consistent velocities horizontally than the $v_v$ model.

153x80mm (300 x 300 DPI)
12a. Elastic VTI RTM images for (a) the starting models, (b) the inverted models with $v_h$, $\varepsilon$, $\delta$ parameterization, and (c) the inverted models with $v_h$, $\varepsilon$, $\eta$ parameterization. For (c), we use the smooth starting model of $\varepsilon$ in Figure 9a instead of the inverted one in Figure 11d.

153x84mm (300 x 300 DPI)
12b. Elastic VTI RTM images for (a) the starting models, (b) the inverted models with $v_h$, $\epsilon$, $\delta$ parameterization, and (c) the inverted models with $v_h$, $\epsilon$, $\eta$ parameterization. For (c), we use the smooth starting model of $\epsilon$ in Figure 9a instead of the inverted one in Figure 11d.

153x84mm (300 x 300 DPI)
12c. Elastic VTI RTM images for (a) the starting models, (b) the inverted models with \( v_n, \epsilon, \delta \) parameterization, and (c) the inverted models with \( v_n, \epsilon, \eta \) parameterization. For (c), we use the smooth starting model of \( \epsilon \) in Figure 9a instead of the inverted one in Figure 11d.
13a. Surface-offset gathers (a) before and (b) after inversion using the $\nu$, $\varepsilon$, $\eta$ parameterization only. Notice how the gathers flatten after inversion for the chalk layer and shallow sediments. Arrows point to locations where gathers are generally flatter after inversion.

153x84mm (300 x 300 DPI)
13b. Surface-offset gathers (a) before and (b) after inversion using the $v_n$, $\varepsilon$, $\eta$ parameterization only. Notice how the gathers flatten after inversion for the chalk layer and shallow sediments. Arrows point to locations where gathers are generally flatter after inversion.

153x84mm (300 x 300 DPI)
14a. Surface-offset gathers above the chalk layer after (a) inversion using the $v_u, \varepsilon, \delta$ parameterization and (b) inversion using the $v_h, \varepsilon, \eta$ parameterization. Arrows point to locations where gathers are flatter using the $v_h, \varepsilon, \eta$ parameterization.

153x84mm (300 x 300 DPI)
14b. Surface-offset gathers above the chalk layer after (a) inversion using the $v$, $\varepsilon$, $\delta$ parameterization and (b) inversion using the $v$, $\varepsilon$, $\eta$ parameterization. Arrows point to locations where gathers are flatter using the $v$, $\varepsilon$, $\eta$ parameterization.