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Mobility-Aware User Association in Uplink Cellular Networks

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***Abstract*—This letter studies the mobility aware user-to-BS association policies, within a stochastic geometry framework, in two tier uplink cellular networks with fractional channel inversion power control. Particularly, we model the base stations' locations using the widely accepted poisson point process and obtain the coverage probability and handover cost expressions for the coupled and decoupled uplink and downlink associations. To this end, we compute the average throughput for the mobile users and study the merits and demerits of each association strategy.**

***Index Terms*—Stochastic Geometry, Mobility Management, Handover Cost, Decoupled uplink downlink association (DUDe).**

I. Introduction

Heterogeneity and spectrum sharing are the fundamentals of current and future cellular networks. In such heterogenous and interference limited networks, efficient user to base station (BS) association is crucial to maximally utilize the deployed small BSs. For instance, downlink range expansion, that favors load balancing over maximum received signal strength (RSS), is considered within the 3GPP standard [1]. Hence, avoiding overloading macro BSs and under-utilizing the small BSs. Another important association challenge in heterogenous networks is that the optimal uplink and downlink associations may not always coincide together, which calls for a decoupled uplink and downlink associations (DUDe) [2]. Due to the homogeneity in the transmission powers of users equipment (UE), the UE may favor uplink association to a closer small BS than a farther macro BS despite their downlink transmission power discrepancy. Such association strategies are studied and their performances are characterized in the literature for cellular networks with stationary UEs.

Since mobility is an intrinsic element of cellular networks, characterizing the performance of a given association strategy for stationary users is not sufficient. Mobile users perform several handovers (HOs) along their trajectories. With the increase in BS intensity, the HO rate increases, which imposes service delays and affects the user throughput. The impact of mobility is further signified in 5G cellular networks due to the foreseen ultra network densification. Consequently, mobility aware network design is mandatory in the context of 5G cellular networks. For instance, the work in [3] proposes a mobility aware range expansion for small BSs' downlink association in order to maximize the throughput of mobile UEs. The work in [4] sheds light on the negative impact of mobility on network densification gains in multi-tier cellular networks, in which the concept of Phantom small BSs is utilized to mitigate such negative impact. The work in [5] shows that the RSS based association may impose excessive HOs in dense cellular environments, in which handover skipping is proposed to alleviate unnecessary handovers. However, to the best of the authors' knowledge, the impact of mobility on the uplink association is never addressed in the literature. In contrast to the existing literature, this letter models an interplay between HO delay and BS intensity for different user-to-BS uplink associations.

To draw general insights on the performance of cellular networks, stochastic geometry is widely utilized in the literature (cf. [6] and the references therein). It captures the irregularity in the BSs deployment, accounts for the network heterogeneity, is not tailored to a certain network realization, and intrinsically incorporates the effect of network interference into the analysis. This letter utilizes stochastic geometry to develop a mobility aware mathematical model for uplink association in a two tier cellular network. Particularly, the developed framework characterizes the uplink throughput for stationary and mobile UEs under the RSS and DUDe association strategies.

II. System Model

This letter considers a two tier cellular network where the BSs belonging to each tier are modeled via an independent homogenous poisson point process (PPP) Φ_k with intensity λ_k and transmission power P_k, where $k \in \{1, 2\}$. The macro and small BSs are denoted by $k = 1$ and $k = 2$, respectively. A power-law path-loss model with path-loss exponent $\eta_k > 2$ is considered. For simplicity, we consider same path loss exponents for both tiers. The extension to different path-loss exponents is straightforward, but comes at the expense of more involved expressions. Rayleigh fading is assumed such that channel power gains are i.i.d. with unity mean exponential distributions, i.e. $h \sim \exp(1)$. UEs are distributed via a homogeneous PPP Ψ with intensity λ_u. Universal frequency reuse is assumed along with independent UE scheduling at each BS. Hence, intra-cell interference is avoided and inter-cell interference exists. UEs transmit with the fractional power control of the form $P_u x^{\eta\epsilon}$, where P_u is the power control parameter and $\epsilon \in \{0, 1\}$ is the path-loss compensation factor (PCF). The cell boundaries in each association scenario can be visualized via a weighted Voronoi tessellation (cf. Fig. 1). Let $u \in \Psi$ be a randomly selected UE, then the uplink user-to-BS association policies in the RSS and DUDe architectures are defined by $b_r = \arg\max_{b\in\Phi} P_k \|u - b\|^{-\eta}$ and $b_d = \arg\min_{b\in\Phi} \|u - b\|$, respectively, where $\|.\|$ denotes the Euclidean norm.

For the mobile UE scenario, we consider the average throughput over an arbitrary long trajectory with a constant velocity

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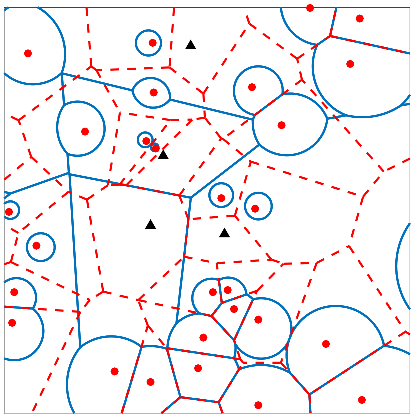

Fig. 1: Association regions for (a) DUDe (red-dotted lines) and (b) RSS (blue-solid lines) based policies modeled via weighted Voronoi tessellation. Macro and small BSs are represented by black triangles and red circles, respectively.

v, where handovers are executed when the UE crosses the cell boundaries. It is assumed that the HOs are always successful and each HO results in a delay of d seconds, where data transmission is interrupted and only HO related signaling is exchanged.

III. Stationary Analysis

For the sake of organized exposition, we first characterize the uplink coverage probability and throughput for the RSS and DUDe associations for stationary UEs scenario. The effect of mobility is incorporated into the analysis in the next section. Without loss of generality, we conduct the stationary analysis on a test user located at the origin.
We start by characterizing the UE transmission power distribution, which is given in the following lemma.

Lemma 1 (Transmission Power Distribution): *The distribution of the power P_i transmitted by the i^{th} user at the distance $r_i \in \Phi$ from the test BS is given by*

$$f_{P_i}(z|r_i) = \frac{2\pi\lambda_t z^{\frac{2}{\epsilon\eta}-1} e^{-\pi\lambda_t\left(\frac{z}{P_u}\right)^{2/\eta}}}{\epsilon\eta P_u^{2/\epsilon\eta}\gamma\left(1, \pi\lambda_t r^2\right)}; \quad 0 \le z \le P_u r_i^{\epsilon\eta} \tag{1}$$

where $\gamma(a,b) = \int_0^b t^{a-1}e^{-t}dt$ is the lower incomplete gamma function and λ_t is the total intensity given by

$$\lambda_t = \begin{cases} \lambda_1 P_1^{2/\eta} + \lambda_2 P_2^{2/\eta}, & \text{for RSS} \\ \Lambda, & \text{for DUDe} \end{cases}$$

Proof: We evaluate the truncated power distribution $f_{P_i}(z|r_i)$ using the service distance distribution $f_r(x) = 2\pi\lambda_t x e^{-\pi\lambda_t x^2}$, evaluated using the null probability of PPP and the transmit power of a generic UE given by $P = P_u x^{\epsilon\eta}$. The total intensity in the DUDe architecture is calculated by the fact that the test user sees the heterogeneous network as a homogenous network with intensity $\Lambda = \lambda_1 + \lambda_2$. For the total intensity in the RSS architecture, we exploit the mapping theorem [7, Theorem 2.34] and follow [5, Lemma 3] and map the two dimensional PPPs into an equivalent one dimensional non-homogenous PPP with the resultant intensity $\lambda_t = \lambda_1 P_1^{2/\eta} + \lambda_2 P_2^{2/\eta}$. ■

Due to association and scheduling, the interfering UEs point process, denoted by $\Psi_I \subset \Psi$, is not a PPP. Furthermore, the correlated sizes of adjacent Voronoi cells impose a correlation among the transmission powers of proximate UEs. For tractability, the UEs transmission powers correlations are ignored and the aggregate interference seen at the test BS is approximated by the interference seen from a non-homogeneous PPP with the intensity function $\lambda(x) = \lambda_t'(1 - e^{-\pi\lambda_t' x^2})$, where the intensity $\lambda_t' \le \lambda_t$ accounts for the UE activity[1] and the thinning factor $(1-e^{-\pi\lambda_t' x^2})$ accounts for the panorama of interferers seen from the test BS perspective. Similar approximations are used and validated in [8]. We also validate these approximations in this letter. For notational and mathematical convenience, we present the following lemma that approximates the interference seen at the test BS.

Lemma 2 (Aggregate interference approximation): *Let $I_{agg} \triangleq \sum_{x_i \in \Psi_I} P_i h_i \|x_i\|^{-\eta}$ be the aggregate interference seen at the test BS. Then, $I_{agg} \overset{d}{\approx} \sum_{x_i \in \widetilde{\Psi}_I} h_i/x_i$, where $\widetilde{\Psi}_I \in \mathbb{R}^+$ is a PPP with intensity function*

$$\widetilde{\lambda}(w) = \frac{2\pi^{1-\epsilon}\lambda_t' P_u^{2/\eta}}{\eta w^{1-\frac{2}{\eta}}\lambda_t^{\epsilon}} \cdot \int_0^{\pi\lambda_t (P_u w)^{\frac{2}{\eta(1-\epsilon)}}} \frac{y^{\epsilon} e^{-y}\gamma\left(1, \pi^{1-\epsilon}\lambda_t'(P_u w)^{2/\eta}(\frac{y}{\lambda_t})^{\epsilon}\right)}{\gamma\left(1, (\pi\lambda_t)^{1-\epsilon}(P_u w)^{2/\eta} y^{\epsilon}\right)} dy, \tag{2}$$

where $\overset{d}{=}$ denotes the equality in distribution and λ_t' is given by

$$\lambda_t' = \begin{cases} \rho_1\lambda_1 P_1^{2/\eta} + \rho_2\lambda_2 P_2^{2/\eta}, & \text{for RSS} \\ \rho\Lambda, & \text{for DUDe} \end{cases}$$

Proof: First, we note that the approximation is due to ignoring the correlations among the transmission powers of the UEs and approximating the interference from Ψ_I with the interference from a PPP with intensity function $\lambda(x) = \lambda_t'(1-e^{-\pi\lambda_t' x^2})$. The lemma then follows in two steps; i) applying the Mapping theorem, with the mapping function $w_i = \|x_i\|^{\eta}$, to achieve the one dimensional PPP with inverse distance path-loss; and ii) applying the displacement theorem [7, Theorem 2.33], with the PDF given by (1)*, to achieve the unity transmission powers [9]. Thus the resulting intensity after mapping and displacement can be obtained as $\widetilde{\lambda}(w) = \int_0^{\infty}\zeta(r,w)\lambda(r)dr$, where $\zeta(r,w)$ is the displacement kernel given by $\zeta(r,w) = \frac{r}{w^2} f_{P_i}\left(\frac{r}{w}, r^{1/\eta}\right)$. The load factors in λ_t' (i.e., ρ, ρ_1, and ρ_2) are obtained by first writing the probability of having n number of users in a cell with cell size A given by $P_A(n) = \frac{(\lambda_u A)^n}{n!}e^{-\lambda_u A}$, solving it for $n \neq 0$ and then integrating it over the distribution of A, given in [10, equations 2, 10, & 11.].* ■

Corollary 1 (Intensity Function with Full Load): *For the special case of fully loaded network $\left(\rho = \rho_1 = \rho_2 = 1\right)$, the intensity function $\widetilde{\lambda}(w)$ boils down to a simpler expression given by*

$$\widetilde{\lambda}(w) = \frac{2(\pi\lambda_t)^{1-\epsilon}P_u^{2/\eta}}{\eta w^{1-\frac{2}{\eta}}}\gamma\left(1+\epsilon, \pi\lambda_t(P_u w)^{\frac{2}{\eta(1-\epsilon)}}\right) \tag{3}$$

Exploiting the results in Lemma 2, a unified definition for the coverage probability of the RSS and DUDe association scenarios can be expressed as

$$\mathcal{C} = \mathbb{P}\left[\frac{P_u h r^{\eta(\epsilon-1)}}{I_{agg}+\sigma^2} > T\right]. \tag{4}$$

where $I_{agg} = \sum_{w_i \in \widetilde{\Psi}_I} h_i w_i^{-1}$ is the approximated aggregate interference seen at the test BS and σ^2 is the noise power. The

[1] There may exist some BSs with no UEs to serve

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$$\mathcal{C}=\int_0^\infty \exp\left(-z-\int_{\mathbf{1}\{\epsilon=1\}}^\infty \frac{2(\lambda_t'/\lambda_t)z^{1-\epsilon}}{\eta x^{1-2/\eta}(1+\frac{x}{T})}\int_0^{zx^{\frac{2}{\eta(1-\epsilon)}}} y^\epsilon e^{-y}\frac{\gamma\left(1,(\lambda_t'/\lambda_t)z^{1-\epsilon}x^{2/\eta}y^\epsilon\right)}{\gamma\left(1,z^{1-\epsilon}x^{2/\eta}y^\epsilon\right)}dydx\right)dz. \quad (5)$$

uplink coverage probability in DUDe and RSS associations is characterized by the following theorem.

***Theorem** 1 (**Coverage Probability**): The uplink coverage probability in the DUDe and RSS associations in two tier cellular networks with fractional channel inversion power control is given by* (5).

Proof: We prove this theorem by first writing the coverage probability conditioned on the serving BS distance and then applying the probability generating functional (PGFL) for PPP [7] with the intensity function given in (2) *and integrating it over the service distance distribution given by* $f_r(x)=2\pi\lambda_t xe^{-\pi\lambda_t x^2}$. ■

Note that (5) is unified for both RSS and DUDe association scenarios,[2] where the association rule is captured in λ_t that has different values for each association strategy as shown in Lemma 1.

***Corollary** 2 (**Coverage Prob. with Full Load**): The uplink coverage probability in a fully loaded network with full path-loss compensation* ($\epsilon=1$) *can be expressed in the terms of Gauss hypergeometric function* ${}_2F_1(.,.,.,.)$ *as*

$$\mathcal{C}=\exp\left(-\frac{2T}{\eta-2}{}_2F_1\left(1,1-\frac{2}{\eta},2-\frac{2}{\eta},-T\right)\right) \quad (6)$$

For the special case at $\eta=4$ *and full path-loss compensation* $\epsilon=1$, *the uplink coverage probability in a fully loaded network boils down to a simpler expression given by*

$$\mathcal{C}=\exp\left(-\sqrt{T}\arctan\left(\sqrt{T}\right)\right) \quad (7)$$

As shown in Corollary 2, the DUDe and RSS based associations offer equivalent spatially averaged coverage probability. However, the superiority of DUDe is observed in balancing the loads of UEs served by macro and small BSs [11]. Furthermore, DUDe association offers better utilization for the small cells by enlarging their coverage regions, and hence, decreasing the probability of having idle or under-utilized small BSs. The discrepancy among the DUDe and the RSS performance is captured via the utilization factors shown in (5). Furthermore, the outperformance of DUDe is better visualized in terms of the load-aware network throughout, which is defined as

$$\mathcal{T}=\rho_L W\mathcal{R}. \quad (8)$$

where ρ_L is the utilized BS intensity ($\rho_L=\rho_1\lambda_1+\rho_2\lambda_2$ for RSS and $\rho_L=\rho\Lambda$ for DUDe), W is the overall bandwidth of the channel, and $\mathcal{R}$ is the effective throughput per unit bandwidth (i.e., nats/sec/Hz). The effective throughput is defined by a constant transmission rate that is subject to outage due to SINR fluctuations, which is expressed as $\mathcal{R}=\ln(1+T^*)\mathbb{P}[SINR>T^*]$. Note that $T^*=\arg\max_T\mathcal{R}(T)$ is the numerically obtained optimum value that maximizes the effective throughput.

IV. User Mobility Analysis

In this section, we incorporate user mobility and handover rates in the user rate analysis. The effective throughput averaged over the user trajectory is approximated via the spatially averaged throughput given in Section III, which is validated in Section V. The average uplink throughput experienced by a test mobile user moving with velocity v is given by

$$\mathcal{T}^{(u)}=\rho_L W\mathcal{R}(1-D_{HO}). \quad (9)$$

where $D_{HO}=\mathcal{H}*d$ is the normalized time consumed in HO signaling, which is a function of overall HO rate $\mathcal{H}$ and delay per HO d. The HO rates are characterized in [12] for multi-tier PPP based cellular networks with the RSS association policy as

$$H_{ij}=\begin{cases}\frac{v\lambda_i\lambda_j F(x_{ij})}{2\pi(\sum_{n=1}^K\lambda_n x_{nk}^2)^{\frac{3}{2}}}+\frac{v\lambda_i\lambda_j F(x_{ji})}{2\pi(\sum_{n=1}^K\lambda_n x_{nj}^2)^{\frac{3}{2}}} & \text{if } i\neq j,\\ \frac{v\lambda_i^2 F(1)}{\pi(\sum_{n=1}^K\lambda_n x_{nk}^2)^{\frac{3}{2}}} & \text{if } i=j.\end{cases} \quad (10)$$

where H_{ij} is the number of tier i to tier j HOs per unit time, v is the user velocity, and $x_{11}=x_{22}=1$, $x_{12}=\left(\frac{P_1}{P_2}\right)^{1/\eta}$, $x_{21}=\frac{1}{x_{12}}$,$F(x)=\frac{1}{x^2}\int_0^\pi\sqrt{(x^2+1)-2xcos(\theta)}d\theta$.

As shown in Fig. 1, the uplink and downlink handovers are synchronous in the RSS strategy (i.e., both happen simultaneously when crossing the blue boundaries), and hence, the overall HO rate $\mathcal{H}$ is obtained by the summation of all inter and intra-tier HOs. On the other hand, the inter-tier uplink and downlink HOs are asynchronous in the DUDe architecture (i.e., the uplink [downlink] HOs occur when crossing the red [blue] boundaries), and hence, the downlink and uplink HOs should be considered separately to calculate the overall HO rate. Since the uplink association in DUDe is based on the user-to-BS distance, the additional uplink HOs can be calculated by enforcing $P_1=P_2$ in (10). The handover process involves several handshaking messages to be exchanged between the UE and the BS. Since the UE does not send the control and data bits simultaneously, such handover signaling will disrupt the data transmission. Assuming that the UE data transmission is disrupted with each downlink and/or uplink HO, the total HO rate in DUDe architecture is given by

$$\mathcal{H}^{DUDe}=\mathcal{H}^{DL}+\mathcal{H}^{UL}-H_{11}^{UL}-H_{22}^{DL} \quad (11)$$

where (11) follows from the fact that intra-tier (i.e., macro-to-macro or small-to-small) uplink and downlink HOs are still synchronous in the DUDe architecture as shown in Fig. 1. Therefore, H_{11}^{UL} and H_{22}^{DL} are eliminated from the overall HO rate to alleviate duplicity in the HO count.

V. Numerical Results

This section provides numerical results according to the parameters highlighted in Table 1. We first note that the results for the effective throughput shown in Figs. 2(a) and 2(b) are supported via independent simulations that account for the UE mobility, and hence, validating the approximations related to the interference in Lemma 2 and mobility throughput in Section IV. Figs. 2(a) and 2(b) confirm that the spatially averaged throughput for both association strategies are comparable. However, the DUDe outperformance appears in the network throughout due to the better small BS utilization as shown in Fig. 2(c). It is important to note that DUDe association also offers a balanced performance among the macro and small BSs as shown in [11]. The effect of UE mobility is depicted in Fig. 3. Fig. 3(a) shows the increased HO cost in the DUDe due to the disjoint uplink and

[2]The extension to the maximum user transmit power constraint is straightforward but comes at the expense of more involved expressions.

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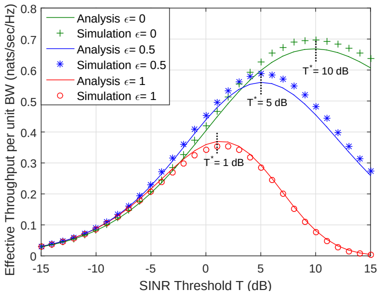
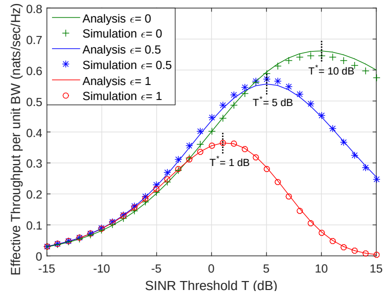
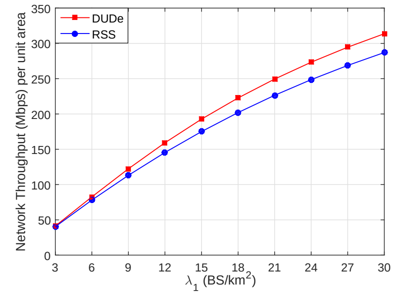


(a) $\mathcal{R}_{\mathrm{RSS}}$ (nats/sec/Hz)

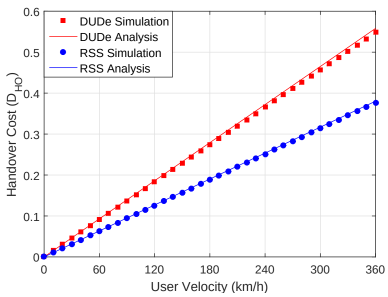


(b) $\mathcal{R}_{\mathrm{DUDe}}$ (nats/sec/Hz)

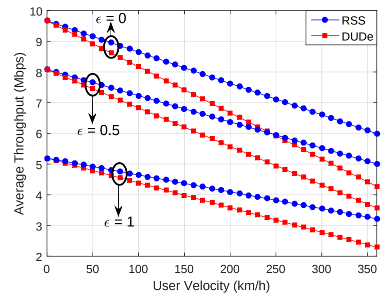


(c) $\mathcal{T}$ (Mb/sec/km^2) with $\epsilon = 1$, $\lambda_2 = 2\lambda_1$

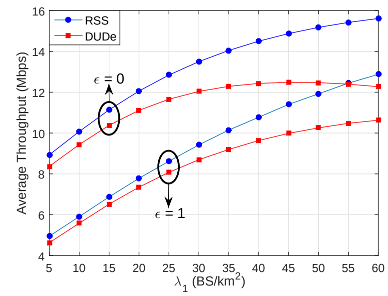
Fig. 2: Stationary Effective & Network Throughput for RSS and DUDe architectures



(a) HO cost vs. velocity



(b) $\mathcal{T}^{(u)}$ vs. velocity



(c) $\mathcal{T}^{(u)}$ vs. intensity with $\lambda_2 = 3\lambda_1$

Fig. 3: Handover Cost & Average Throughput for mobile UE employing RSS and DUDe associations.

TABLE I: Simulation parameters

Parameter	Value	Parameter	Value
Macros Power P_1:	10 watt	Small BSs Power P_2:	P_1/6 watt
Macros intensity λ_1:	3 BS/km^2	Small BSs intensity λ_2:	15 BS/km^2
Users intensity λ_u:	50 BS/km^2	PCF ϵ:	0, 0.5, 1
User baseline power P_u:	1 watt	Path-loss exponent η:	4
Channel BW W:	10 MHz	HO delay d:	0.7 s

downlink associations. The results in Fig. 3(a) is supported via Monte Carlo simulations to validate the analysis. Figs. 3(b) and 3(c) show that the RSS outperforms the DUDe association when accounting for mobility and handover cost. The figures show that the throughput gap between the RSS and DUDe depends on the UE velocity and BS intensity. Hence, compromising between the two association strategies based on the UE and network conditions is necessary. For instance, stationary and low mobility UE can employ the DUDe association to enhance the small BS utilization. However, at moderate and high mobility, UEs should switch to the RSS association to avoid excessive handovers. Note that the switching velocities among the RSS and DUDe are functions of the BS intensity, which can be computed via the proposed framework.

VI. Conclusion

Using stochastic geometry, we develop mobility-aware mathematical model for uplink cellular networks. The developed model is utilized to study the effect of mobility on coupled (RSS) and decoupled (DUDe) uplink association strategies. For stationary users, it is shown that the DUDe outperforms the RSS association in terms of network throughput due to load balancing. Accounting for the mobility, the DUDe imposes higher handover rate, which degrades the throughput, compared to the RSS strategy. Hence, mobility-aware user association is advocated where the stationary and nomadic [moderate and high mobility] users follow the DUDe [RSS] strategy.

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